Propagation factor of electromagnetic concentric rings Schell-model beams in non-Kolmogorov turbulence*

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We derive an analytical expression for the propagation factor (known as M^2-factor) of electromagnetic concentric rings Schell-model (EM CRSM) beams in non-Kolmogorov turbulence by utilizing the extended Huygens–Fresnel diffraction integral formula and the second-order moments of the Wigner distribution function (WDF). Our results show that the EM CRSM beam has advantage over the scalar CRSM beam for reducing the turbulence-induced degradation under suitable conditions. The EM CRSM beam with multi-rings far-fields in free space is less affected by the turbulence than the one with dark-hollow far-fields or the electromagnetic Gaussian Schell-model (EGSM) beam. The dependence of the M^2-factor on the beam parameters and the turbulence are investigated in detail.

Keywords: atmospheric propagation, partial coherence, atmospheric turbulence

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1. Introduction

Since the important application in free-space and atmospheric optical communication, the propagation properties of laser beams through atmospheric turbulence have been widely investigated.[1–11] Studies show that partially coherent beams on propagation in atmospheric turbulence are less affected than spatially coherent beams. Superposition rule for random sources presented in Ref. [12] has provided an effective approach to generate new genuine sources. Those sources can radiate beam-like fields with special intensity profiles, such as self-focusing far-fields,[13–15] flat-top far-fields,[16] ring-shaped far-fields,[17,18] four-petal far-fields,[19,20] lattice-like far-fields,[21,22] etc. The behavior of these scalar beams (regardless of the polarization properties) in free space and atmospheric turbulence are closely related to their spatial correlation functions.[13–22] Based on the unified theory of coherence and polarization, the electromagnetic partially coherent beams can be described by the cross-spectral density (CSD) matrix in space-frequency domain,[1] and can be obtained as an extension of the corresponding scalar partially coherent beams. However, due to the constraint of non-negative definiteness of the CSD matrix,[23] only several kinds of electromagnetic partially coherent Schell-model beams have been proposed, such as electromagnetic Gaussian Schell-model (EGSM) beam,[24] electromagnetic multi-Gaussian Schell-model (EM MGSM) beam,[25,26] electromagnetic cosine-Gaussian Schell-model (EM CGSM) beam,[27] electromagnetic sinc Schell-model (EM SSM) beam,[28] which all demonstrate special distributions of intensity and polarization properties.

The propagation factor (known as M^2-factor) of laser beams is an important parameter for characterizing the beam quality in many practical applications.[29] Studies of the M^2-factor of several partially coherent Schell-model beams in atmospheric turbulence[30–35] indicate that the EGSM beam and partially coherent Schell-model beams with nonconventional correlation functions are all less affected by the turbulence than the classical GSM beams, and the beam quality is closely related to the spatial correlation function and the intensity profile.

Recently, an electromagnetic concentric rings Schell-model (EM CRSM) beam has been proposed.[36] The intensity profile of this beam demonstrates multi-rings or dark-hollow (single ring) far-fields by the modulating of the special correlation function, which is a sum of series of cosine function. Since the properties of coherence and polarization have been studied, this paper tends to investigate the beam quality of EM CRSM beams on propagation in non-Kolmogorov turbulence. By employing the extended Huygens-Fresnel diffraction integral formula and second-order moments of the WDF, we derive the analytical expression for the M^2-factor of EM CRSM beams propagating in non-Kolmogorov turbulence. By which we numerically analyzed the influence of beam parameters and

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turbulence exert on the M^2-factor of EM CRSM beams, and the classical GSM beam as well for a reference.

2. Propagation factor of EM CRSM beams in non-Kolmogorov turbulence

The second-order statistical properties of an EM CRSM beam can be characterized by the 2×2 cross-spectral density (CSD) matrix[36]

$$W_{ij}(\boldsymbol{r}_1,\boldsymbol{r}_2,0)=A_iA_jB_{ij}\exp\left(-\frac{\boldsymbol{r}_1^2+\boldsymbol{r}_2^2}{4\sigma^2}\right)\exp\left[-\frac{(\boldsymbol{r}_1-\boldsymbol{r}_2)^2}{2\delta_{ij}^2}\right]\times\frac{1}{N}\sum_{n=-(N-1)/2}^{(N-1)/2}\cos\left[\frac{2\pi nR(\boldsymbol{r}_1-\boldsymbol{r}_2)}{\delta_(ij)}\right] \quad (1)$$

with $i=x,y$ and $j=x,y$, where $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ are two points at the initial plane $z=0$, N is a positive integer, A_i, A_j, and R are positive real constants, $B_{xx}=B_{yy}=1$, $B_{xy}=B_{yx}^*$ is the complex correlation coefficient, σ is the beam waist width, δ_{ij} denotes the correlation width. The real parameters N, R, A_x, A_y, $|B_{xy}|$, σ, δ_{xx}, and δ_{yy} satisfying several specific restrictions could guarantee that the mathematical model (1) describes a physically realizable field.[36] When $N=1$ or $R=0$, equation (1) corresponds to the CSD matrix of the classic electromagnetic Gaussian Schell-model (EGSM) beams.[24] The trace of the CSD matrix of an EM CRSM beam is expressed in the following formula

$$\boldsymbol{W}_{\rm tr}(\boldsymbol{r}_1,\boldsymbol{r}_2,0)=\mathrm{Tr}\overleftrightarrow{\boldsymbol{W}}(\boldsymbol{r}_1,\boldsymbol{r}_2,0)=\boldsymbol{W}_{xx}(\boldsymbol{r}_1,\boldsymbol{r}_2,0)+\boldsymbol{W}_{yy}(\boldsymbol{r}_1,\boldsymbol{r}_2,0). \quad (2)$$

Within the validity of the paraxial approximation, the trace of the CSD of an EM CRSM beam propagating in non-Kolmogorov turbulence can be calculated by the following generalized Huygens–Fresnel integral[37]

$$W_{\rm tr}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,z)=\left(\frac{k}{2\pi z}\right)^2\iiiint W_{\rm tr}(\boldsymbol{r}_1,\boldsymbol{r}_2,0)\exp\left[-\mathrm{i}k\frac{(\boldsymbol{\rho}_1-\boldsymbol{r}_1)^2+(\boldsymbol{\rho}_2-\boldsymbol{r}_2)^2}{2z}\right]\times\exp\left\{-\frac{k^2zT}{3}\left[(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2)^2+(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2)(\boldsymbol{r}_1-\boldsymbol{r}_2)+(\boldsymbol{r}_1-\boldsymbol{r}_2)^2\right]\right\}\mathrm{d}^2\boldsymbol{r}_1\,\mathrm{d}^2\boldsymbol{r}_2, \quad (3)$$

where $\boldsymbol{\rho}_1$ and $\boldsymbol{\rho}_2$ are two spatial points at the propagation distance z, $k=2\pi/\lambda$ is the wave number with λ being the wavelength, and the term T is the intensity of the turbulence as follows:

$$T=\pi^2\int_0^\infty\kappa^3\Phi(\kappa,\alpha)\,d\kappa, \quad (4)$$

where $\Phi(\kappa,\alpha)$ is the spatial power spectrum of the refractive index fluctuations of the turbulent medium, κ is the magnitude of two-dimensional spatial frequency, and α is the power-law exponent. For covering a wide scope of atmospheric conditions, we adopt a generalized power spectrum model valid in non-Kolmogorov turbulence[38]

$$\Phi(\kappa,\alpha)=A(\alpha)\tilde{C}_n^2\left[\exp\left(-\kappa^2/\kappa_m^2\right)\right]/\left(\kappa^2+\kappa_0^2\right)^{\alpha/2},$$

$$0\le\kappa<\infty,3<\alpha<4, \quad (5)$$

where the term $\tilde{C}_n^2$ is a generalized index-of-refraction structure constant with units $\mathrm{m}^{3-\alpha}$, $\kappa_0=2\pi/L_0$, and $\kappa_m=c(\alpha)/l_0$, in which L_0 and l_0 are the outer and inner scales of the turbulence,

$$A(\alpha)=(2\pi)^{-2}\Gamma(\alpha-1)\cos(\alpha\pi/2),$$
$$c(\alpha)=\left[2\pi\Gamma(5-\alpha/2)A(\alpha)/3\right]^{1/\alpha-5},$$

$\Gamma(\cdot)$ is the Gamma function. Substituting Eq. (5) to Eq. (4), we obtain the formula

$$T=\frac{\pi^2}{3(\alpha-2)}A(\alpha)\tilde{C}_n^2\left[\beta\kappa_m^{2-\alpha}\exp\left(\frac{\kappa_0^2}{\kappa_m^2}\right)\Gamma\left(2-\frac{\alpha}{2},\frac{\kappa_0^2}{\kappa_m^2}\right)-2\kappa_0^{4-\alpha}\right], \quad (6)$$

where $\beta=(\alpha-2)\kappa_m^2+2\kappa_0^2$, and $\Gamma(\cdot,\cdot)$ is the incomplete Gamma function.

Substituting Eqs. (1) and (2) into Eq. (3), and using $\boldsymbol{\rho}_1=\boldsymbol{\rho}_2=\boldsymbol{\rho}$, the spectral density of the EM CRSM beam in free space with $T=0$ can be derived as

$$S(\boldsymbol{\rho},z)=\overleftrightarrow{\boldsymbol{W}}_{\rm tr}(\boldsymbol{\rho},\boldsymbol{\rho},z)=\frac{k^2\sigma^2}{2Nz^2}\left[\frac{A_x^2}{\Delta_{xx}}\exp\left(-\frac{k^2\boldsymbol{\rho}^2}{4z^2\Delta_{xx}}\right)\sum_{n=-(N-1)/2}^{(N-1)/2}\exp\left(-\frac{\pi^2n^2R^2}{\delta_{xx}^2\Delta_{xx}}\right)\times\cosh\left(\frac{\pi nkR\boldsymbol{\rho}}{z\delta_{xx}\Delta_{xx}}\right)+\frac{A_y^2}{\Delta_{yy}}\exp\left(-\frac{k^2\boldsymbol{\rho}^2}{4z^2\Delta_{yy}}\right)\sum_{n=-(N-1)/2}^{(N-1)/2}\exp\left(-\frac{\pi^2n^2R^2}{\delta_{yy}^2\Delta_{yy}}\right)\cosh\left(\frac{\pi nkR\boldsymbol{\rho}}{z\delta_{yy}\Delta_{yy}}\right)\right] \quad (7)$$

with

$$\Delta_{ii} = \frac{1}{8\sigma^2} + \frac{k^2\sigma^2}{2z^2} + \frac{1}{2\delta_{ii}^2}.$$

Since the spectral density of an electromagnetic beam does not depend on the off-diagonal components of the CSD matrix, we only consider the case that the CSD matrix of the EM CRSM beam has the following form

$$\overleftrightarrow{\boldsymbol{W}}(\boldsymbol{r}_1,\boldsymbol{r}_2,0) = \begin{bmatrix} \boldsymbol{W}_{xx}(\boldsymbol{r}_1,\boldsymbol{r}_2,0) & 0 \\ 0 & \boldsymbol{W}_{yy}(\boldsymbol{r}_1,\boldsymbol{r}_2,0) \end{bmatrix}. \quad (8)$$

Utilizing Eqs. (1) and (5), the spectral degree of polarization at the initial plane can be expressed as

$$P(\boldsymbol{r},0) = \sqrt{1 - \frac{4\,\mathrm{Det}\,\overleftrightarrow{\mathrm{W}}(\mathrm{r}_1,\mathrm{r}_2,0)}{\left[\mathrm{Tr}\,\overleftrightarrow{\mathrm{W}}(\mathrm{r}_1,\mathrm{r}_2,0)\right]^2}} = \left|\frac{A_x^2 - A_y^2}{A_x^2 + A_y^2}\right|. \quad (9)$$

Because the spectral degree of polarization is a constant, the polarization properties across the initial plane are uniform. Under the condition of $A_x = 0$ or $A_y = 0$, the EM CRSM beam is simplified as a scalar CRSM beam with $P(\boldsymbol{r},0) = 1$ (fully polarized CRSM beam).

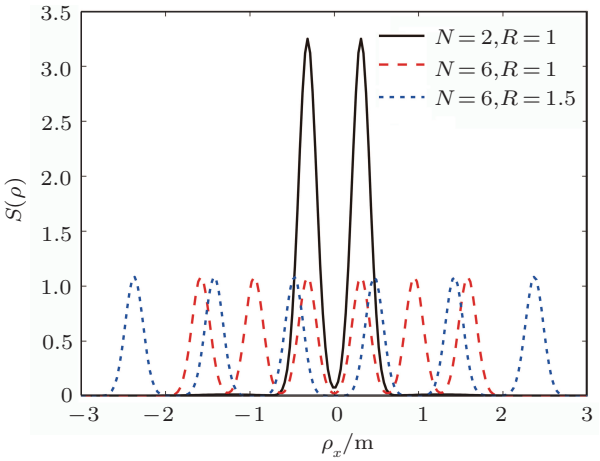


Fig. 1. (color online) The cross line (ρ=0) of the spectral density of EM CRSM beams for several values of N and R at the distance z=10km in free space.

Figure 1 plots the cross line ($\rho_y = 0$) of the spectral density of EM CRSM beams for several values of N and R at the distance $z = 10$ km in free space. The beam parameters are $\lambda = 632.8$ nm, $\sigma = 0.02$ m, $\delta_{xx} = 0.01$ m, $\delta_{yy} = 0.25\delta_{xx}$, $A_x = 1$, $A_y = 0.5$. One can clearly see that the number of rings is determined by N, and the radius of each concentric ring can be adjusted by R. When $N = 2$, the far field has dark-hollow (single ring) profile. The multi-rings profile appears along with the increase of N, and the radius of each concentric ring extends as R increases.

The WDF of an EM CRSM beam in non-Kolmogorov turbulence can be expressed in terms of the trace of the CSD as follows:[30]

$$h_{\mathrm{tr}}(\boldsymbol{\rho},\boldsymbol{\Theta},z) = \left(\frac{k}{2\pi}\right)^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} W_{\mathrm{tr}}(\boldsymbol{\rho},\boldsymbol{\rho}_d,z)\exp\left(-\mathrm{i}k\boldsymbol{\Theta}\cdot\boldsymbol{\rho}_d\right)\mathrm{d}^2\boldsymbol{\rho}_d, \quad (10)$$

where $\boldsymbol{\rho} = (\boldsymbol{\rho}_1 + \boldsymbol{\rho}_2)/2$, $\boldsymbol{\rho}_d = \boldsymbol{\rho}_1 - \boldsymbol{\rho}_2$, vector $\boldsymbol{\Theta} = (\theta_x, \theta_y)$ denotes an angle which the vector of interest makes with the z direction, $k\theta_x$ and $k\theta_y$ are the wave vector components along the x axis and y axis, respectively.

The M^2-factor is regarded as a beam quality factor in many practical applications, which is defined by the second-order moments of the WDF as follows:[39,40]

$$M^2(z) = k\left(\langle\boldsymbol{\rho}^2\rangle\langle\boldsymbol{\Theta}^2\rangle - \langle\boldsymbol{\rho}\cdot\boldsymbol{\Theta}\rangle^2\right)^{1/2}, \quad (11)$$

with

$$\langle\rho_x^{n_1}\rho_y^{n_2}\theta_x^{m_1}\theta_y^{m_2}\rangle = \frac{1}{Q}\iiiint_{-\infty}^{\infty}\rho_x^{n_1}\rho_y^{n_2}\theta_x^{m_1}\theta_y^{m_2}h_{\mathrm{tr}}(\boldsymbol{\rho},\boldsymbol{\Theta},z)\,\mathrm{d}^2\boldsymbol{\rho}\mathrm{d}^2\boldsymbol{\Theta}, \quad (12)$$

$$Q = \iiiint_{-\infty}^{\infty} h_{\mathrm{tr}}(\boldsymbol{\rho},\boldsymbol{\Theta},z)\,\mathrm{d}^2\boldsymbol{\rho}\mathrm{d}^2\boldsymbol{\Theta}. \quad (13)$$

For partially coherent Schell beams, the propagation of the second-order moments of the WDF in the turbulence obey the general formulae as follows:[41]

$$\langle\boldsymbol{\rho}^2\rangle = \langle\boldsymbol{\rho}^2\rangle_0 + \langle\boldsymbol{\Theta}^2\rangle_0 z^2 + 4Tz^3/3, \quad (14)$$

$$\langle\boldsymbol{\Theta}^2\rangle = \langle\boldsymbol{\Theta}^2\rangle_0 + 4Tz, \quad (15)$$

$$\langle\boldsymbol{\rho}\cdot\boldsymbol{\Theta}\rangle = \langle\boldsymbol{\Theta}^2\rangle_0 z + 2Tz^2, \quad (16)$$

where $\langle\boldsymbol{\Theta}^2\rangle_0 = \langle\theta_x^2\rangle_0 + \langle\theta_y^2\rangle_0$ and $\langle\boldsymbol{\rho}^2\rangle_0 = \langle\rho_x^2\rangle_0 + \langle\rho_y^2\rangle_0$ are the second-order moments of WDF at the initial plane.

Substituting Eqs. (1) and (10) into Eq. (12), the second-order moments of WDF of EM CRSM beams at the initial plane $z = 0$ can be derived as

$$\langle\boldsymbol{\rho}^2\rangle_0 = 2\sigma^2, \quad (17)$$

$$\langle\boldsymbol{\Theta}^2\rangle_0 = \frac{2}{k^2}\left[\frac{1}{4\sigma^2} + \frac{A_x^2}{A_x^2+A_y^2}\left(\frac{1}{\delta_{xx}^2} + \frac{1}{N-1}\sum_{n=-(N-1)/2}^{(N-1)/2}\frac{4\pi^2n^2R^2}{\delta_{xx}^2}\right) + \frac{A_y^2}{A_x^2+A_y^2}\left(\frac{1}{\delta_{yy}^2} + \frac{1}{N-1}\sum_{n=-(N-1)/2}^{(N-1)/2}\frac{4\pi^2n^2R^2}{\delta_{yy}^2}\right)\right]. \quad (18)$$

So the analytical expressions of the M^2-factor of EM CRSM beams can be obtained

$$M^2(z) = \left\{\left(4\sigma^2 + 4Tz^3\right)\left[\frac{1}{4\sigma^2} + \frac{A_x^2}{A_x^2+A_y^2}\left(\frac{1}{\delta_{xx}^2} + \frac{1}{N-1}\sum_{n=-(N-1)/2}^{(N-1)/2}\frac{4\pi^2n^2R^2}{\delta_{xx}^2}\right)\right.\right.$$

$$+\frac{A_y^2}{A_x^2+A_y^2}\left(\frac{1}{\delta_{yy}^2}+\frac{1}{N-1}\sum_{n=-(N-1)/2}^{(N-1)/2}\frac{4\pi^2n^2R^2}{\delta_{yy}^2}\right)\Bigg]+8k^2\sigma^2Tz+\frac{4k^2T^2z^2}{3}\Bigg\}^{1/2}. \tag{19}$$

For the case of $N=1$, equation (19) is simplified as the M^2-factor of the EGSM beam, which is expressed as

$$M_G^2(z)=\left\{\left(4\sigma^2+4Tz^3\right)\left[\frac{1}{4\sigma^2}+\frac{A_x^2}{\left(A_x^2+A_y^2\right)\delta_{xx}^2}+\frac{A_y^2}{\left(A_x^2+A_y^2\right)\delta_{yy}^2}\right]+8k^2\sigma^2Tz+\frac{4k^2T^2z^2}{3}\right\}^{1/2}. \tag{20}$$

When $A_x=0$ or $A_y=0$, equation (19) reduces to the following expression for the M^2-factor of a scalar CRSM beam (i.e., fully polarized CRSM beam)

$$M^2(z)=\left[\left(4\sigma^2+4Tz^3\right)\left(\frac{1}{4\sigma^2}+\frac{1}{\delta_{ii}^2}+\frac{1}{N-1}\sum_{n=-(N-1)/2}^{(N-1)/2}\frac{4\pi^2n^2R^2}{\delta_{ii}^2}\right)+8k^2\sigma^2Tz+\frac{4k^2T^2z^2}{3}\right]^{1/2},\quad (i=x,y). \tag{21}$$

3. Numerical results and analysis

In this section, we will investigate the M^2-factor of an EM CRSM beam in non-Kolmogorov turbulence by applying the formulae derived in above section. Without loss of generality, the parameters of the EM CRSM beam are the same as those given in Fig. 1, and parameters of the non-Kolmogorov turbulence are $\tilde{C}_n^2=10^{-14}$ m$^{3-\alpha}$, $\alpha=3.1$, $L_0=1$ m, $l_0=1$ mm unless stated otherwise.

Firstly, we study the influence of the initial degree of polarization on the normalized M^2-factor of the EM CRSM beam through the non-Kolmogorov turbulence. Figure 2 shows the evolution of the normalized M^2-factor of the EM CRSM beam with $N=6$ and $R=1$ in the non-Kolmogorov turbulence for several values of the initial degree of polarization P. The turbulence with $\tilde{C}_n^2=10^{-13}$ m$^{3-\alpha}$ is stronger than that with $\tilde{C}_n^2=10^{-14}$ m$^{3-\alpha}$. One can clearly see that for either case of $A_x\le A_y$ or $A_x\ge A_y$, the normalized M^2-factor increases monotonically with the increase of propagation distance z, however when considering the initial degree of polarization P, it shows different trends. From Fig. 2, the normalized M^2-factor decreases with increasing P in the case of $A_x\le A_y$, and increases in the case of $A_x\ge A_y$. It means a scalar CRSM beam is less affected by the turbulence than an EM CRSM beam for the case of $A_x\le A_y$, while an EM CRSM beam has advantage over a scalar CRSM beam for reducing the turbulence-induced degradation for the case of $A_x\ge A_y$, and the advantage will be further improved by decreasing P. In the following analyses, we set $A_x=1$, $A_y=0.5$ for the case of partially polarized source with $P=0.6$.

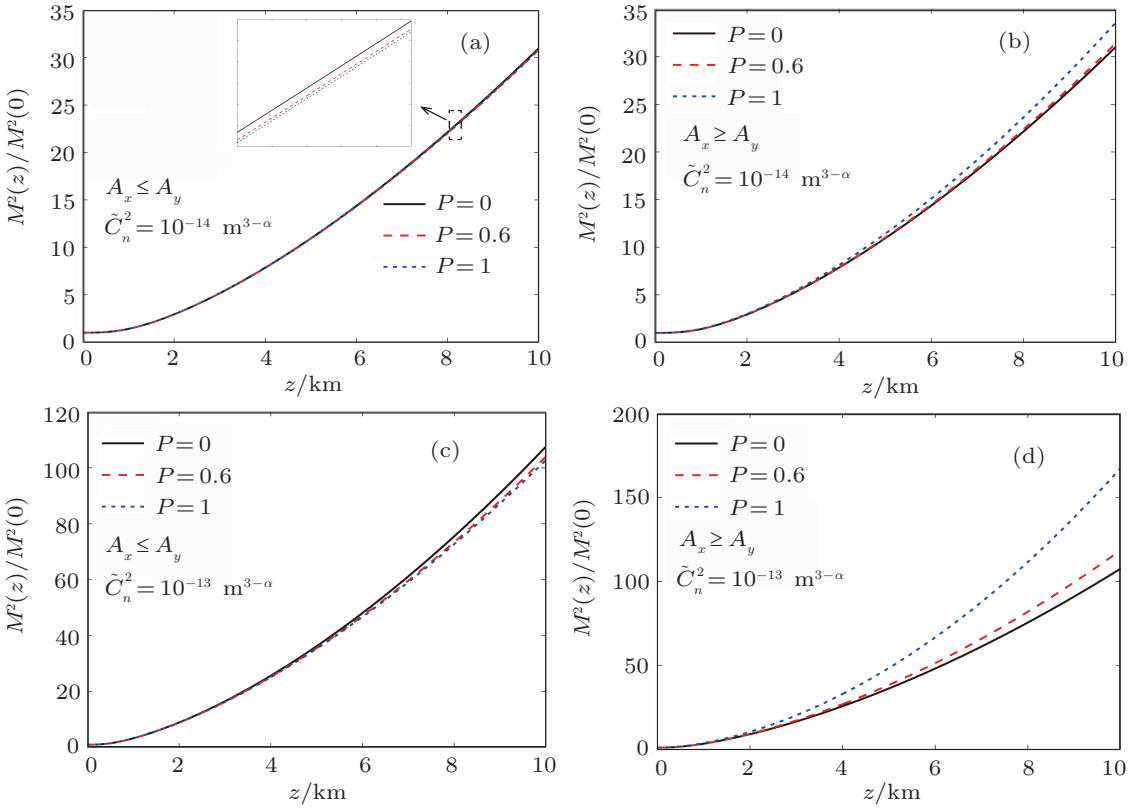


Fig. 2. (color online) Normalized M^2-factor of the EM CRSM beam with $N=6$ and $R=1$ in non-Kolmogorov turbulence for several values of the initial degree of polarization P.

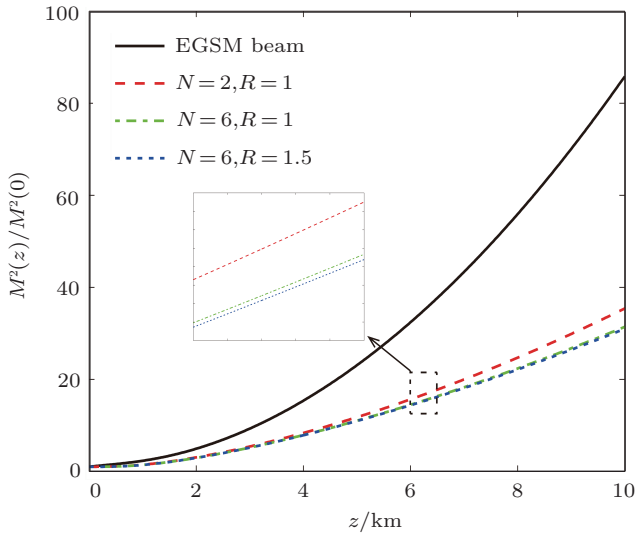


Fig. 3. (color online) Normalized M^2-factor of EM CRSM beams for several values of N and R versus propagation distance z in the non-Kolmogorov turbulence.

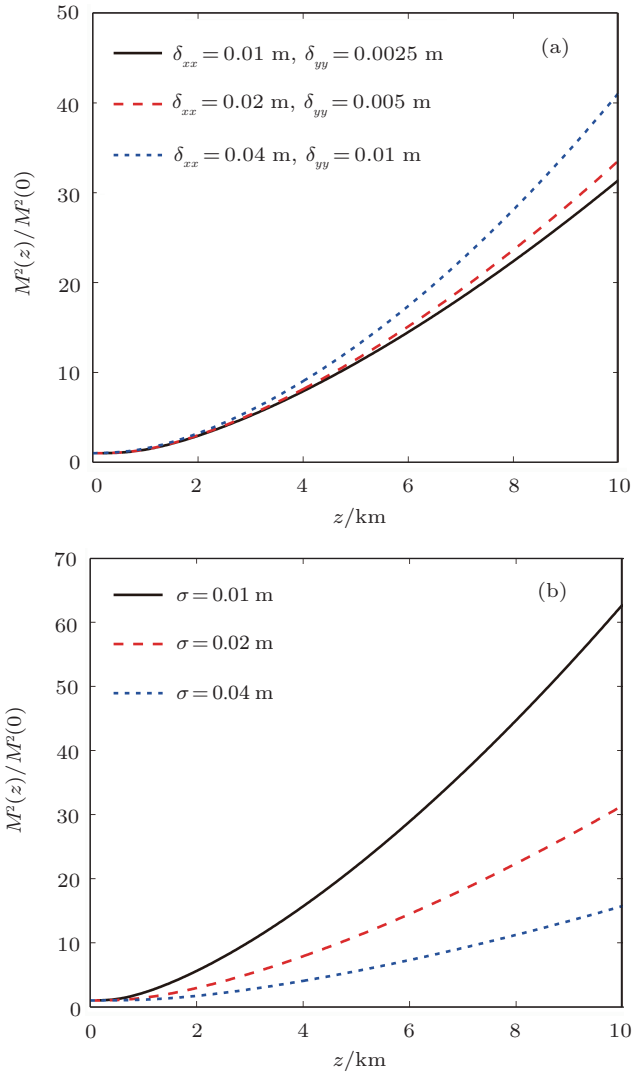


Fig. 4. (color online) Normalized M^2-factor of the EM CRSM beam with $N=6$ and $R=1$ versus propagation distance z in the non-Kolmogorov turbulence for several values of initial spatial correlation factors δ_{xx}, δ_{yy}, and beam width σ.

Figure 3 illustrates the normalized M^2-factor of EM CRSM beams for several values of N and R versus propagation distance z in the non-Kolmogorov turbulence. For comparison, the case of the EGSM beam is also shown under the same condition. From Fig. 3, the normalized M^2-factor of EM CRSM beams is obviously smaller than the EGSM beam, and the difference enlarges with increasing N and R. It means that EM CRSM beams are less affected by the turbulence than the EGSM beam, and the beam quality of EM CRSM beams with multi-rings far-fields in free space is better than the one with dark-hollow far-fields. Increasing the radius of each concentric ring can further improve the robustness against the destructive effect of the turbulence.

The effect of initial spatial correlation factors δ_{xx}, δ_{yy}, beam width σ, and wavelength λ of EM CRSM beams on the normalized M^2-factor is considered in Figs. 4 and 5. Figure 4 shows the evolution of the normalized M^2-factor of the EM CRSM beam with $N=6$ and $R=1$ in the non-Kolmogorov turbulence for several values of initial spatial correlation factors δ_{xx}, δ_{yy}, and beam width σ. It can be concluded that the normalized M^2-factor increases more slowly on propagation as δ_{xx} and δ_{yy} decrease or σ increases, which is to say, the EM CRSM beam with lower spatial coherence or larger beam width is less affected by the turbulence.

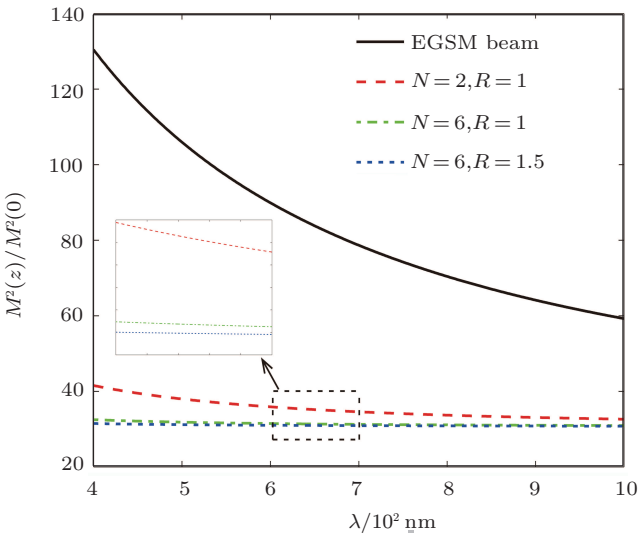


Fig. 5. (color online) Normalized M^2-factor of EM CRSM beams for several values of N and R versus wavelength λ at propagation distance $z=10$ km in the non-Kolmogorov turbulence.

The influence of wavelength λ on the normalized M^2-factor of EM CRSM beams in the non-Kolmogorov turbulence is shown in Fig. 5. The normalized M^2-factor of EM CRSM beams decreases with the increase of λ, implying that the beam quality of the EM CRSM beam can be improved by increasing wavelength.

In the following section, we analyze the influence of the turbulent parameters on the normalized M^2-factor of an EM CRSM beam in the non-Kolmogorov turbulence. Figure 6 plots the normalized M^2-factor of EM CRSM beams and the

EGSM beam versus the power-law exponent α at propagation distance $z = 10$ km in the non-Kolmogorov turbulence. The normalized M^2-factor of EM CRSM beams is obviously lower than that of the EGSM beam. The value of the normalized M^2-factor monotonically increases in the region $(3, 3.07]$, and decreases in $[3.07, 4)$, which is similar with the case of turbulent intensity T versus α. Hence, the effect of the turbulence on EM CRSM beams compared to the EGSM beam is significantly less. And under stronger turbulence, the EM CRSM beam with multi-rings far-fields has obvious advantage over the one with dark-hollow far-fields for reducing the turbulence-induced degradation.

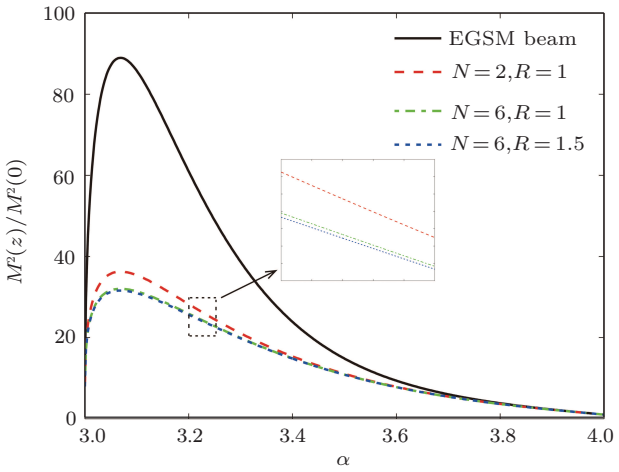


Fig. 6. (color online) Normalized M^2-factor of EM CRSM beams for several values of N and R versus the power-law exponent α at propagation distance $z = 10$ km in non-Kolmogorov turbulence.

Figure 7 shows the normalized M^2-factor of the EM CRSM beam with $N = 6$ and $R = 1$ for several values of the generalized index-of-refraction structure constant $\tilde{C}_n^2$ in non-Kolmogorov turbulence. The normalized M^2-factor of the EM CRSM beam increases more slowly with the decrease of $\tilde{C}_n^2$. Therefore, the beam quality of the EM CRSM beam is less affected by the turbulence with smaller $\tilde{C}_n^2$.

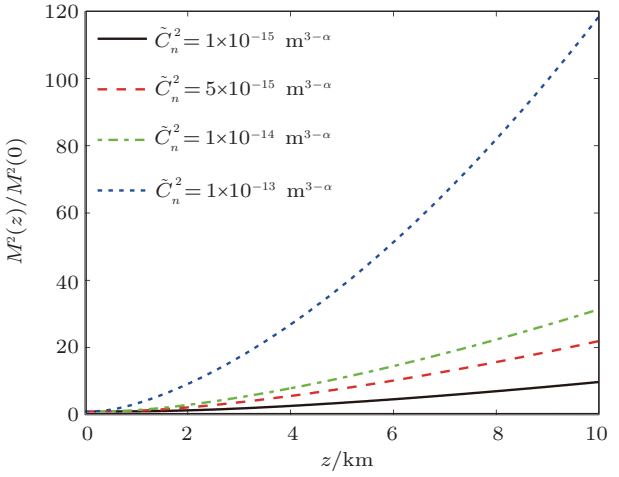


Fig. 7. (color online) Normalized M^2-factor of the EM CRSM beam with $N = 6$ and $R = 1$ in the non-Kolmogorov turbulence for several values of the generalized index-of-refraction structure constant $\tilde{C}_n^2$.

The effect of outer scale L_0 and inner scale l_0 of the non-Kolmogorov turbulence on the normalized M^2-factor of the EM CRSM beam is shown in Fig. 8. The turbulence with $\alpha = 3.07$ is stronger than the case of $\alpha = 3.67$. It can be deduced that the influence of the turbulence with longer l_0 or smaller L_0 on the EM CRSM beam is less. And the influence of l_0 on the normalized M^2-factor is greater than that of L_0, especially in stronger turbulence. Our results may find applications in free-space and atmospheric optical communications.

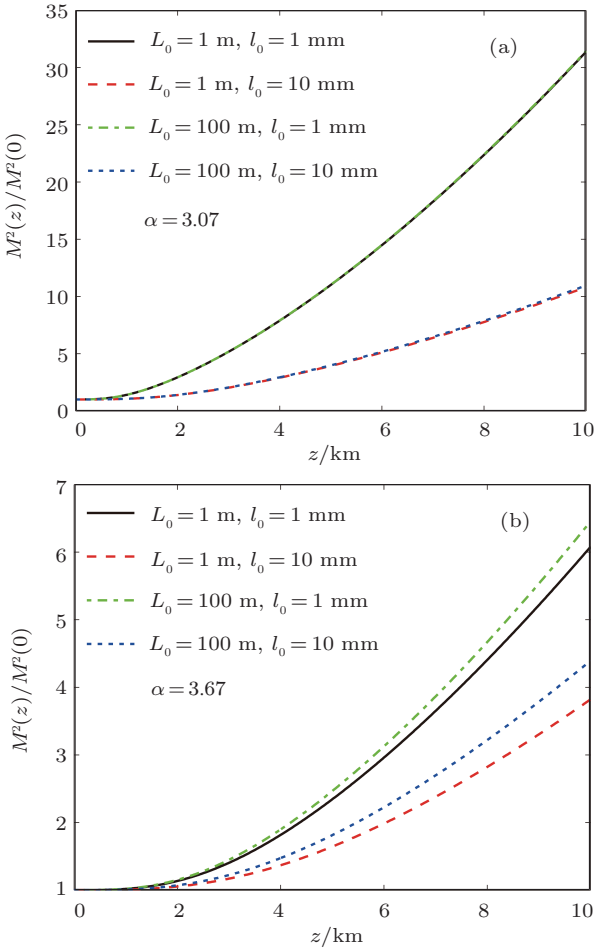


Fig. 8. (color online) Normalized M^2-factor of the EM CRSM beam with $N = 6$ and $R = 1$ versus propagation distance z in the non-Kolmogorov turbulence for several values of power-law exponent α, outer scale L_0, and inner scale l_0.

4. Conclusion

The analytical formula for the M^2-factor of EM CRSM beams on propagation in the non-Kolmogorov turbulence is derived. By which, the dependence of the normalized M^2-factor on beam parameters and non-Kolmogorov turbulence have been investigated numerically. We found that EM CRSM beams are less affected by the turbulence than the scalar CRSM beam under suitable conditions. The beam quality of the EM CRSM beam with multi-rings far-fields in free space is better than the one with dark-hollow far-fields or the EGSM beam, especially for the EM CRSM beam with larger radius of each concentric ring. Furthermore, the effect of the turbulence

on the EM CRSM beam can be reduced by choosing lower initial degree of polarization P, longer wavelength λ, larger beam width σ, or weaker turbulence. Our results will be useful for the practical applications in free-space and atmospheric optical communications.

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