Macro-performance of multilayered thermoelectric medium*

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The effective properties of thermoelectric composites are well known to depend on boundary conditions, which causes the macro performance of thermoelectric composite to be difficult to assess. The overall macro-performance of multilayered thermoelectric medium is discussed in this paper. The analytical solutions are obtained, including the heat flux, temperature, electric potential, and the overall energy conversion efficiency. The results show that there are unique relationships between the temperature/electric potential and the electric current/energy flux in the material, and whether the material is independent of or embedded in thermoelectric composites. Besides, the Peltier effect at the interface can significantly improve the overall energy conversion efficiency of thermoelectric composites. These results provide a powerful tool to analyze the effective behaviors of thermoelectric composites.

Keywords: multi-layered, thermoelectric material, conversion efficiency, Peltier effect, heat flux, electric energy

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1. Introduction

Owing to the fact that its specific performance in converting heat into electricity, thermoelectric material has been widely used in many areas, such as aerospace,[1–4] vehicle,[5,6] ship,[7,8] smart robots,[9–11] and waste heat recovery.[12–17] However, conversion efficiency is one of the important factors which hiders the application of thermoelectric materials extending. It is well known that the conversion efficiency in thermoelectric material is governed by the figure of merit ZT by[18]

$$ZT = \frac{\sigma \varepsilon^2 T}{\kappa}, \tag{1}$$

where σ and κ are electric and thermal conductivity, respectively, and ε is the Seebeck coefficient, a specific parameter which relates the electron flux to heat flux. Obviously, a high conversion efficiency needs high electric conductivity and low thermal conductivity at a given temperature T. However, it is hard to raise the value of ZT in a single thermoelectric material since there is an internal relation among these parameters.[19] Hence, composite thermoelectric materials have attracted more and more attention.

There have been carried out a number of experiments about nanostructured composite thermoelectric materials. For example, in 2008, Poudel *et al.* showed that a peak ZT of 1.4 at 100 °C can be achieved in a p-type nanocrystalline BiSbTe bulk alloy.[20] Soon after that, a significantly reduced thermal conductivity led to the figure of merit $ZT = 1.56$ at 300 K in p-type $Bi_{0.52}Sb_{1.48}Te_3$ bulk material embedded in amorphous matrix and 5 nm–15 nm nanocrystals with coherent grain boundary;[21] and a maximum ZT of 1.60 at 750 K was achieved at the composition $K_{1-x}Pb_{m+\delta}Sb_{1+\gamma}Te_{m+2}$ materials system.[22] Furthermore, Heremans *et al.* observed a very large enhancement of the thermoelectric power of composites containing bismuth nanowires with diameters of 9 nm and 15 nm, embedded in porous alumina and porous silica.[23] A series of Bi_2Te_3 nanocomposite samples has been experimented by incorporating nanoparticle concentrations of 5 mol%–50 mol% into a bulk matrix via a mixing process and subsequently hot pressing into highly densified pellets.[24] Many other researchers have also made outstanding contributions to high performance thermoelectric materials.[25–27] These studies highlight the importance of thermoelectric composites and nanostructures, and quite a few atomistic models have been developed for their understanding. For example, David and Levy pointed out that the effective figure of merit of the composite can never exceed the largest value of ZT in any component.[28] More rigorous continuum analysis has been carried out for layered thermoelectrics, which demonstrated that the energy conversion efficiency of layered medium can be higher than those of both constituents, and the effective properties of thermoelectric composites are dependent on boundary conditions.[29] Because of this, there is no stable ZT for thermoelectric composite, and the macro performance of thermoelectric composite is still difficult to assess. It is thus highly desirable to be able to accurately evaluate overall macro-performance of multilayered thermoelectric medium.

The method used in the literature is dealing with the inter-

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face conditions one by one.[29,30] In each interface there are 4 interface conditions (the temperature, electric potential, electric current density, and heat flux are continuous across the interface), so it is needed to solve $4n$ equations (including 4 boundary conditions) simultaneously for an n-layered thermoelectric medium by traditional methods. It is difficult to obtain analytic solutions for multilayered thermoelectric medium. A new evaluation method is proposed based on the internal relations among the distributions of temperature/electric potential and electric current/energy flux in this paper. The new method simplifies the solving system into only 2 equations for multilayered thermoelectric medium, which reduces the solving processes dramatically. Besides, the Peltier effect at the interface, which is ignored by many researchers, is taken into consideration, and it has a noteworthy influence on overall energy conversion efficiency. These results provide a powerful tool to analyze the effective behaviors of thermoelectric composites.

2. Governing equations

We consider that the heat flux $\boldsymbol{J}_Q$ and electron flux $\boldsymbol{J}_N$ in thermoelectric material are related by the transport equations as follows:[18]

$$-\boldsymbol{J}_N = \left(\frac{T\sigma}{e^2}\right)\frac{1}{T}\nabla\mu - \left(\frac{T^2\sigma\varepsilon}{e}\right)\nabla\frac{1}{T}, \tag{2}$$

$$\boldsymbol{J}_Q = -\left(\frac{T^2\sigma\varepsilon}{e}\right)\frac{1}{T}\nabla\mu - \left(T^3\sigma\varepsilon^2 + T^2\kappa\right)\nabla\frac{1}{T}, \tag{3}$$

where e and μ are the charge and electrochemical potential of the electron, respectively. Noticing that

$$\mu = e\phi, \tag{4}$$

where ϕ is the electric potential, and the electric current density $\boldsymbol{J}$ can be derived from electron flux,

$$\boldsymbol{J} = e\boldsymbol{J}_N. \tag{5}$$

The transport equations governing electric current density and heat flux can be expressed as

$$-\boldsymbol{J} = \sigma\nabla\phi + \sigma\varepsilon\nabla T, \tag{6}$$

$$\boldsymbol{J}_Q = T\varepsilon\boldsymbol{J} - \kappa\nabla T. \tag{7}$$

Since the energy are transported by both heat and electrons, the energy flux $\boldsymbol{J}_E$ can be derived from the current density and heat flux as

$$\boldsymbol{J}_E = \boldsymbol{J}_Q + \boldsymbol{J'}_E, \tag{8}$$

where $\boldsymbol{J'}_E$ is the electric energy, which can be expressed as

$$\boldsymbol{J'}_E = \phi\boldsymbol{J}. \tag{9}$$

Substituting Eq. (7) into Eq. (8) and using Eq. (9), we have

$$\boldsymbol{J}_E = (\phi + \varepsilon T)\boldsymbol{J} - \kappa\nabla T. \tag{10}$$

We assume that both the charges and energy are conserved in thermoelectric materials, and then both current density and energy flux are divergence-free,

$$\nabla\cdot\boldsymbol{J} = 0, \tag{11}$$

$$\nabla\cdot\boldsymbol{J}_E = 0. \tag{12}$$

Therefore, the heat flux is not divergence-free, which is different from the scenario of uncoupled problem.

3. One-dimensional problems

Now we consider a multilayered thermoelectric medium. All the variables in this medium are dependent on coordinate x, while independent of coordinates y and z. It is clear that both the current density and energy flux are constants due to Eqs. (11) and (12).

3.1. Relationship between temperature/electric potential and electric/energy flux

Considering a homogeneous thermoelectric medium with the length of L. The temperature and electric potential at two ends of the medium are T_l, ϕ_l, and T_h, ϕ_h, respectively as shown in Fig. 1.

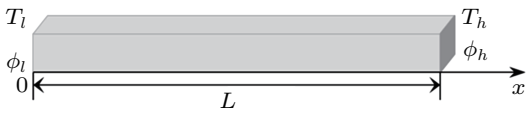


Fig. 1. Homogeneous thermoelectric medium.

Assuming that the material parameters are independent of temperature, and substituting Eq. (10) into Eq. (12), we have

$$\frac{\partial^2 T}{\partial x^2} = -\frac{J^2}{\sigma\kappa}. \tag{13}$$

The solution of Eq. (13) is

$$T = -\frac{J^2}{2\sigma\kappa}x^2 + c_1 x + c_2. \tag{14}$$

Substituting Eq. (14) into Eq. (6), yields

$$\frac{\partial\phi}{\partial x} = \varepsilon\left(\frac{J^2}{\sigma\kappa}x - c_1\right) - \frac{J}{\sigma}. \tag{15}$$

So we have

$$\phi = \frac{\varepsilon J^2}{2\sigma\kappa}x^2 - \left(\varepsilon c_1 + \frac{J}{\sigma}\right)x + c_3. \tag{16}$$

Noticing Eq. (10), the undetermined coefficient c_1 can be derived as

$$c_1 = \frac{bJ - J_E}{\kappa}, \tag{17}$$

where $b = \phi|_{x=0} + \varepsilon\, T|_{x=0} = \phi_l + \varepsilon T_l$.

Then the temperature field and electric current field can be expressed as

$$T = -\frac{J^2}{2\sigma\kappa}x^2 + \frac{bJ - J_E}{\kappa}x + c_2, \tag{18}$$

$$\phi = \frac{\varepsilon J^2}{2\sigma\kappa}x^2 - \left[\frac{\varepsilon(bJ - J_E)}{\kappa} + \frac{J}{\sigma}\right]x + c_3. \tag{19}$$

Equations (18) and (19) reveal a physical image that the distributions of temperature and electric potential are determined by electric current density J and energy flux J_E, if we ignore the constants c_2 and c_3 that represent uniform temperature and electric potential field, which means that the diversification of temperature/electric potential is only dependent on the electric current/energy flux for a certain thermoelectric material. More importantly, equations (18) and (19) still hold even the component is embedded in thermoelectric composites, as they are obtained directly from governing equations (6)–(12).

On the other hand, electric current and energy flux can be determined by boundary conditions as

$$J = -\frac{\sigma}{L}\left[(\phi_h - \phi_l) + \varepsilon(T_h - T_l)\right], \tag{20}$$

$$J_E = -\frac{\sigma}{2L}\left[\varepsilon^2\left(T_h^2 - T_l^2\right) + 2\varepsilon\left(T_h\phi_h - T_l\phi_l\right) + \left(\phi_h^2 - \phi_l^2\right)\right] - \frac{\kappa}{L}(T_h - T_l). \tag{21}$$

3.2. Multilayered thermoelectric medium with traditional interface conditions

Now consider a multilayered thermoelectric medium with material parameters σ_i, ε_i, and κ_i, and length L_i ($i = 1, 2, \ldots, n$). The temperature and electric potential are continuous at the interface, and represented as T_i and ϕ_i at the right end of each layer, as shown in Fig. 2.

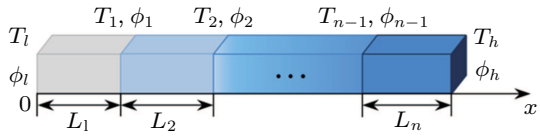


Fig. 2. (color online) Multilayered thermoelectric medium.

Obviously, the total temperature and electric potential differences at two ends of the multilayered thermoelectric medium consist of temperature and electric potential differences between layers, thus

$$\begin{cases} \sum_{i=1}^{n}(T_i - T_{i-1}) = T_h - T_l, \\ \sum_{i=1}^{n}(\phi_i - \phi_{i-1}) = \phi_h - \phi_l. \end{cases} \tag{22}$$

As shown in the above section, the temperature and electric potential differences between layers are only dependent on the electric current and energy flux. Noting that electric current and energy flux are uniform in one-dimensional multilayered thermoelectric medium, the differences in temperature and electric potential at two ends of phase-i can be expressed from Eqs. (18) and (19) as

$$T_i - T_{i-1} = -\frac{J^2}{2\sigma_i\kappa_i}L_i^2 + \frac{b_iJ - J_E}{\kappa_i}L_i, \tag{23}$$

$$\phi_i - \phi_{i-1} = \frac{\varepsilon_iJ^2}{2\sigma_i\kappa_i}L_i^2 - \left[\frac{\varepsilon_i(b_iJ - J_E)}{\kappa_i} + \frac{J}{\sigma_i}\right]L_i, \tag{24}$$

where the parameters b_i can be calculated from

$$\begin{aligned} b_1 &= \phi_l + \varepsilon_1 T_l, \\ b_2 &= (\varepsilon_1 - \varepsilon_2)\left[\frac{J^2}{2\sigma_1\kappa_1}L_1^2 - \frac{b_1J - J_E}{\kappa_1}L_1\right] \\ &\quad - \frac{J}{\sigma_1}L_1 + \phi_l + \varepsilon_2 T_l, \\ b_{i+1} &= b_i - \frac{JL_i}{\sigma_i} \\ &\quad + (\varepsilon_{i+1} - \varepsilon_i)\left[\frac{(b_iJ - J_E)L_i}{\kappa_i} - \frac{J^2L_i^2}{2\sigma_i\kappa_i}\right. \\ &\quad \left. + \frac{\sigma_{i-1}(b_i - b_{i-1}) + JL_{i-1}}{\sigma_{i-1}(\varepsilon_i - \varepsilon_{i-1})}\right], \quad (i \geq 2). \end{aligned} \tag{25}$$

It is seen that b_i also depends on J and J_E as well as the boundary conditions T_l and ϕ_l.

Finally, substituting Eqs. (23)–(24) into Eq. (22), yields

$$\begin{cases} \sum_{i=1}^{n}\left(-\frac{J^2}{2\sigma_i\kappa_i}L_i^2 + \frac{b_iJ - J_E}{\kappa_i}L_i\right) = T_h - T_l, \\ \sum_{i=1}^{n}\left(\frac{\varepsilon_iJ^2}{2\sigma_i\kappa_i}L_i^2 - \left[\frac{\varepsilon_i(b_iJ - J_E)}{\kappa_i} + \frac{J}{\sigma_i}\right]L_i\right) = \phi_h - \phi_l. \end{cases} \tag{26}$$

The only two unknown quantities J and J_E can be obtained from Eq. (26), then temperature and electric potential in each layer can be derived from Eqs. (18) and (19). This set of equations solves the problems completely, leading to the full determination of the field distributions in each layer.

For bilayered medium, equation (26) can be reduced to

$$\begin{cases} \sum_{i=1}^{2}\left(-\frac{J^2}{2\sigma_i\kappa_i}L_i^2 + \frac{b_iJ - J_E}{\kappa_i}L_i\right) = T_h - T_l, \\ \sum_{i=1}^{2}\left(\frac{\varepsilon_iJ^2}{2\sigma_i\kappa_i}L_i^2 - \left[\frac{\varepsilon_i(b_iJ - J_E)}{\kappa_i} + \frac{J}{\sigma_i}\right]L_i\right) = \phi_h - \phi_l. \end{cases} \tag{27}$$

Solving Eq. (27), the electric current density J and energy flux J_E can be deduced as

$$J = -\frac{\Psi_1 + \Psi_3 - \sqrt{\Psi_4}}{(\varepsilon_2 - \varepsilon_1)L_1L_2(L_1\sigma_2 + L_2\sigma_1)}, \tag{28}$$

$$J_E = -\frac{\frac{L_1\Psi_2}{\sigma_1}J^3 + \Psi_5J^2 - \Psi_6J + 2\sigma_1\sigma_2\kappa_1\kappa_2(T_h - T_l)}{2\sigma_1\sigma_2[L_1L_2(\varepsilon_2 - \varepsilon_1)J + L_1\kappa_2 + L_2\kappa_1]}, \tag{29}$$

where

$$\begin{aligned} \Psi_1 &= (L_1\sigma_2 + L_2\sigma_1)(L_1\kappa_2 + L_2\kappa_1), \\ \Psi_2 &= L_1L_2\sigma_1\sigma_2(\varepsilon_2 - \varepsilon_1), \\ \Psi_3 &= \Psi_2(\phi_h - \phi_l + \varepsilon_2T_h - \varepsilon_1T_l), \end{aligned}$$

$$\begin{aligned}\Psi_4 &= -2\Psi_2\left[\Psi_1\left(\phi_h-\phi_l\right)+\left(L_1\sigma_2+L_2\sigma_1\right)\right.\\ &\quad\left.\times\left(L_1\varepsilon_1\kappa_2+L_2\varepsilon_2\kappa_1\right)\left(T_h-T_l\right)\right]+\left(\Psi_1+\Psi_3\right)^2,\\ \Psi_5 &= L_1\sigma_2\left(L_1\kappa_2+L_2\kappa_1\right)+L_2\kappa_1\left(L_1\sigma_2+L_2\sigma_1\right)\\ &\quad-2\left(\phi_l+\varepsilon_1T_l\right)\Psi_2,\\ \Psi_6 &= 2\sigma_1\sigma_2\left[\left(L_1\kappa_2+L_2\kappa_1\right)\phi_l+\left(L_1\varepsilon_1\kappa_2+L_2\varepsilon_2\kappa_1\right)T_l\right].\end{aligned}$$

The results are consistent with the Yang Y *et al.*'s results.[29]

3.3. Multilayered thermoelectric medium with Peltier effect at the interface

The Peltier effect implies the heating or cooling existing at the junction of two different thermoelectric materials, which can be considered as the back-action counterpart to the Seebeck effect. When electric current flows through a junction between two thermoelectric materials, heat may be absorbed or released at the junction as shown in Fig. 3. The heat flux changed by Peltier effect is equal to[31]

$$J_{Qi}^{*}=\left(\Pi_i-\Pi_{i+1}\right)J, \tag{30}$$

where Π_i is the Peltier coefficient of phase-i, and the relationship between Peltier and Seebeck coefficients is

$$\Pi=\varepsilon T. \tag{31}$$

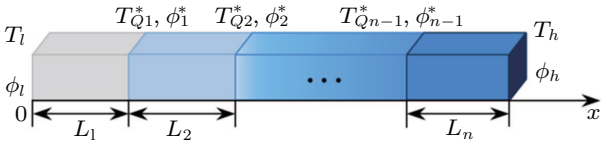


Fig. 3. (color online) Two-layered thermoelectric medium.

As both the charges and energy are conserved in a current system, the change of heat flux J_{Qi}^{*} at the junction should be provided by electric energy J'_E. Substituting Eqs. (30) and (31) into Eqs. (8) and (9) yields

$$\left(\varepsilon_i-\varepsilon_{i+1}\right)T_iJ+\phi_i^{*}J=0, \tag{32}$$

where ϕ_i^{*} is the electric potential difference at the junction. Therefore the electric potential difference caused by Peltier effect at interface can be expressed as

$$\phi_i^{*}=-\left(\varepsilon_i-\varepsilon_{i+1}\right)T_i. \tag{33}$$

Noticing this, the inherent relation of multilayered thermoelectric medium in Eq. (22) can be rewritten as

$$\begin{cases}\sum\limits_{i=1}^{n}\left(T_i-T_{i-1}\right)=T_h-T_l,\\ \sum\limits_{i=1}^{n}\left(\phi_i-\phi_{i-1}\right)=\phi_h-\phi_l+\sum\limits_{j=1}^{n-1}\phi_j^{*}.\end{cases} \tag{34}$$

From Eq. (33), the total electric potential jump generated by Peltier effect at junction can be expressed as

$$\sum_{j=1}^{n-1}\phi_j^{*}=\sum_{j=1}^{n-1}\left(\varepsilon_{j+1}-\varepsilon_j\right)T_j=\varepsilon_nT_n-\varepsilon_1T_0$$

$$-\sum_{j=1}^{n}\left(-\frac{\varepsilon_jJ^2}{2\sigma_j\kappa_j}L_j^2+\frac{b_jJ-J_E}{\kappa_j}L_j\right). \tag{35}$$

Like Eq. (25), the parameter b_i while considering Peltier effect can be calculated by

$$\begin{cases}b_1=\phi_l+\varepsilon_1T_l,\\ b_{i+1}=b_i-\dfrac{JL_i}{\sigma_i},\quad(i\geq1).\end{cases} \tag{36}$$

Substituting Eqs. (23), (24), and (35) into Eq. (34), and noticing the boundary condition, we have

$$\begin{cases}\sum\limits_{i=1}^{n}\left(-\dfrac{J^2}{2\sigma_i\kappa_i}L_i^2+\dfrac{b_iJ-J_E}{\kappa_i}L_i\right)=T_h-T_l,\\ \sum\limits_{i=1}^{n}\left(-\dfrac{J}{\sigma_i}L_i\right)=\phi_h-\phi_l+\varepsilon_nT_h-\varepsilon_1T_l.\end{cases} \tag{37}$$

Similarly, the only two unknown quantities J and J_E can be obtained from Eq. (37), then temperature and electric potential in each layer can be derived from Eqs. (18) and (19) and interface conditions. Thus temperature is continuous at the interface while electric potential difference at interface is $\phi_i^{*}=-\left(\varepsilon_i-\varepsilon_{i+1}\right)T_i$. This set of equations solves the problems completely, leading to the full determination of the field distribution in each layer.

The conversion efficiency η in a thermoelectric medium is[32]

$$\eta=\frac{\Delta\phi\cdot J}{J_{Q0}}, \tag{38}$$

where J_{Q0} is the initial heat flux. The η can be optimized with respect to current density for a given temperature,[32] and the maximum conversion efficiency in homogeneous thermoelectric material can be expressed as

$$\eta_{\text{opt}}=\frac{T_l-T_h}{T_l}\frac{\sqrt{1+Z\dfrac{T_l+T_h}{2}}-1}{\sqrt{1+Z\dfrac{T_l+T_h}{2}}+\dfrac{T_h}{T_l}}. \tag{39}$$

The maximum energy conversion efficiency for an arbitrary multilayered thermoelectric medium is too complicated to be expressed analytically. However, for the case of attaching a thin thermoelectric film to the right end of the homogeneous material, if the interface Peltier effect is taken into consideration, the maximum energy conversion efficiency can be obtained as

$$\eta_{\text{opt}}=\frac{T_l-\dfrac{\varepsilon'}{\varepsilon}T_h}{T_l}\frac{\sqrt{1+Z\dfrac{T_l^2+\left(\dfrac{\varepsilon'}{\varepsilon}\right)^2T_h^2}{2\left(T_h-T_l\right)}}-1}{\sqrt{1+Z\dfrac{T_l^2+\left(\dfrac{\varepsilon'}{\varepsilon}\right)^2T_h^2}{2\left(T_l-T_h\right)}}+\dfrac{\varepsilon'}{\varepsilon}\dfrac{T_h}{T_l}}, \tag{40}$$

where ε' is the Seebeck coefficient of the attached film.

The optimal electric current density of this medium can also be obtained as

$$J_{\text{opt}} = \frac{\sqrt{4\kappa^2 (T_l - T_h)^2 + 2\kappa\sigma (T_l - T_h)\left(\varepsilon^2 T_l^2 - \varepsilon'^2 T_h^2\right)} - 2\kappa (T_l - T_h)}{(\varepsilon T_l + \varepsilon' T_h) L}, \tag{41}$$

where L is the length of the medium

By comparing Eq. (39) with Eqs. (40) and (41), we can find out that the Peltier effect has a remarkable influence on both the maximum conversion efficiency and the optimal electric current density.

3.4. Numerical results and discussion

A three-layered thermoelectric medium is used to demonstrate the analysis by using the material parameters shown in Table 1. The temperature loads at two ends of the medium are taken to be $T_l = 600$ K and $T_h = 300$ K.

Table 1. Material parameters of thermoelectric samples.[33–35]

Sample	σ/S·m^{-1}	ε/μV·K^{-1}	κ/W·m^{-1}·K^{-1}
BiSbTe	27700	213	0.7
Bi_2Te_3	110000	200	1.6
$Bi_{0.5}Sb_{1.5}Te_3$	38000	250	0.85

The energy conversion efficiency of three-layered thermoelectric medium is dependent on the electric flux density while the temperatures at two ends are fixed. The ϕ_l is taken to be zero without loss of generality, then the energy conversion efficiency in Eq. (38) can be rewrite as

$$\eta = \frac{\phi_h \cdot J}{J_E}. \tag{42}$$

The maximum conversion efficiency of a three-layered thermoelectric medium and its constituents are analyzed in Fig. 4. As we can see, when considering the interface Peltier effect, the maximum conversion efficiency of the composite is significantly higher than those of all the constituents under the same temperature difference.

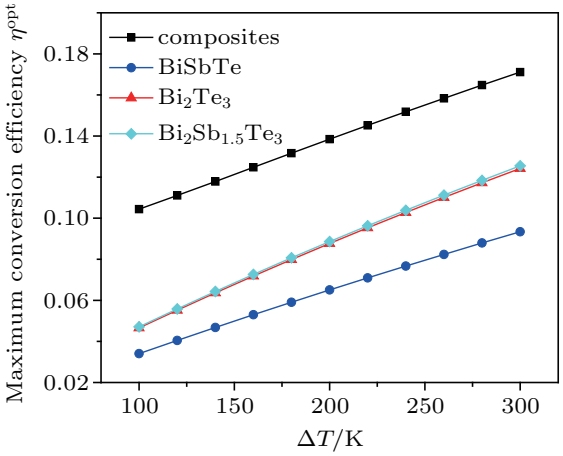


Fig. 4. (color online) Maximum conversion efficiencies of a thermoelectric composite and its constituents.

The plots of energy conversion efficiency versus electric current density under two different interface conditions are shown in Fig. 5. It can be seen that the energy conversion efficiency is significantly improved when considering Peltier effect at the interface, and the relevant maximum efficiency increases from 0.11 to 0.17. Besides, the optimal electric current density increases from $J = 0.38$ (A/cm^2) to $J = 0.48$ (A/cm^2).

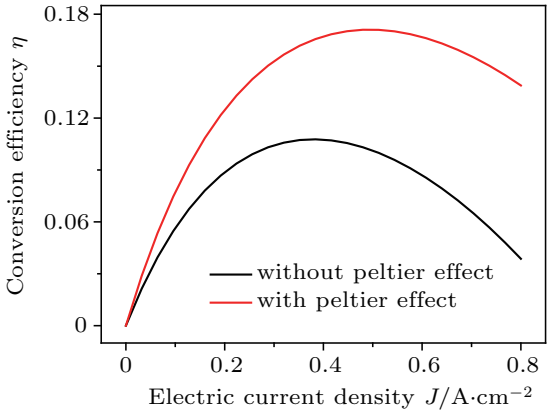


Fig. 5. (color online) Energy conversion efficiencies versus electric current density.

To evaluate such observations, we also show the field distributions under two different interface conditions, including the electric potential, temperature, electric energy and heat flux in Fig. 6. Numerical simulations is also made to verify the correctness of our results, since interface conditions in software cannot be modified, we only give the simulation results under traditional interface conditions. From Fig. 6(a), it is seen that the consideration of Peltier effect at the interface results in much bigger electric potential difference at the same electric current density, which leads to much higher electric energy and energy conversion efficiency. On the other hand, there is little difference in temperature distribution when considering Peltier effect at the interface as shown in Fig. 6(b). From Figs. 6(c) and 6(d) we can see that both the electric energy and thermal flux jump at the interface when considering the interface Peltier effect. The electric energy jumps up at the interface, leading to higher energy conversion efficiency, while the thermal flux jumps down due to energy conservation. Besides, numerical simulations under traditional interface conditions fit well to our analytical solutions.

Finally we point out that a thermoelectric material with temperature dependent properties can be approximated as a multilayered thermoelectric composite, and each layer has constant property. Therefore the solutions obtained above also offer a path to the analysis of temperature dependent properties, especially when the materials are subjected to large temperature gradient so that the material properties cannot be considered as being unchanged.

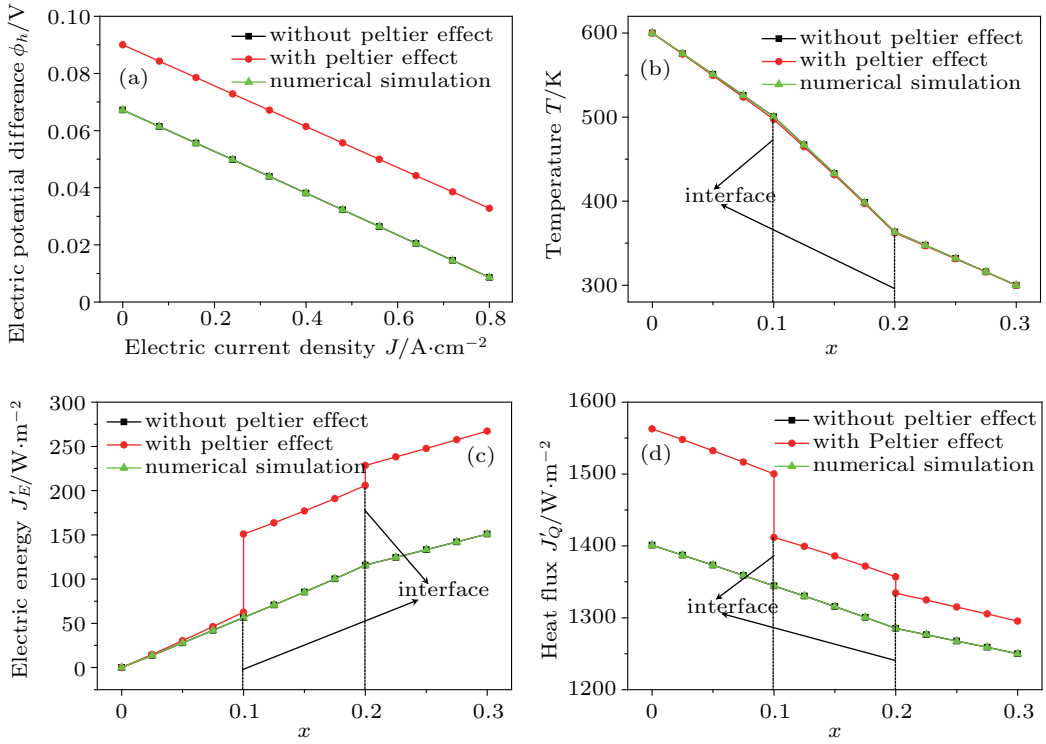


Fig. 6. (color online) Comparisons among fields under two different interface conditions and numerical simulations. (a) optimum electric current density, (b)temperature, (c) electric energy, (d) thermal flux.

4. Conclusions

The overall macro-performances of multilayered thermoelectric medium are discussed in this paper. The analytical solutions are obtained, including the heat flux, temperature, electric potential, and the total energy conversion efficiency. The results show that there are unique relationships between the temperature/electric potential and the electric current/energy flux in the material, and whether the material is independent of or embedded in thermoelectric composites, which can simplify the process of solving J and J_E effectively. Besides, the Peltier effect at the interface has a noteworthy influence on overall energy conversion efficiency. These results provide a powerful tool to analyze the effective behavior of thermoelectric composites.

References

[1] Liu L, Lu X S, Shi M L, Ma Y K and Shi J Y 2016 *Solar Energy* **132** 386
[2] Ting H 2016 *7th International Conference on IEEE Mechanical and Aerospace Engineering (ICMAE)*, p. 35
[3] Bankston C P, Cole T, Jones R and Ewell R 1983 *J. Energy* **7** 442
[4] Yang J and Caillat T 2006 *MRS Bulletin* **31** 224
[5] He W, Wang S, Li Y and Zhao Y 2016 *Energy Conversion and Management* **129** 240
[6] De Leon M T, Chong H and Kraft M 2012 *Procedia Engineering* **47** 76
[7] Schirrmacher S, Ullmann G and Overmeyer L 2015 *Energy Harvesting and Systems* **2** 81
[8] Kristiansen L N R and Nielsen H K 2010 *J. Electron. Mater.* **39** 1746
[9] Luan Y Q, Yang W, Xiao P, Ma Z F and Wang H P 2015 *Appl. Mech. Mater.* **716** 1457
[10] Yip M C and Niemeyer G 2015 2015 *IEEE International Conference Robotics and Automation (ICRA)*, p. 2313
[11] De Backer J, Bolmsjö G and Christiansson A K 2014 *Int. J. Adv. Manuf. Technol.* **70** 375
[12] Ming T, Yang W, Huang X, Wu Y, Li X and Liu J 2017 *Energy Conversion and Management* **132** 261
[13] Demir M E and Dincer I 2017 *Desalination* **404** 59
[14] Zhang J and Xuan Y 2016 *Energy Conversion and Management* **129** 1
[15] Kossyvakis D N, Vassiliadis S G, Vossou C G, Mangiorou E E, Potirakis S M and Hristoforou E V 2016 *J. Electron. Mater.* **45** 2957
[16] Crépieux L A and Michelini F 2014 *J. Phys.: Condens. Matter* **27** 015302
[17] Bell L E 2008 *Science* **321** 1457
[18] Harman T C and Honig J M 1967 *Thermoelectric and Thermomagnetic Effects and Applications* (New York: McGraw-Hill) p. 377
[19] Nolas G S, Morelli D T and Tritt T M 1999 *Ann. Rev. Mater. Sci.* **29** 89
[20] Poudel B, Hao Q, Ma Y, Lan Y, Minnich A, Yu B, Yan X, Wang D Z, Muto A, Vashaee D, Chen X Y, Liu J M, Dresselhaus M S, Chen G and Ren Z F 2008 *Science* **320** 634
[21] Xie W, Tang X, Yan Y, Zhang Q and Tritt T M 2009 *Appl. Phys. Lett.* **94** 102111
[22] Poudeu P F, Guéguen A, Wu C I, Hogan T and Kanatzidis M G 2009 *Chem. Mater.* **22** 1046
[23] Heremans J P, Thrush C M, Morelli D T and Wu M C 2002 *Phys. Rev. Lett.* **88** 216801
[24] Gothard N, Ji X, He J andTritt T M 2008 *J. Appl. Phys.* **103** 054314
[25] Qi X K, Zeng H R, Yu H Z, Zhao K Y, LiG R, Song J Q, Shi X and Chen L D 2014 *Chin. Phys. Lett.* **31** 127201
[26] Xu Q Q, Xu J, Li M, Liu J X and Li H L 2016 *Acta Phys. Sin.* **65** 2372017 (in Chinese)
[27] Zhang H, Chen S P, Long Y, Fan W H, Wang W X and Meng Q S 2015 *Acta Phys. Sin.* **64** 2473028 (in Chinese)
[28] Bergman D J and Levy O 1991 *J. Appl. Phys.* **70** 6821
[29] Yang Y, Xie S H, Ma F Y and Li J Y 2012 *J. Appl. Phys.* **111** 013510
[30] ZhangA B and Wang B L 2016 *Int. J. Therm. Sci.* **104** 396
[31] Spanner D C 1951 *J. Experimental Botany* **2** 145
[32] Yang Y, Ma F Y, Lei C H, Liu Y Y and Li J Y 2013 *Appl. Phys. Lett.* **102** 053905
[33] Ahmad K, Wan C and Al-Eshaikh M A 2017 *J. Electron. Mater.* **46** 1348
[34] Antonova E E and Looman D C 24*th International Conference on Thermoelectrics*, June 19–23, 2005, p. 215
[35] Seo S, Oh M W, Jeong Y and Yoo B 2017 *J. Alloys Compd.* **696** 1151