Seismoelectric wave propagation modeling in a borehole in water-saturated porous medium having an electrochemical interface*

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Water-saturated porous media often exhibit a seismoelectric effect due to the existence of an electrical double layer and a relative flow of pore fluid. Here we consider the seismoelectric waves in an open borehole surrounded by water-saturated porous formation which exhibits discontinuity of electrochemical properties at a cylindrical interface. We carefully analyze the seismoelectric interface response since these signals show sensitivity to contrasts in electrochemical properties across an interface. Both coupled and approximate methods are used to compute borehole seismoelectric fields. The simulation results show that the radiated electromagnetic wave from the electrochemical interface is generated due to the change of salinity in pore fluid in the porous formation. However, the elastic properties of the formation remain unchanged across such an electrochemical interface. As a result it is difficult to recognize such a change in electrochemical properties using only elastic waves. Therefore, the seismoelectric interface response is potentially used to detect the changes of the electrochemical properties in the formation.

Keywords: seismoelectric effect, interface response, porous formation, borehole

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1. Introduction

The seismoelectric effect is generated by the porous medium consisting of a solid matrix and the electrolyte in the pores. The ensemble is generally electrically neutral. The fluid in porous formation can be considered as an electrolyte solution. Usually, the surface of the solid matrix has the ability to adsorb the anions, so there is an electric double layered structure that exists at the interface between solid phase and liquid phase. The electric double layered structure is related to the seismoelectric effect. When an acoustic wave travels through a porous medium the diffuse layer is free to move with the pore fluid. Separation of the positively charged ions in electrolyte solution due to this movement generates an electric current and electromagnetic wave fields.[1–3] The seismoelectric effect in porous media can cause three different types of electromagnetic fields. The first one is related to the seismic source itself;[4] the second one is the induced electric field that exists in the region of the acoustic wave-generated disturbance in the porous medium, it is caused by the conductivity change when an acoustic wave travels through a porous medium. It is called the coseismic electric field, its apparent velocity is associated with the speed of the seismic wave. The third kind is generated at the interface and consists of independently propagating electromagnetic waves when an interface between two porous media exists due to the discontinuity of physical-chemical properties. The seismoelectric interface response is independent of the acoustic wave propagation, and its apparent velocity is the velocity of electromagnetic wave.[5,6] The seismoelectric effect in porous formation has potential applications in geophysical problems, observation and forecasting of earthquakes. The seismoelectric effect can provide information about the electrical potential and the resistivity of formation for deducing the distributions of the oil and gas storage. It can also give the vibration frequency characteristics of electric field which is similar to the excitation frequency of the seismic wave field.

The coseismic electric field and seismoelectric interface response have been measured in the field and in laboratory studies. Butler *et al.*,[7] Mikhailov *et al.*[8] and Garambois and Dietrich[9] have recorded two kinds of electromagnetic signals through exploration experiment on seismoelectric effect. Zhu and Toksöz [10] have shown that interface response fields in laboratory crosshole models are sensitive to fracture aperture and orientation. Kulessa and Hubbard[11] recorded the electrokinetic conversion signals at different interfaces on the glacier, indicating that the glacial fracture can be detected by the seismoelectric effect. Bordes *et al.*[12] quantified the amplitude of the coseismic magnetic fields within the seismic shear waves. Wang *et al.*[13] has experimentally measured seismoelectric effects in fluid-saturated porous medium, and detected electrokinetic conversion signal, which provided a theoretical base for designing the electrokinetic logging tools.

Generation and propagation mechanisms of seismoelec-

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tric effects in the borehole have been studied by several investigators. The coseismic electric field and interface response effects have been predicted by the numerical model of Haartsen and Pride.[14] Hu and Liu,[15] Hu *et al.*,[16] and Hu and Liu[17] proposed a simplified approach to simulating the seismoelectric logs. Haines and Pride[18] presented a finite difference algorithm for seismoelectric wave propagation. Cui *et al.*[19] simulated borehole seismoelectric field excited by eccentric source. Cui *et al.*[20] used two methods to simulate the full waveforms of acoustic waves and electromagnetic wave-induced SH waves excited by vertical magnetic dipole source. Guan *et al.*[21] and Ding *et al.*[22] used numerical simulation to invert compressional velocities and transverse wave from electric measurement in porous formations through the seismoelectric logging while drilling. Wang *et al.*[23] found that there are direct collar waves and indirect collar waves in acoustic logging while drilling, and indirect collar waves could be relatively strong in the full wave data. Zheng *et al.*[24] obtained that formation compressional and shear velocities form seismoelectric LWD signals. Wang *et al.*[25] conducted seismoelectric logging while drilling experiment to verify the feasibility of electrokinetic effect on weakening collar waves. Guan *et al.*[26] theoretically calculated the ratio of the electric field amplitude to pressure amplitude for the low-frequency Stoneley wave of different formations. Talebitooti *et al.*[27] proposed an extension of the full method to the investigating of sound transmission through a poroelastic cylindrical shell. Liu *et al.*[28] investigated the characteristics of the seismoelectric interface response at the interface outside the borehole during the seismoelectric logging while drilling. Gao *et al.*[29] studied the propagation of seismoelectric wave in a poroelastic hollow cylinder. Zhao *et al.*[30] investigated the seismoelectric waves in cylindrical double-layered porous formation based on full Pride coupled theory.

In this paper, we will pay attention to the case of porous formation exhibiting discontinuity of electrochemical properties at the interface. The purpose of the current work is to provide the numerical evidence that borehole seismoelectric logging can be used to monitor the change of the electrochemical properties in the formation. We first review highly cited Pride's seismoelectric theory and assume ideal double cylindrical porous formation. Then we make an assumption that electrochemical cylindrical interface occurs in infinite porous formation outside the borehole. Here we present for the first time how to use the approximate solution method to simulate this scenario. The simulation results by coupled and approximate methods are compared. The numerical examples will show seismoelectric interface response associated with discontinuities of electrochemical properties due to the change of salinity of pore fluid in the porous formation.

2. Brief review of seismoelectric wave theory

In order to model the current problem, we will adopt Pride's governing-equations which are combinations of the basic principle of mechanics and electromagnetism,[2] and based on Biot's theory[31] and Maxwell's electromagnetic equation. According to Pride's theory, the following governing-equations express the electrokinetic coupling in an isotropic, homogeneous fluid-saturated porous medium without a current source or a stress source; time dependence is assumed to be $\mathrm{e}^{-\mathrm{i}\omega t}$

$$\nabla \times \boldsymbol{E} = \mathrm{i}\omega \boldsymbol{B}, \tag{1}$$

$$\nabla \times \boldsymbol{H} = -\mathrm{i}\omega \boldsymbol{D} + \boldsymbol{J}, \tag{2}$$

$$\nabla \cdot \boldsymbol{\tau} = -\omega^2(\rho \boldsymbol{u} + \rho_{\mathrm{f}} \boldsymbol{w}), \tag{3}$$

$$\boldsymbol{J} = \sigma(\omega)\boldsymbol{E} + L(\omega)(-\nabla p + \omega^2 \rho_{\mathrm{f}} \boldsymbol{u}), \tag{4}$$

$$-\mathrm{i}\omega \boldsymbol{w} = L(\omega)\boldsymbol{E} + (-\nabla p + \omega^2 \rho_{\mathrm{f}} \boldsymbol{u})\kappa(\omega)/\eta, \tag{5}$$

$$\boldsymbol{\tau} = (H - 2G)(\nabla \cdot \boldsymbol{u})\boldsymbol{I} + C(\nabla \cdot \boldsymbol{w})\boldsymbol{I} + G(\nabla \boldsymbol{u} + \nabla \boldsymbol{u}^{\mathrm{T}}), \tag{6}$$

$$-p = C\nabla \cdot \boldsymbol{u} + M\nabla \cdot \boldsymbol{w}, \tag{7}$$

$$\boldsymbol{B} = \mu \boldsymbol{H}, \tag{8}$$

$$\boldsymbol{D} = \varepsilon \boldsymbol{E}, \tag{9}$$

where $\boldsymbol{J}$ is the electric-current density, $\boldsymbol{E}$ is the electric field, $\boldsymbol{B}$ is the magnetic induction, $\boldsymbol{H}$ is the magnetic field, $\boldsymbol{D}$ is the electric displacement, $\boldsymbol{\tau}$ is the bulk stress in the porous medium, p is the pore fluid pressure, C, M, H, and G are four elastic moduli of the porous media, $\boldsymbol{u}$ is the solid-phase displacement, $\boldsymbol{w}$ is the relative fluid–solid displacement, ω is the angular frequency, η and ρ_{f} are the viscosity and density of the pore fluid, $\sigma(\omega)$ and $k(\omega)$ are electrical conductivity and the dynamic permeability, $L(\omega)$ is the electrokinetic coupling coefficient, the $L(\omega)$ reflects the strength of the electrokinetic coupling. When $L(\omega)$ is not zero, the seismic and electromagnetic fields are coupled, electric field can produce relative fluid motion and pressure gradients, and electric current can be caused by the pressure gradient of pore fluid as well as the inertial force of the solid phase. When $L(\omega)$ is set to be zero, Darcy's law and Ohm's law are uncoupled, then Pride's governing-equations would be separated into Biot's equations for elastic field and Maxwell's equations for electromagnetic field.

3. Seismoelectric waves in double cylindrical porous formation

In this section, we will present the mathematical formulations for the propagations of seismoelectric waves in double cylindrical porous formation. The frequency wavenumber integral expressions for the acoustic and electromagnetic waves in the borehole fluid are formulated. A borehole in the double cylindrical porous formation is shown in Fig. 1. It is a cylindrically layered structure (r, θ, z), inside the borehole is fluid

layer, the radius of borehole is 'a', the interlayer is porous formation whose outer radius is 'b', the outermost layer is an infinite porous formation.

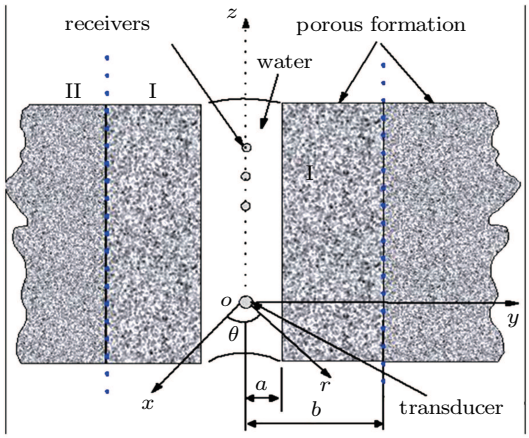


Fig. 1. (color online) A borehole in the cylindrical porous formation.

3.1. Seismoelectric waves in double cylindrical porous formation

From full Pride's theory, we briefly review the equations applicable to the configuration of an open borehole surrounded by concentric double cylindrical porous formation.[30] Based on the full Helmholtz potential functions of the seismoelectric wave fields excited by a monopole acoustic source in cylindrical layered medium one obtains[15,21,29,32]

$$\begin{aligned} U^{\mathrm{I}} &= \nabla\varphi^{\mathrm{I}}_{\mathrm{pf}} + \nabla\varphi^{\mathrm{I}}_{\mathrm{ps}} + \nabla\times\nabla\times(\Gamma^{\mathrm{I}}_{\mathrm{sv}} + \Gamma^{\mathrm{I}}_{\mathrm{em}})e_z, \\ W^{\mathrm{I}} &= \alpha^{\mathrm{I}}_{\mathrm{pf}}\nabla\varphi^{\mathrm{I}}_{\mathrm{pf}} + \alpha^{\mathrm{I}}_{\mathrm{ps}}\nabla\varphi^{\mathrm{I}}_{\mathrm{ps}} + \alpha^{\mathrm{I}}_{\mathrm{sv}}\nabla\times\nabla\times\Gamma^{\mathrm{I}}_{\mathrm{sv}}e_z \\ &\quad + \alpha^{\mathrm{I}}_{\mathrm{em}}\nabla\times\nabla\times\Gamma^{\mathrm{I}}_{\mathrm{em}}e_z, \\ U^{\mathrm{II}} &= \nabla\varphi^{\mathrm{II}}_{\mathrm{pf}} + \nabla\varphi^{\mathrm{II}}_{\mathrm{ps}} + \nabla\times\nabla\times(\Gamma^{\mathrm{II}}_{\mathrm{sv}} + \Gamma^{\mathrm{II}}_{\mathrm{em}})e_z, \\ W^{\mathrm{II}} &= \alpha^{\mathrm{II}}_{\mathrm{pf}}\nabla\varphi^{\mathrm{II}}_{\mathrm{pf}} + \alpha^{\mathrm{II}}_{\mathrm{ps}}\nabla\varphi^{\mathrm{II}}_{\mathrm{ps}} + \alpha^{\mathrm{II}}_{\mathrm{sv}}\nabla\times\nabla\times\Gamma^{\mathrm{II}}_{\mathrm{sv}}e_z \\ &\quad + \alpha^{\mathrm{II}}_{\mathrm{em}}\nabla\times\nabla\times\Gamma^{\mathrm{II}}_{\mathrm{em}}e_z. \end{aligned} \tag{10}$$

Here, the φ_j (j = pf, ps) and Γ_j (j = sv, em) are the potential functions with respect to the compressional wave and vertically polarized shear wave, respectively, the superscripts I and II denote the variables of the interlayer and the outermost porous formation, respectively, e_z is the unit vector in the z direction, a_j (j = pf, ps, sv, em) are coefficients each of which is a dimensionless complex, and they reflect the amplitude ratio between the two kinds of displacements $\boldsymbol{u}$ and $\boldsymbol{w}$ in porous media,[3] β_j (j = pf, ps, sv, em) are coefficients and reflect the ratios between the electric potential and displacements.

The potential functions in frequency-axial wavenumber domain can be formulated as follows:

$$\begin{aligned} \varphi^{\mathrm{I}} &= A^{\mathrm{I}}_{\mathrm{pf}}\mathrm{I}_0(\eta^{\mathrm{I}}_{\mathrm{pf}}r) + B^{\mathrm{I}}_{\mathrm{pf}}\mathrm{K}_0(\eta^{\mathrm{I}}_{\mathrm{pf}}r) \\ &\quad + A^{\mathrm{I}}_{\mathrm{ps}}\mathrm{I}_0(\eta^{\mathrm{I}}_{\mathrm{ps}}r) + B^{\mathrm{I}}_{\mathrm{ps}}\mathrm{K}_0(\eta^{\mathrm{I}}_{\mathrm{ps}}r), \\ \Gamma^{\mathrm{I}} &= A^{\mathrm{I}}_{\mathrm{sv}}\mathrm{I}_0(\eta^{\mathrm{I}}_{\mathrm{sv}}r) + B^{\mathrm{I}}_{\mathrm{sv}}\mathrm{K}_0(\eta^{\mathrm{I}}_{Sv}r) \\ &\quad + A^{\mathrm{I}}_{\mathrm{em}}\mathrm{I}_0(\eta^{\mathrm{I}}_{\mathrm{em}}r) + B^{\mathrm{I}}_{\mathrm{em}}\mathrm{K}_0(\eta^{\mathrm{I}}_{\mathrm{em}}r), \\ \varphi^{\mathrm{II}} &= B^{\mathrm{II}}_{\mathrm{pf}}\mathrm{K}_0(\eta^{\mathrm{II}}_{\mathrm{pf}}r) + B^{\mathrm{II}}_{\mathrm{ps}}\mathrm{K}_0(\eta^{\mathrm{II}}_{\mathrm{ps}}r), \\ \Gamma^{\mathrm{II}} &= B^{\mathrm{II}}_{\mathrm{sv}}\mathrm{K}_0(\eta^{\mathrm{II}}_{\mathrm{sv}}r) + B^{\mathrm{II}}_{\mathrm{em}}\mathrm{K}_0(\eta^{\mathrm{II}}_{\mathrm{sv}}r), \end{aligned} \tag{11}$$

where k_z is the axial wavenumber, $\eta_j = \sqrt{k_z^2 - k_j^2}$ (j = pf, ps, sv, em) are the radial wavenumbers, k_j is the wavenumber, I_0 and K_0 are the modified Bessel functions, $A^{\mathrm{I}}_{\mathrm{pf}}$, $A^{\mathrm{I}}_{\mathrm{ps}}$, $A^{\mathrm{I}}_{\mathrm{sv}}$, $A^{\mathrm{I}}_{\mathrm{em}}$, $B^{\mathrm{I}}_{\mathrm{pf}}$, $B^{\mathrm{I}}_{\mathrm{ps}}$, $B^{\mathrm{I}}_{\mathrm{sv}}$, $B^{\mathrm{I}}_{\mathrm{em}}$, $B^{\mathrm{II}}_{\mathrm{pf}}$, $B^{\mathrm{II}}_{\mathrm{ps}}$, $B^{\mathrm{II}}_{\mathrm{sv}}$, $B^{\mathrm{II}}_{\mathrm{em}}$ are the unknown coefficients, and they characterize the outgoing and incoming seismoelectric waves and can be determined by solving the boundary value problem. When the water-saturated porous formation is homogeneous and infinite, the model gives the classical solution.[15,32]

3.2. Acoustic field and electromagnetic field in fluid

The solutions of potential of acoustic field in borehole fluid in the frequency-axial wavenumber domain can be described as[29]

$$\varphi_b = A_b\mathrm{I}_0(\eta_m r) + \mathrm{K}_0(\eta_m r), \tag{12}$$

where k_z is the axial wavenumber and $\eta_m = \sqrt{k_z^2 - k_m^2}$, k_m is the wavenumber of fluid. The expression of the displacement field in fluid is given by $U^{\mathrm{B}} = \nabla\varphi_b$.

The constraint equations of electric field intensity satisfy the Helmholtz-type wave equations without electric current sources in the borehole fluid, and we take the z-component of vector E to investigate for convenience

$$\nabla^2 E_z + k_{\mathrm{e}}^2 E_z = 0, \tag{13}$$

where $k_{\mathrm{e}}^2 = \omega^2\mu\varepsilon_b$, $\varepsilon_b = \varepsilon_{\mathrm{f}}(1 + \mathrm{i}\sigma_{\mathrm{f}}/\omega\varepsilon_{\mathrm{f}})$, ε_{f} and σ_{f} are the electrical permittivity and electrical conductivity of borehole fluid, respectively.

In the borehole fluid, because the axial electric field should be finite, according to the relationship between $\boldsymbol{E}$ and $\boldsymbol{H}$, the expressions of component of vector $\boldsymbol{E}$ and $\boldsymbol{H}$ in the frequency-axial wavenumber are

$$E_z = \frac{\eta_{\mathrm{e}}}{\mathrm{i}k} B_{\mathrm{e}}\mathrm{I}_0(\eta_{\mathrm{e}}r), \tag{14}$$

$$H_z = \left(k_z - \frac{\eta_{\mathrm{e}}}{\mathrm{i}k}\right) B_{\mathrm{e}}\mathrm{I}_0(\eta_{\mathrm{e}}r), \tag{15}$$

where $\eta_{\mathrm{e}}^2 = k_z^2 - k_{\mathrm{e}}^2$.

3.3. Solution of the unknown coefficients

The unknown coefficients A_b, B_{e}, $A^{\mathrm{I}}_{\mathrm{pf}}$, $A^{\mathrm{I}}_{\mathrm{ps}}$, $A^{\mathrm{I}}_{\mathrm{sv}}$, $A^{\mathrm{I}}_{\mathrm{em}}$, $B^{\mathrm{I}}_{\mathrm{pf}}$, $B^{\mathrm{I}}_{\mathrm{ps}}$, $B^{\mathrm{I}}_{\mathrm{sv}}$, $B^{\mathrm{I}}_{\mathrm{em}}$, $B^{\mathrm{II}}_{\mathrm{pf}}$, $B^{\mathrm{II}}_{\mathrm{ps}}$, $B^{\mathrm{II}}_{\mathrm{sv}}$, $B^{\mathrm{II}}_{\mathrm{em}}$ which appear in the expressions for the field are determined by satisfying the boundary conditions.

The boundary conditions of seismoelectric waves for a cylindrical interface between a fluid and interlayer porous media at inner interface ($r = a$) are as follows:

$$-p^{\mathrm{B}} = \tau^{\mathrm{I}}_{rr}, \quad 0 = \tau^{\mathrm{I}}_{rz}, \quad p^{\mathrm{B}} = p^{\mathrm{I}}, \quad u^{\mathrm{B}}_r = u^{\mathrm{I}}_r + w^{\mathrm{I}}_r,$$

$$E_z^{\mathrm{B}} = E_z^{\mathrm{I}}, \quad H_\theta^{\mathrm{B}} = H_\theta^{\mathrm{I}}. \tag{16}$$

The boundary conditions between the interlayer porous medium and the infinite porous formation at the outer interface ($r = b$) are

$$\begin{aligned} &u_r^{\mathrm{I}} = u_r^{\mathrm{II}}, \quad u_z^{\mathrm{I}} = u_z^{\mathrm{II}}, \quad w_r^{\mathrm{I}} = w_r^{\mathrm{II}}, \quad p^{\mathrm{I}} = p^{\mathrm{II}}, \\ &\tau_{rr}^{\mathrm{I}} = \tau_{rr}^{\mathrm{II}}, \quad \tau_{rz}^{\mathrm{I}} = \tau_{rz}^{\mathrm{II}}, \quad E_z^{\mathrm{I}} = E_z^{\mathrm{II}}, \quad H_\theta^{\mathrm{I}} = H_\theta^{\mathrm{II}}, \end{aligned} \tag{17}$$

where the superscripts B, I, and II denote the variables of the borehole fluid, the interlayer, and the outmost porous medium, respectively.

Boundary conditions may be written in matrix form as

$$[m_{ij}]_{14\times 14} \cdot \{X_j\}_{14\times 1} = \{Y_i\}_{14\times 1}, \tag{18}$$

where the matrix $\boldsymbol{M}$ is given in Appendix A, and

$$\{X_j\}_{14\times 1} = \{A_{\mathrm{b}}, B_{\mathrm{e}}, A_{\mathrm{pf}}^{\mathrm{I}}, A_{\mathrm{ps}}^{\mathrm{I}}, A_{\mathrm{sv}}^{\mathrm{I}}, A_{\mathrm{em}}^{\mathrm{I}}, B_{\mathrm{pf}}^{\mathrm{I}}, B_{\mathrm{ps}}^{\mathrm{I}}, B_{\mathrm{sv}}^{\mathrm{I}}, B_{\mathrm{em}}^{\mathrm{I}}, B_{\mathrm{pf}}^{\mathrm{II}}, B_{\mathrm{ps}}^{\mathrm{II}}, B_{\mathrm{sv}}^{\mathrm{II}}, B_{\mathrm{em}}^{\mathrm{II}}\}^{\mathrm{T}}, \tag{19}$$

$$\{Y_i\}_{14\times 1} = \{\eta_m \mathrm{K}_1(\eta_m a)/\pi, -\omega^2\rho_0 \mathrm{K}_0(\eta_m a)/\pi, 0,0,0,0,0,0,0,0,0,0,0,0\}^{\mathrm{T}}. \tag{20}$$

From the above fourteen boundary conditions, the fourteen unknown coefficients can be solved, so the acoustic field and electromagnetic field in each region can be obtained.

4. Approximate method

In this section, we propose an approximate method to simulate borehole seismoelectric wave when only electrochemical properties in the two layers of the double layered structure are different, i.e., the elastic properties of the water-saturated porous formation are still the same (in such a case, $B_j^{\mathrm{I}} = B_j^{\mathrm{II}} = B_j$, and A_j^{I} disappears). That means that only discontinuities in electrochemical properties exist at the interface of the infinite porous formation. The distance between electrochemical interface and axis of the borehole is b, and the model is shown in Fig. 1. Here we can refer to Hu's quasi-static simplified method.[16,17,21,29] First, the effect of the conversion of the electric field on elastic wave propagation is ignored. Then in the simplified solution $L(\omega) = 0$ in Eq. (5). In such a case, the acoustic field in porous formation is independent of electromagnetic field and the acoustic field is described by Biot's equations. The acoustic field can be obtained from Rosenbaum's work,[33] also see Hu *et al.*'s work.[15,16] Once the acoustic field is known, the electromagnetic field arising from the seismoelectric effect can be determined. Secondly, we ignore the terms involving the time derivatives in the electric field, i.e., regardless of conduction current, so the electromagnetic field is a quasi-static field. The divergence is zero on condition that quasi-static electric field, i.e., the electric field of the fluid in the formation and borehole can be expressed as a negative gradient of the electric potential

$$\boldsymbol{E} = -\nabla\boldsymbol{\Phi}. \tag{21}$$

The solution of the electric potential in the inner porous formation can be expressed as

$$\begin{aligned} \Phi^{\mathrm{I}} = {} & A_{\mathrm{E}}^{\mathrm{I}} \mathrm{K}_0(k_z r) + B_{\mathrm{E}}^{\mathrm{I}} \mathrm{I}_0(k_z r) + \frac{L^{\mathrm{I}}}{\sigma^{\mathrm{I}}} \\ & \times \sum_{j=\mathrm{pf,ps}} [\omega^2\rho_{\mathrm{f}} - (C + \alpha_j M)k_j^2] B_j \mathrm{K}_0(\eta_j r), \end{aligned} \tag{22}$$

where σ^{I} and L^{I} are the electrical conductivity and electrokinetic coupling coefficient in the inner porous formation of the interface. The first two terms on the right-hand side of Eq. (22) are the general solution of the Laplace equation (homogeneous equation), while the third term is the particular solution arising from the seismic field in the formation.

The expression for the current density component is given by Eq. (4)

$$\begin{aligned} J_r^{\mathrm{I}} = {} & \sigma^{\mathrm{I}} k_z [A_{\mathrm{E}}^{\mathrm{I}} \mathrm{K}_1(k_z r) - B_{\mathrm{E}}^{\mathrm{I}} \mathrm{I}_1(k_z r)] \\ & - \mathrm{i} k_z \omega^2 \rho_{\mathrm{f}} B_{\mathrm{sv}} \mathrm{K}_1(\eta_{\mathrm{sv}}). \end{aligned} \tag{23}$$

Since outside the borehole there is an infinite formation of porous structure, these expressions should not contain modified Bessel function of the first kind, so the expression is

$$\begin{aligned} \Phi^{\mathrm{II}} = {} & A_{\mathrm{E}}^{\mathrm{II}} \mathrm{K}_0(k_z r) + \frac{L^{\mathrm{II}}}{\sigma^{\mathrm{II}}} \\ & \times \sum_{j=\mathrm{pf,ps}} [\omega^2\rho_{\mathrm{f}} - (C + \alpha_j M)k_j^2] B_j \mathrm{K}_0(\eta_j r), \end{aligned} \tag{24}$$

$$J_r^{\mathrm{II}} = \sigma^{\mathrm{II}} k_z A_{\mathrm{E}}^{\mathrm{II}} \mathrm{K}_1(k_z r) - \mathrm{i} k_z \omega^2 \rho_{\mathrm{f}} B_{\mathrm{sv}} \mathrm{K}_1(\eta_{\mathrm{sv}} r), \tag{25}$$

where Φ^{II} and L^{II} are the electrical conductivity and electrokinetic coupling coefficient in the outermost porous formation of the interface.

In the borehole fluid, because the axial electric field should be finite, the expressions for the electric potential and the current density component are expressed as

$$\Phi^{\mathrm{B}} = A_{\mathrm{E}}^{\mathrm{B}} \mathrm{I}_0(k_z r), \tag{26}$$

$$J_r^{\mathrm{B}} = -\sigma_{\mathrm{f}} \frac{\partial \Phi^{\mathrm{B}}}{\partial r} = -\sigma_{\mathrm{f}} k_z A_{\mathrm{E}}^{\mathrm{B}} \mathrm{I}_1(k_z r). \tag{27}$$

Here, the unknown coefficients $A_{\mathrm{E}}^{\mathrm{B}}$, $A_{\mathrm{E}}^{\mathrm{I}}$, $B_{\mathrm{E}}^{\mathrm{I}}$, $A_{\mathrm{E}}^{\mathrm{II}}$ in the formation and borehole fluid can be determined by solving the continuity conditions, and they are continuities of electric current density and the electric potential for approximate method as follows:

At inner interface $r = a$,

$$\Phi^{\mathrm{B}} = \Phi^{\mathrm{I}}, \quad J_r^{\mathrm{B}} = J_r^{\mathrm{I}}. \tag{28}$$

At outer interface $r = b$,

$$\Phi^{\mathrm{I}} = \Phi^{\mathrm{II}}, \quad J_r^{\mathrm{I}} = J_r^{\mathrm{II}}. \tag{29}$$

From the above four boundary conditions, the four unknown coefficients can be solved and the electric field in each region can be obtained after the acoustic field has been determined independently.

Therefore, the approximate method needs to solve two sets of four equations, otherwise the coupled method needs to solve one set of fourteen equations. Also, the acoustic field by the approximate method is very easy to obtain, and will simulate the results more efficiently.

5. Numerical simulation

In this section, we simulate the acoustic field and electric field obtained from the above coupled method and the approximate method. The physical parameters of the borehole fluid and porous formation are listed in Table 1. The formation in this example is assumed to have a P wave velocity (about 3220 m·s^{-1}), and it is a fast formation whose S wave velocity (about 1760 m·s^{-1}) is larger than the acoustic velocity in the borehole fluid (1500 m·s^{-1}). The radius of borehole is 0.1 m, the distance between electrochemical interface and axis of the borehole is 2.5 m. Relative permittivity of the pore fluid is the same as that in the porous medium, the magnetic permeability of the porous formation is equal to permeability of vacuum μ_0 throughout the paper. In the following simulation, a monopole pressure is employed which has a cosine wave oscillating with an envelope, the source-to-receiver distance is 6 m in the axial direction, the centre frequency of source is 6 kHz, and the acoustic pulse length of source is 0.5 ms. In view of the similarity between the waveforms of different electric field components, we will only show the acoustic pressure and electric field waveforms in the axial direction. The acoustic and electric signal are assumed to be recorded at the same place with an offset $r = 0.05$ m in the r direction.

Table 1. Parameters of the borehole fluid and porous formation.

Parameter	Property	Formation	Fluid
φ	Porosity	0.345	–
κ_0	Permeability/μm^2	3.1	–
α_∞	Tortuosity	2.1	–
k_{b}	Frame bulk modulus/Pa	6.6×10^9	–
G_{b}	Frame shear modulus/Pa	5.5×10^9	–
ρ_{s}	Solid density/(kg/m^3)	2212	–
k_{s}	Solid bulk modulas/Pa	50×10^9	–
ε_s	Solid permittivity	$4\varepsilon_0$	–
k_{f}	Fluid bulk modulas/Pa	2.2×10^9	–
ρ_{f}	Fluid density/(kg/m^3)	1000	1000
η	Fluid viscosity/Pa·s	0.001	–
c_0	Salinity/(mol/L)	0.001	0.01
ε_{f}	Fluid permittivity	$80\varepsilon_0$	$80\varepsilon_0$

5.1. Acoustic field and electric field

Using the above-mentioned Pride's theory we first investigate the theoretical mode by the coupled method. The formation outside the borehole is extended to infinity and has no electrochemical interface. For such a case the model reduces to the classical model.[15,32] The borehole mechanisms involved in seismoelectric coupling have been summarized by Hu and Liu,[15] Hu *et al.*,[16] and Hu and Liu.[17] Figure 2(a) shows the waveforms of acoustic field and electric field in the borehole excited by a monopole source, and two wave groups in the acoustic field waveforms: in the order of their arrival times, they are the P waves, the S waves and pseudo-Rayleigh waves. Figure 2(b) shows stationary electric field in the axial direction accompanying acoustic waves groups, both frequency and arrival time of acoustic field and electric field are very close, so the propagation velocity of coseismic electric field is equal to corresponding acoustic velocity. These coseismic electric fields exist in the region of the acoustic wave disturbance in the porous medium, and originated from the wall through converting the acoustic energy into the electromagnetic energy. Meanwhile, it can be observed that there is an electric pulse arriving earlier than the coseismic electric field: this electric pulse has a propagation velocity larger than the coseismic electric fields in the porous formation, and it is actually independent propagation of electromagnetic waves generated when the acoustic wave impinges on the borehole interface, and we call it "EM" wave.

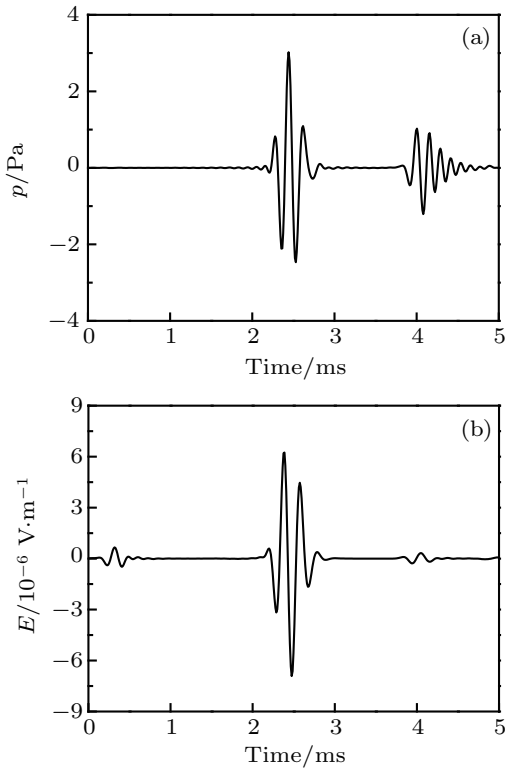


Fig. 2. Full waveforms of (a) acoustic field and (b) axial electric field from infinite formation model without electrochemical interface, obtained from Pride's theory.

In order to verify the correctness of simulation results of our problem by different calculation methods, we make the quantitative comparison between the acoustic field and the electric field obtained by the coupled method and the approximate method. The theoretical mode is the configuration of an open borehole surrounded by concentric double cylindrical porous formation. We select a set of parameters of the interlayer and the outermost porous formation listed in Table 1. We set the input parameters of interlayer and outermost formation to be equal except for the salinity. The salinities of borehole fluid, interlayer and outermost formation are 1×10^{-2} mol·L^{-1}, 1×10^{-3} mol·L^{-1}, and 3×10^{-3} mol·L^{-1}, respectively. Thus an electrochemical interface exists in such a case. Figure 3 shows the waveforms of acoustic field and electric field. They are calculated based on the coupled method and the approximate method. Solid and dashed lines denote the results obtained by coupled method and the approximate method, respectively. It can be seen from the figure that there is almost no difference between the results from the approximate method and the coupled method. Therefore the problem of infinite formation having an electrochemical interface can be solved by the approximate method to improve computing efficiency. Figure 3(a) shows the waveform curve of acoustic field is practically the same as simulated result shown in Fig. 2(a). As shown in Fig. 3(b), by comparing the waveforms of electric field with simulated result shown in Fig. 2(b), and enlarging the localized region of the electric field 10 times one can see that there is a wave group generated, which possesses the smallest amplitude: it is smaller than those of EM wave and coseismic electric field. It arrives earlier than the formation P wave of the conversional electric field. Based on the above simulation it can be concluded that the acoustic waveform is the same as that of single infinite porous formation, one can conclude that the elastic properties of the formation have no change. But the salinity of outermost porous formation is different from that of the interlayer, so it is a radiated electromagnetic wave generated when the acoustic wave propagates at the interface with different electrochemical properties, in order to distinguish a similar response (EM wave) generated at the borehole wall, here we denote it as seismoelectric interface response (SIR). As shown in Fig. 3(c), we enlarge the distance between electrochemical interface and axis of the borehole to 3.5 m, comparing with the case of interface distance 2.5 m, the arrival time of SIR is delayed by about 0.31 ms. One can find that the time difference is travel time for the formation P wave to propagate 1 meter in the porous formation. The existence of SIR can be used to extrapolate whether the electrochemical properties have been changed in the formation.

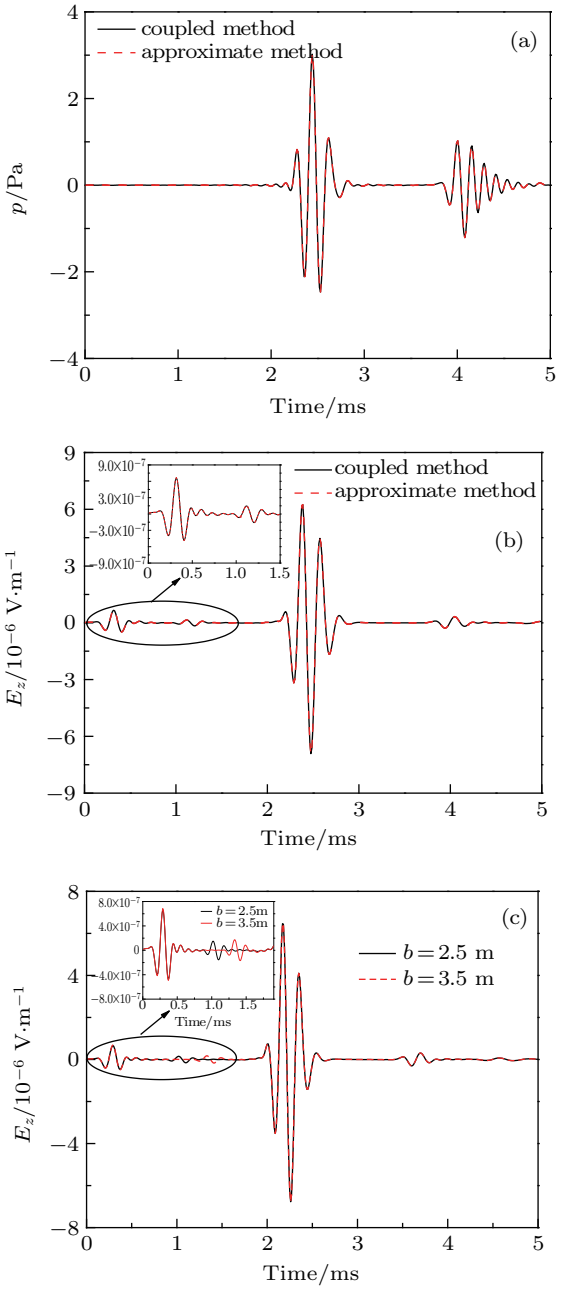


Fig. 3. (color online) The acoustic fields and the axial electric fields obtained from the coupled method (solid line) and approximate method (dashed line). (a) The acoustic field. (b) The axial electric field. (c) The axial electric field when the distance from electrochemical interface to borehole axis are 2.5 m and 3.5 m respectively.

5.2. Influence of salinity

The formation having discontinuities of electrochemical properties at an interface is further investigated. For this purpose, we investigate first the response characteristics of electric field as the salinity of outermost porous formation is changed. We set the salinity of each layer medium: the salinity of borehole fluid is 1×10^{-2} mol·L^{-1}, the salinity of interlayer is 1×10^{-3} mol·L^{-1} and change the salinity in the outermost formation into 0.25×10^{-3} mol·L^{-1}, 0.5×10^{-3} mol·L^{-1}, 2×10^{-3} mol·L^{-1}, and 4×10^{-3} mol·L^{-1} respectively. For all these four cases an electrochemical interface will exist. The other input parameters of formations are listed in Table 1. Fig-

ure 4(a) shows that if the acoustic field is simulated on condition that the porous formation outside the borehole exhibits the same acoustic impedance but different salinity, its acoustic waveforms have no difference from those of the single infinite porous formation model. However, figure 4(b) shows the change of SIR amplitude with salinity in the outermost porous medium. This amplitude first decreases and then increases as the salinity of the outer formation increases. This simulation result verifies the work by Liu *et al.*[28] for the logging while drilling case. Meanwhile, it can be observed that the variation of EM wave amplitude with salinity is relatively small, and influence of the salinity on coseismic electric amplitude is very small.

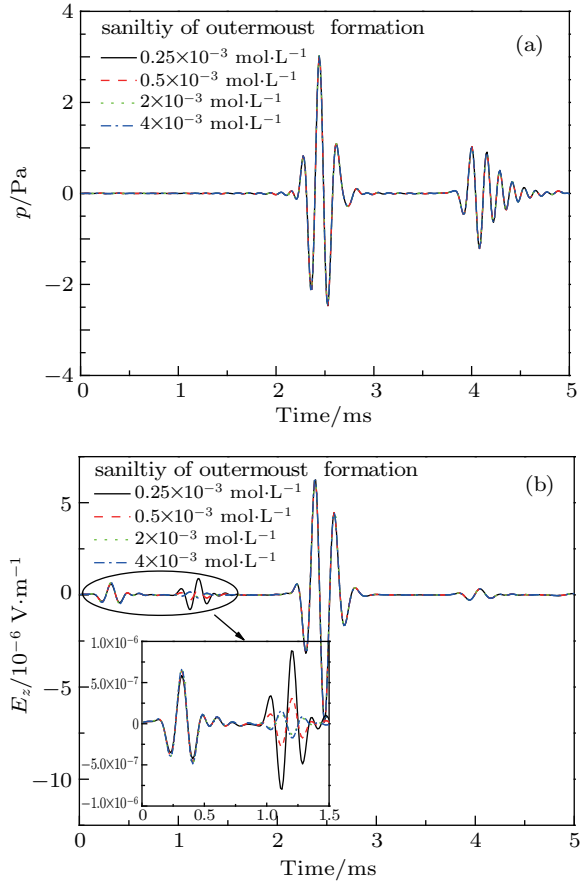


Fig. 4. (color online) Influences of salinity on synthetic waveforms of (a) acoustic field and (b) axial electric field.

Furthermore, we also investigate the response characteristics of electric field as the salinity of interlayer formation is changed. We set the salinity of the borehole fluid to be 1×10^{-2} mol·L^{-1}, the salinity in the outermost porous formation is 1×10^{-3} mol·L^{-1}, and change the salinity in the interlayer formation into 0.25×10^{-3} mol·L^{-1}, 0.5×10^{-3} mol·L^{-1}, 2×10^{-3} mol·L^{-1}, 4×10^{-3} mol·L^{-1} separately. Figure 5(a) shows the waveform of acoustic field, which does not change with the salinity. However, the amplitude of electric field presents distinct variations with the salinity change. As shown in Fig. 5(b), the SIR amplitude first decreases and then increases with the salinity of the inner formation increasing. The influence of salinity on the EM wave amplitude is also noticeable. It means that the amplitude of the EM wave decreases significantly with the increase of salinity. The variation tendency of coseismic electric amplitude with the salinity is roughly similar to that of EM wave, but the change of amplitude is relatively small.

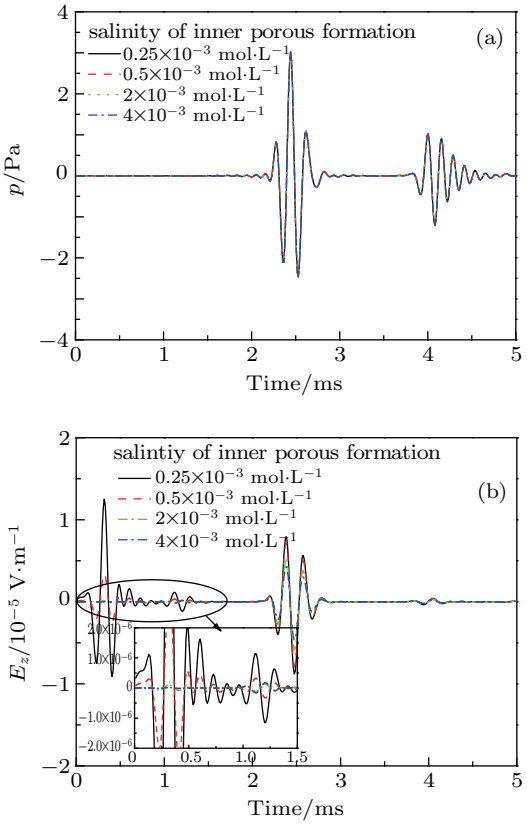


Fig. 5. (color online) Influences of salinity on synthetic waveforms of (a) acoustic field and (b) axial electric field.

6. Discussion and conclusions

In this paper, seismoelectric wave propagation in an infinite porous formation having discontinuities of electrochemical properties at an interface is investigated. Theoretical models are analyzed and simulated by both couple method and approximate method. The mathematical equations to study the acoustic field and the electric field obtained by the two methods are presented. We verify that the acoustic field and the electric field obtained by the approximate method are the same as those calculated by the complete coupled method. It means that the problem of infinite formation having an electrochemical interface can be solved by approximate method to improve computing efficiency. For comparison, we simulate first the acoustic and electric fields when the infinite formation has no electrochemical interface using the reduced coupling equations. Then we consider the case where the formation has discontinuities of electrochemical properties at the interface. A comparison between full waveforms of the acoustic field shows that the acoustic waveform is the same as the classical

model solution for single infinite porous formation. Therefore, it can be concluded that the change of electrochemical properties in porous formation does not influence the acoustic field.

Effect of the change in electrochemical properties on the electric field is then investigated. This investigation shows that for the porous formation having an electrochemical interface there is a wave group that has the smallest amplitude, smaller than those of EM wave and coseismic electric field, and it arrives before the P wave of the converted electric field. This wave is denoted as SIR. Furthermore, when the salinity of outermost porous formation increases, the amplitude of SIR first decreases and then increases. This is because the salinity mismatch between the inner porous layer and outer porous layer first decreases and then increases as the outer layer salinity increases. The influences of the salinity change on the coseismic electric and EM wave are relatively small. When the salinity of the interlayer or the inner porous formation increases, the amplitude of SIR first decreases and then increases for the same reason. The salinity mismatch at the interface first decreases and then increases as the interlayer salinity increases. For this case the EM wave decreases more significantly with the increase of salinity. The overall trend of coseismic electric wave amplitude varying with the salinity is approximately similar to that of EM wave, but the change of amplitude is relatively small.

One potential application of this investigation is to monitor the water pollution during environment monitoring. Since the acoustic pressure waveform is the same as that for the single infinite porous formation, it can be concluded that such an electrochemical property change cannot be detected by acoustic waves. This is because the elastic properties of the formation do not change. The change of the interface electromagnetic wave is due to the change of electrochemical property. Such an electrochemical property change can be due to the water pollution in the formation, thus the water pollution can be monitored by this technique. The model in this article is idealized, the actual formation and water pollution can be very heterogeneous, and therefore further study is needed to extend this concept of water pollution to the real life problems.

Appendix A

The matrix elements of $\boldsymbol{M}$ are as follows:

$$
\begin{aligned}
&m_{11} = \eta_m I_1(\eta_m a)/\pi, \quad m_{12} = 0,\\
&m_{13} = \eta_{\mathrm{pf}}^{\mathrm{I}}(1+\alpha_{\mathrm{pf}}^{\mathrm{I}})\mathrm{K}_1(\eta_{\mathrm{pf}}^{\mathrm{I}}a),\\
&m_{14} = \eta_{\mathrm{pf}}^{\mathrm{I}}(1+\alpha_{\mathrm{pf}}^{\mathrm{I}})\mathrm{K}_1(_{\mathrm{pf}}^{\mathrm{I}}a),\\
&m_{15} = \mathrm{i}k(1+\alpha_{\mathrm{pf}}^{\mathrm{I}})\mathrm{K}_1(\eta_{\mathrm{pf}}^{\mathrm{I}}a),\\
&m_{16} = \mathrm{i}k(1+\alpha_{\mathrm{em}}^{\mathrm{I}})\mathrm{K}_1(\eta_{\mathrm{em}}^{\mathrm{I}}a),\\
&m_{17} = -\eta_{\mathrm{pf}}^{\mathrm{I}}(1+\alpha_{\mathrm{pf}}^{\mathrm{I}})\mathrm{I}_1(\eta_{\mathrm{pf}}^{\mathrm{I}}a),\\
&m_{18} = -\eta_{\mathrm{ps}}^{\mathrm{I}}(1+\alpha_{\mathrm{ps}}^{\mathrm{I}})\mathrm{I}_1(\eta_{\mathrm{ps}}^{\mathrm{I}}a),\\
&m_{19} = \mathrm{i}k(1+\alpha_{\mathrm{sh}}^{\mathrm{I}})\mathrm{I}_1(\eta_{\mathrm{sh}}^{\mathrm{I}}a),\\
&m_{1,10} = \mathrm{i}k(1+\alpha_{\mathrm{em}}^{\mathrm{I}})\mathrm{I}_1(\eta_{\mathrm{em}}^{\mathrm{I}}a),\\
&m_{1,11} = m_{1,12} = m_{1,13} = m_{1,14} = 0,\\
&m_{21} = \rho_0\omega^2\mathrm{I}_0(\eta_m a)/\pi, \quad m_{22} = 0,\\
&m_{23} = -{k_{\mathrm{pf}}^{\mathrm{I}}}^2(C^{\mathrm{I}} + M^{\mathrm{I}}\alpha_{\mathrm{pf}}^{\mathrm{I}})\mathrm{K}_0(\eta_{\mathrm{pf}}^{\mathrm{I}}a),\\
&m_{24} = -{k_{\mathrm{ps}}^{\mathrm{I}}}^2(C^{\mathrm{I}} + M^{\mathrm{I}}\alpha_{\mathrm{ps}})\mathrm{K}_0(\eta_{\mathrm{ps}}^{\mathrm{I}}a),\\
&m_{25} = 0, \quad m_{26} = 0,\\
&m_{27} = -{k_{\mathrm{pf}}^{\mathrm{I}}}^2(C^{\mathrm{I}} + M^{\mathrm{I}}\alpha_{\mathrm{pf}}^{\mathrm{I}})\mathrm{I}_0(\eta_{\mathrm{pf}}^{\mathrm{I}}a),\\
&m_{28} = -{k_{\mathrm{ps}}^{\mathrm{I}}}^2(C^{\mathrm{I}} + M^{\mathrm{I}}\alpha_{\mathrm{ps}}^{\mathrm{I}})\mathrm{I}_0(\eta_{\mathrm{ps}}^{\mathrm{I}}a),\\
&m_{2,9} = m_{2,10} = m_{2,11} = m_{2,12} = m_{2,13} = m_{2,14} = 0,\\
&m_{31} = \rho_0\omega^2 I_0(\eta_m a)/\pi, \quad m_{32} = 0,\\
&m_{33} = 2G^{\mathrm{I}}\eta_{\mathrm{pf}}^{\mathrm{I}}\mathrm{K}_1(\eta_{\mathrm{pf}}^{\mathrm{I}}a)/a\\
&\qquad + \left[(2G^{\mathrm{I}} - H^{\mathrm{I}} - C^{\mathrm{I}}\alpha_{\mathrm{pf}}^{\mathrm{I}}){k_{\mathrm{pf}}^{\mathrm{I}}}^2 + 2G^{\mathrm{I}}{\eta_{\mathrm{pf}}^{\mathrm{I}}}^2\right]\cdot\mathrm{K}_0(\eta_{\mathrm{pf}}^{\mathrm{I}}a),\\
&m_{34} = 2G^{\mathrm{I}}\eta_{\mathrm{ps}}^{\mathrm{I}}\mathrm{K}_1(\eta_{\mathrm{ps}}^{\mathrm{I}}a)/a\\
&\qquad + \left[(2G^{\mathrm{I}} - H^{\mathrm{I}} - C^{\mathrm{I}}\alpha_{\mathrm{ps}}^{\mathrm{I}}){k_{\mathrm{ps}}^{\mathrm{I}}}^2 + 2G^{\mathrm{I}}{\eta_{\mathrm{ps}}^{\mathrm{I}}}^2\right]\cdot\mathrm{K}_0(\eta_{\mathrm{ps}}^{\mathrm{I}}a),\\
&m_{35} = 2\mathrm{i}kG^{\mathrm{I}}\left[\eta_{\mathrm{sh}}^{\mathrm{I}}\mathrm{K}_0(\eta_{\mathrm{sh}}^{\mathrm{I}}a) + \mathrm{K}_1(\eta_{\mathrm{sh}}^{\mathrm{I}}a)/a\right],\\
&m_{36} = 2\mathrm{i}kG^{\mathrm{I}}\left[\eta_{\mathrm{em}}^{\mathrm{I}}\mathrm{K}_0(\eta_{\mathrm{em}}^{\mathrm{I}}a) + \mathrm{K}_1(\eta_{\mathrm{em}}^{\mathrm{I}}a)/a\right],\\
&m_{37} = -2G^{\mathrm{I}}\eta_{\mathrm{pf}}^{\mathrm{I}}\mathrm{I}_1(\eta_{\mathrm{pf}}^{\mathrm{I}}a)/a\\
&\qquad + \left[(2G^{\mathrm{I}} - H^{\mathrm{I}} - C^{\mathrm{I}}\alpha_{\mathrm{pf}}^{\mathrm{I}}){k_{\mathrm{pf}}^{\mathrm{I}}}^2 + 2G^{\mathrm{I}}{\eta_{\mathrm{pf}}^{\mathrm{I}}}^2\right]\cdot\mathrm{I}_0(\eta_{\mathrm{pf}}^{\mathrm{I}}a),\\
&m_{38} = -2G^{\mathrm{I}}\eta_{\mathrm{ps}}^{\mathrm{I}}\mathrm{I}_1(\eta_{\mathrm{ps}}^{\mathrm{I}}a)/a\\
&\qquad + \left[(2G^{\mathrm{I}} - H^{\mathrm{I}} - C^{\mathrm{I}}\alpha_{\mathrm{ps}}^{\mathrm{I}}){k_{\mathrm{ps}}^{\mathrm{I}}}^2 + 2G^{\mathrm{I}}{\eta_{\mathrm{ps}}^{\mathrm{I}}}^2\right]\cdot\mathrm{I}_0(\eta_{\mathrm{ps}}^{\mathrm{I}}a),\\
&m_{39} = 2\mathrm{i}kG^{\mathrm{I}}\left[-\eta_{\mathrm{ps}}^{\mathrm{I}}\mathrm{I}_0(\eta_{\mathrm{ps}}^{\mathrm{I}}a) + \mathrm{I}_1(\eta_{\mathrm{ps}}^{\mathrm{I}}a)/a\right],\\
&m_{3,10} = 2\mathrm{i}kG^{\mathrm{I}}\left[-\eta_{\mathrm{em}}^{\mathrm{I}}\mathrm{I}_0(\eta_{\mathrm{em}}^{\mathrm{I}}a) + \mathrm{I}_1(\eta_{\mathrm{em}}^{\mathrm{I}}a)/a\right],\\
&m_{3,11} = m_{3,12} = m_{3,13} = m_{3,14} = 0, \quad m_{4,1} = m_{4,2} = 0,\\
&m_{43} = -2\mathrm{i}k\eta_{\mathrm{pf}}^{\mathrm{I}}\mathrm{K}_1(\eta_{\mathrm{pf}}^{\mathrm{I}}a), \quad m_{44} = -2\mathrm{i}k\eta_{\mathrm{ps}}^{\mathrm{I}}\mathrm{K}_1(\eta_{\mathrm{ps}}^{\mathrm{I}}a),\\
&m_{45} = (k^2 + {\eta_{\mathrm{sh}}^{\mathrm{I}}}^2)\mathrm{K}_1(\eta_{\mathrm{sh}}^{\mathrm{I}}a), \quad m_{46} = (k^2 + {\eta_{\mathrm{em}}^{\mathrm{I}}}^2)\mathrm{K}_1(\eta_{\mathrm{em}}^{\mathrm{I}}a),\\
&m_{47} = 2\mathrm{i}k\eta_{\mathrm{pf}}^{\mathrm{I}}\mathrm{I}_1(\eta_{\mathrm{pf}}^{\mathrm{I}}a), \quad m_{48} = 2\mathrm{i}k\eta_{\mathrm{ps}}^{\mathrm{I}}\mathrm{I}_1(\eta_{\mathrm{ps}}^{\mathrm{I}}a),\\
&m_{49} = \left(k^2 + {\eta_{\mathrm{sh}}^{\mathrm{I}}}^2\right)\mathrm{I}_1\left(\eta_{\mathrm{sh}}^{\mathrm{I}}a\right),\\
&m_{4,10} = \left(k^2 + {\eta_{\mathrm{em}}^{\mathrm{I}}}^2\right)\mathrm{I}_1\left(\eta_{\mathrm{em}}^{\mathrm{I}}a\right),\\
&m_{4,11} = m_{4,12} = m_{4,13} = m_{4,14} = 0,\\
&m_{5,1} = 0, \quad m_{52} = \eta_{\mathrm{e}}\mathrm{I}_0(\eta_{\mathrm{e}}a)/\mathrm{i}k,\\
&m_{53} = \mathrm{i}k\beta_{\mathrm{pf}}^{\mathrm{I}}\mathrm{K}_0(\eta_{\mathrm{pf}}^{\mathrm{I}}a), \quad m_{54} = \mathrm{i}k\beta_{\mathrm{ps}}^{\mathrm{I}}\mathrm{K}_0(\eta_{\mathrm{ps}}^{\mathrm{I}}a),\\
&m_{55} = -\beta_{\mathrm{sh}}^{\mathrm{I}}\eta_{\mathrm{sh}}^{\mathrm{I}}\mathrm{K}_0(\eta_{\mathrm{sh}}^{\mathrm{I}}a), \quad m_{56} = -\beta_{\mathrm{em}}^{\mathrm{I}}\eta_{\mathrm{em}}^{\mathrm{I}}\mathrm{K}_0(\eta_{\mathrm{em}}^{\mathrm{I}}a),\\
&m_{57} = \mathrm{i}k\beta_{\mathrm{pf}}^{\mathrm{I}}\mathrm{I}_0\left(\eta_{\mathrm{pf}}^{\mathrm{I}}a\right), \quad m_{58} = \mathrm{i}k\beta_{\mathrm{ps}}^{\mathrm{I}}\mathrm{I}_0\left(\eta_{\mathrm{ps}}^{\mathrm{I}}a\right),\\
&m_{59} = \beta_{\mathrm{sh}}^{\mathrm{I}}\eta_{\mathrm{sh}}^{\mathrm{I}}\mathrm{I}_0\left(\eta_{\mathrm{sh}}^{\mathrm{I}}a\right), \quad m_{5,10} = \beta_{\mathrm{em}}^{\mathrm{I}}\eta_{\mathrm{em}}^{\mathrm{I}}\mathrm{I}_0\left(\eta_{\mathrm{em}}^{\mathrm{I}}a\right),\\
&m_{5,11} = m_{5,12} = m_{5,13} = m_{5,14} = 0,\\
&m_{61} = 0, \quad m_{62} = \mathrm{i}\left(\frac{\eta_{\mathrm{e}}^2}{k} - k\right)\mathrm{I}_1(\eta_{\mathrm{e}}a),\\
&m_{63} = m_{64} = 0,\\
&m_{65} = \beta_{\mathrm{sh}}^{\mathrm{I}}{k_{\mathrm{sh}}^{\mathrm{I}}}^2\mathrm{K}_1(\eta_{\mathrm{sh}}^{\mathrm{I}}a), \quad m_{66} = \beta_{\mathrm{em}}^{\mathrm{I}}{k_{\mathrm{em}}^{\mathrm{I}}}^2\mathrm{K}_1(\eta_{\mathrm{em}}^{\mathrm{I}}a),\\
&m_{67} = m_{68} = 0,\\
&m_{69} = \beta_{\mathrm{sh}}^{\mathrm{I}}{k_{\mathrm{sh}}^{\mathrm{I}}}^2\mathrm{I}_1\left(\eta_{\mathrm{sh}}^{\mathrm{I}}a\right), \quad m_{6,10} = \beta_{\mathrm{em}}^{\mathrm{I}}{k_{\mathrm{em}}^{\mathrm{I}}}^2\mathrm{I}_1\left(\eta_{\mathrm{em}}^{\mathrm{I}}a\right),\\
&m_{6,11} = m_{6,12} = m_{6,13} = m_{6,14} = 0,
\end{aligned}
$$

$$m_{71}=m_{72}=0,$$
$$m_{73}=-\eta_{\rm pf}^{\rm I}{\rm K}_1\left(\eta_{\rm pf}^{\rm I}b\right),\quad m_{74}=-\eta_{\rm ps}^{\rm I}{\rm K}_1\left(\eta_{\rm ps}^{\rm I}b\right),$$
$$m_{75}=-{\rm i}k{\rm K}_1\left(\eta_{\rm sh}^{\rm I}b\right),\quad m_{76}=-{\rm i}k{\rm K}_1\left(\eta_{\rm em}^{\rm I}b\right),$$
$$m_{77}=\eta_{\rm pf}^{\rm I}{\rm I}_1\left(\eta_{\rm pf}^{\rm I}b\right),\quad m_{78}=\eta_{\rm ps}^{\rm I}{\rm I}_1\left(\eta_{\rm ps}^{\rm I}b\right),$$
$$m_{79}=-{\rm i}{\rm I}_1\left(\eta_{\rm sh}^{\rm I}b\right),\quad m_{7,10}=-{\rm i}k{\rm I}_1\left(\eta_{\rm em}^{\rm I}b\right),$$
$$m_{7,11}=\eta_{\rm pf}{\rm K}_1(\eta_{\rm pf}^{\rm II}b),\quad m_{7,12}=\eta_{\rm ps}^{\rm II}{\rm K}_1(\eta_{\rm ps}^{\rm II}b),$$
$$m_{7,13}={\rm i}k{\rm K}_1(\eta_{\rm sh}^{\rm II}b),\quad m_{7,14}={\rm i}k{\rm K}_1(\eta_{\rm em}^{\rm II}b),$$
$$m_{81}=m_{82}=0,$$
$$m_{83}={\rm i}k{\rm K}_0\left(\eta_{\rm pf}^{\rm I}b\right),\quad m_{84}={\rm i}k{\rm K}_0\left(\eta_{\rm ps}^{\rm I}b\right),$$
$$m_{85}=-\eta_{\rm sh}^{\rm I}{\rm K}_0\left(\eta_{\rm sh}^{\rm I}b\right),\quad m_{86}=-\eta_{\rm em}^{\rm I}{\rm K}_0\left(\eta_{\rm em}^{\rm I}b\right),$$
$$m_{87}={\rm i}k{\rm I}_0\left(\eta_{\rm pf}^{\rm I}b\right),\quad m_{88}={\rm i}k{\rm I}_0\left(\eta_{\rm ps}^{\rm I}b\right),$$
$$m_{89}=\eta_{\rm sh}^{\rm I}{\rm I}_0\left(\eta_{\rm sh}^{\rm I}b\right),\quad m_{8,10}=\eta_{\rm em}^{\rm I}{\rm I}_0\left(\eta_{\rm em}^{\rm I}b\right),$$
$$m_{8,11}=-{\rm i}k{\rm K}_0\left(\eta_{\rm pf}^{\rm II}b\right),\quad m_{8,12}=-{\rm i}k{\rm K}_0\left(\eta_{\rm ps}^{\rm II}b\right),$$
$$m_{8,13}=\eta_{\rm sh}^{\rm II}{\rm K}_0\left(\eta_{\rm sh}^{\rm II}b\right),\quad m_{8,14}=\eta_{\rm em}^{\rm II}{\rm K}_0\left(\eta_{\rm em}^{\rm II}b\right),$$
$$m_{91}=m_{92}=0,$$
$$m_{93}=-\alpha_{\rm pf}^{\rm I}\eta_{\rm pf}^{\rm I}{\rm K}_1\left(\eta_{\rm pf}^{\rm I}b\right),\quad m_{94}=-\alpha_{\rm ps}^{\rm I}\eta_{\rm ps}^{\rm I}{\rm K}_1\left(\eta_{\rm ps}^{\rm I}b\right),$$
$$m_{95}=-{\rm i}k\alpha_{\rm sh}^{\rm I}{\rm K}_1\left(\eta_{\rm sh}^{\rm I}b\right),\quad m_{96}=-{\rm i}k\alpha_{\rm em}^{\rm I}{\rm K}_1\left(\eta_{\rm em}^{\rm I}b\right),$$
$$m_{97}=\alpha_{\rm pf}^{\rm I}\eta_{\rm pf}^{\rm I}{\rm I}_1\left(\eta_{\rm pf}^{\rm I}b\right),\quad m_{98}=\alpha_{\rm ps}^{\rm I}\eta_{\rm ps}^{\rm I}{\rm I}_1\left(\eta_{\rm ps}^{\rm I}b\right),$$
$$m_{99}=-{\rm i}k\alpha_{\rm sh}^{\rm I}{\rm I}_1\left(\eta_{\rm sh}^{\rm I}b\right),\quad m_{9,10}=-{\rm i}k\alpha_{\rm em}^{\rm I}{\rm I}_1\left(\eta_{\rm em}^{\rm I}b\right),$$
$$m_{9,11}=\alpha_{\rm pf}^{\rm II}\eta_{\rm pf}^{\rm II}{\rm K}_1\left(\eta_{\rm pf}^{\rm II}b\right),\quad m_{9,12}=\alpha_{\rm ps}^{\rm II}\eta_{\rm ps}^{\rm II}{\rm K}_1\left(\eta_{\rm ps}^{\rm II}b\right),$$
$$m_{9,13}={\rm i}k\alpha_{\rm sh}^{\rm II}{\rm K}_1\left(\eta_{\rm sh}^{\rm II}b\right),\quad m_{9,14}={\rm i}k\alpha_{\rm em}^{\rm II}{\rm K}_1\left(\eta_{\rm em}^{\rm II}b\right),$$
$$m_{10,1}=m_{10,2}=0,$$
$$m_{10,3}=k_{\rm pf}^{\rm I\,2}\left(C^{\rm I}+M^{\rm I}\alpha_{\rm pf}^{\rm I}\right){\rm K}_0\left(\eta_{\rm pf}^{\rm I}b\right),$$
$$m_{10,4}=k_{\rm ps}^{\rm I\,2}\left(C^{\rm I}+M^{\rm I}\alpha_{\rm ps}^{\rm I}\right){\rm K}_0\left(\eta_{\rm ps}^{\rm I}b\right),$$
$$m_{10,7}=k_{\rm pf}^{\rm I\,2}\left(C^{\rm I}+M^{\rm I}\alpha_{\rm pf}^{\rm I}\right){\rm I}_0\left(\eta_{\rm pf}^{\rm I}b\right),$$
$$m_{10,8}=k_{\rm ps}^{\rm I\,2}\left(C^{\rm I}+M^{\rm I}\alpha_{\rm ps}^{\rm I}\right){\rm I}_0\left(\eta_{\rm ps}^{\rm I}b\right),$$
$$m_{10,11}=-k_{\rm pf}^{\rm II\,2}\left(C^{\rm II}+M^{\rm II}\alpha_{\rm pf}^{\rm II}\right){\rm K}_0\left(\eta_{\rm pf}^{\rm II}b\right),$$
$$m_{10,12}=-k_{\rm ps}^{\rm II\,2}\left(C^{\rm II}+M^{\rm II}\alpha_{\rm ps}^{\rm II}\right){\rm K}_0\left(\eta_{\rm ps}^{\rm II}b\right),$$
$$m_{10,5}=m_{10,6}=m_{10,9}=m_{10,10}=m_{10,13}=m_{10,14}=0,$$
$$m_{11,1}=m_{11,2}=0,$$
$$m_{11,3}=\left[\left(2G^{\rm I}-H^{\rm I}-C^{\rm I}\alpha_{\rm pf}^{\rm I}\right)k_{\rm pf}^{\rm I\,2}+2G^{\rm I}\eta_{\rm pf}^{\rm I\,2}\right]\cdot{\rm K}_0(\eta_{\rm pf}^{\rm I}b)+2G^{\rm I}\eta_{\rm pf}^{\rm I}{\rm K}_1(\eta_{\rm pf}^{\rm I}b)/b,$$
$$m_{11,4}=\left[\left(2G^{\rm I}-H^{\rm I}-C^{\rm I}\alpha_{\rm ps}^{\rm I}\right)k_{\rm ps}^{\rm I\,2}+2G^{\rm I}\eta_{\rm ps}^{\rm I\,2}\right]\cdot{\rm K}_0(\eta_{\rm ps}^{\rm I}b)+2G^{\rm I}\eta_{\rm ps}^{\rm I}{\rm K}_1(\eta_{\rm ps}^{\rm I}b)/b,$$
$$m_{11,5}=2{\rm i}kG^{\rm I}\left[\eta_{\rm sh}^{\rm I}{\rm K}_0(\eta_{\rm sh}^{\rm I}b)+{\rm K}_1(\eta_{\rm sh}^{\rm I}b)/b\right],$$
$$m_{11,6}=2{\rm i}kG^{\rm I}\left[\eta_{\rm em}^{\rm I}{\rm K}_0(\eta_{\rm em}^{\rm I}b)+{\rm K}_1(\eta_{\rm em}^{\rm I}b)/b\right],$$
$$m_{11,7}=\left[\left(2G^{\rm I}-H^{\rm I}-C^{\rm I}\alpha_{\rm ps}^{\rm I}\right)k_{\rm pf}^{\rm I\,2}+2G^{\rm I}\eta_{\rm pf}^{\rm I\,2}\right]\cdot{\rm I}_0(\eta_{\rm pf}^{\rm I}b)-2G^{\rm I}\eta_{\rm pf}^{\rm I}{\rm I}_1(\eta_{\rm pf}^{\rm I}b)/b,$$
$$m_{11,8}=\left[\left(2G^{\rm I}-H^{\rm I}-C^{\rm I}\alpha_{\rm ps}^{\rm I}\right)k_{\rm ps}^{\rm I\,2}+2G^{\rm I}\eta_{\rm ps}^{\rm I\,2}\right]\cdot{\rm I}_0(\eta_{\rm ps}^{\rm I}b)-2G^{\rm I}\eta_{\rm ps}^{\rm I}{\rm I}_1(\eta_{\rm ps}^{\rm I}b)/b,$$
$$m_{11,9}=2{\rm i}kG^{\rm I}\left[-\eta_{\rm ps}^{\rm I}{\rm I}_0(\eta_{\rm ps}^{\rm I}b)+{\rm I}_1(\eta_{\rm ps}^{\rm I}b)/b\right],$$
$$m_{11,10}=2{\rm i}kG^{\rm I}\left[-\eta_{\rm em}^{\rm I}{\rm I}_0(\eta_{\rm em}^{\rm I}b)+{\rm I}_1(\eta_{\rm em}^{\rm I}b)/b\right],$$
$$m_{11,11}=-\left[\left(2G^{\rm II}-H^{\rm II}-C^{\rm II}\alpha_{\rm pf}^{\rm II}\right)k_{\rm pf}^{\rm II\,2}+2G^{\rm II}\eta_{\rm pf}^{\rm II\,2}\right]\cdot{\rm K}_0(\eta_{\rm pf}^{\rm II}b)-2G^{\rm II}\eta_{\rm pf}^{\rm II}{\rm K}_1(\eta_{\rm pf}^{\rm II}b)/b,$$
$$m_{11,12}=-\left[\left(2G^{\rm II}-H^{\rm II}-C^{\rm II}\alpha_{\rm ps}^{\rm II}\right)k_{\rm ps}^{\rm II\,2}+2G^{\rm II}\eta_{\rm ps}^{\rm II\,2}\right]\cdot{\rm K}_0(\eta_{\rm ps}^{\rm II}b)-2G^{\rm II}\eta_{\rm ps}^{\rm II}{\rm K}_1(\eta_{\rm ps}^{\rm II}b)/b,$$
$$m_{11,13}=-2{\rm i}kG^{\rm II}\left[\eta_{\rm sh}^{\rm II}{\rm K}_0(\eta_{\rm sh}^{\rm II}b)+{\rm K}_1(\eta_{\rm sh}^{\rm II}b)/b\right],$$
$$m_{11,14}=-2{\rm i}kG^{\rm II}\left[\eta_{\rm em}^{\rm II}{\rm K}_0(\eta_{\rm sh}^{\rm II}b)+{\rm K}_1(\eta_{\rm sh}^{\rm II}b)/b\right],$$
$$m_{12,1}=m_{12,2}=0,$$
$$m_{12,3}=-2{\rm i}k\eta_{\rm pf}^{\rm I}{\rm K}_1\left(\eta_{\rm pf}^{\rm I}b\right),\quad m_{12,4}=-2{\rm i}k\eta_{\rm ps}^{\rm I}{\rm K}_1\left(\eta_{\rm ps}^{\rm I}b\right),$$
$$m_{12,5}=\left(k^2+\eta_{\rm sh}^{\rm I\,2}\right){\rm K}_1\left(\eta_{\rm sh}^{\rm I}b\right),$$
$$m_{12,6}=\left(k^2+\eta_{\rm em}^{\rm I\,2}\right){\rm K}_1\left(\eta_{\rm em}^{\rm I}b\right),$$
$$m_{12,7}=2{\rm i}k\eta_{\rm pf}^{\rm I}{\rm I}_1\left(\eta_{\rm pf}^{\rm I}b\right),\quad m_{12,8}=2{\rm i}k\eta_{\rm ps}^{\rm I}{\rm I}_1\left(\eta_{\rm ps}^{\rm I}b\right),$$
$$m_{12,9}=\left(k^2+\eta_{\rm sh}^{\rm I\,2}\right){\rm I}_1\left(\eta_{\rm sh}^{\rm I}b\right),$$
$$m_{12,10}=\left(k^2+\eta_{\rm em}^{\rm I\,2}\right){\rm I}_1\left(\eta_{\rm em}^{\rm I}b\right),$$
$$m_{12,11}=2{\rm i}k\eta_{\rm pf}^{\rm II}{\rm K}_1\left(\eta_{\rm pf}^{\rm II}b\right),\quad m_{12,12}=2{\rm i}k\eta_{\rm ps}^{\rm II}{\rm K}_1\left(\eta_{\rm ps}^{\rm II}b\right),$$
$$m_{12,13}=-\left(k^2+\eta_{\rm sh}^{\rm II\,2}\right){\rm K}_1\left(\eta_{\rm sh}^{\rm II}b\right),$$
$$m_{12,14}=-\left(k^2+\eta_{\rm em}^{\rm II\,2}\right){\rm K}_1\left(\eta_{\rm em}^{\rm II}b\right),$$
$$m_{13,1}=m_{13,2}=0,$$
$$m_{13,3}={\rm i}k\beta_{\rm pf}^{\rm I}{\rm K}_0\left(\eta_{\rm pf}^{\rm I}b\right),\quad m_{13,4}={\rm i}k\beta_{\rm ps}^{\rm I}{\rm K}_0\left(\eta_{\rm ps}^{\rm I}b\right),$$
$$m_{13,5}=-\beta_{\rm sh}^{\rm I}\eta_{\rm sh}^{\rm I}{\rm K}_0\left(\eta_{\rm sh}^{\rm I}b\right),\quad m_{13,6}=-\beta_{\rm em}^{\rm I}\eta_{\rm em}^{\rm I}{\rm K}_0\left(\eta_{\rm em}^{\rm I}b\right),$$
$$m_{13,7}={\rm i}k\beta_{\rm pf}^{\rm I}{\rm I}_0\left(\eta_{\rm pf}^{\rm I}b\right),\quad m_{13,8}={\rm i}k\beta_{\rm ps}^{\rm I}{\rm I}_0\left(\eta_{\rm ps}^{\rm I}b\right),$$
$$m_{13,9}=\beta_{\rm sh}^{\rm I}\eta_{\rm sh}^{\rm I}{\rm I}_0\left(\eta_{\rm sh}^{\rm I}b\right),\quad m_{13,10}=\beta_{\rm em}^{\rm I}\eta_{\rm em}^{\rm I}{\rm I}_0\left(\eta_{\rm em}^{\rm I}b\right),$$
$$m_{13,11}=-{\rm i}k\beta_{\rm pf}^{\rm II}{\rm K}_0\left(\eta_{\rm pf}^{\rm II}b\right),\quad m_{13,12}=-{\rm i}k\beta_{\rm ps}^{\rm II}{\rm K}_0\left(\eta_{\rm ps}^{\rm II}b\right),$$
$$m_{13,13}=\beta_{\rm sh}^{\rm II}\eta_{\rm sh}^{\rm II}{\rm K}_0\left(\eta_{\rm sh}^{\rm II}b\right),\quad m_{13,14}=\beta_{\rm em}^{\rm II}\eta_{\rm em}^{\rm II}{\rm K}_0\left(\eta_{\rm em}^{\rm II}b\right),$$
$$m_{14,1}=m_{14,2}=0,$$
$$m_{14,5}=\beta_{\rm sh}^{\rm I}k_{\rm sh}^{\rm I\,2}{\rm K}_1\left(\eta_{\rm sh}^{\rm I}b\right),\quad m_{14,6}=\beta_{\rm em}^{\rm I}k_{\rm em}^{\rm I\,2}{\rm K}_1\left(\eta_{\rm em}^{\rm I}b\right),$$
$$m_{14,9}=\beta_{\rm sh}^{\rm I}k_{\rm sh}^{\rm I\,2}{\rm I}_1\left(\eta_{\rm sh}^{\rm I}b\right),\quad m_{14,10}=\beta_{\rm em}^{\rm I}k_{\rm em}^{\rm I\,2}{\rm I}_1\left(\eta_{\rm em}^{\rm I}b\right),$$
$$m_{14,13}=-\beta_{\rm sh}^{\rm II}k_{\rm sh}^{\rm II\,2}{\rm K}_1\left(\eta_{\rm sh}^{\rm II}b\right),\quad m_{14,14}=\beta_{\rm em}^{\rm II}k_{\rm em}^{\rm II\,2}{\rm K}_1\left(\eta_{\rm em}^{\rm II}b\right),$$
$$m_{14,3}=m_{14,4}=m_{14,7}=m_{14,8}=m_{14,11}=m_{14,12}=0.$$

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