Dynamic localization of two electrons in AC-driven triple quantum dots and quantum dot shuttles*

Jin-Xian Qu(屈晋先)[1], Su-Qing Duan(段素青)[2], and Ning Yang(杨宁)[2,†]

[1] *Beijing Computational Science Research Center, Beijing* 100193, *China*
[2] *Institute of Applied Physics and Computational Mathematics, Beijing* 100088, *China*

(Received 8 May 2017; revised manuscript received 16 August 2017; published online 20 October 2017)

We analyze the dynamic localization of two interacting electrons induced by alternating current electric fields in triple quantum dots and triple quantum dot shuttles. The calculation of the long-time averaged occupation probability shows that both the intra- and inter-dot Coulomb interaction can increase the localization of electrons even when the AC field is not very large. The mechanical oscillation of the quantum dot shuttles may keep the localization of electrons at a high level within a range if its frequency is quite a bit smaller than the AC field. However, the localization may be depressed if the frequency of the mechanical oscillation is the integer times of the frequency of the AC field. We also derive the analytical condition of two-electron localization both for triple quantum dots and quantum dot shuttles within the Floquet formalism.

Keywords: dynamic localization, quantum dot shuttles

PACS: 73.63.–b, 74.55.+v, 73.23.Hk **DOI:** 10.1088/1674-1056/26/12/127308

1. Introduction

In recent years, there has been great research interest on the dynamic localization in quantum dot systems.[1–7] In particular, it has been shown that localization and entanglement in a two-electron coupled quantum dot system[2] or in finite quantum-dot superlattices[3] can be achieved by applying an alternating current (AC) electric field. Then a method of quantitatively studying the dynamical localization of the quantum system driven by a periodic field is found by Zhe Jiang *et al*.[4] It was found that the dynamic localization emerges when the ratios of the strength to the frequency of the external driving field correspond to zero points of a Bessel function whose order is determined by the electric fields and the effective Coulomb interaction.[6,7]

Besides traditional quantum dots, the transport of electrons by movable quantum dots, i.e., the quantum dot shuttles has also drawn a great deal of attention in recent years.[8–10] If the reciprocating motion of a quantum dot affects the electronic transport greatly, the system may be regard as an electron shuttle. There have been some experiments realizing electron shuttles. For example, the vibrating Si quantum dots (nanopillars) can realize the electron shuttle between two electrodes.[11] Based on the dielectric properties of the GaAs material, the surface acoustic wave excited by a microwave field can form a moving electronic bound potential between two quantum dots, then an electron shuttle between two quantum dots that are apart by a few microns can be achieved.[12,13] Triple quantum dots with a movable quantum dot in the center is also a kind of electric shuttle.[14] It is found that the electric current is greatly increased by the shuttle system. In the Coulomb blockade regime, magnetic control of electron shuttling can be realized.[15] In a quadruple-quantum-dot system, when one electron in a spin-singlet pair is shuttled to a distant dot, a coherent singlet-triplet oscillation occurs.[16] In the Kondo regime, the destruction of the Kondo resonance by the mechanical oscillation of the quantum dot is also investigated.[17] Villavicencio *et al*. analyzed the quasi-energy spectrum and tunneling current in AC-driven triple quantum dot shuttles with a single electron and showed that the dynamic localization may also exist in electron shuttle systems.[18] However, the localization of interacting electrons in quantum dot shuttles has not been studied systematically, especially the analytical condition of the localization of electrons with interaction has not been obtained. In this paper, we will compare the behaviors of two interacting electrons in triple quantum dots and quantum dot shuttles to reveal the effects of Coulomb interaction and the mechanical oscillation on the localization of electrons under AC field.

The paper is organized as follows. The second section is the model for the triple quantum dots and quantum dot shuttles with two electrons under an AC electric field and the derivation of the long-time averaged occupation probability. Then in Section 3, we study the dynamic localization and the analytical conditions for two-electron localization in triple quantum dots and quantum dot shuttles. Finally, we make a summary in the last section.

*Project supported by the National Natural Science Foundation of China (Grant No. 11204016).
†Corresponding author. E-mail: yang_ning@iapcm.ac.cn
© 2017 Chinese Physical Society and IOP Publishing Ltd

2. Model and methods

We consider a one-dimensional array of three quantum dots with two electrons, and a time-dependent electric field applied between the left and right quantum dots. We use the Hubbard model of a single-band system, so the system Hamiltonian can be obtained as

$$\begin{aligned}\hat{H}(t) = &\sum_{\sigma,i=1,2,3} \varepsilon_i(t) d_{i\sigma}^{\dagger} d_{i\sigma} \\ &+ W \sum_{\sigma} \left(d_{1\sigma}^{\dagger} d_{2\sigma} + d_{2\sigma}^{\dagger} d_{1\sigma} + d_{2\sigma}^{\dagger} d_{3\sigma} + d_{3\sigma}^{\dagger} d_{2\sigma} \right) \\ &+ U_1 \sum_{i} n_{i\uparrow} n_{i\downarrow} + U_2 \sum_{\sigma_1,\sigma_2,\sigma_3} \left(n_{1\sigma} n_{2\sigma} + n_{2\sigma} n_{3\sigma} \right). \end{aligned} \quad (1)$$

In the above equation, $d_{i\sigma}^{\dagger}$ ($d_{i\sigma}$) creates (annihilates) an electron with spin σ on the i-th site, $n_{i\sigma}$ is the number operator, W is the tunneling between the neighbor dots, U_1 is the inner-dot Coulomb interaction, and U_2 is the inter-dot Coulomb interaction. There has been more research of the inner-dot Coulomb interaction impacts on the localization,[4,19] but the inter-dot Coulomb interaction also has significant effects on the localization.[7,20] Thus we consider both the inner-dot and inter-dot Coulomb interactions in our calculation. For the AC field, each dot has one level $\varepsilon_i(t)$ and is presented as

$$\varepsilon_1 = \frac{V}{2}\cos(\omega_{\rm ac} t), \quad \varepsilon_2 = 0, \quad \varepsilon_3 = -\frac{V}{2}\cos(\omega_{\rm ac} t), \quad (2)$$

where V is the amplitude of the AC field, and $\omega_{\rm ac}$ is the frequency of the AC field. We set the period $T = 2\pi/\omega_{\rm ac}$.

In this paper, we just consider the single subspace, as in the triplet subspace the Pauli principle forbids double occupation of a quantum dot, and the Coulomb interaction described by the Hubbard$-U$ term is consequently irrelevant. In triple quantum dots, there are two electrons, so we have nine basis states $|\uparrow,\downarrow,0\rangle$, $|\downarrow,\uparrow,0\rangle$, $|\uparrow,0,\downarrow\rangle$, $|\downarrow,0,\uparrow\rangle$, $|0,\uparrow,\downarrow\rangle$, $|0,\downarrow,\uparrow\rangle$, $|2,0,0\rangle$, $|0,2,0\rangle$, $|0,0,2\rangle$. The nine bases are divided into three groups: inner-dot states (the electrons in the same quantum dot $|2,0,0\rangle$, $|0,2,0\rangle$, $|0,0,2\rangle$), inter-dot states (the electrons in neighboring quantum dots $|\uparrow,\downarrow,0\rangle$, $|\downarrow,\uparrow,0\rangle$), and wide separated states (the electrons on the 1-th and 3-th sites $|\uparrow,0,\downarrow\rangle$, $|\downarrow,0,\uparrow\rangle$, $|0,\uparrow,\downarrow\rangle$, $|0,\downarrow,\uparrow\rangle$). Thus we can obtain the matrix of the Hamiltonian (1) as follows:

$$\begin{bmatrix}
\varepsilon_1+\varepsilon_2+U_2 & 0 & W & 0 & 0 & 0 & W & W & 0 \\
0 & \varepsilon_1+\varepsilon_2+U_2 & 0 & W & 0 & 0 & -W & -W & 0 \\
W & 0 & \varepsilon_1+\varepsilon_3 & 0 & W & 0 & 0 & 0 & 0 \\
0 & W & 0 & \varepsilon_1+\varepsilon_3 & 0 & W & 0 & 0 & 0 \\
0 & 0 & W & 0 & \varepsilon_2+\varepsilon_3+U_2 & 0 & 0 & W & W \\
0 & 0 & 0 & W & 0 & \varepsilon_2+\varepsilon_3+U_2 & 0 & -W & -W \\
W & -W & 0 & 0 & 0 & 0 & 2\varepsilon_1+U_1 & 0 & 0 \\
W & -W & 0 & 0 & W & -W & 0 & 2\varepsilon_2+U_1 & 0 \\
0 & 0 & 0 & 0 & W & -W & 0 & 0 & 2\varepsilon_3+U_1
\end{bmatrix}. \quad (3)$$

For the quantum system with the time periodic field, we can analyze the dynamics of a system in terms of Floquet formalism. For the Hamiltonian $H(t)$, the quasi-energy wave function of a Floquet eigenstate is $|\phi_\alpha(t)\rangle = \exp(-{\rm i}\varepsilon_\alpha t)\,|u_\alpha(t)\rangle$, and $u(t) = u(t+T)$, ε_α is the quasi-energy. So the Schrödinger equation can be written as

$$\left(H(t) - {\rm i}\frac{\partial}{\partial t} \right) |u_\alpha(t)\rangle = \varepsilon_\alpha\,|u_\alpha(t)\rangle\,. \quad (4)$$

With the time evolution operator, $\psi(t) = U(t,0)\psi(0)$, the equation can be written as

$${\rm i}\frac{\partial}{\partial t} U(t,0) = H(t) U(t,0). \quad (5)$$

Then we can obtain the quasi-energy and the Floquet state $u(0)$ by diagonalizing $U(T,0)$, and the evolution of the system can be given as

$$|\psi(t)\rangle = \sum_{\alpha=1}^{M} \exp(-{\rm i}\varepsilon_\alpha t)\,|u_\alpha(t)\rangle\,\langle u_\alpha(0)|\psi(0)\rangle. \quad (6)$$

In terms of Eq. (5), one can find that

$$U(t,0) = \sum_{\alpha=1}^{M} \exp(-{\rm i}\varepsilon_\alpha t)\,|u_\alpha(t)\rangle\,\langle u_\alpha(0)|\,. \quad (7)$$

Then $|u_\alpha(t)\rangle$ is given by

$$|u_\alpha(t)\rangle = \sum_{m=1,|\varphi\rangle\neq|\psi(0)\rangle}^{M} c_{\alpha,m}(t)|\varphi_m\rangle + d_\alpha(t)|\psi(0)\rangle. \quad (8)$$

And $|\varphi_m\rangle$ $(m = 1,\ldots,M)$ is the M-dimensional real-space basis vector and the initial state is $|\psi(0)\rangle$. From the above equations we can obtain

$$|\psi(t)\rangle = \sum_{\alpha=1}^{M} d_\alpha^*(0) d_\alpha(t) \exp(-{\rm i}\varepsilon_\alpha t)|\psi(0)\rangle + \sum_{\alpha=1}^{M} \sum_{m=1,|\varphi\rangle\neq|\psi(0)\rangle}^{M} d_\alpha^*(0) c_{\alpha,m}(t) \exp(-{\rm i}\varepsilon_\alpha t)|\varphi_m\rangle. \quad (9)$$

The occupation probability $P(t)$ of the component of state $|\psi(0)\rangle$ in the state $|\psi(t)\rangle$ can be obtained by

$$P(t)=\sum_{\alpha=1}^{M}|d_\alpha(0)|^2|d_\alpha(t)|^2+2\mathrm{Re}\sum_{\alpha=1}^{M}\sum_{\beta=\alpha+1}^{M}d_\alpha(0)d_\beta^*(0)d_\alpha^*(t)d_\beta(t)\exp(-\mathrm{i}(\varepsilon_\beta-\varepsilon_\alpha)t). \tag{10}$$

Then we fix $\Delta t_i=T/Q$, the expression of the long-time averaged occupation probability P_a can be seen as follows:

$$\begin{aligned}P_\mathrm{a}&=\lim_{\tau\to\infty}\frac{1}{\tau}\int_0^\tau P(t)\,\mathrm{d}t=\lim_{N\to\infty}\frac{1}{NT}\sum_{n=1}^{N}\int_0^T P(t+(n-1)T)\,\mathrm{d}t\\&=\lim_{\lambda\to0}\frac{1}{T}\sum_{i=1}^{Q}\sum_{\alpha=1}^{M}|d_\alpha(0)|^2|d_\alpha(\xi_i)|^2\Delta t_i+2\mathrm{Re}\lim_{\lambda\to0}\frac{1}{T}\sum_{i=1}^{Q}\sum_{\alpha=1}^{M-1}\sum_{\beta=\alpha+1}^{M}\delta(\varepsilon_\beta-\varepsilon_\alpha)d_\alpha(0)d_\beta^*(0)d_\alpha^*(\xi_i)d_\beta(\xi_i)\Delta t_i.\end{aligned} \tag{11}$$

In this paper, we set $Q=10$ which is enough for convergence.

If the central dot of the triple quantum dots is movable, the system will be a quantum dot shuttle with the two static dots at fixed positions $\pm x_0$ as shown in Fig. 1.

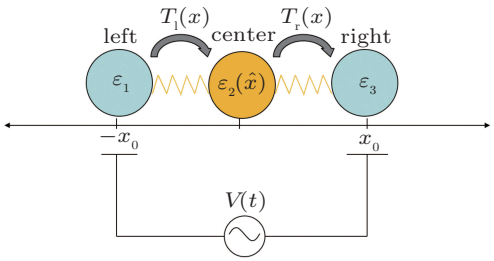


Fig. 1. (color online) Triple quantum dots with oscillating central dot, and a voltage difference (detuning) ε_b between left and right quantum dots. The system is driven by $V(t)=V\cos(\omega_\mathrm{ac}t)$.

The Hamiltonian of two electrons in such a quantum dot shuttle driven by an AC field can be written with two parts

$$\hat{H}=\hat{H}_\mathrm{e}+\hat{H}_\mathrm{os}, \tag{12}$$

where $\hat{H}_\mathrm{e}$ is the Hamiltonian of two electrons in triple quantum dots,

$$\begin{aligned}\hat{H}_\mathrm{e}=&\sum_{\sigma,i=1,2,3}\varepsilon_i(t)d_{i\sigma}^\dagger d_{i\sigma}\\&+T_\mathrm{l}(\hat{x})\sum_\sigma\left(d_{1\sigma}^\dagger d_{2\sigma}+d_{2\sigma}^\dagger d_{1\sigma}\right)\\&+T_\mathrm{r}(\hat{x})\sum_\sigma\left(d_{2\sigma}^\dagger d_{3\sigma}+d_{3\sigma}^\dagger d_{2\sigma}\right)\\&+U_1\sum_i n_{i\uparrow}n_{i\downarrow}+U_2\sum_{\sigma_1,\sigma_2,\sigma_3}(n_{1\sigma}n_{2\sigma}+n_{2\sigma}n_{3\sigma}).\end{aligned} \tag{13}$$

In Eq. (13), the meanings of the parameters are the same as those in Eq. (1). In order to simplify the calculation, we will not consider the dependency of U_2 on x. If we set $x=0$ as the equilibrium position of the center dot, ε_i is presented as

$$\begin{aligned}&\varepsilon_1=\frac{\varepsilon_\mathrm{b}}{2}+\frac{V}{2}\cos(\omega_\mathrm{ac}t),\quad \varepsilon_2=\varepsilon_1-\frac{\varepsilon_\mathrm{b}}{2x_0}(\hat{x}+x_0),\\&\varepsilon_3=-\frac{\varepsilon_\mathrm{b}}{2}-\frac{V}{2}\cos(\omega_\mathrm{ac}t),\end{aligned} \tag{14}$$

where ε_b is the voltage difference at $t=0$,

$$\hat{x}=\sqrt{\hbar/(2m\omega)}(d^\dagger+d).$$

ω is the oscillation frequency of the center dot. $T_{\mathrm{l(r)}}(\hat{x})$ is the position-dependent tunneling between the center and left (right) dots, and they can be written as

$$T_\mathrm{l}(\hat{x})=-\nu\,\mathrm{e}^{-\alpha(x_0+\hat{x})},\quad T_\mathrm{r}(\hat{x})=-\nu\,\mathrm{e}^{-\alpha(x_0-\hat{x})}, \tag{15}$$

where α is the inverse of the tunneling length, and ν is the tunneling amplitude.[18] $\hat{H}_\mathrm{os}$ is the Hamiltonian of the oscillation of the center dot,

$$\hat{H}_\mathrm{os}=\hbar\omega d^\dagger d. \tag{16}$$

where $d^\dagger$ (d) is the creation (annihilation) operator of the phonon. With respect to the oscillation states, we will just consider two oscillator states $(i,j=0,1)$ in this work. The nonzero matrix elements of the Hamiltonian (13) are expressed as follows:

$$\begin{aligned}H_{11}^{ij}&=H_{22}^{ij}=\left(\varepsilon_1+j\hbar\omega+\frac{V}{2}\cos(\omega_\mathrm{ac}t)\right)\delta_{ij}+\Delta x\frac{\varepsilon_\mathrm{b}}{2x_0}\left(\sqrt{j+1}\delta_{i,j+1}+\sqrt{j}\delta_{i,j-1}\right),\\H_{33}^{ij}&=H_{44}^{ij}=(\varepsilon_1+\varepsilon_2+j\hbar\omega)\delta_{ij},\\H_{55}^{ij}&=H_{66}^{ij}=\left(\varepsilon_3+j\hbar\omega-\frac{V}{2}\cos(\omega_\mathrm{ac}t)\right)\delta_{ij}+\Delta x\frac{\varepsilon_\mathrm{b}}{2x_0}\left(\sqrt{j+1}\delta_{i,j+1}+\sqrt{j}\delta_{i,j-1}\right),\\H_{77}^{ij}&=(2\varepsilon_1+U_1+j\hbar\omega+V\cos(\omega_\mathrm{ac}t))\delta_{ij},\\H_{88}^{ij}&=(U_1+j\hbar\omega)\delta_{ij}-\Delta x\frac{\varepsilon_\mathrm{b}}{2x_0}\left(\sqrt{j+1}\delta_{i,j+1}+\sqrt{j}\delta_{i,j-1}\right),\\H_{99}^{ij}&=(2\varepsilon_3+U_1+j\hbar\omega-V\cos(\omega_\mathrm{ac}t))\delta_{ij},\end{aligned}$$

$$H^{ij}_{13(31)} = H^{ij}_{24(42)} = H^{ij}_{58(85)} = H^{ij}_{59(95)} = H^{ij}_{68(86)} = H^{ij}_{69(96)} = T^{ij}_{\rm l}(\hat{x}),$$
$$H^{ij}_{17(71)} = H^{ij}_{18(81)} = H^{ij}_{27(72)} = H^{ij}_{28(82)} = H^{ij}_{35(53)} = H^{ij}_{46(64)} = T^{ij}_{\rm r}(\hat{x}), \quad (17)$$

where $\Delta x = \sqrt{\hbar/(2m\omega)}$, and

$$T^{ij}_{\rm l} = -\nu[2^{n-m} i!\, j!]^{1/2}\, {\rm e}^{-\alpha x_0}\, {\rm e}^{(\alpha_0^2/4)} [-\alpha_0]^{i-j} \sum_{k=0}^{j} \frac{[\alpha_0]^{2k}}{2^k k!(k+i-j)!(j-k)!}, \quad i \geq j$$

with $T^{ij}_{\rm l} = (-1)^{j-i} T^{ij}_{\rm r}$, and $\alpha_0 = \sqrt{2}\Delta x \alpha$. Similar to the case of triple quantum dots, we can also calculate the long-time average occupation using the Floquet formalism.

3. Results and discussion

3.1. The localization in triple quantum dots and the effects of Coulomb interaction

In the following discussions, we set $\hbar = 2m = e = 1$ and $\omega_{\rm ac}$ as the unit. We will first discuss the main features of localization of interacting electrons in triple quantum dots to reveal the effect of the Coulomb interaction.

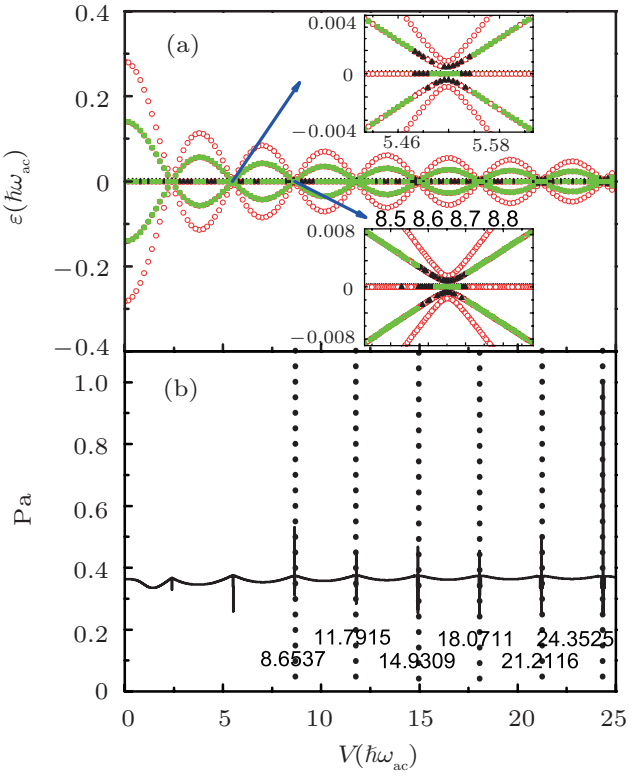


Fig. 2. (color online) (a) Quasi-energy spectrum for triple quantum dots with $U_1 = 0$ and $W = 0.1$. Symbols indicate the sorting of the corresponding Floquet state: red circles, green squares, and black triangles represent inner-dot states, inter-dot states, and wide separated states, respectively. (b) The long-time averaged occupation probability $P_{\rm a}$ for the triple quantum dots with electrons, initialized in state $|2,0,0\rangle$. The dash line indicates the values of the peak corresponding to the zero point of the Bessel function J_0.

We have calculated the long-time averaged occupation probability of two electrons in triple quantum dots with the initial state $|2,0,0\rangle$. We show the exact quasi-energy spectra as the function of V with parameters $W = 0.1$, $U_1 = 0$, and $U_2 = 0$ in Fig. 2(a). For such a case without any Coulomb interaction, the probability value $P_{\rm a}$ is shown in Fig. 2(b). It is found that the electrons can be localized, where the quasi-energies collapse. Moreover, with the increase of the value of V, the peak values of localization become larger.

To reveal the effect of Coulomb interaction on the dynamic localization, we show the long-time averaged occupation probabilities of two electrons in triple quantum dots as the function of V with the initial state $|2,0,0\rangle$ and inner-dot Coulomb interaction $U_1 = 8.0$ in Fig. 3(a). From the comparison of Fig. 2(b) and Fig. 3(a), it can be found that the introduction of the inner-dot Coulomb interaction of the electrons increases the values of the peaks of $P_{\rm a}$, thus the electrons are more localized. When we add the inter-dot Coulomb interaction U_2 in Figs. 3(b) and 3(c), the dynamic localization will increase further. Those phenomena are the results of the competition between the Coulomb interaction and the tunneling.

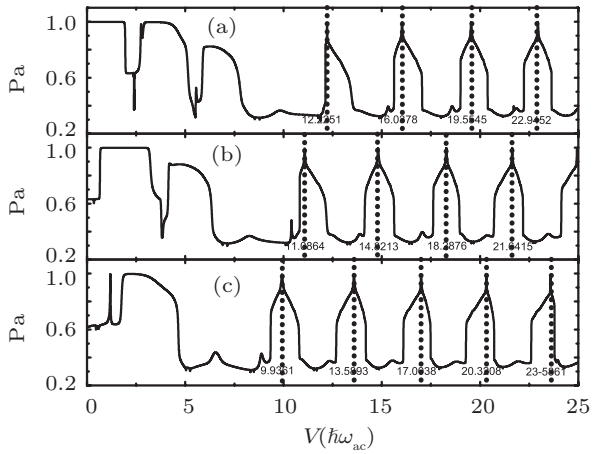


Fig. 3. (a) $P_{\rm a}$ for triple quantum dots, initialized in state $|2,0,0\rangle$, for $U_1 = 8.0$, $U_2 = 0.0$, and $W = 0.1$. The dash line indicates the values of the peak corresponding to the zero point of the Bessel function J_8; (b) $P_{\rm a}$ for triple quantum dots, initialized in state $|2,0,0\rangle$, for $U_1 = 8.0$, $U_2 = 1.0$, and $W = 0.1$. The dash line indicates the values of the peak corresponding to the zero point of the Bessel function J_7; (c) $P_{\rm a}$ for triple quantum dots, initialized in state $|2,0,0\rangle$, for $U_1 = 8.0$, $U_2 = 2.0$, and $W = 0.1$. The dash line indicates the values of the peak corresponding to the zero point of the Bessel function J_6.

It has been shown that the condition of dynamic localization can be derived analytically for quantum dot[21] and quantum dot array[22] at least with the assumption that $V \gg U$. For triple quantum dots with two electrons, the derivation is quite similar. The Hamiltonian (1) can be divided into two parts H_0 and H'. H_0 only includes the diagonal term. H' only includes the tunneling term and is treated as the perturbation. For the

diagonal terms the Floquet equation can be written as

$$\left(H_0 - \mathrm{i}\frac{\partial}{\partial t}\right)|u_j(t)\rangle = \varepsilon_j|u_j(t)\rangle.$$

The quasi-energies are

$$\begin{aligned}
&\varepsilon_1 = \varepsilon_2 = \frac{2\pi n}{T} + U_2, \quad \varepsilon_3 = \varepsilon_4 = \frac{2\pi n}{T}, \\
&\varepsilon_5 = \varepsilon_6 = \frac{2\pi n}{T} + U_2, \quad \varepsilon_7 = \varepsilon_8 = \varepsilon_9 = \frac{2\pi n}{T} + U_1. \qquad (18)
\end{aligned}$$

The corresponding Floquet states are

$$\begin{aligned}
u_1(t) &= \exp\left(-\mathrm{i}\left[-\frac{V}{2\omega_{\mathrm{ac}}}\sin\omega_{\mathrm{ac}}t + (U_2-\varepsilon_1)t\right]\right), \\
u_2(t) &= \exp\left(-\mathrm{i}\left[-\frac{V}{2\omega_{\mathrm{ac}}}\sin\omega_{\mathrm{ac}}t + (U_2-\varepsilon_2)t\right]\right), \\
u_3(t) &= \exp\left(\mathrm{i}\varepsilon_3 t\right), \\
u_4(t) &= \exp\left(\mathrm{i}\varepsilon_4 t\right), \\
u_5(t) &= \exp\left(-\mathrm{i}\left[\frac{V}{2\omega_{\mathrm{ac}}}\sin\omega_{\mathrm{ac}}t + (U_2-\varepsilon_5)t\right]\right), \\
u_6(t) &= \exp\left(-\mathrm{i}\left[\frac{V}{2\omega_{\mathrm{ac}}}\sin\omega_{\mathrm{ac}}t + (U_2-\varepsilon_6)t\right]\right), \\
u_7(t) &= \exp\left(-\mathrm{i}\left[-\frac{V}{2\omega_{\mathrm{ac}}}\sin\omega_{\mathrm{ac}}t + (U_1-\varepsilon_7)t\right]\right), \\
u_8(t) &= \exp\left[-\mathrm{i}(U_1-\varepsilon_8)t\right], \\
u_9(t) &= \exp\left(-\mathrm{i}\left[\frac{V}{2\omega_{\mathrm{ac}}}\sin\omega_{\mathrm{ac}}t + (U_1-\varepsilon_9)t\right]\right), \qquad (19)
\end{aligned}$$

where $n = 0, \pm1, \pm2, \ldots$. At the position where the quasi-energies collapse, i.e., $\varepsilon_1 = \varepsilon_3$, or $\varepsilon_1 = \varepsilon_7$, or $\varepsilon_1 = \varepsilon_8$, or $\varepsilon_2 = \varepsilon_4$, or $\varepsilon_2 = \varepsilon_7$, or $\varepsilon_2 = \varepsilon_8$, or $\varepsilon_3 = \varepsilon_5$, or $\varepsilon_4 = \varepsilon_6$, or $\varepsilon_5 = \varepsilon_8$, or $\varepsilon_5 = \varepsilon_9$, or $\varepsilon_6 = \varepsilon_8$, or $\varepsilon_6 = \varepsilon_9$, $U_1 - U_2 = 2\pi n'/T$, $(n' = 0, 1, 2, \ldots)$ must be satisfied. Then the tunneling probability can be calculated analytically. Taking P_{17} for example,

$$\begin{aligned}
P_{17} &= \left\langle\left\langle u_1(t)|H'|u_7(t)\right\rangle\right\rangle \\
&= \frac{1}{T}\int_0^T \mathrm{d}t\left\langle u_1(t)|H'|u_7(t)\right\rangle \\
&= W\frac{1}{T}\int_0^T \mathrm{d}t \exp\left(\mathrm{i}\left[-\frac{V}{2\omega_{\mathrm{ac}}}\sin\omega_{\mathrm{ac}}t + (U_2-\varepsilon_1)t\right]\right) \\
&\quad \cdot \exp\left(-\mathrm{i}\left[-\frac{V}{2\omega_{\mathrm{ac}}}\sin\omega_{\mathrm{ac}}t + (U_1-\varepsilon_7)t\right]\right). \qquad (20)
\end{aligned}$$

When we use the identity

$$\mathrm{e}^{\pm i\xi\sin\omega_{\mathrm{ac}}t} = \sum_{n=-\infty}^{\infty} \mathrm{J}_n(\xi)\,\mathrm{e}^{\pm in\omega_{\mathrm{ac}}t}, \qquad (21)$$

$$\begin{aligned}
P_{17} &= W\sum_{n=-\infty}^{\infty}\mathrm{J}_n\left(\frac{V}{2\omega_{\mathrm{ac}}}\right)\frac{1}{T}\int_0^T \mathrm{d}t\,\mathrm{e}^{\mathrm{i}(n\omega_{\mathrm{ac}}-(U_1-U_2))t} \\
&= W\,\mathrm{J}_{n'}(V/2\omega_{\mathrm{ac}}). \qquad (22)
\end{aligned}$$

So the tunneling will be depressed when $V/2\omega_{\mathrm{ac}}$ equals to the zero points of $\mathrm{J}_{n'}$. We have indicated the zero points of corresponding Bessel functions $\mathrm{J}_{n'}$ in both Fig. 2 and Fig. 3. From the top to the bottom of Fig. 3, the Coulomb interactions are $U_1 = 8.0$ and $U_2 = 0.0$, 1.0, and 2.0, then the peaks correspond to the zero point of the Bessel functions J_8, J_7, and J_6 respectively. It can be seen that the positions of the localization fit well with the analytical results.

3.2. Localization in quantum dot shuttle and the effect of the mechanical oscillation

In this section, we will compare the main feature of the long-time averaged occupation probability of triple quantum dots and quantum dot shuttles.

To simplify the calculation, we will take $U_2 = 0$. We calculate the long-time averaged occupation probability with the initial state $|2,0,0\rangle$ for triple quantum dots and $|2,0,0;0\rangle$ (the last number indicates the oscillation state) for quantum dot shuttles in Fig. 4. The parameters are both $U_1 = \omega_{\mathrm{ac}}$, $\omega_{\mathrm{ac}} = 0.5$. From the comparison of Fig. 4, it can be found that the introduction of vibration of the quantum dot changes the behaviors of the localization greatly. For the smaller oscillation frequency of the shuttles, the tunneling rate is greatly depressed. Then the localization will not be limited at specific values of the AC field, but can maintain at a high level within a large range, see Fig. 4(b) where $\omega = 0.03\omega_{\mathrm{ac}}$ as an example. However, with the increase of the oscillation frequency, the tunneling rate will gradually increase. Then P_{a} will recover to the more common behavior, i.e., the emergence of peaks at specific values of V, see Fig. 4(c) where $\omega = 0.3\omega_{\mathrm{ac}}$ as an example. Especially when the oscillation frequency is just the integer times of the AC field frequency, as in the case of Fig. 4(d) where $\omega = 2\omega_{\mathrm{ac}}$, the mechanical vibration forms an effective tunneling channel. Then the localization will be greatly depressed. We also calculate the long-time averaged occupation probability with the oscillation frequencies of the shuttles between the four frequencies above; we find that the behavior is gradually transited from Fig. 4(a) to Fig. 4(b). Similarly, the behavior is gradually transited from Fig. 4(b) to Fig. 4(c).

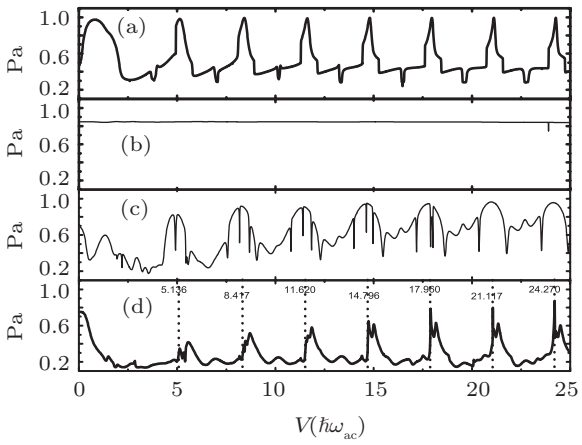


Fig. 4. (a) P_{a} for triple quantum dots, initialized in state $|2,0,0\rangle$, $\omega = 0$; (b) P_{a} for quantum dot shuttles, initialized in state $|2,0,0;0\rangle$, $\omega = 0.03\omega_{\mathrm{ac}}$; (c) P_{a} for quantum dot shuttles, initialized in state $|2,0,0;0\rangle$, $\omega = 0.3\omega_{\mathrm{ac}}$; (d) P_{a} for quantum dot shuttles, initialized in state $|2,0,0;0\rangle$, $\omega = 2\omega_{\mathrm{ac}}$. The parameters are $\{U_1, \varepsilon_{\mathrm{b}}\} = \{0.5, 1.0\}$. The dash line indicates the values of the peak corresponding to the zero point of the Bessel function J_2. In the calculations of $T_{l(r)}$, the parameters $\{v; x_0; \alpha\} = \{0.5; 5.0; 0.4\}$.

When the frequency of the mechanical oscillation is the integer times of the AC field, the analytical condition of the localization can also be derived. Again we divide the Hamiltonian (13) into two parts H_0 and H'. H_0 only includes the diagonal term. H' only includes the tunneling term as the perturbation. For H_0 the Floquet equation can be written as

$$\left(H_0 - \mathrm{i}\frac{\partial}{\partial t}\right)|u_\alpha^\beta(t)\rangle = \varepsilon_\alpha^\beta |u_\alpha^\beta(t)\rangle.$$

Be aware that, we only consider two oscillator states ($\beta = 0, 1$) for the center quantum dot, then the quasi-energies can be obtained as

$$\begin{aligned}
&\varepsilon_1^0 = \varepsilon_2^0 = \frac{2\pi n}{T} + \frac{\varepsilon_\mathrm{b}}{2}, \quad \varepsilon_1^1 = \varepsilon_2^1 = \frac{2\pi n}{T} + \frac{\varepsilon_\mathrm{b}}{2} + \hbar\omega,\\
&\varepsilon_3^0 = \varepsilon_4^0 = \frac{2\pi n}{T} + \frac{\varepsilon_\mathrm{b}}{2}, \quad \varepsilon_3^1 = \varepsilon_4^1 = \frac{2\pi n}{T} + \frac{\varepsilon_\mathrm{b}}{2} + \hbar\omega,\\
&\varepsilon_5^0 = \varepsilon_6^0 = \frac{2\pi n}{T} - \frac{\varepsilon_\mathrm{b}}{2}, \quad \varepsilon_5^1 = \varepsilon_6^1 = \frac{2\pi n}{T} - \frac{\varepsilon_\mathrm{b}}{2} + \hbar\omega,\\
&\varepsilon_7^0 = \frac{2\pi n}{T} + \varepsilon_\mathrm{b} + U_1, \quad \varepsilon_7^1 = \frac{2\pi n}{T} + \varepsilon_\mathrm{b} + U_1 + \hbar\omega,\\
&\varepsilon_8^0 = \frac{2\pi n}{T} + U_1, \qquad\quad \varepsilon_8^1 = \frac{2\pi n}{T} + U_1 + \hbar\omega,\\
&\varepsilon_9^0 = \frac{2\pi n}{T} - \varepsilon_\mathrm{b} + U_1, \quad \varepsilon_9^1 = \frac{2\pi n}{T} - \varepsilon_\mathrm{b} + U_1 + \hbar\omega, \qquad (23)
\end{aligned}$$

and the corresponding Floquet states are

$$\begin{aligned}
&u_1^0(t) = \exp\left(-\mathrm{i}\left[\frac{V}{2\omega_\mathrm{ac}}\sin\omega_\mathrm{ac}t + (\varepsilon_1 - \varepsilon_1)t\right]\right), && u_1^1(t) = \exp\left(-\mathrm{i}\left[\frac{V}{2\omega_\mathrm{ac}}\sin\omega_\mathrm{ac}t + (\varepsilon_1 + \hbar\omega - \varepsilon_2)t\right]\right),\\
&u_2^0(t) = \exp\left(-\mathrm{i}\left[\frac{V}{2\omega_\mathrm{ac}}\sin\omega_\mathrm{ac}t + (\varepsilon_1 - \varepsilon_3)t\right]\right), && u_2^1(t) = \exp\left(-\mathrm{i}\left[\frac{V}{2\omega_\mathrm{ac}}\sin\omega_\mathrm{ac}t + (\varepsilon_1 + \hbar\omega - \varepsilon_4)t\right]\right),\\
&u_3^0(t) = \exp\left(-\mathrm{i}(\varepsilon_1 + \varepsilon_2 - \varepsilon_5)t\right), && u_3^1(t) = \exp\left(-\mathrm{i}(\varepsilon_1 + \varepsilon_2 + \hbar\omega - \varepsilon_6)t\right),\\
&u_4^0(t) = \exp\left(-\mathrm{i}(\varepsilon_1 + \varepsilon_2 - \varepsilon_7)t\right), && u_4^1(t) = \exp\left(-\mathrm{i}(\varepsilon_1 + \varepsilon_2 + \hbar\omega - \varepsilon_8)t\right),\\
&u_5^0(t) = \exp\left(-\mathrm{i}\left[-\frac{V}{2\omega_\mathrm{ac}}\sin\omega_\mathrm{ac}t + (\varepsilon_3 - \varepsilon_9)t\right]\right), && u_5^1(t) = \exp\left(-\mathrm{i}\left[-\frac{V}{2\omega_\mathrm{ac}}\sin\omega_\mathrm{ac}t + (\varepsilon_3 + \hbar\omega - \varepsilon_{10})t\right]\right),\\
&u_6^0(t) = \exp\left(-\mathrm{i}\left[-\frac{V}{2\omega_\mathrm{ac}}\sin\omega_\mathrm{ac}t + (\varepsilon_3 - \varepsilon_{11})t\right]\right), && u_6^1(t) = \exp\left(-\mathrm{i}\left[-\frac{V}{2\omega_\mathrm{ac}}\sin\omega_\mathrm{ac}t + (\varepsilon_3 + \hbar\omega - \varepsilon_{12})t\right]\right),\\
&u_7^0(t) = \exp\left(-\mathrm{i}\left[\frac{V}{\omega_\mathrm{ac}}\sin\omega_\mathrm{ac}t + (2\varepsilon_1 + U_1 - \varepsilon_{13})t\right]\right), && u_7^1(t) = \exp\left(-\mathrm{i}\left[\frac{V}{\omega_\mathrm{ac}}\sin\omega_\mathrm{ac}t + (2\varepsilon_1 + U_1 + \hbar\omega - \varepsilon_{14})t\right]\right),\\
&u_8^0(t) = \exp\left[-\mathrm{i}(U_1 - \varepsilon_{15})t\right], && u_8^1(t) = \exp\left[-\mathrm{i}(U_1 + \hbar\omega - \varepsilon_{16})t\right],\\
&u_9^0(t) = \exp\left(-\mathrm{i}\left[-\frac{V}{\omega_\mathrm{ac}}\sin\omega_\mathrm{ac}t + (2\varepsilon_3 + U_1 - \varepsilon_{17})t\right]\right), && u_9^1(t) = \exp\left(-\mathrm{i}\left[-\frac{V}{\omega_\mathrm{ac}}\sin\omega_\mathrm{ac}t + (2\varepsilon_3 + U_1 + \hbar\omega - \varepsilon_{18})t\right]\right), \qquad (24)
\end{aligned}$$

where $n = 0, \pm1, \pm2, \ldots$. At the position where the quasi-energies collapse, i.e., $\varepsilon_1^i = \varepsilon_3^i$, or $\varepsilon_1^i = \varepsilon_7^i$, or $\varepsilon_1^i = \varepsilon_8^i$, or $\varepsilon_2^i = \varepsilon_4^i$, or $\varepsilon_3^i = \varepsilon_5^i$, or $\varepsilon_4^i = \varepsilon_6^i$, or $\varepsilon_5^i = \varepsilon_8^i$, or $\varepsilon_5^i = \varepsilon_9^i$, or $\varepsilon_6^i = \varepsilon_8^i$, or $\varepsilon_6^i = \varepsilon_9^i$, where $i = 0, 1$, the conditions

$$\omega = n_1\omega_\mathrm{ac}, \quad \varepsilon_\mathrm{b} = 2n_2\hbar\omega_\mathrm{ac}, \quad \frac{\varepsilon_\mathrm{b}}{2} \pm U_1 = n_3\omega_\mathrm{ac}, \qquad (25)$$

where $(n_1, n_2, n_3, \ldots = 0, 1, 2, \ldots)$ must be satisfied. Then the tunneling probability can be calculated analytically. Taking P_{71}^{00} for example,

$$\begin{aligned}
P_{71}^{00} &= \left\langle\left\langle u_7^0(t)|H'|u_1^0(t)\right\rangle\right\rangle = \frac{1}{T}\int_0^T \mathrm{d}t\left\langle u_7^0(t)|H'|u_1^0(t)\right\rangle\\
&= T_1^{00}\frac{1}{T}\int_0^T \mathrm{d}t \exp\left(\mathrm{i}\left[\frac{V}{\omega_\mathrm{ac}}\sin\omega_\mathrm{ac}t + (2\varepsilon_1 + U_1 - \varepsilon_7^0)t\right]\right)\\
&\quad\cdot\exp\left(-\mathrm{i}\left[\frac{V}{2\omega_\mathrm{ac}}\sin\omega_\mathrm{ac}t + (\varepsilon_1 - \varepsilon_1^0)t\right]\right). \qquad (26)
\end{aligned}$$

By using Eq. (21), we have

$$\begin{aligned}
P_{71}^{00} &= T_1^{00}\sum_{n=-\infty}^{\infty}\mathrm{J}_n\left(\frac{V}{2\omega_\mathrm{ac}}\right)\frac{1}{T}\int_0^T \mathrm{d}t\,\mathrm{e}^{\mathrm{i}n\omega_\mathrm{ac}t}\,\mathrm{e}^{-\mathrm{i}n_3\omega_\mathrm{ac}t}\\
&= T_1^{00}\mathrm{J}_{n_3}(V/2\omega_\mathrm{ac}). \qquad (27)
\end{aligned}$$

So the tunneling will be depressed when $V/2\omega_\mathrm{ac}$ equals the zero point of the Bessel function J_{n_3}. It can be seen from Fig. 4(d) that the positions of the localization fit well with the analytical results.

4. Conclusion

In summary, we have studied the dynamic localization of two interacting electrons in triple quantum dots and quantum dot shuttles induced by alternating current electric fields. It is found that the introduction of the Coulomb interaction will increase the dynamic localization. The mechanical vibration of quantum dot shuttles will change the pattern of localization greatly if its frequency is quite a bit smaller than the frequency of the AC field. If the frequency of the mechanical oscillation is the integer times of the frequency of the AC field, the localization may be depressed since the mechanical vibration just supplies an effective tunneling channel under such condition. We also derive the analytical condition of two-electron localization both for triple quantum dots and quantum dot shuttles within the Floquet formalism. Our results can be useful

for controlling the transport and the dynamics of electrons in nanoelectro-mechanical systems.

References

[1] Terzis A F and Kosionis S G and Paspalakis E 2007 *J. Phys. B* **40** S331
[2] Zhang P, Xue Q K, Zhao X G and Xie X C 2002 *Phys. Rev. A* **66** 022117
[3] Madureira J R, Schulz P A and Maialle M Z 2004 *Phys. Rev. B* **70** 033309
[4] Jiang Z and Duan S Q and Zhao X G 2004 *Phys. Lett. A* **330** 260
[5] Zhang P and Zhao X G 2000 *J. Phys.: Condens. Matter* **12** 2351
[6] Zhong G H and Wang L M 2010 *Chin. Phys. B* **19** 107202
[7] Zhong G H and Wang L M 2015 *Chin. Phys. B* **24** 047306
[8] Gorelik L Y, Isacsson A, Voinova M V, Kasemo B, Shekhter R I and Jonson M 1998 *Phys. Rev. Lett* **80** 4526
[9] Shekhter R I, Galperin Yu, Gorelik L Yu, Isacsson A and Jonson M 2003 *J. Phys.: Condens. Matter* **15** R441
[10] Shekhter R I, Gorelik L Y, Jonson M, Galperin Yu M and Vinokur V M 2007 *J. Comput. Theor. Nanosci.* **4** 860
[11] Kim C, Prada M and Blick R H 2012 *ACS Nano* **6** 651
[12] Sylvain H, Shintaro T, Michihisa Y, Seigo T, Andreas D W, Laurent S, Christopher B and Tristan M 2011 *Nature* **477** 435
[13] McNeil R P G, Kataoka M, Ford C J B, Barnes C H W, Anderson D, Jones G A C, Farrer I and Ritchie D A 2011 *Nature* **477** 439
[14] Armour A D and MacKinnon A 2002 *Phys. Rev. B* **66** 035333
[15] Ilinskaya O A, Kulinich S I, Krive I V, Shekhter R I and Jonson M 2015 *Low Temp. Phys.* **41** 70
[16] Fujita T, Baart T A, Reichl C, Wegscheider W and Vandersypen L M K 2017 arXiv preprint arXiv:1701.00815
[17] Zhang R, Chu W D, Duan S Q and Yang N 2013 *Chin. Phys. B* **22** 117305
[18] Villavicencio J, Maldonado I, Cota E and Platero G 2011 *New J. Phys.* **13** 023032
[19] Zhang P and Zhao X G 2000 *Phys. Lett. A* **271** 419
[20] Creffield C E and Platero G 2002 *Phys. Rev. B* **66** 235303
[21] Grossmann F, Dittrich T, Jung P and Hanggi P 1991 *Phys. Rev. Lett.* **67** 516
[22] Creffield C E and Platero G 2004 *Phys. Rev. B* **69** 165312