Sub-Rayleigh imaging via undersampling scanning based on sparsity constraints*

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(Received 29 August 2016; revised manuscript received 11 October 2016; published online 10 January 2017)

We demonstrate that, by undersampling scanning object with a reconstruction algorithm related to compressed sensing, an image with the resolution exceeding the finest resolution defined by the numerical aperture of the system can be obtained. Experimental results show that the measurements needed to achieve sub-Rayleigh resolution enhancement can be less than 10% of the pixels of the object. This method offers a general approach applicable to point-by-point illumination super-resolution techniques.

Keywords: super-resolution, image reconstruction techniques

PACS: 42.40.Lx, 42.30.Va **DOI:** 10.1088/1674-1056/26/2/024203

1. Introduction

A widely used criterion to define the spatial resolution of a diffraction-limited imaging system is the so-called Rayleigh limit which sets the minimum separation for two points to be distinguishable in the image. The Rayleigh diffraction bound is related to the width of the point spread function (PSF) of the optical instrument, and the image is obtained by convolving the PSF with the transmission function of the object. The super-resolution, which refers to methods for improving the resolution of optical imaging system beyond the Rayleigh limit, has been a topic of great interest for more than a century. The use of scanning is very common in super-resolution methods such as confocal scanning microscopy.[1,2] Generally in confocal scanning fluorescence microscopy systems, the light source should be powerful so that it can still reach the detector when a pinhole is placed to limit the radius of the light. The accurate movement of the pinhole based on the focused beam cannot be achieved under particular circumstances.[3] The scanning time also is unfavorable for achieving real-time observation.

Recently, an information processing technique known as compressed sensing (CS),[4,5] which employs optimization to detect a sparse n-dimensional signal with $m < n$ measurements, has been utilized in many optical schemes.[6–8] The sparsity-based concepts of CS have also been taken into the super-resolution domain, and dramatic performances have been revealed.[9–13] The implementations of the single-pixel camera,[6] which is a successful application of CS, in fluorescence microscopy benefits from many advantages such as high dynamic range, facilitated multiplexing, and wide spectral range (from ultraviolet range to infra-red range).[14]

In this paper we demonstrate another approach that combines focused scanning with a reconstruction algorithm related to CS to achieve super-resolution images borne on incoherent light with theoretical analysis and experimental proof. In this method, different from the conventional focused illumination super-resolution methods which mainly try to narrow the PSF of the system, the sparsity of the object and the PSF of optical imaging system are utilized as priori knowledge to recover the information of the object. The measurements needed to achieve sub-Rayleigh resolution enhancement can be less than 10% of the pixels of the object. The conventional compressive fluorescence microscopy configurations mentioned above also can benefit from this scheme since it does not require modifying the experimental apparatus.

2. Image reconstruction methods

If we denote the object as an n-dimensional vector x, then the data acquisition by scanning can be described as

$$y = A \cdot x, \tag{1}$$

where A is the scanning matrix, namely, an n-by-n identity matrix, and y is the intensities measured. In an ideal imaging

*Project supported by the National Natural Science Foundation of China (Grant Nos. 61605218 and 61601442), the National Defense Science and Technology Innovation Foundation of the Chinese Academy of Sciences (Grant No. CXJJ-16S047), and the National Major Scientific Instruments Development Project of China (Grant No. 2013YQ030595).

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system, y should be the same with x, so an ideal image of x is retrieved. When the light is fully spatially incoherent, the linear, space-invariant model for imaging is

$$I_{\mathrm{i}}(x,y) = h(x,y) \otimes I_{\mathrm{g}}(x,y), \tag{2}$$

where $h(x,y)$ is the PSF of the system, $I_{\mathrm{g}}(x,y)$ is the ideal geometric irradiance image, $I_{\mathrm{i}}(x,y)$ is the image intensity, and $\otimes$ denotes convolution. In a diffraction-limited scanning imaging system, the scanning process can be describe as

$$y = M \cdot x, \tag{3}$$

where $M = h \otimes A$ is the measurement matrix obtained by A convoluted with the PSF of the system $h(x,y)$.

We suppose there exists a transform matrix Ψ to the sparse basis such that $x = \Psi \cdot x'$ where x' is sparse. The priori knowledge that the object can be sparsely expressed in a known basis is quite general, because many natural objects are indeed sparse in an appropriate basis. The basic result of CS is that the information of object can be obtained by searching for the sparsest solution under suitable measurements. We take the PSF of optical imaging system as another priori and pursue the sparsest solution consistent with the Rayleigh limit scanning measurements, i.e., we seek the x' of the minimum $l1$-norm that yields a good agreement with the measurements

$$\begin{aligned} \hat{x}' &= \arg\min \left\{ \left\| M \cdot \Psi \cdot x' - y \right\|_2^2 - \tau \left\| x' \right\|_1 \right\}, \\ \hat{x} &= \Psi \cdot \hat{x}', \end{aligned} \tag{4}$$

where $\hat{x}$ is the reconstructed image and τ is a constant scalar weighting the relative strength of the two terms. This super-resolution image reconstruction can be achieved via the optimization just as the optimization process in CS.

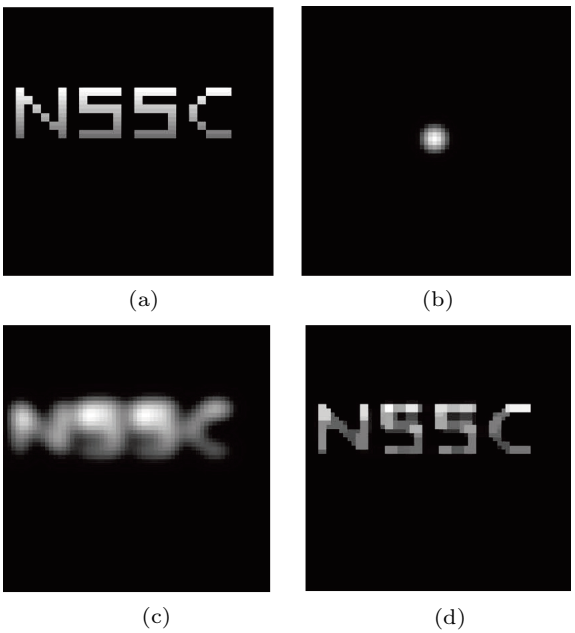


Fig. 1. The simulated demonstration of sub-Rayleigh imaging via undersampling scanning. (a) The object. (b) The PSF of optical imaging system. (c) The image obtained by the conventional scanning. (d) The image reconstructed by our method.

To demonstrate this idea theoretically, a simulation is performed in which an image of continuous varying grayscales is used as the object (Fig. 1(a)). The the wavelength of the incoherent incident light is 550 nm. The resolution of the imaging system is limited to by setting a certain PSF (Fig. 1(b)). The blurred image of the object is shown in Fig. 1(c). The undersampling scanning is executed by scanning with an interval of one pixel. The total number of the measurements of the is a half of that of the pixels of the object. The result yielded from our method is shown in Fig. 1(d), in which the detail displays again.

3. Experimental results

The experimental setup is given in Fig. 2. The illumination source was a halogen lamp with center wavelength 550 nm. An area on a digital micromirror device (DMD) in the window of size 875.52 μm × 875.52 μm was combined with a 4-F imaging system to provide scan coverage over the entire object. In the experiment, the focus length of the lens in the 4-F imaging system is 50.4 mm, and the diameter of the lens is 25.4 mm. The DMD consists of a 1024 × 768 array of individually addressable mirrors, and the size of each mirror is 13.68 μm × 13.68 μm. Each mirror can be turned to two different directions, and the light can be reflected in or out the system accordingly. The scanning process was achieved by turning the mirrors on the DMD to reflect light into the 4-F imaging system (magnification = 1) in sequence. A low-pass spatial filter was positioned at the Fourier plane of the imaging system to impose a stringent diffraction limit. A charge-coupled device (CCD) camera played as a bucket detector to measure the total intensity transmitted from the object.

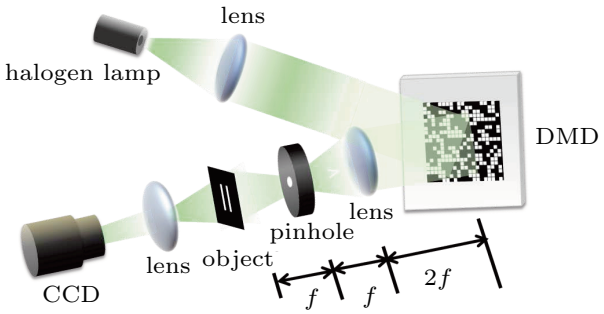


Fig. 2. (color online) Experimental setup.

The scanning was realized by group clusters of 2 × 2 mirrors, so the size of one "pixel" is 27.36 μm × 27.36 μm. A double slit (slit width 100 μm and center-to-center separation 225 μm) which is shown in Fig. 3(a) was used as the object. The object was covered by the images of 32 × 32 pixels, so the total pixels of the object is 1024. As shown in Fig. 3(c), since a low-pass spatial filter was inserted, the blurred image

of one pixel after this diffraction-limited 4-F imaging system on the object plane was much larger than the size of one pixel. The normalized cross section curve of the blurred image is shown in Fig. 3(d), which stands for the PSF of the 4-F imaging system. The FWHM (full width at half maximum) of the blurred pixel image is about 300 μm which is larger than the center-to-center separation of the double slit. From the conventional scanning result shown in Fig. 3(b), it can be seen that the double slit is not resolved, as expected. To implement sub-Rayleigh imaging, the TVAL3 algorithm[15] and the sparsity of the gradient of object were utilized to solve Eq. (4). Some other algorithms also can be used. The reconstructed result is shown in Fig. 3(a). The double slit is well resolved, and an improvement of resolution by a factor of ~ 2 is shown.

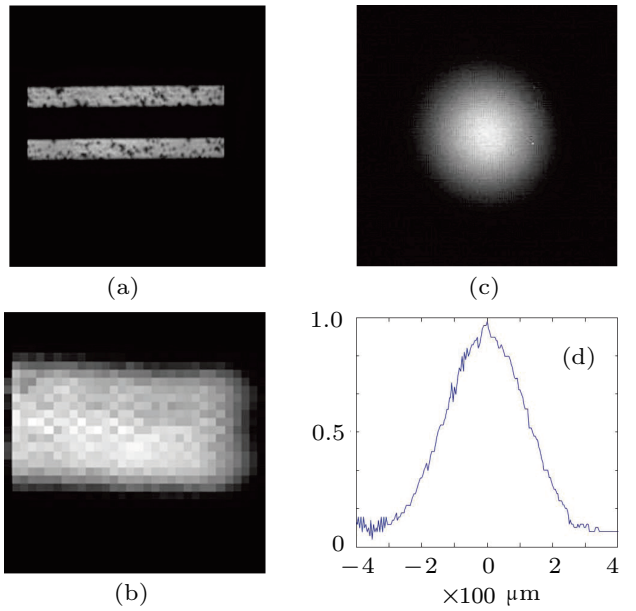


Fig. 3. (color online) (a) Image of the object obtained by a resolved imaging system. (b) The reconstructed image by scanning. (c) The diffraction-limited image of a pixel. (d) Cross section through the center of panel (c).

Different from the point-by-point reconstruction in conventional scanning, the measurement in CS is an information perception of the entire object. As shown in Fig. 3(c), the blurred image of one single pixel is much larger than the size of one pixel, so there exists the possibility of reducing the number of measurements. In the process of taking measurements for reconstruction, we reduced the number of the measurements to half of the total pixels of the object by scanning with an interval of one pixel. From the reconstructed result with 512 measurements shown in Fig. 4(a), it can be seen that the double slit is clearly resolved. The reconstructed result with 256 measurements obtained by scanning with an interval of three pixels is shown in Fig. 4(b), and the double silt also can be resolved. Since the diffraction-limited images of scan pixels can cover the entire object, it would then be possibility to retrieve the object by undersampling scanning. In the end, we took measurements only once in every region of 4×4 pixel, and the positions of the pixels scanned is shown in Fig. 4(d) (these positions are located on the upper left of the total area because the scanning begin with the pixel at the upper left corner). In this way, only 64 pixels were scanned. Though the measurements have been reduced to 6.25% of the pixels of the object, the double slit is still clearly separated in the image shown in Fig. 4(c). In an ideal scanning system, which is free from the Rayleigh diffraction bound, the image of the object cannot be retrieved by undersampling scanning. The diffraction-limited scanning system provides the superiority to achieve undersampling reconstruction via CS. Through undersampling scanning, the sampling time is reduced greatly, which is beneficial to achieve real-time observation.

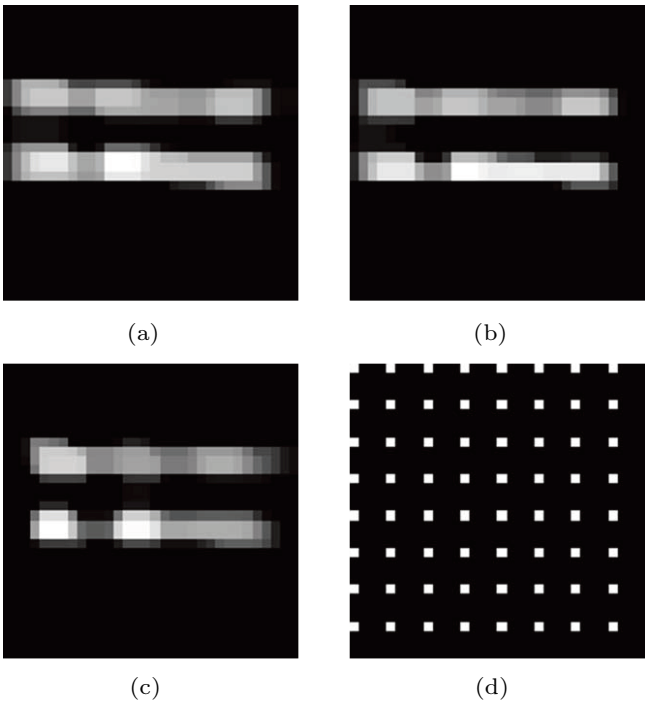


Fig. 4. (a) Sub-Rayleigh image reconstructed with 512 measurements. (b) Sub-Rayleigh image reconstructed with 256 measurements. (c) Sub-Rayleigh image reconstructed with 64 measurements. (d) The positions of the 64 pixels scanned.

As this sub-Rayleigh imaging structure is similar to the single-pixel camera,[6] we drew a comparison between these two imaging protocols. The reconstructed result through undersampling scanning with 512 measurements is shown in Fig. 5(a), which is the same with Fig. 4(a). The active illumination single-pixel camera setup[16] was the same as shown in Fig. 2, in which 1024 Bernoulli distributed random binary patterns were projected on the object in sequence. The TVAL3 algorithm and the sparsity of the gradient of object were utilized equally. The retrieved image is shown in Fig. 5(b). Though the PSF has also been considered in the process of reconstruction, the double slit cannot be resolved. In the Donoho and Elads spark theory of matrix,[17] the spark value of matrix is

defined as the smallest possible number such that there exists a subgroup of columns in a matrix that are linearly dependent. The sparsity of the object is defined as the number of nonzero pixels in object. When the half of the spark value of the measurement matrix is larger than the sparsity of the object the solution to Eq. (3) exists. The binary patterns on the DMD were smeared into nearly-uniform intensity patterns by the diffraction-limited imaging system, and as a result the spark value of the measurement matrix M is very small so that it is difficult to obtain the accurate solution. On the other hand, the scanning which is not affected by the disturbance from adjoining pixels kept fair performance in diffraction-limited imaging system.

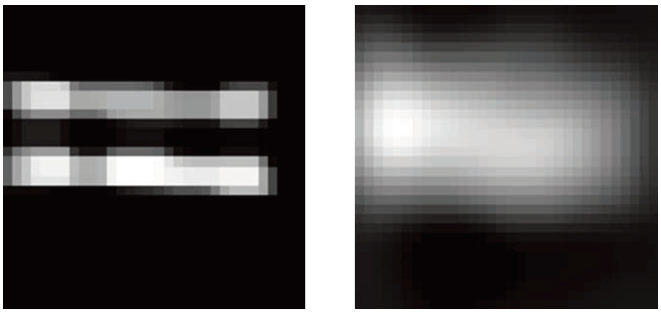


Fig. 5. (a) Sub-Rayleigh image reconstructed with 1024 measurements. (b) Image obtained by active illumination single-pixel camera.

4. Scanning strategy

In conventional CS single-pixel camera configurations, random binary patterns are used to achieve the measurements. We define K as the sparsity of the measurement matrix, which is the number of non-zero elements in a random binary pattern. In Fig. 6, the images of random binary patterns with different sparsity after a diffraction-limited imaging system are shown. When the value of K is small, the speckles distribute relative discretely. However, when the value of K increase, the speckles tend to superimpose, which leads to a smooth pattern. As a result the correlations between columns in measurement matrix A is large so that it is difficult to obtain the accurate solution.

A simulation is performed to estimate the influence of K on resolution. We use the FWHM of PSF to define the resolution of the the imaging system. The resolution of the diffraction-limited imaging system is 182.5 μm. Random binary patterns with different K are used and TVAL3 algorithm is utilized to solve Eq. (4). The number of total pixels is 4096. The total number of measurements is 2500. The resolution varies with K is shown in Table 1. From the result, we can see that: sub-Rayleigh imaging can be obtained via random binary patterns with different K and sparsity constraints, which is consistent with the proposal by Han *et al.*,[13] when $K = 1$, the finest resolution is achieved. So scanning ($K = 1$) is employed in our configuration to gain a better resolution.

Table 1. Resolution with different K.

Sparsity K	1	10	100	200	500
Resolution/μm	32.7	84.0	120.1	134.1	144.4

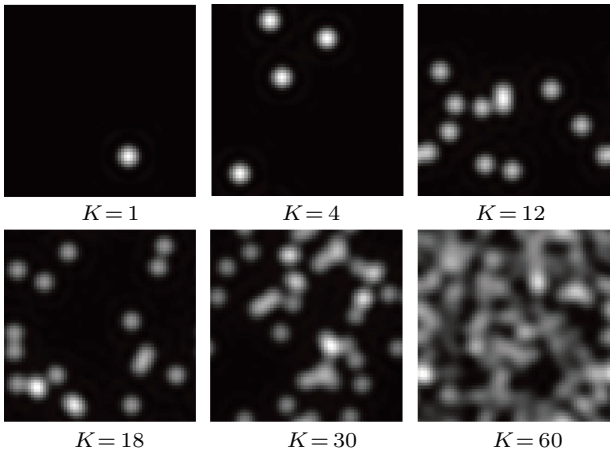


Fig. 6. Random binary patterns with different sparsity after a diffraction-limited imaging system.

5. Conclusion

In conclusion, we experimentally demonstrated a sub-Rayleigh imaging technique via a reconstruction algorithm related to CS with undersampling scanning in which an incoherent light source is employed. An image with the resolution exceeding the finest resolution of the system can be achieved with the number of measurements equal to 6.25% of the pixels of the object. The difficulties of super-resolution techniques via point-by-point illumination can be relieved to some extent by this sub-Rayleigh imaging scheme, because no limitation is placed on the detector and the CCD in this system is used as a bucket detector which measures all the transmitted light that is fixed on the collecting plane. Moreover, this method is not sensitive of the wavelength of light, so it can be applied under broadband illumination. The undersampling scanning can reduce the sampling time, which represents a step forward towards real-time applications.

Acknowledgments

We thank Zhai Yu-Ping for the critical reading of the manuscript.

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