Probe gain via four-wave mixing based on spontaneously generated coherence

Hong Yang(杨红)[1], Ting-gui Zhang(张廷桂)[2], and Yan Zhang(张岩)[3,†]

[1] College of Physics and Electronic Engineering, Hainan Normal University, Haikou 571158, China
[2] School of Mathematics and Statistics, Hainan Normal University, Haikou 571158, China
[3] School of Physics, Northeast Normal University, Changchun 130024, China

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We have studied the probe gain via a double-Λ atomic system with a pair of closely lying lower levels in the presence of two probe and two coherent pump fields. The inversionless gain can be realized by using nondegenerate four-wave mixing under the condition of spontaneously generated coherence (SGC) owing to near-degenerate lower levels. Note that by using SGC, two probe fields can be amplified with more remarkable amplitudes, and the gain spectra of an extremely narrow linewidth can be obtained. Last but not least, our results show that the probe gain is quite sensitive to relative phases due to the SGC presence which allows one to modulate the gain spectra periodically by phase modulation, and can also be influenced by all laser field intensities and frequencies, and the angles between dipole elements.

Keywords: probe gain, four-wave mixing, spontaneously generated coherence, narrow spectra linewidth

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1. Introduction

In recent years, lasing without population inversion (LWI) has been demonstrated experimentally and theoretically, which provides us with an important and interesting alternative[1–8] depending on the incoherent pumping process. Thus the high-frequency laser (i.e., the ultraviolet or x-ray range) can be realized which is impractical in conventional lasering with population inversion. The essence of LWI achieved with applied fields attributed to either inversion between dressed states or coherence among these states.[9–12] Due to the multiple potential applications of LWI, i.e., from ultrahigh-sensitive magnetometers to the generation of giant pulses of laser light and from cooling to isotope separation,[13–15] various schemes to realize inversionless gain have been proposed and attracted extensive attention.[16,17] Then, some factors have been considered in the LWI scheme, such as the phase of external fields which can modulate the probe gain periodically.[18–21] Besides, the effect of spontaneously generated coherence (SGC) depending on nonorthogonality of dipole matrix elements[24–26] has been discussed in the systems of light amplification without population inversion as shown in Refs. [18]–[23]. In those systems, SGC may destroy the dark state for achieving laser-gain, as the coherence process of spontaneous emission arises from two interference channels (either from two closely lying excited levels to a common ground level or from a common excited level to two closely lying ground levels.[27,28])

In this article, we study inversionless probe gain in a coherently driven four-level double-Λ atomic system. Being distinct from previous schemes depending on an incoherent pump to destroy dark states, our scheme's probe fields are amplified by common contributions of the resonant four-wave mixing (FWM) process and the electromagnetically induced transparency (EIT) effect. The resonant FWM process can be used to amplify probe fields and the EIT results in the reduction of probe absorption.[29–37] Moreover, the atomic system in our study has two closely lying lower levels, so SGC should be taken into account. It is interesting that the probe fields can be enhanced approximately 2500 times owing to the coherence arising from the spontaneous emission. Meanwhile, a narrow gain spectral linewidth can be obtained by decreasing the coherent pump field Rabi frequencies. An obvious advantage of the high and narrow gain spectral line is that it can be used to improve the measuring accuracy. Besides, we further study the probe gain modulation by adjusting the relative phase between applied fields. Our numerical results show that the probe gain is sensitive to the phase, and restricted by the angle between dipole elements.

2. Theoretical models and equations

We consider a four-level double-Λ atomic system of two far-spaced upper levels $|3\rangle$ and $|4\rangle$ and two closely lying lower

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†Corresponding author. E-mail: zhangy345@nenu.edu.cn

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levels $|1\rangle$ and $|2\rangle$, as shown in Fig. 1.

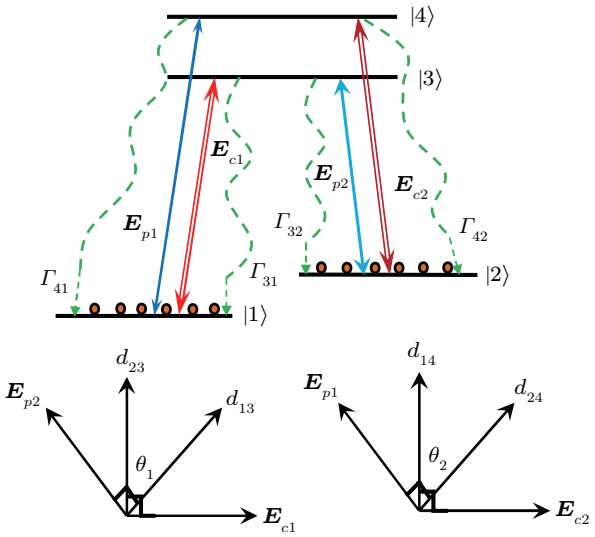


Fig. 1. (color online) Schematic diagram of a four-level double-Λatomic system driven by two weak probe fields $\boldsymbol{E}_{p1}$, $\boldsymbol{E}_{p2}$ and two strong pumping fields $\boldsymbol{E}_{c1}$, $\boldsymbol{E}_{c2}$. One laser field drives only a transition for the polarizations of laser fields are chosen; the arrangement of the applied electric fields $\boldsymbol{E}_{p2}$ and $\boldsymbol{E}_{c1}$ ($\boldsymbol{E}_{p1}$ and $\boldsymbol{E}_{c2}$) and relevant dipole moments $\boldsymbol{d}_{23}$ and $\boldsymbol{d}_{13}$ ($\boldsymbol{d}_{14}$ and $\boldsymbol{d}_{24}$).

The electric dipole-allowed transitions $|1\rangle \leftrightarrow |3\rangle$ and $|2\rangle \leftrightarrow |4\rangle$ are coupled by double strong laser fields with amplitudes (frequencies) $\boldsymbol{E}_{c1}$ and $\boldsymbol{E}_{c2}$ (ω_{c1} and ω_{c2}), respectively. Two weak laser fields with amplitudes (frequencies) $\boldsymbol{E}_{p1}$ and $\boldsymbol{E}_{p2}$ (ω_{p1} and ω_{p2}) probe the gain on transitions $|1\rangle \leftrightarrow |4\rangle$ and $|2\rangle \leftrightarrow |3\rangle$, respectively. The single-photon detunings (Rabi frequencies) of fields are denoted by

$$\Delta_{p1} = \omega_{41} - \omega_{p1}, \quad \Delta_{p2} = \omega_{32} - \omega_{p2}, \quad \Delta_{c1} = \omega_{31} - \omega_{c1}$$

and

$$\begin{aligned}
&\Delta_{c2} = \omega_{42} - \omega_{c2}, \quad \Omega_{p1} = \boldsymbol{E}_{p1} \cdot \boldsymbol{d}_{14}/2\hbar, \\
&\Omega_{p2} = \boldsymbol{E}_{p2} \cdot \boldsymbol{d}_{23}/2\hbar, \quad \Omega_{c1} = \boldsymbol{E}_{c1} \cdot \boldsymbol{d}_{13}/2\hbar, \\
&\Omega_{c2} = \boldsymbol{E}_{c2} \cdot \boldsymbol{d}_{24}/2\hbar,
\end{aligned}$$

respectively. Here, ω_{ij} ($\boldsymbol{d}_{ij}$) is the transition frequency (diploe moment) on transition $|i\rangle \leftrightarrow |j\rangle$. It is worth stressing that $\boldsymbol{d}_{14}$.

Under the rotating-wave and electric-dipole approximations, with the assumption of $\hbar = 1$, we can write down the atom–field interaction Hamiltonian

$$\begin{aligned}
H_I = &(\Delta_{c1} - \Delta_{p2})\,|2\rangle\langle 2| + \Delta_{c1}\,|3\rangle\langle 3| + (\Delta_{c1} + \Delta_{c2} - \Delta_{p2})\,|4\rangle\langle 4| \\
&-[\Omega_{c1}\,|3\rangle\langle 1| + \Omega_{p2}\,|3\rangle\langle 2| + \Omega_{p1}\,|4\rangle\langle 1| + \Omega_{c2}\,|4\rangle\langle 2| + \text{h.c.}].
\end{aligned} \tag{1}$$

Then, by substituting H_I into the master equation of motion for the density operator by using the Weisskopf–Wigner theory of spontaneous emission,[23] we obtain the following density matrix elements

$$\begin{aligned}
\partial_t\rho_{11} &= \mathrm{i}\Omega_{c1}^*\rho_{31} - \mathrm{i}\Omega_{c1}\rho_{13} + \mathrm{i}\Omega_{p1}^*\rho_{41} - \mathrm{i}\Omega_{p1}\rho_{14} + \Gamma_{31}\rho_{33} + \Gamma_{41}\rho_{44}, \\
\partial_t\rho_{12} &= -(\gamma_{21} + \mathrm{i}\Delta_{p2} - \mathrm{i}\Delta_{c1})\rho_{12} + \mathrm{i}\Omega_{c1}^*\rho_{32} - \mathrm{i}\Omega_{p2}\rho_{13} + \mathrm{i}\Omega_{p1}^*\rho_{42} - \mathrm{i}\Omega_{c2}\rho_{14} - (\eta_1\sqrt{\Gamma_{32}\Gamma_{31}}\cos\theta_1\rho_{33} + \eta_2\sqrt{\Gamma_{42}\Gamma_{41}}\cos\theta_2\rho_{44}), \\
\partial_t\rho_{13} &= -(\gamma_{31} - \mathrm{i}\Delta_{c1})\rho_{13} - \mathrm{i}\Omega_{c1}^*\rho_{11} - \mathrm{i}\Omega_{p2}^*\rho_{12} + \mathrm{i}\Omega_{c1}^*\rho_{33} + \mathrm{i}\Omega_{p1}^*\rho_{43}, \\
\partial_t\rho_{14} &= -(\gamma_{41} - \mathrm{i}\Delta_{c1} - \mathrm{i}\Delta_{c2} + \mathrm{i}\Delta_{p2})\rho_{14} - \mathrm{i}\Omega_{p1}^*\rho_{11} - \mathrm{i}\Omega_{c2}^*\rho_{12} + \mathrm{i}\Omega_{p1}^*\rho_{44} + \mathrm{i}\Omega_{c1}^*\rho_{34}, \\
\partial_t\rho_{22} &= \mathrm{i}\Omega_{p2}^*\rho_{32} - \mathrm{i}\Omega_{p2}\rho_{23} + \mathrm{i}\Omega_{c2}^*\rho_{42} - \mathrm{i}\Omega_{c2}\rho_{24} + \Gamma_{32}\rho_{33} + \Gamma_{42}\rho_{44}, \\
\partial_t\rho_{23} &= -(\gamma_{32} - \mathrm{i}\Delta_{p2})\rho_{23} - \mathrm{i}\Omega_{c1}^*\rho_{21} - \mathrm{i}\Omega_{p2}^*\rho_{22} + \mathrm{i}\Omega_{p2}^*\rho_{33} + \mathrm{i}\Omega_{c2}^*\rho_{43}, \\
\partial_t\rho_{24} &= -(\gamma_{42} - \mathrm{i}\Delta_{c2})\rho_{24} - \mathrm{i}\Omega_{p1}^*\rho_{21} - \mathrm{i}\Omega_{c2}^*\rho_{22} + \mathrm{i}\Omega_{p2}^*\rho_{34} + \mathrm{i}\Omega_{c2}^*\rho_{44}, \\
\partial_t\rho_{33} &= \mathrm{i}\Omega_{c1}\rho_{13} - \mathrm{i}\Omega_{c1}^*\rho_{31} + \mathrm{i}\Omega_{p2}\rho_{23} - \mathrm{i}\Omega_{p2}^*\rho_{32} - (\Gamma_{31} + \Gamma_{32} + \Gamma_{43})\rho_{33} + \Gamma_{43}\rho_{44}, \\
\partial_t\rho_{34} &= -(\gamma_{43} + \mathrm{i}\Delta_{p2} - \mathrm{i}\Delta_{c2})\rho_{34} + \mathrm{i}\Omega_{c1}\rho_{14} - \mathrm{i}\Omega_{p1}^*\rho_{31} + \mathrm{i}\Omega_{p2}\rho_{24} - \mathrm{i}\Omega_{c2}^*\rho_{32},
\end{aligned} \tag{2}$$

which are constrained by the trace condition $\rho_{11} + \rho_{22} + \rho_{33} + \rho_{44} = 1$ and the conjugation condition $\rho_{ij} = \rho_{ji}^*$. Here θ_1 (θ_2) is the angle between the two induced dipole moments as shown in Fig. 1, θ_1 (θ_2) could not be changed if the atomic level structure has been chosen, because the induced dipole moments depend on the atomic transitions. Γ_{31} and Γ_{32} (Γ_{41} and Γ_{42}) are the spontaneous emission rates from level $|3\rangle$ ($|4\rangle$) to levels $|1\rangle$ and $|2\rangle$, respectively. To be more specific, we use $\gamma_{ij} = (\Gamma_i + \Gamma_j)/2$ with $\Gamma_i = \sum_m \Gamma_{im}$, $\Gamma_j = \sum_n \Gamma_{jn}$. $\eta_1\sqrt{\Gamma_{32}\Gamma_{31}}\cos\theta_i$ ($\eta_2\sqrt{\Gamma_{42}\Gamma_{41}}\cos\theta_2$) represents the quantum interference effect arising from the cross coupling between spontaneous emission $|3\rangle \to |1\rangle$ and $|3\rangle \to |2\rangle$ ($|4\rangle \to |1\rangle$ and $|4\rangle \to |2\rangle$), respectively, referring to SGC. Note that the existence of SGC requires that dipole moments $d_{13}(d_{14})$ and $d_{23}(d_{24})$ are not orthogonal, which is denoted by $\eta_1 = 1$ ($\eta_2 = 1$) otherwise $\eta_1 = 0$ ($\eta_2 = 0$). In other words, SGC may become remarkable only for the small energy spacing between two closely lying levels, if $\omega_{21} \gg \Gamma_{31}, \Gamma_{41}, \Gamma_{32}, \Gamma_{42}$ there is no SGC effect.

By taking into account the field phases required by SGC, we rewrite the complex Rabi frequencies as $\Omega_{pi} = G_{pi}\,\mathrm{e}^{\mathrm{i}\Phi_{pi}}$ ($G_{p1} = G_{p10}\sin\theta_2$, $G_{p2} = G_{p20}\sin\theta_1$), and $\Omega_{ci} = G_{ci}\,\mathrm{e}^{\mathrm{i}\Phi_{ci}}$ ($G_{ci} = G_{ci0}\sin\theta_i$) where G_{pi} (G_{ci}) is real. We redefine atomic variables in Eqs. (2) as $\rho_{ii} = \sigma_{ii}$, $\rho_{12} = \sigma_{12}\exp[\mathrm{i}\,(\Phi_{p2} - \Phi_{c1})]$, $\rho_{21} = \sigma_{21}\exp[\mathrm{i}\,(\Phi_{p1} - \Phi_{c2})]$, $\rho_{34} = \sigma_{34}\exp[\mathrm{i}\,(\Phi_{c2} - \Phi_{p2})]$,

$\rho_{43} = \sigma_{43}\exp[\mathrm{i}(\Phi_{c1}-\Phi_{p1})]$, $\rho_{13} = \sigma_{13}\exp(\mathrm{i}\Phi_{c1})$, $\rho_{14} = \sigma_{14}\exp(\mathrm{i}\Phi_{p1})$, $\rho_{23} = \sigma_{23}\exp(\mathrm{i}\Phi_{p2})$, and $\rho_{24} = \sigma_{24}\exp(\mathrm{i}\Phi_{c2})$, where the relative phases are $\Phi_1 = \Phi_{p2} - \Phi_{c1}$ and $\Phi_2 = \Phi_{c2} - \Phi_{p1}$, satisfying $\Phi_{p2} - \Phi_{c1} = -(\Phi_{p1} - \Phi_{c2})$, $\Phi_{c2} - \Phi_{p2} = -(\Phi_{c1} - \Phi_{p1})$. With the redefined density-matrix elements, we obtain the following new equations

$$
\begin{aligned}
&\partial_t\sigma_{11} = \mathrm{i}G_{c1}^*\sigma_{31} - \mathrm{i}G_{c1}\sigma_{13} + \mathrm{i}G_{p1}^*\sigma_{41} - \mathrm{i}G_{p1}\sigma_{14} + \Gamma_{31}\sigma_{33} + \Gamma_{41}\sigma_{44},\\
&\partial_t\sigma_{12} = -(\gamma_{21} + \mathrm{i}\Delta_{p2} - \mathrm{i}\Delta_{c1})\sigma_{12} + \mathrm{i}G_{c1}^*\sigma_{32} - \mathrm{i}G_{p2}\sigma_{13} + \mathrm{i}G_{p1}^*\sigma_{42} - \mathrm{i}G_{c2}\sigma_{14} - (\eta_1'\sqrt{\Gamma_{32}\Gamma_{31}}\cos\theta_1\sigma_{33} + \eta_2'\sqrt{\Gamma_{42}\Gamma_{41}}\cos\theta_2\sigma_{44}),\\
&\partial_t\sigma_{13} = -(\gamma_{31} - \mathrm{i}\Delta_{c1})\sigma_{13} - \mathrm{i}G_{c1}^*\sigma_{11} - \mathrm{i}G_{p2}^*\sigma_{12} + \mathrm{i}G_{c1}^*\sigma_{33} + \mathrm{i}G_{p1}^*\sigma_{43},\\
&\partial_t\sigma_{14} = -(\gamma_{41} - \mathrm{i}\Delta_{c1} - \mathrm{i}\Delta_{c2} + \mathrm{i}\Delta_{p2})\sigma_{14} - \mathrm{i}G_{p1}^*\sigma_{11} - \mathrm{i}G_{c2}^*\sigma_{12} + \mathrm{i}G_{p1}^*\sigma_{44} + \mathrm{i}G_{c1}^*\sigma_{34},\\
&\partial_t\sigma_{22} = \mathrm{i}G_{p2}^*\sigma_{32} - \mathrm{i}G_{p2}\sigma_{23} + \mathrm{i}G_{c2}^*\sigma_{42} - \mathrm{i}G_{c2}\sigma_{24} + \Gamma_{32}\sigma_{33} + \Gamma_{42}\sigma_{44},\\
&\partial_t\sigma_{23} = -(\gamma_{32} - \mathrm{i}\Delta_{p2})\sigma_{23} - \mathrm{i}G_{c1}^*\sigma_{21} - \mathrm{i}G_{p2}^*\sigma_{22} + \mathrm{i}G_{p2}^*\sigma_{33} + \mathrm{i}G_{c2}^*\sigma_{43},\\
&\partial_t\sigma_{24} = -(\gamma_{42} - \mathrm{i}\Delta_{c2})\sigma_{24} - \mathrm{i}G_{p1}^*\sigma_{21} - \mathrm{i}G_{c2}^*\sigma_{22} + \mathrm{i}G_{p2}^*\sigma_{34} + \mathrm{i}G_{c2}^*\sigma_{44},\\
&\partial_t\sigma_{33} = \mathrm{i}G_{c1}\sigma_{13} - \mathrm{i}G_{c1}^*\sigma_{31} + \mathrm{i}G_{p2}\sigma_{23} - \mathrm{i}G_{p2}^*\sigma_{32} - (\Gamma_{31} + \Gamma_{32} + \Gamma_{43})\sigma_{33} + \Gamma_{43}\sigma_{44},\\
&\partial_t\sigma_{34} = -(\gamma_{43} + \mathrm{i}\Delta_{p2} - \mathrm{i}\Delta_{c2})\sigma_{34} + \mathrm{i}G_{c1}\sigma_{14} - \mathrm{i}G_{p1}^*\sigma_{31} + \mathrm{i}G_{p2}\sigma_{24} - \mathrm{i}G_{c2}^*\sigma_{32}.
\end{aligned}
\tag{3}
$$

Note that equations (3) including phases are identical to Eqs. (2) except that η_i is replaced by $\eta_i' = \eta_i\mathrm{e}^{\mathrm{i}\Phi_i}$ while the complex Rabi frequencies are replaced by real ones. In the following discussion, we set $\Phi_1 = \Phi_2 = \Phi$ and $\theta_1 = \theta_2 = \theta$ due to the balance FWM process that $\Delta_{p2} = \Delta_{c1} + \Delta_{c2} - \Delta_{p1}$ in our symmetric scheme.

3. Numerical results and analytical discussions

With EIT and nondegenerate four-wave mixing nonlinearities, the inversionless gain has been discussed recently,[38] where the probe fields can transmit through the medium due to the laser induced quantum interference, and amplified in the process of resonant nondegenerate four-wave mixing, however, the gain amplitude is often small. Here, we find that with considering SGC, it is more interesting that the probe fields can be amplified with a much larger amplitude due to the interference between spontaneous emission channels via the four-wave mixing. Meanwhile, we can achieve inversionless gain because most atomic populations distribute on ground levels $|1\rangle$ and $|2\rangle$ as shown in Fig. 2(a).

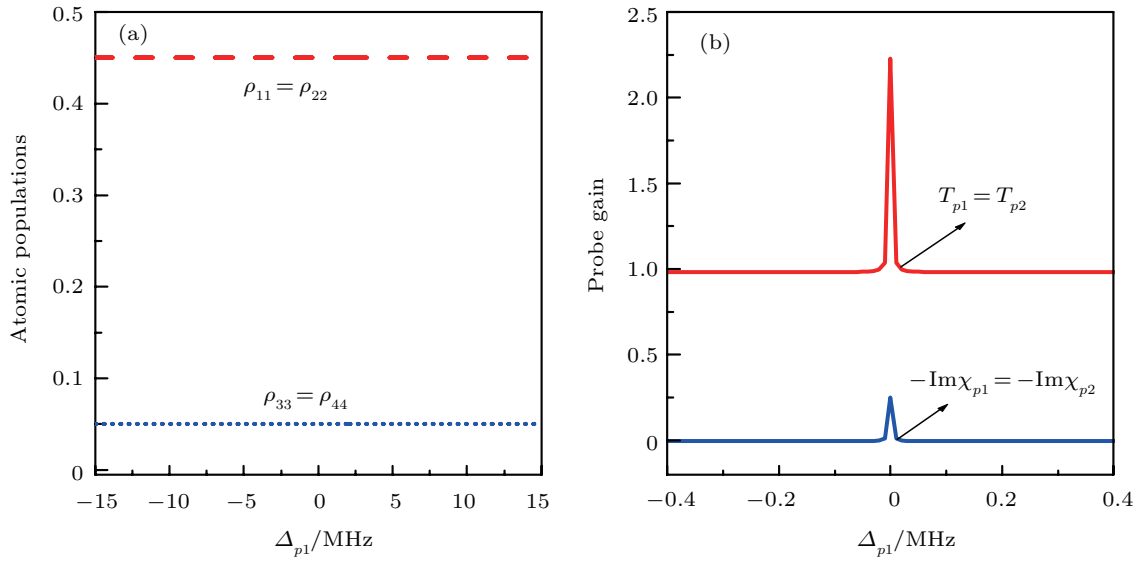


Fig. 2. (color online) (a) Atomic populations as functions of probe detuning Δ_{p1} with $\theta = \pi/2$, $G_{c10} = G_{c20} = 3.0$ MHz, $\Delta_{c1} = \Delta_{c2} = 0.0$ MHz. (b) The probe gain as functions of probe detuning Δ_{p1}, with $\theta = \pi/4$, $G_{c10} = G_{c20} = 0.1$ MHz, $z = 3$ mm. The relative parameters are $\Gamma_{31} = \Gamma_{32} = \Gamma_{41} = \Gamma_{42} = 6.0$ MHz, $\Gamma_{34} = \Gamma_{43} = 0$, $\Gamma_{12} = \Gamma_{21} = 1.0$ kHz, $G_{p10} = G_{p20} = 1.0$ kHz.

The probe field susceptibility is

$$\chi_{p1} = \frac{N_0|d_{14}|}{2\varepsilon_0\hbar}\frac{\rho_{41}}{G_{p1}}\left(\chi_{p2} = \frac{N_0|d_{23}|}{2\varepsilon_0\hbar}\frac{\rho_{32}}{G_{p2}}\right),$$

which also can be written as $\chi_{p1} = \chi_{p1}' + \mathrm{i}\chi_{p1}''$ ($\chi_{p2} = \chi_{p2}' + \mathrm{i}\chi_{p2}''$). Then it is easy to examine the complex refractive index $n_{p1} = \sqrt{1+\chi_{p1}}$, ($n_{p2} = \sqrt{1+\chi_{p2}}$). The imaginary and real parts of the refractive index govern local absorption (gain) and dispersive properties of the probe, respectively, and the gain coefficient for the light transmitted[39] can be expressed as

$$G_T = |T|^2, \tag{4}$$

where the transmissivity of probes can be expressed as

$$T_{pi} = \frac{I_{pi}}{I_{pi}(0)} = \frac{E_{pi}^2}{E_{pi}(0)^2} = \exp\left(\frac{-2\pi}{\lambda}\chi''_{pi}z\right), \qquad (5)$$

for $I_{pi}(0)$ ($E_{pi}(0)$) being the intensity (amplitude) of the incident and I_{pi} (E_{pi}) being the output intensity (amplitude). By numerically solving Eqs. (3) for the probe(s) susceptibility we can examine with the help of Eqs. (4) the probe gain (absorption). We plot the imaginary parts of two probe susceptibilities governing the responses of probe absorption for $t \to \infty$, as functions of probe detuning Δ_{p1} with different θ and $\Phi = 0$ as shown in Fig. 3.

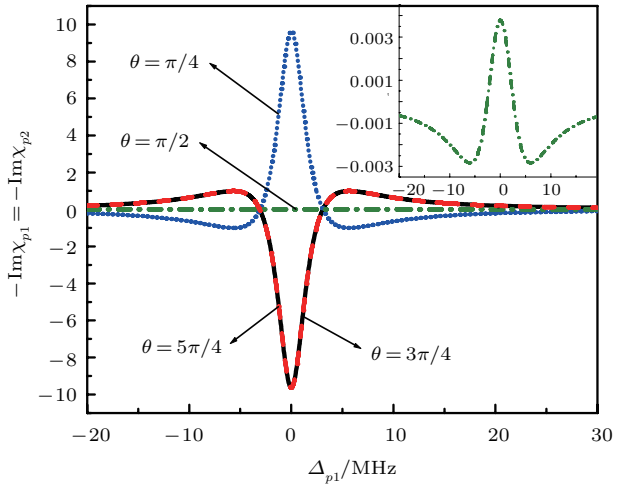


Fig. 3. (color online) Probe gain spectra (imaginary parts of probe susceptibilities) as functions of probe detuning Δ_{p1} with various θ. The relative parameters are $\Phi = 0$, $\Gamma_{31} = \Gamma_{32} = \Gamma_{41} = \Gamma_{42} = 6.0$ MHz, $\Gamma_{34} = \Gamma_{43} = 0$, $\Gamma_{12} = \Gamma_{21} = 1.0$ kHz, $G_{p10} = G_{p20} = 1.0$ kHz. $G_{c10} = G_{c20} = 3.0$ MHz, $\Delta_{c1} = \Delta_{c2} = 0.0$ MHz, and $\eta_1 = \eta_2 = 1$. The inset without considering SGC ($\eta_1 = \eta_2 = 0$) is the same as the line of $\theta = \pi/2$ in the main part.

When $\theta_i = \pi/4$ (then $\eta_i = 1$ meaning the SGC presence), around the frequency range of $\Delta_{p1} = 0$, the imaginary part of the probe susceptibility can reach to 10, which is approximative 2500 times as big as that when $\theta = \pi/2$ (then $\eta_i = 0$) referring to the absence of SGC resulting from the fact that dipole matrix elements are orthogonal. Then, we find that the single gain peak splits into double peaks appearing on both sides of $\Delta_{p1} = 0$ with $\theta = 3\pi/4$ ($5\pi/4$), where the maximum imaginary part value of the probe susceptibility is smaller compared with $\theta = \pi/4$ but much bigger compared with $\theta = \pi/2$. The results imply that the SGC presence has contributed enormously to the probe gain, whose efficiency can be increased drastically by using SGC in the FWM process. Meanwhile, the contribution of SGC is much more than that of the dynamically induced coherence arising from the coherent fields, especially in our resonant FWM atomic system. In addition, the results show that the amplitude and frequency position of the gain can be modulated by θ. In addition, we also study the case of $\theta = \pi/4$ and $3\pi/4$ ($5\pi/4$) but $\Phi = \pi$ (not shown here) and find that the gain spectra are contrary to those while $\Phi = 0$, which means that when we modulate the probe gain by adjusting θ, Φ should be properly chosen. This is because this system is very sensitive to θ and Φ due to the SGC presence, which can be seen in Eqs. (3). In experiments we know that, we do not often adjust θ, but we can make the weak probe field and the strong coherent field propagate in parallel to maximize the SGC effect.

However, the gain amplitude may be enhanced with increasing the coherent field intensity. In Fig. 4, we examine the gain spectral lines by changing Rabi frequencies of two coherent fields simultaneously with $\theta = \pi/4$ and $\Phi = 0$. In Fig. 4(a), compared with that when the coherent field Rabi frequency $\Omega_{ci} = 3.0$ MHz with SGC, the imaginary part of the probe susceptibility decreases a little when Ω_{ci} is reduced to 0.5 MHz. However, its value is still much larger than that when $\Omega_{ci} = 3.0$ MHz without SGC. Meanwhile, the linewidth of the gain spectrum will become quite narrow with the decreasing Ω_{ci}. Then, we further reduce two coherent field Rabi frequencies simultaneously.

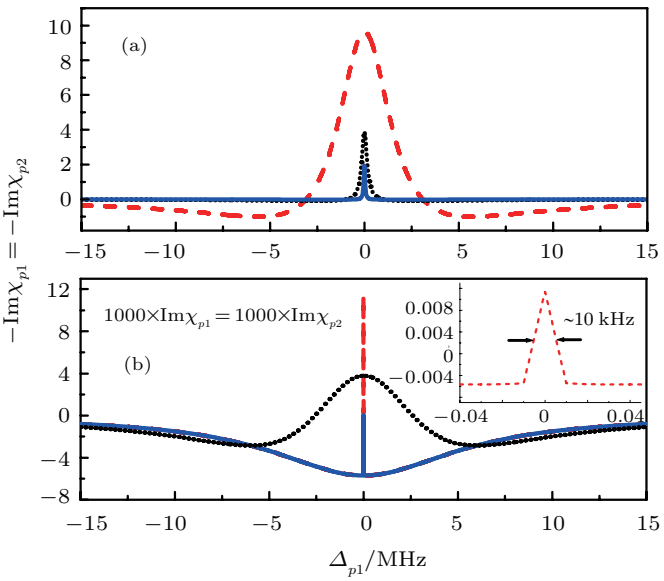


Fig. 4. (color online) Probe gain spectra versus probe detuning Δ_{p1} with the same reduced Rabi frequencies $G_{c1} = G_{c2} = G_{ci}$ and $\theta = \pi/4$. (a) Red-dashed, black-short-dotted, and blue-solid lines represent $G_{ci0} = 3.0$ MHz, 1.0 MHz, and 0.5 MHz, respectively. (b) Red-dashed, blue-solid, and black-short-dotted lines represent $G_{ci0} = 30$ kHz and $\eta_i = 1$, $G_{ci0} = 20$ kHz, and $\eta_i = 1$, $G_{ci0} = 3.0$ MHz, and $\eta_i = 0$, respectively. Other parameters are the same as those in Fig. 3.

Figure 4(b) shows that when the coherent field Rabi frequencies are reduced to $1/150 \sim 1/100$ with SGC, the gain amplitude is as big as that with $\Omega_{ci} = 3.0$ MHz and the SGC absence. However, the results show that an ultra narrow spectral line of probe gain can be obtained due to the relatively weak coherent fields. So our results imply that by using SGC there are two main advantages in this system which can realize the inversionless gain in the resonant FWM process. First, we do not need to use very strong coherent fields to amplify the probe fields with obvious amplitudes. Second, an ultra narrow probe spectrum can be obtained with the linewidth of

10KHz as shown in the inset of Fig. 4(b). The spectral feature as narrow as one wants is usually obtained in the numerics, because it is hard in experiments to reduce the relative phase noise between the involved lasers to reach this level of coherence. Some experimental results have tried to decrease the spectral linewidth by using SGC.[40]

In general, phase dependence is expected for a closed-loop excitation structure where pairs of modes interact through a common spin coherence.[41–45] In the single Λ system, there is no influence of the relative phase, except for the existence of SGC.[46] So there is no doubt that in our regime the probe gain is sensitive to the relative phase Φ while depending on θ due to the SGC presence, which can be seen in Eqs. (3). As described in Fig. 5, we plot the imaginary parts of probe susceptibilities as functions of Φ with $\theta = \pi/4$. When all the laser fields detunings are zero, Figure 5(a) shows that the gain of two probe fields can be periodically modulated with the period of 2π by the relative phases simultaneously, and can reach the maximum value while $\Phi = 2k\pi$. That means we can control the probe gain by adjusting the relative phases, which can be modulated by using a linear phase shifter.

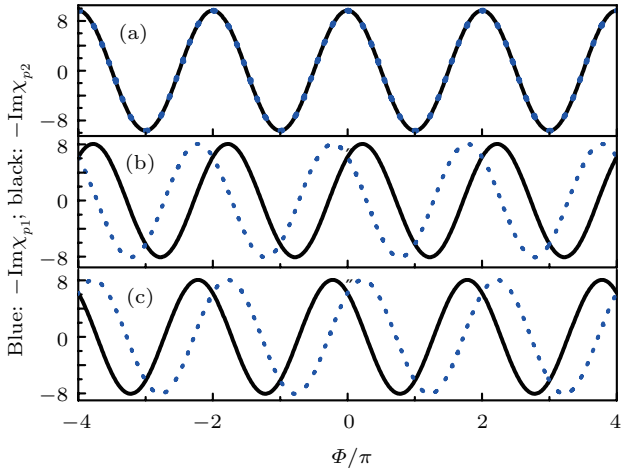


Fig. 5. (color online) Probe gain spectra versus relative phase Φ for various laser field detunings with $\theta = \pi/4$: (a) $\Delta_{pi} = \Delta_{ci} = 0$ MHz; (b) $\Delta_{pi} = \Delta_{ci} = 5$ MHz; (c) $\Delta_{p1} = -5$ MHz, $\Delta_{p2} = 15$ MHz, and $\Delta_{ci} = 5$ MHz. Other parameters are the same as those in Fig. 3.

As shown in Fig. 5(b), when we simultaneously change the detunings $\Delta_{p1} = \Delta_{p2} = \Delta_{c1} = \Delta_{c2} = 5$ MHz in the condition of $\Delta_{p1} + \Delta_{p2} = \Delta_{c1} + \Delta_{c2}$, the gain spectra of two probe fields can be separated and vary separately with the increasing Φ. Whereas, figure 5(c) shows that when $\Delta_{p1} = -5$ MHz, $\Delta_{p2} = 15$ MHz, and $\Delta_{c1} = \Delta_{c2} = 5$ MHZ, the phase position of two gain spectra can be changed by adjusting the detunings.

In order to examine the relationship between θ and Φ, we plot gain spectra as functions of Φ with $\theta = 3\pi/4$ in Fig. 6. Compared with Fig. 5, figure 6 shows that the maximum value appears at $\Phi = (2k+1)\pi$, which means that the probe gain spectra are a reversal. The results imply that if θ is varied by $\pi/2$, the phase position of the probe gain will be changed by π. However, the period is invariant.

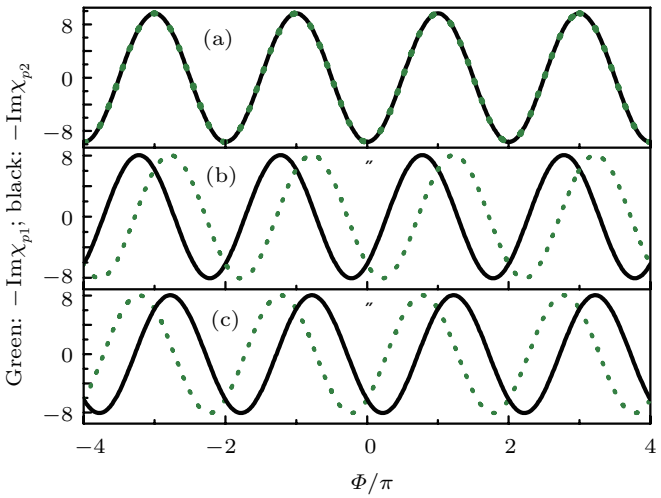


Fig. 6. (color online) Probe gain spectra versus the phase Φ for various laser field detunings with $\theta = 3\pi/4$: (a) $\Delta_{pi} = \Delta_{ci} = 0$ MHz; (b) $\Delta_{pi} = \Delta_{ci} = 5$ MHz; (c) $\Delta_{p1} = -5$ MHz, $\Delta_{p2} = 15$ MHz, and $\Delta_{ci} = 5$ MHz. Other parameters are the same as those in Fig. 3.

Our work is based on the theoretical model which provides meaningful and surprising results. The probe gains can be improved largely by considering SGC, and a narrow probe spectrum can be obtained by reducing the Rabi frequency of coupling fields. To the best of our knowledge, the linewidth of the probe spectrum can be modulated easily in the region between 1.5 MHz and 15 MHz in the existing experimental level.[47–49] Moreover, in experiment, the SGC is rather difficult to be achieved in a real atomic system. This rigorous condition can be realized in charged quantum dots and the atomic system with atoms placed near a metallic nanostructure or managed in the dressed-state picture.[50–52] For considering the experimental realization, the two close levels can be chosen as well separated Zeeman levels which belong to different hyperfine states of our model.[53]

4. Conclusions

In summary, we elaborately studied the inversionless gain with nondegenerate FWM linearities under the condition of EIT and SGC. We find that the two probe fields are simultaneously amplified with the same extent in a spectral range around $\Delta = 0$ with $\Omega_{c1} = \Omega_{c2}$ due to the balance FWM. In addition, two pairs of coherent channels between Γ_{31} and Γ_{32}, Γ_{41}, and Γ_{42} may exist due to the near-degenerate doublets $|1\rangle$ and $|2\rangle$, so SGC should be considered. With numerical solutions and qualitative analyses in hand, some advantages of our system are clearly shown in comparison with the previous systems, i.e., greatly enhanced the amplitude of the probe gain which is approximately 2500 times as large as that without considering SGC. Another noteworthy feature is that an extremely narrow gain spectral line can be obtained with weak coherent fields. Especially, with the laser field detunings of zero, we can obtain the two-color probe gain. Besides, we find that the gain

spectrum structure can be modulated by adjusting the phase with a fixed θ ($\theta \neq \pi/2$) required by the SGC existence and the laser field detunings.

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