Band gaps structure and semi-Dirac point of two-dimensional function photonic crystals*

Si-Qi Zhang(张斯淇)[1], Jing-Bin Lu(陆景彬)[1], Yu Liang(梁禹)[2], Ji Ma(马季)[2], Hong Li(李宏)[3], Xue Li(李雪)[1], Xiao-Jing Liu(刘晓静)[2], Xiang-Yao Wu(吴向尧)[2,†], and Xiang-Dong Meng(孟祥东)[2]

[1] *Institute of Physics, Jilin University, Changchun* 130012, *China*
[2] *Institute of Physics, Jilin Normal University, Siping* 136000, *China*
[3] *Institute of Physics, Northeast Normal University, Changchun* 130012, *China*

(Received 28 April 2016; revised manuscript received 15 October 2016; published online 20 December 2016)

Two-dimensional function photonic crystals, in which the dielectric constants of medium columns are the functions of space coordinates $\boldsymbol{r}$, are proposed and studied numerically. The band gaps structures of the photonic crystals for TE and TM waves are different from the two-dimensional conventional photonic crystals. Some absolute band gaps and semi-Dirac points are found. When the medium column radius and the function form of the dielectric constant are modulated, the numbers, width, and position of band gaps are changed, and the semi-Dirac point can either occur or disappear. Therefore, the special band gaps structures and semi-Dirac points can be achieved through the modulation on the two-dimensional function photonic crystals. The results will provide a new design method of optical devices based on the two-dimensional function photonic crystals.

Keywords: two-dimensional photonic crystals, function dielectric constants, band gaps structures

PACS: 42.70.Qs, 78.20.Bh, 73.20.At **DOI:** 10.1088/1674-1056/26/2/024208

1. Introduction

Photonic crystals, periodic structures that control photons in a way comparable to the way semiconductors control electrons, have inspired extensive study since their emergence in the late 1980's.[1–5] The most important property of the photonic crystals is the photonic band gap. In the two-dimensional photonic crystals, there are absolute band gaps and Dirac points that create many applications such as polarization beam splitter, waveguides, self-collimation beams, defects, all-optical logic gates, and different functional optical devices[6–8] for photonic crystals structures.

Recently, Dirac point in photonic and phononic crystals have been found at the corners of the Brillouin zones of triangular and honeycomb lattices where two bands meet,[9] leading to the observation of many novel wave transport properties, such as classical analogs of Zitterbewegung and pseudodiffusion. It was reported that linear dispersions can also occur at the Brillouin zone center of a square lattice photonic crystal, induced by simultaneous zero permittivity ($\varepsilon_{\rm eff} = 0$) and permeability ($\mu_{\rm eff} = 0$), and the linear dispersions could be understood from an effective medium perspective. Various theoretical approaches, such as multiple scattering,[10] tight binding,[11] and perturbation,[12,13] have also been developed to analyze the properties of the Dirac point in photonic crystals. The Dirac point produces at the Brillouin zone boundary, which has also been found in elastic/acoustic waves[14] and in the simple cubic lattice. The Dirac-like points are created by the bands intersecting at one point at the Brillouin zone center, and the band dispersion relation near is point are linear, which is obtained by accidental degeneracy. When the material parameters and structure parameters are changed, the bands of the Dirac-like point should be separated and are not linear. The various transport properties of zero-refractive-index material have been studied near the Dirac-like point, which are the focus of much of the recent work.[15–18] This dispersion relation is analogues to the Dirac point in electron systems, where two linear bands touch so that there is no band gap. Owing to this special property, remarkable wave transport behaviors and interesting applications in electromagnetic waves have been reported.[15–18] The semi-Dirac point, a type of unique and unprecedented electronic band dispersion, was discovered[19] and studied.[20] It was found that, near a point in the Fermi surface in the two-dimensional Brillouin zone, the dispersion relation is linear along the symmetry line (it is a direction of lattice, i.e., (1,1) direction) but quadratic in the perpendicular direction.[19–21] It is reported that this semi-Dirac point is associated with the topological phase transition between a semimetallic phase and a band insulator.[21]

The semi-Dirac points are formed by the bands intersecting at one point at the Brillouin zone center or boundary; the dispersion relations are linear at one direction and are quadratic at its vertical direction. The Dirac points are formed

*Project supported by the National Natural Science Foundations of China (Grant No. 61275047), the Research Project of Chinese Ministry of Education (Grant No. 213009A), and the Scientific and Technological Development Foundation of Jilin Province, China (Grant No. 20130101031JC).

†Corresponding author. E-mail: wuxy2066@163.com

© 2017 Chinese Physical Society and IOP Publishing Ltd

by the bands intersecting at one point at the Brillouin zone center or boundary, and the band dispersion relation near the point are linear, which is obtained by doubly degeneracy. When the material parameters and structure parameters are changed, the bands of the Dirac point should not be separated.

In Refs. [22]–[27], we have proposed one-dimensional function photonic crystals, which consist of two media: A and B. Their refractive indices are the functions of space position, unlike conventional photonic crystal (PCs), which is constituted by the constant refractive index media A and B. We have studied the transmissivity and the electric field distribution with and without defect layer. In this paper, we have proposed two-dimensional function photonic crystals, in which the medium column dielectric constants are the functions of the space coordinate, and calculated the band gaps structures of TE and TM waves, different from the two-dimensional conventional photonic crystals band gaps structures. We also found there are absolute band gaps and semi-Dirac points in the two-dimensional function photonic crystals. The findings have wide-ranging application potential. The absolute band gaps can be designed into the polarization selection device, and the semi-Dirac points can be used to achieve zero refractive index materials. Using our calculations, as the medium column radius and the function form of the dielectric constant changes, we find the numbers, width, and position of band gaps are changed, and the semi-Dirac point can either occur or disappear. They can design the needed band gaps structures for the two-dimensional function photonic crystals. Hence, the two-dimensional function photonic crystals can provide a new design method of optical devices.

2. The Fourier transform of dielectric constant for two-dimensional function photonic crystals

For the two-dimensional function photonic crystals, the medium column dielectric constants are the functions of space coordinates $\boldsymbol{r}$, different from the two-dimensional conventional photonic crystals, whose dielectric constants are constant. The medium column dielectric constants become the functions of space coordinates $\boldsymbol{r}$, which can be realized easily in physics. We know that the external electric field can cause the change of the medium refractive index, called the electro-optical effect,[28] i.e., $n(E) = n_0 + aE + bE^2$, where E is the external electric field intensity, n_0 is the medium refractive index without an external electric field, $n(E)$ is the medium refractive index with external electric field, a and b are the electro–optical coefficients. When the external electric field is $E = E(x,y,z)$, the medium refractive index is $n(E) = n(x,y,z)$. Moreover, in nonlinear optics, the medium refractive index is the linear function of external light intensity I, which is called the optical Kerr effect,[29] i.e., $n(I) = n_0 + n_2 I$, where n_0 represents the usual weak-field refractive index, $n_2 = (3/4n_0^2\varepsilon_0 c)\chi^{(3)}$ is the optical Kerr coefficient, and $\chi^{(3)}$ is the third-order nonlinear optical susceptibility. When the external light intensity is $I = I(x,y,z)$, the medium refractive index is $n(I) = n(x,y,z)$. So the medium column dielectric constants can easily become the function of space coordinates $\boldsymbol{r}$, i.e., $\varepsilon = \varepsilon(x,y,z)$, when the medium columns are imposed on the external electric field $E(x,y,z)$ and external light intensity $I(x,y,z)$.

The dielectric constant of the cylindrical medium column can be written as

$$\varepsilon(\boldsymbol{r}) = \begin{cases} \varepsilon_a(\boldsymbol{r}), & r \le r_a, \\ \varepsilon_b, & r > r_a, \end{cases} \tag{1}$$

or

$$\frac{1}{\varepsilon(\boldsymbol{r})} = \begin{cases} \dfrac{1}{\varepsilon_a(\boldsymbol{r})}, & r \le r_a, \\ \dfrac{1}{\varepsilon_b}, & r > r_a, \end{cases} \tag{2}$$

equation (2) can be written as

$$\frac{1}{\varepsilon(\boldsymbol{r})} = \frac{1}{\varepsilon_b} + \left(\frac{1}{\varepsilon_a(\boldsymbol{r})} - \frac{1}{\varepsilon_b}\right) s(r), \tag{3}$$

where

$$s(r) = \begin{cases} 1, & r \le r_a, \\ 0, & r > r_a. \end{cases} \tag{4}$$

The Fourier inverse transform of $1/\varepsilon(\boldsymbol{r})$ is

$$\varepsilon^{-1}(\boldsymbol{G}) = \frac{1}{V_0}\int_{V_0} \mathrm{d}\boldsymbol{r}\frac{1}{\varepsilon(\boldsymbol{r})}\mathrm{e}^{-\mathrm{i}\boldsymbol{G}\cdot\boldsymbol{r}}, \tag{5}$$

in the two-dimensional reciprocal space, it is

$$\varepsilon^{-1}(\boldsymbol{G}_{||}) = \frac{1}{V_0^{(2)}}\int_{V_0^{(2)}} \mathrm{d}\boldsymbol{r}_{||}\frac{1}{\varepsilon(\boldsymbol{r}_{||})}\mathrm{e}^{-\mathrm{i}\boldsymbol{G}_{||}\cdot\boldsymbol{r}_{||}}, \tag{6}$$

where $\boldsymbol{G}_{||} = m_1\boldsymbol{b}_1 + m_2\boldsymbol{b}_2$, $\boldsymbol{r}_{||} = x\boldsymbol{i} + y\boldsymbol{j}$, $V_0^{(2)}$ represents the unit cell area in the two dimensional lattice space.

Substituting Eq. (3) into Eq. (6), we obtain

$$\begin{aligned}
\varepsilon^{-1}(\boldsymbol{G}_{||}) &= \frac{1}{V_0^{(2)}}\int_{V_0^{(2)}} \mathrm{d}\boldsymbol{r}_{||}\left[\frac{1}{\varepsilon_b} + \left(\frac{1}{\varepsilon_a} - \frac{1}{\varepsilon_b}\right)s(\boldsymbol{r}_{||})\right]\mathrm{e}^{-\mathrm{i}\boldsymbol{G}_{||}\cdot\boldsymbol{r}_{||}} \\
&= \frac{1}{\varepsilon_b}\frac{1}{V_0^{(2)}}V_0^{(2)}\delta_{\boldsymbol{G}_{||},0} \\
&\quad + \frac{1}{V_0^{(2)}}\int_{V_0^{(2)}} \mathrm{d}\boldsymbol{r}_{||}\left(\frac{1}{\varepsilon_a} - \frac{1}{\varepsilon_b}\right)s(\boldsymbol{r}_{||})\,\mathrm{e}^{-\mathrm{i}\boldsymbol{G}_{||}\cdot\boldsymbol{r}_{||}} \\
&= \frac{1}{\varepsilon_b}\delta_{m,0}\delta_{n,0} + \frac{1}{V_0^{(2)}} \\
&\quad \times \int_{V_0^{(2)}} \mathrm{d}\boldsymbol{r}_{||}\left(\frac{1}{\varepsilon_a} - \frac{1}{\varepsilon_b}\right)s(\boldsymbol{r}_{||})\,\mathrm{e}^{-\mathrm{i}\boldsymbol{G}_{||}\cdot\boldsymbol{r}_{||}} \\
&= \frac{1}{\varepsilon_b}\delta_{m,0}\delta_{n,0} + I,
\end{aligned} \tag{7}$$

where

$$I = \frac{1}{V_0^{(2)}}\int_{V_0^{(2)}} \mathrm{d}\boldsymbol{r}_{||}\left(\frac{1}{\varepsilon_a} - \frac{1}{\varepsilon_b}\right)s(\boldsymbol{r}_{||})\,\mathrm{e}^{-\mathrm{i}\boldsymbol{G}_{||}\cdot\boldsymbol{r}_{||}}$$

$$
\begin{aligned}
&= \frac{1}{V_0^{(2)}}\int_{V_0^{(2)}} \mathrm{d}\boldsymbol{r}_{||}\frac{1}{\varepsilon_a}s(\boldsymbol{r}_{||})\,\mathrm{e}^{-\mathrm{i}\boldsymbol{G}_{||}\cdot\boldsymbol{r}_{||}} \\
&\quad - \frac{1}{V_0^{(2)}}\int_{V_0^{(2)}} \mathrm{d}\boldsymbol{r}_{||}\frac{1}{\varepsilon_b}s(\boldsymbol{r}_{||})\,\mathrm{e}^{-\mathrm{i}\boldsymbol{G}_{||}\cdot\boldsymbol{r}_{||}} \\
&= I_1 - I_2, \qquad (8)
\end{aligned}
$$

where

$$
\begin{aligned}
I_2 &= \frac{1}{V_0^{(2)}}\int_{V_0^{(2)}} \mathrm{d}\boldsymbol{r}_{||}\frac{1}{\varepsilon_b}s(\boldsymbol{r}_{||})\,\mathrm{e}^{-\mathrm{i}\boldsymbol{G}_{||}\cdot\boldsymbol{r}_{||}} \\
&= \frac{1}{\varepsilon_b}\frac{1}{V_0^{(2)}}\int_{V_0^{(2)}} \mathrm{d}\boldsymbol{r}_{||}s(\boldsymbol{r}_{||})\,\mathrm{e}^{-\mathrm{i}\boldsymbol{G}_{||}\cdot\boldsymbol{r}_{||}} \\
&= \frac{1}{\varepsilon_b}\frac{1}{V_0^{(2)}}\int_0^{r_a} r\mathrm{d}r\int_0^{2\pi}\mathrm{d}\theta\,\mathrm{e}^{-\mathrm{i}G_{||}\cdot r\cdot\cos\theta} \\
&= \frac{1}{\varepsilon_b}\frac{1}{V_0^{(2)}}\int_0^{r_a} r\mathrm{d}r\int_0^{2\pi}\mathrm{d}\theta\,\mathrm{e}^{\mathrm{i}G_{||}\cdot r\cdot\sin(\theta-\pi/2)}, \qquad (9)
\end{aligned}
$$

where $|\boldsymbol{r}_{||}| = r_{||} = r$, $|\boldsymbol{G}_{||}| = G_{||}$, $\mathrm{d}\boldsymbol{r}_{||} = \mathrm{d}s = r\mathrm{d}r\mathrm{d}\theta$, and θ is the included angle of $\boldsymbol{r}_{||}$ and $\boldsymbol{G}_{||}$.

By the formulas

$$
\mathrm{e}^{\mathrm{i}\omega\cdot\sin\theta} = \sum_{l=-\infty}^{\infty} J_l(\omega)\,\mathrm{e}^{\mathrm{i}l\theta}, \qquad (10)
$$

$$
\int_0^{2\pi}\mathrm{d}\theta\,\mathrm{e}^{\mathrm{i}l(\theta-\pi/2)} = \begin{cases} 0, & (l\neq 0), \\ 2\pi, & (l=0), \end{cases} \qquad (11)
$$

and

$$
\int x^m J_{m-1}(x)\mathrm{d}x = x^m J_m(x) + c, \qquad (12)
$$

we have

$$
\begin{aligned}
I_2 &= \frac{1}{\varepsilon_b}\frac{1}{V_0^{(2)}}\int_0^{r_a} r\mathrm{d}r\sum_{l=-\infty}^{\infty} J_l(G_{||}\cdot r)\int_0^{2\pi}\mathrm{d}\theta\,\mathrm{e}^{\mathrm{i}l(\theta-\pi/2)} \\
&= \frac{1}{\varepsilon_b}\frac{1}{V_0^{(2)}}\frac{2\pi r_a}{G_{||}}\cdot J_1(G_{||}\cdot r_a), \quad (\boldsymbol{G}_{||}\neq 0), \qquad (13)
\end{aligned}
$$

when $G_{||}\to 0$ ($m\to 0$, $n\to 0$), we have

$$
\begin{aligned}
I_2(m=0,n=0) &= \lim_{G_{||}\to 0}\frac{1}{\varepsilon_b}\frac{1}{V_0^{(2)}}2\pi r_a\frac{J_1(G_{||}\cdot r_a)}{G_{||}} \\
&= \frac{1}{\varepsilon_b}\frac{1}{V_0^{(2)}}2\pi r_a\lim_{G_{||}\to 0}\frac{(J_1(G_{||}\cdot r_a))'}{(G_{||})'} \\
&= \frac{1}{\varepsilon_b}\frac{1}{V_0^{(2)}}2\pi r_a\lim_{G_{||}\to 0}J_1'(G_{||}\cdot r_a)\cdot r_a \\
&= \frac{1}{\varepsilon_b}\frac{\pi r_a^2}{V_0^{(2)}} = \frac{f}{\varepsilon_b}, \quad (\boldsymbol{G}_{||}=0), \qquad (14)
\end{aligned}
$$

where $J_1'(0) = 1/2$ and $f = \pi r_a^2/V_0^{(2)}$ is the filling ratio.

Where

$$
\begin{aligned}
I_1 &= \frac{1}{V_0^{(2)}}\int_{V_0^{(2)}} \mathrm{d}\boldsymbol{r}_{||}\frac{1}{\varepsilon_a(r,\theta)}s(\boldsymbol{r}_{||})\,\mathrm{e}^{-\mathrm{i}\boldsymbol{G}_{||}\cdot\boldsymbol{r}_{||}} \\
&= \frac{1}{V_0^{(2)}}\int_0^{r_a} r\mathrm{d}r\int_0^{2\pi}\frac{1}{\varepsilon_a(r,\theta)}\,\mathrm{e}^{\mathrm{i}G_{||}\cdot r_{||}\cdot\sin(\theta-\pi/2)}\,\mathrm{d}\theta \\
&= \frac{1}{V_0^{(2)}}\int_0^{r_a} r\mathrm{d}r\int_0^{2\pi}\mathrm{d}\theta\frac{1}{\varepsilon_a(r,\theta)}\sum_{l=-\infty}^{\infty} J_l(G_{||}\cdot r)\,\mathrm{e}^{\mathrm{i}l(\theta-\pi/2)} \\
&= \frac{1}{V_0^{(2)}}\int_0^{r_a} r\frac{1}{\varepsilon_a(r)}\sum_{l=-\infty}^{\infty} J_l(G_{||}\cdot r)\mathrm{d}r\int_0^{2\pi}\mathrm{d}\theta\,\mathrm{e}^{\mathrm{i}l(\theta-\pi/2)} \\
&= \frac{2\pi}{V_0^{(2)}}\int_0^{r_a} r\frac{1}{\varepsilon_a(r)}J_0(G_{||}\cdot r)\mathrm{d}r, \quad (\boldsymbol{G}_{||}\neq 0). \qquad (15)
\end{aligned}
$$

In Eq. (15), we have considered $\varepsilon_a(r,\theta) = \varepsilon_a(r)$.

When $G_{||} = 0$, as $J_0(0) = 1$, we have

$$
I_1 = \frac{2\pi}{V_0^{(2)}}\int_0^{r_a} r\frac{1}{\varepsilon_a(r)}\mathrm{d}r, \quad (\boldsymbol{G}_{||}=0), \qquad (16)
$$

substituting I_1, I_2, and I into Eq. (7), we obtain

$$
\varepsilon^{-1}(\boldsymbol{G}_{||}) = \begin{cases} \dfrac{1}{\varepsilon_b}(1-f) + \dfrac{2f}{r_a^2}\displaystyle\int_0^{r_a} r\frac{1}{\varepsilon_a(r)}\mathrm{d}r, & (\boldsymbol{G}_{||}=0) \\ \dfrac{2f}{r_a^2}\displaystyle\int_0^{r_a} r\frac{1}{\varepsilon_a(r)}J_0(G_{||}\cdot r)\mathrm{d}r - \dfrac{2f}{\varepsilon_b}\dfrac{J_1(G_{||}\cdot r_a)}{G_{||}\cdot r_a}, & (\boldsymbol{G}_{||}\neq 0). \end{cases} \qquad (17)
$$

Equation (18) is the Fourier transform of the dielectric constant for two-dimensional function photonic crystals. When $\varepsilon_a(r) = \varepsilon_a$, the ε_a is a constant, then equation (17) becomes

$$
\varepsilon^{-1}(\boldsymbol{G}_{||}) = \begin{cases} \dfrac{1}{\varepsilon_b} + \left(\dfrac{1}{\varepsilon_a} - \dfrac{1}{\varepsilon_b}\right)f, & (\boldsymbol{G}_{||}=0), \\ 2f\left(\dfrac{1}{\varepsilon_a} - \dfrac{1}{\varepsilon_b}\right)\dfrac{J_1(G_{||}\cdot r_a)}{G_{||}\cdot r_a}, & (\boldsymbol{G}_{||}\neq 0). \end{cases} \qquad (18)
$$

Equation (18) is the Fourier transform dielectric constant of two-dimensional conventional photonic crystals. So, the two-dimensional conventional photonic crystal is the special case of two-dimensional function photonic crystals.

In Refs. [30] and [31], the authors have given the eigenvalue equations of TM and TE waves with the plane-wave expansion method, they are

$$
\sum_{\boldsymbol{G}'}(\boldsymbol{k}+\boldsymbol{G}')\cdot(\boldsymbol{k}+\boldsymbol{G})\varepsilon^{-1}(\boldsymbol{G}-\boldsymbol{G}')H_{\boldsymbol{k}}(\boldsymbol{G}') = \frac{\omega^2}{c^2}H_{\boldsymbol{k}}(\boldsymbol{G}), \qquad (19)
$$

and

$$
\sum_{\boldsymbol{G}'}|\boldsymbol{k}+\boldsymbol{G}'||\boldsymbol{k}+\boldsymbol{G}|\varepsilon^{-1}(\boldsymbol{G}-\boldsymbol{G}')E_{\boldsymbol{k}}(\boldsymbol{G}') = \frac{\omega^2}{c^2}E_{\boldsymbol{k}}(\boldsymbol{G}). \qquad (20)
$$

Substituting Eq. (17) into Eqs. (19) and (20), we can obtain the band gaps structure of two-dimensional function photonic crystals.

3. Numerical result

In this section, we will report our numerical results of band gaps structures for the two-dimensional function photonic crystals. In order to compare the band gaps structures of two-dimensional conventional and function photonic crystals, we will first calculate the band gaps structures of the two-dimensional conventional photonic crystals with Eqs. (18), (19), and (20). The structure is the triangle lattice, that is shown in Fig. 1, and the cylindrical medium columns are located in the air, whose dielectric constants are $\varepsilon_a = 9$ and medium column radius is $r_a = 0.65a$, where $a = 10^{-6}$ m is the lattice constant. The band gaps structures of TE and TM waves are shown in Figs. 2(a) and 2(b), respectively. In the frequency range from 0 to 0.8 (in unit of $a/2\pi c$), there are three band gaps for the TE and TM waves.

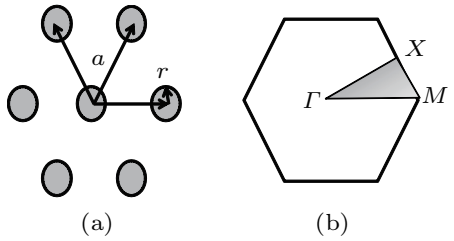


Fig. 1. (a) The triangular lattice structure of two-dimensional photonic crystals, (b) the first Brillouin zone of the triangle grid.

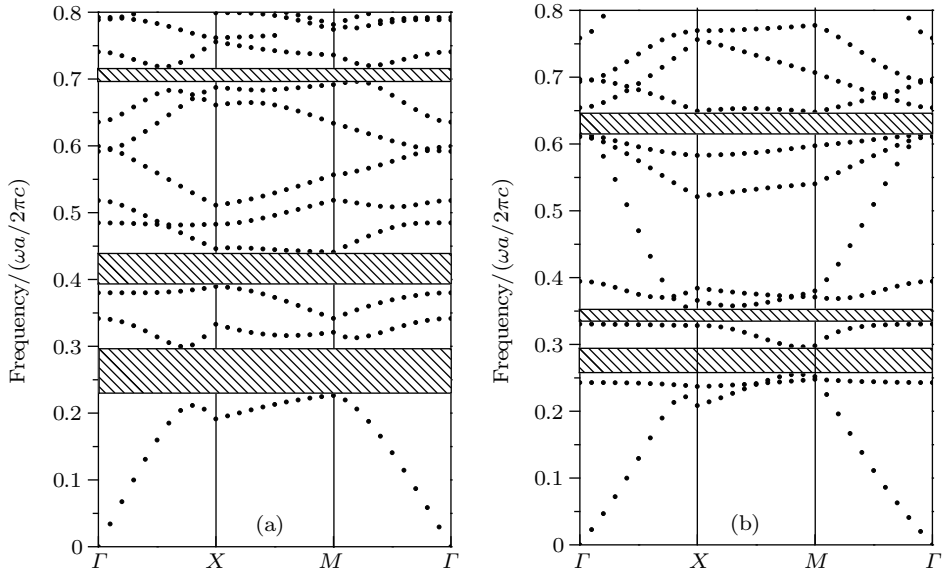


Fig. 2. The band gaps structure of two-dimensional conventional photonic crystals for the triangle lattice, $\varepsilon_b = 1$, $\varepsilon_a = 9$, medium column radius $r_a = 0.65a$. (a) TE wave, (b) TM wave.

In the following, we calculated the band gaps structures of the two-dimensional function photonic crystals with Eqs. (17), (19), and (20). The structure is also the triangle lattice, and the cylindrical medium columns are located in the air, whose dielectric constants are the function of the space coordinate, i.e. $\varepsilon_a(\boldsymbol{r}) = kr + b$ $(0 \le r \le r_a)$, where r_a is the medium column radius, and k is called the function coefficient. When $k = 0$, it is conventional photonic crystals, when $k \neq 0$, it is function photonic crystals. From Fig. 3 to Fig. 6, we take $k = 1.8 \times 10^6$, $b = 9$, and study the medium column radius r_a effect on the band gaps structures in the frequency range from 0 to 0.8. In Figs. 3(a) and 3(b), we give the band gaps structures of TE and TM waves with the medium column radius $r_a = 0.26a$. There are two wider band gaps for the TE wave, and one band gap for the TM wave. In the vicinity of 0.55 frequency, the band gaps of TE and TM waves overlap to form an absolute band gap, which can be designed into the polarization selection device. Figures 4(a) and (b) are the band gaps structures of TE and TM waves with the medium column radius $r_a = 0.3a$. There are three band gaps for the TE wave, and one band gap for the TM wave. In the vicinity of 0.5 frequency, there is an absolute band gap, and one semi-Dirac point A for TM wave, which is because the dispersion relation is linear along the XM direction and quadratic along the $X\Gamma$ direction in the vicinity of the A point. In Figs. 5(a) and 5(b), we give the band gaps structures of TE and TM waves with the medium column radius $r_a = 0.65a$. There are four band gaps for the TE wave, and two band gaps for the TM wave. In the vicinity of 0.32 and 0.76 frequencies, there are two absolute band gaps, and one semi-Dirac point A for the TE wave. In the vicinity of the A point, the dispersion relation is linear along the XM direction and quadratic along the $X\Gamma$ direction. In Figs. 6(a) and 6(b), we give the band gaps structures of TE and TM waves with the medium column radius $r_a = 0.8a$. There are two band gaps for the TE wave, and one band gap for the TM wave.

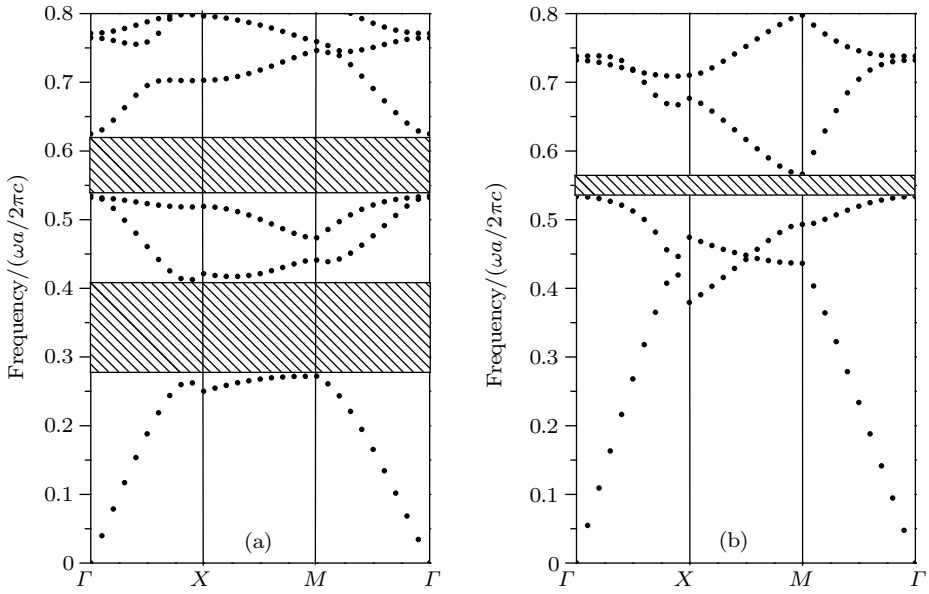


Fig. 3. The band gaps structure of two-dimensional function photonic crystals for the triangle lattice, $\varepsilon_b = 1$, $\varepsilon_a = k \cdot r + 9$, function coefficient $k = 1.8 \times 10^6$, medium column radius $r_a = 0.26a$. (a) TE wave, (b) TM wave.

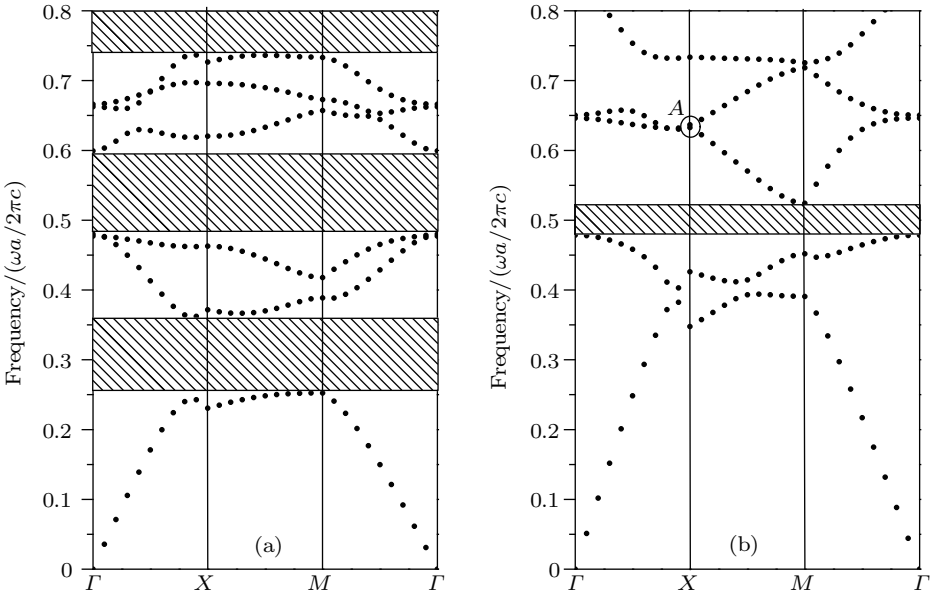


Fig. 4. The band gaps structure of two-dimensional function photonic crystals for the triangle lattice, $\varepsilon_b = 1$, $\varepsilon_a = k \cdot r + 9$, function coefficient $k = 1.8 \times 10^6$, medium column radius $r_a = 0.3a$. (a) TE wave, (b) TM wave, there is one semi-Dirac point A for TM wave. In the vicinity of the A point, the dispersion relation is linear along the XM direction and quadratic along the $X\Gamma$ direction.

In Figs. 7 and 8, we take $r_a = 0.65a$, $b = 9$ and research the function coefficient k effect on the band gaps structures in the frequency range from 0 to 0.8. In Figs. 7(a) and 7(b), we give the band gaps structures of TE and TM waves with the function coefficient $k = 3.6 \times 10^6$. There are three band gaps for the TE wave, and one band gap for the TM wave. In the vicinity of 0.31 frequency, there is an absolute band gap. There are two semi-Dirac points A and B for the TE wave, and one semi-Dirac point C for the TM wave. In the vicinity of the A point, the dispersion relation is linear along the XM direction and quadratic along the $X\Gamma$ direction. In the shadow of B and C points, the dispersion relation is linear along the MX direction and quadratic along the $M\Gamma$ direction. In Figs. 8(a) and 8(b), we give the band gaps structures of TE and TM waves with function coefficient $k = -1.8 \times 10^6$. There are two band gaps for the TE and TM waves. In the vicinity of 0.26 frequency, there is an absolute band gap, and one semi-Dirac point A for the TM wave. In the vicinity of the A point, the dispersion relation is linear along the XM direction and quadratic along the $X\Gamma$ direction. From Fig. 3 to Fig. 8, we find the band gaps numbers, width, and position are changed, and the semi-Dirac point can either occur or disappear with the medium column radius r_a and function coefficient k changing. We can obtain the appropriate band gaps structures by select-

ing the different medium column radius and function form of the dielectric constant for the two-dimensional function photonic crystals. Comparing the band gaps structure of conventional photonic crystals (shown in Fig. 2) and function photonic crystals (shown in Figs. 3–8), we can find the band gaps numbers, width and position are different.

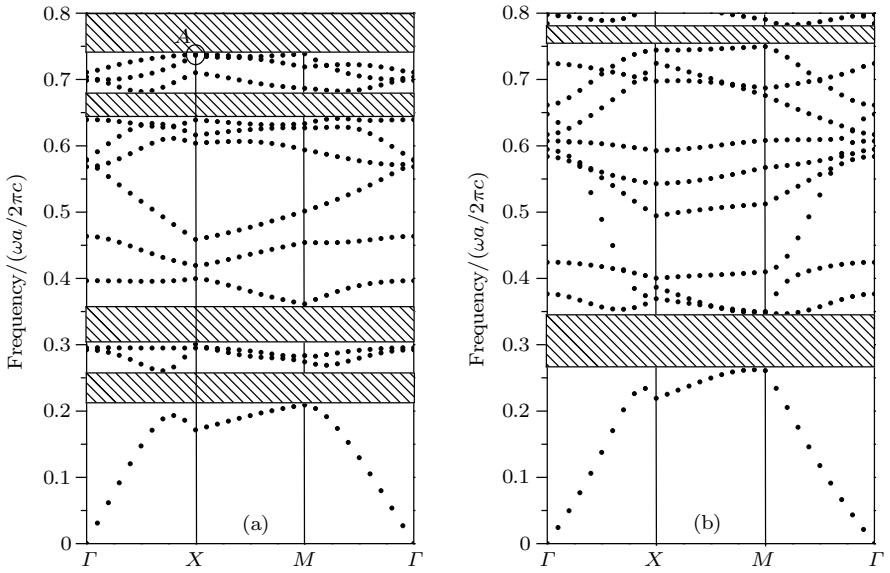


Fig. 5. The band gaps structure of two-dimensional function photonic crystals for the triangle lattice, $\varepsilon_b = 1$, $\varepsilon_a = k \cdot r + 9$, function coefficient $k = 1.8 \times 10^6$, and medium column radius $r_a = 0.65a$. (a) TE wave, there is one semi-Dirac point A for the TE wave. In the vicinity of the A point, the dispersion relation is linear along the XM direction and quadratic along the $X\Gamma$ direction. (b) TM wave.

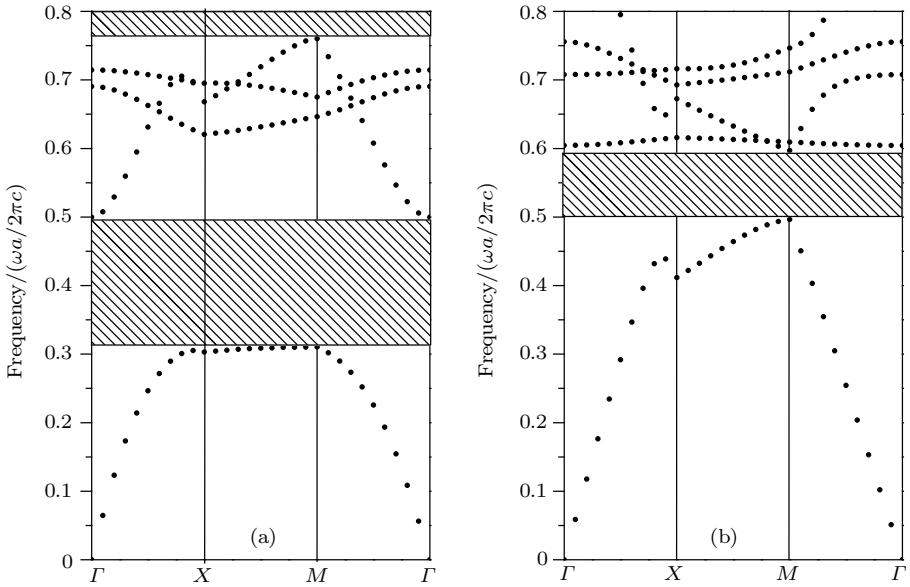


Fig. 6. The band gaps structure of two-dimensional function photonic crystals for the triangle lattice, $\varepsilon_b = 1$, $\varepsilon_a = k \cdot r + 9$, function coefficient $k = 1.8 \times 10^6$, and medium column radius $r_a = 0.8a$. (a) TE wave, (b) TM wave.

For the two-dimensional conventional photonic crystals, it has a certain structure, and its band gaps structure are determined. If we need a new band gaps structure, the two-dimensional conventional photonic crystals should be remade. For the two-dimensional function photonic crystals, by changing the external light intensity distribution, the function forms of the medium column dielectric constants are changed, and the band gaps structure will be changed, i.e., through adjusting the light intensity distribution, we can obtain the determinate band gaps structure; the two-dimensional photonic crystals need not be remade and can be used repeatedly. Therefore, the two-dimensional function photonic crystals can both adjust the band gaps structure conveniently and reduce the manufacturing cost. A photonic crystal band gap's width is related to the lattice symmetry degree. The two-dimensional photonic crystal has the point group symmetry,[32] it can form a semi-Dirac point. In this paper, the two-dimensional triangle lattice function photonic crystal met the creation conditions of the semi-Dirac point, since the two-dimensional function photonic crystal keep the point group symmetry, but translation

symmetry is decreased, the crystal symmetry reduces, which is advantageous to the formation of the anisotropic dispersion relation of the Dirac point and semi-Dirac point. Since the symmetry decrease makes the band gap width narrow, at a special condition, the bands intersect at one point, to form a Dirac point or semi-Dirac point. The two-dimensional conventional photonic crystals possess both point group symmetry and translation symmetry, it has higher symmetry and wider band gaps, which make the possibility of forming a semi-Dirac point reduce. Therefore, the two-dimensional function photonic crystals form a semi-Dirac point more easily than the two-dimensional conventional photonic crystals.

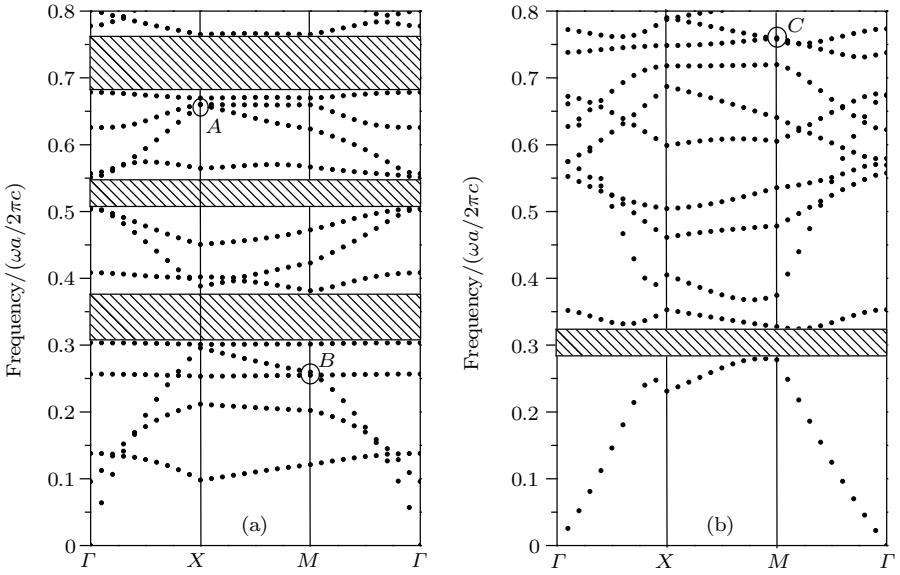


Fig. 7. The band gaps structure of two-dimensional function photonic crystals for the triangle lattice, $\varepsilon_b = 1$, $\varepsilon_a = k \cdot r + 9$, function coefficient $k = 3.6 \times 10^6$, medium column radius $r_a = 0.65a$. (a) TE wave, there are two semi-Dirac points A and B for the TE wave. In the vicinity of the A point, the dispersion relation is linear along the XM direction and quadratic along the $X\Gamma$ direction. In the vicinity of the B point, the dispersion relation is linear along the MX direction and quadratic along the $M\Gamma$ direction. (b) TM wave, there is one semi-Dirac point C for the TM wave. In the vicinity of the C point, the dispersion relation is linear along the MX direction and quadratic along the $M\Gamma$ direction.

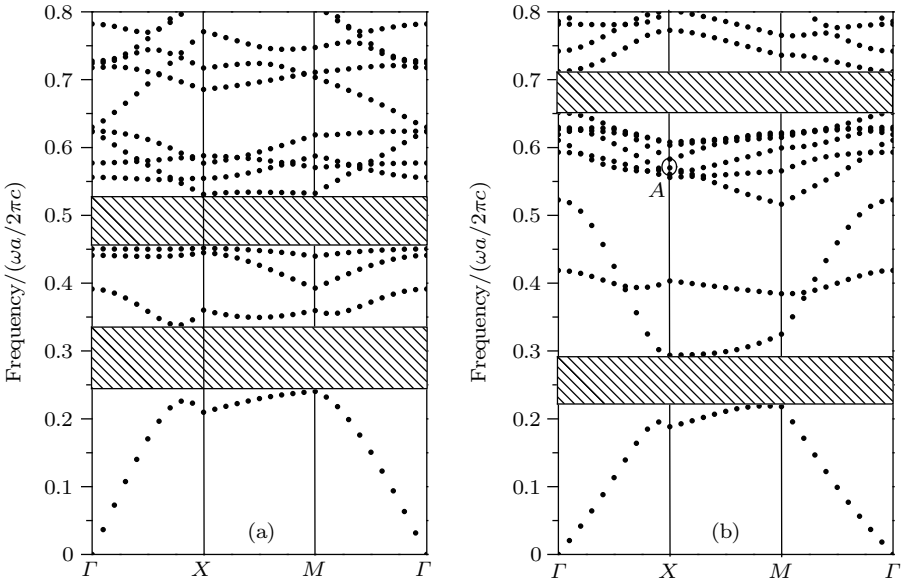


Fig. 8. The band gaps structure of two-dimensional function photonic crystals for the triangle lattice, $\varepsilon_b = 1$, $\varepsilon_a = k \cdot r + 9$, function coefficient $k = -1.8 \times 10^6$, medium column radius $r_a = 0.65a$. (a) TE wave, (b) TM wave, there is one semi-Dirac point A for the TM wave. In the vicinity of the A point, the dispersion relation is linear along the XM direction and quadratic along the $X\Gamma$ direction.

4. Conclusion

In this paper, we have studied the two-dimensional function photonic crystals through calculating the band gaps structures of TE and TM waves and found that they are different from the two-dimensional conventional photonic crystals. Some absolute band gaps and semi-Dirac points occur in the two-dimensional function photonic crystals. With the changing of the external light intensity distribution, the function forms of the medium column dielectric constants are changed, and the numbers, width, and position of band gaps

are changed, and a semi-Dirac point can either occur or disappear. All the results will provide a new design method for optical devices.

References

[1] Sun Y X, Kong X T and Fang Y T 2016 *Opt. Commun.* **376** 115
[2] Francesco M and Andrea A 2014 *Chin. Phys. B* **23** 047809
[3] Wang X, Chen L C, Liu Y H, Shi Y L and Sun Y 2015 *Acta Phys. Sin.* **64** 174206 (in Chinese)
[4] Geng T, Wu N, Dong X M and Gao X M 2016 *Acta Phys. Sin.* **65** 014213 (in Chinese)
[5] Lu C, Li W, Jiang X Y and Cao J C 2014 *Chin. Phys. B* **23** 097802
[6] Chen Z H, Wang Y, Yang Y B, Qiao N, Wang Y C and Yu Z Y 2014 *Nanoscale* **6** 14708
[7] Chen Z H, Qiao N, Yang Y B, Ye H, Liu S D, Wang W J and Wang Y C 2015 *Sci. Rep.* **5** 12794
[8] Chen Z H, Qiao N, Wang Y, Liang L, Yang Y B, Ye H and Liu S D 2016 *Appl. Energy* **172** 59
[9] Bittner S, Dietz B, Miski O M, Oria I P, Richter A and Schafer F 2010 *Phys. Rev. B* **82** 014301
[10] Chan C T, Hang Z H and Huang X Q 2012 *Adv. OptoElectron.* **2012** 313984
[11] Sakoda K 2012 *Opt. Express* **20** 3898
[12] Sakoda K 2012 *Opt. Express* **20** 25181
[13] Mei J, Wu Y, Chan C T and Zhang Z Q 2012 *Phys. Rev. B* **86** 035141
[14] Liu F M, Lai Y, Huang X Q and Chan C T 2011 *Phys. Rev. B* **84** 224113
[15] Haldane F D M and Raghu S 2008 *Phys. Rev. Lett.* **100** 013904
[16] Sepkhanov R A, Nilsson J and Beenakker C W J 2008 *Phys. Rev. B* **78** 045122
[17] Khanikaev A B, Mousavi S H, Tse W K, Kargarian M, MacDonald A H and Shvets G 2013 *Nat. Mater.* **12** 233
[18] Rechtsman M C, Zeuner J M, Plotnik Y, Lumer Y, Podolsky D, Dreisow F, Nolte S, Segev M and Szameit A 2013 *Nature* **496** 196
[19] Pardo V and Pickett W E 2009 *Phys. Rev. Lett.* **102** 166803
[20] Banerjee S, Singh R R P, Pardo V and Pickett W E 2009 *Phys. Rev. Lett.* **103** 016402
[21] Goerbig M O 2011 *Rev. Mod. Phys.* **83** 1193
[22] Wu X Y, Zhang B J, Yang J H, Liu X J, Ba N, Wu Y H and Wang Q C 2011 *Physica E* **43** 1694
[23] Wu X Y, Zhang B J, Liu X J, Ba N, Zhang S Q and Wang J 2012 *Physica E* **44** 1223
[24] Wu X Y, Zhang B J, Yang J H, Zhang S Q, Liu X J, Wang J, Ba N, Hua Z and Yin X G 2012 *Physica E* **45** 166
[25] Wu X Y, Zhang B J, Liu X J, Zhang S Q, Wang J, Ba N, Xiao L and Li H 2012 *Physica E* **46** 133
[26] Wu X Y, Zhang S Q, Zhang B J, Liu X J, Wang J, Li H, Ba N, Yin X G and Li J W 2013 *Physica E* **53** 1
[27] Ma J, Wu X Y, Li H B, Li H, Liu X J, Zhang S Q, Chen W J and Wu Y H 2015 *Opt. Mater.* **48** 59
[28] Nye J F 1985 *Physical Properties of Crystals* (Oxford: Oxford University Press) p. 32
[29] Boyd R W 2010 *Nonlinear Optics* (Acad. Press) pp. 207–228
[30] Ho K M, Chan C T and Soukoulis C M 1990 *Phys. Rev. Lett.* **65** 3152
[31] Zhang Z and Satpathy S 1990 *Phys. Rev. Lett.* **65** 2650
[32] Ochiai T and Onoda M 2009 *Phys. Rev. B* **80** 155103