Spin transport in a chain of polygonal quantum rings with Dresselhaus spin–orbit coupling*

Han-Zhao Tang(唐翰昭)[1], Xiao-Teng Yao(要晓腾)[2], and Jian-Jun Liu(刘建军)[2,3,†]

[1] *Department of Mathematics and Physics, Shijiazhuang Tiedao University, Shijiazhuang* 050043, *China*
[2] *Physics Department and Hebei Advanced Thin Film Laboratory, Hebei Normal University, Shijiazhuang* 050024, *China*
[3] *Physics Department, Shijiazhuang University, Shijiazhuang* 050035, *China*

(Received 4 May 2017; revised manuscript received 13 July 2017; published online 29 September 2017)

Using a transfer matrix method, we investigate spin transport through a chain of polygonal rings with Dresselhaus spin–orbit coupling (DSOC). The spin conductance is dependent on the number of sides in the polygons. When DSOC is considered in a chain which also has Rashba spin–orbit coupling (RSOC) of the same magnitude, the total conductance is the same as that for the same chain with no SOC. However, when the two types of SOC have different values, there results a unique anisotropic conductance.

Keywords: quantum ring chain, Dresselhaus spin–orbit coupling, spin transport

PACS: 72.25.Dc, 03.65.Nk, 71.70.Ej **DOI:** 10.1088/1674-1056/26/11/117203

1. Introduction

Over the past few years, due to both experimental and theoretical interest, spintronics has attracted more and more attention,[1–3] in areas such as magnetic memory circuits, quantum computers, magnetic nano-structures, and quasi one-dimensional semiconductor rings. The key focal points in these areas are the effects of quantum coherence in low-dimensions and the generation of spin conductance as well as their applications.[4,5] To obtain a spin conductance, we can, in principle, regulate an external magnetic field to manipulate the spins of the electrons. However, the inefficiency of spin injection from a ferromagnetic lead into a circuit requires a new method to regulate the electron spin. Fortunately, Datta and Das proposed a spin-field-effect transistor device, based on spin–orbit coupling (SOC), to control the electron spin.[3] The mechanisms behind SOC include structure inversion asymmetry, giving Rashba spin–orbit coupling (RSOC),[6] and bulk inversion asymmetry, giving Dresselhaus spin–orbit coupling (DSOC).[7] The precise measurement of both of the two types of SOC is important, both experimentally and theoretically, for designing spin electronic devices.

In 2004, Bercioux *et al.*[8] investigated electron localization phenomena and conductance in a one-dimensional chain of square loops with RSOC, which is the simplest structure for a quantum network. This work resulted in a new theoretical platform for electron spin transport in a quantum network. Inspired by these promising predictions, several theoretical and experimental studies have reported on the spin transport properties of polygonal structures[9–16] and the quantum transport properties of one-dimensional quantum chains.[17–21] The experimental realization of transport through an array of square loops with RSOC has been discussed by Koga *et al.*[9] Bercioux *et al.*[17] studied the quantum transport through a chain with diamond shaped subunits (squares joined at the corners) in the presence of RSOC and a magnetic field. Wang *et al.*[18,19] have also studied quantum transport properties through a quantum network with RSOC.

To date, RSOC has drawn a lot of attention because it can be accurately controlled by an external electric field. However, an experiment designed by Meier *et al.*[10] has shown that both the RSOC and the DSOC can be measured by optically tuning the angular dependence of the electron spin precession on the direction of motion with respect to the crystal lattice. This opens up the possibility of using the DSOC more effectively. Although the transport properties through a one-dimensional chain of polygonal rings with RSOC have been discussed, there has been little discussion of the effects of DSOC on electron transport and little on the interaction between the RSOC and the DSOC in a chain of polygonal rings. The present work addresses spin transport properties through a chain of polygons with DSOC and also considers quantum interference effects when both the DSOC and the RSOC are present. This analysis may contribute to a basic understanding of transport in quantum network structures.

In this paper, employing the standard transfer matrix method, we consider the effects of DSOC and the combined effects of RSOC and DSOC in a chain of polygonal rings. The theoretical model is presented in Section 2. The numerical results and analyses are illustrated in Section 3, and we sum-

*Project supported by the National Natural Science Foundation of China (Grant Nos. 61176089 and 11504083) and the Foundation of Shijiazhuang University, China (Grant No. XJPT002).

†Corresponding author. E-mail: liujj@mail.hebtu.edu.cn

© 2017 Chinese Physical Society and IOP Publishing Ltd

marize our results in Section 4.

2. Model and method

The system under study is a ballistic chain of polygonal rings with two leads, as depicted in Fig. 1. In our system, as illustrated schematically in Fig. 1, there are two specific cases: one is that the polygonal rings of the chain are connected at nodes and the other is that the polygonal rings are connected along segments of the polygons. To simplify the analysis, we use the same "narrow segments" approximation as those in Refs. [22] and [23] for the segments of the quantum ring. Within a tight-binding framework, the Hamiltonian of a segment lying in an arbitrary direction γ in this quantum network in the presence of RSOC and DSOC can then be written as

$$\boldsymbol{H} = \frac{\hbar^2 k_\gamma^2}{2m^*} + \frac{\hbar^2 k_\gamma \alpha_{\rm SO}}{m^*}\boldsymbol{\sigma}_n, \tag{1}$$

in which $k_\gamma = -\mathrm{i}\partial/\partial l$ is the wave vector operator of the electrons along the direction γ, l is the relative coordinate of the electrons with respect to the origin of the segment, $\alpha_{\rm SO} = \sqrt{\alpha_{\rm R}^2 + \alpha_{\rm D}^2 + 2\alpha_{\rm R}\alpha_{\rm D}\sin(2\theta)}$ is the effective strength of the SOC, $\alpha_{\rm R}$ and $\alpha_{\rm D}$ are the strength of the RSOC and the DSOC, respectively, and the operator $\boldsymbol{\sigma}_n = \boldsymbol{\sigma}\cdot\boldsymbol{n}$, where $\boldsymbol{\sigma}$ represents the Pauli matrices. The direction vector

$$\boldsymbol{n} = \left(\frac{\alpha_{\rm R}}{\alpha_{\rm SO}}\sin\theta + \frac{\alpha_{\rm D}}{\alpha_{\rm SO}}\cos\theta, -\frac{\alpha_{\rm R}}{\alpha_{\rm SO}}\cos\theta - \frac{\alpha_{\rm D}}{\alpha_{\rm SO}}\sin\theta, 0\right).$$

When only the DSOC is considered, the Hamiltonian of a segment can be written as

$$\boldsymbol{H} = \frac{\hbar^2 k_\gamma^2}{2m^*} + \frac{\hbar^2 k_\gamma \alpha_{\rm D}}{m^*}\boldsymbol{\sigma}_n, \tag{2}$$

where the direction vector is $\boldsymbol{n} = (\cos\theta, -\sin\theta, 0)$.

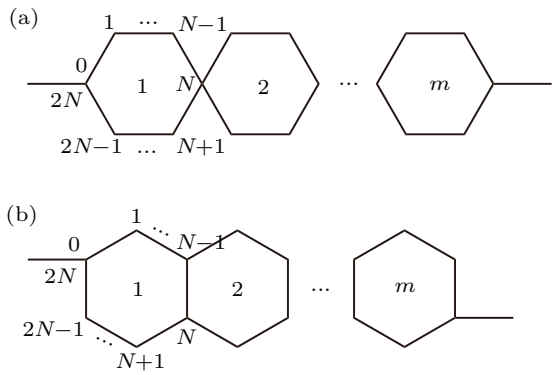


Fig. 1. Schematic diagram of a chain of hexagonal rings (a) when the rings are connected at nodes and (b) when the rings are connected along segments. There are m polygonal ring subunits in the chain. The chain is coupled to two leads and electrons are injected into the first ring and transmit through to the mth ring.

Based on the Hamiltonian of a segment, we can obtain the corresponding eigenstates of an electron in the segment

$$\Psi_{ab}(l) = \frac{\mathrm{e}^{\mathrm{i}\alpha_{\rm D} l\boldsymbol{\sigma}_n}}{\sin(kl_{ab})}\{\sin[k(l_{ab}-l)]\Psi_a + \sin(kl)\,\mathrm{e}^{-\mathrm{i}\alpha_{\rm D} l\boldsymbol{\sigma}_n}\Psi_b\}. \tag{3}$$

Here, Ψ_a and Ψ_b are the wavefunctions of the electrons at the two ends, a and b, of the segment, and l_{ab} is the length of the segment. Then, considering the conservation of probability current density, the current density should satisfy the constraint that the net current in the segment (in at a, out at b) is zero, i.e., $\sum_{\langle a,b\rangle} j_{a\to b} = 0$. Here, $\sum_{\langle a,b\rangle}$ is the sum of node b, taken to be the nearest neighbour of node a. Then, using Eq. (3), the current density can be expressed as

$$\sum_{\langle a,b\rangle} j_{a\to b} = \frac{\hbar}{2m^*\mathrm{i}}\sum_{\langle a,b\rangle}\left[\Psi^\dagger \mathrm{e}^{-\mathrm{i}\alpha_{\rm D} l\boldsymbol{\sigma}_n}\frac{\partial}{\partial l}(\mathrm{e}^{\mathrm{i}\alpha_{\rm D} l\boldsymbol{\sigma}_n}\Psi) - \frac{\partial}{\partial l}(\Psi^\dagger \mathrm{e}^{-\mathrm{i}\alpha_{\rm D} l\boldsymbol{\sigma}_n})\,\mathrm{e}^{\mathrm{i}\alpha_{\rm D} l\boldsymbol{\sigma}_n}\Psi\right]_{l=0}. \tag{4}$$

The boundary condition can then be simplified to

$$\sum_{\langle a,b\rangle}\frac{\partial}{\partial l}[\mathrm{e}^{\mathrm{i}\alpha_{\rm D} l\boldsymbol{\sigma}_n}\Psi_{ab}] = 0. \tag{5}$$

Substituting Eq. (3) into Eq. (5), we obtain

$$\sum_{\langle a,b\rangle}\cos(kl_{ab})\Psi_a - \sum_{\langle a,b\rangle} S_{ab}\Psi_b = 0, \tag{6}$$

where

$$S_{a,b} = \begin{pmatrix} S_{\uparrow\uparrow} & S_{\uparrow\downarrow} \\ S_{\downarrow\uparrow} & S_{\downarrow\downarrow} \end{pmatrix} = \begin{pmatrix} \cos(\alpha_{\rm D} l) & -\mathrm{i}\,\mathrm{e}^{\mathrm{i}\theta_{ab}}\sin(\alpha_{\rm D} l) \\ \mathrm{i}\,\mathrm{e}^{-\mathrm{i}\theta_{ab}}\sin(\alpha_{\rm D} l) & \cos(\alpha_{\rm D} l) \end{pmatrix}.$$

Here, S is the spin-resolved transfer matrix. Substituting the wavefunctions of the electrons in the input/output lead, $\psi_{\rm in} = A\,\mathrm{e}^{\mathrm{i}k_0 r} + B\,\mathrm{e}^{-\mathrm{i}k_0 r}$ and $\psi_{\rm out} = C\,\mathrm{e}^{\mathrm{i}k_0 r}$, into Eq. (5), we can obtain the boundary conditions at the input/output node

$$\sum_{\langle 0,0\rangle}\cos(kl)\Psi_0 - \sum_{\langle 0,b\rangle} S_{0b}\Psi_b = \mathrm{i}\frac{k_0(B-A)}{k}, \tag{7}$$

$$\sum_{\langle m,m\rangle}\cos(kl)\Psi_m - \sum_{\langle m,b\rangle} S_{mb}\Psi_b = \mathrm{i}\frac{k_0 C}{k}. \tag{8}$$

With the help of the Landauer–Büttiker conductance formula,[24,25] the spin conductance through the polygonal quantum ring can then be obtained in the form

$$G = \frac{\mathrm{e}^2}{h}\sum_{s=\uparrow,\downarrow}\frac{C_s^\dagger C_s}{A_s^\dagger A_s} = G_\uparrow + G_\downarrow, \tag{9}$$

where $G_\uparrow$ and $G_\downarrow$ are, respectively, the spin up and spin down conductances in the output lead of the mth polygonal ring.

3. Results and discussion

In this section, we concentrate on the physics of Eq. (9) by numerically analyzing the spin conductance for the two cases where the chain of polygonal rings is connected at nodes and when they are connected along segments. For the case where the chain is connected along segments, the spin conductance

through chains of hexagons with $m = 3$ subunits and chains of octagonal rings with $m = 3$, $m = 4$, and $m = 5$ subunits are shown in Fig. 2. Here, we set the spin direction of the incident electrons to be "up", and only one period, $[\pi, 2\pi]$, is shown for the incident electron energy kl.[26,27] Comparing Figs. 2(a) and 2(b), we note that the spin conductance shows a significant change in the oscillatory pattern when the number of segments increases. For Figs. 2(b)–2(d), it may be seen that the oscillation frequency of the spin conductance through a chain with the DSOC increases as the number of polygonal rings increases. This feature is similar to the conductances for a chain of rings without SOC.[28]

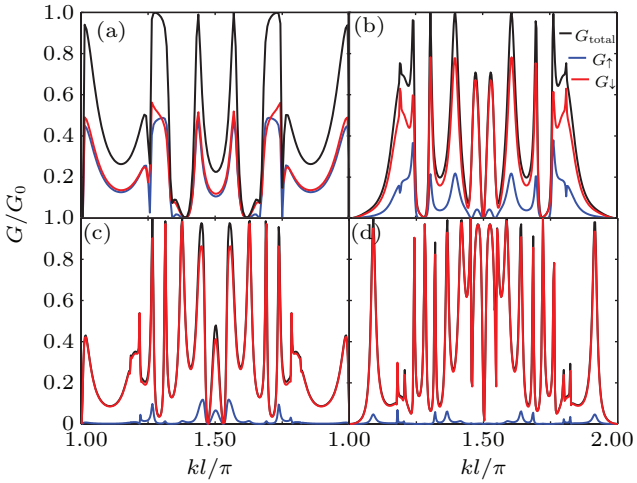


Fig. 2. (color online) The input electrons are taken to have $s = \uparrow$ in all cases. Spin conductance as a function of kl when a chain of hexagonal rings is connected along segments for (a) $m = 3$. Spin conductance as a function of kl when a chain of octagonal rings is connected along segments for (b) $m = 3$, (c) $m = 4$ and (d) $m = 5$ and the strength of the DSOC is $\alpha_{\rm D} = 0.05\pi$. We use $G_0 = 2e^2/h$ as the unit of conductance.

From Fig. 2, it is interesting to note that the conductance for the spin orientation opposite to that of the input surpasses the conductance for the incident spin orientation as the subunit number increases. For a single quantum ring, the frequency of precession of the electron increases with the increase of the strength DSOC. Then, more electrons with the spin down appear in the output terminal.[12] Thus, the Aharonov–Casher phase arises from spin precession due to the DSOC. The differential phase shift between the two spin components depends on the strength of SOC and the size of the system.[3] Then, with the increase of the subunit number, the frequency of precession of the electron increases. The spin flip is enhanced with the increase of the subunit number for the case where the chain is connected along segments. However, with a further increase of the subunit number, the effect of spin flip will be suppressed. The conductances of two spin components should oscillate with the increase of the subunit number. We show the spin conductance of a chain with $m = 5$ subunits in Fig. 2(d). It is of interest to note that the spin of the transmitted electrons is "down" near $kl = 1.5\pi$. This is a result of the precession of the electrons, which is controlled by the DSOC. Therefore, this chain of polygonal rings illustrates a theoretical method for designing a spin flip device.

In addition, when a chain of octagonal rings is connected along segments, the even–odd oscillations for the spin conductance can be seen in Fig. 2. There is a resonance peak in the case of $m = 4$ at $kl = 1.5\pi$, while there are resonance dips for $m = 3$ and $m = 5$. This phenomenon is called "even–odd oscillations" of the conductance. These peaks show a resonance shape due to the interference of electrons passing the chain and the existence of the DSOC. In our model, the phase of the transmitted electrons depends on three factors. The first is the initial phase of the electron injected into a chain. The second is the path difference of the electrons through various parts of the chain. This factor is determined by the geometry of the quantum network, including the type of connection (nodes or segments) between the polygonal rings and the shape of the individual polygonal rings. The third is the spin phase difference due to the precession of the electrons for different spin directions.[31] This factor is controlled by the DSOC. The even–odd oscillations of curves come from the competition of spin precession and quantum interference in the chain.

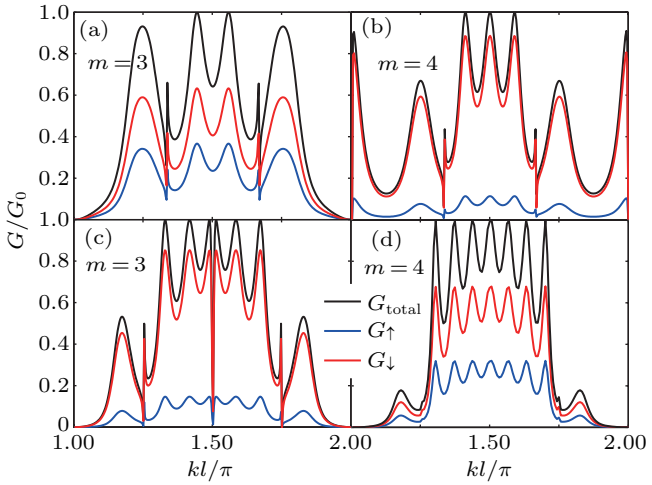


Fig. 3. (color online) The input electrons are taken to have $s = \uparrow$ in all cases. Spin conductance as a function of kl when a chain of hexagonal rings is connected at the nodes for (a) $m = 3$ and (b) $m = 4$. Spin conductance as a function of kl when a chain of octagonal rings is connected at nodes for (c) $m = 3$ and (d) $m = 4$. In all cases, the strength of the DSOC is $\alpha_{\rm D} = 0.05\pi$.

The spin conductances for a chain of hexagonal rings that are connected at nodes are shown in Fig. 3. The spin conductances through a chain of hexagons with $m = 3$ and $m = 4$ are shown in Figs. 3(a) and 3(b), respectively. The spin conductances through a chain of octagonal rings with $m = 3$ and $m = 4$ are shown in Figs. 3(c) and 3(d). Comparing Figs. 3(a) and 3(c), it is interesting to note that the spin conductance shows a significant change in the oscillatory pattern when the number of segments increases. It is interesting to note that the spin conductance through a single polygonal ring with DSOC

is independent of the number of the segments in the ring.[13] Therefore, unlike the spin conductance through a single polygonal ring with DSOC, the spin conductance for a chain with DSOC depends on the number of segments that make up the polygons.

In this part, we concentrate on the effect of DSOC on a chain of polygonal quantum rings which are also subject to RSOC. Firstly, we consider the case where the strength of the RSOC, α_R, and the strength of the DSOC, α_D, have the same value. The spin conductance versus kl when the polygonal rings of the chain are connected at nodes and when the polygonal rings of the chain are connected along segments are shown in Figs. 4(a) and 4(b) respectively. The spin phase difference induced by the DSOC is different from that induced by the RSOC. The effects of the DSOC are, in fact, like an effective magnetic field.[31] For a quantum ring, the eigenstates for the RSOC case alone and the DSOC case alone can be connected by a unitary transformation.[32] But the spin orientations of the corresponding eigenstates are different. When there is RSOC alone, the local spin orientation exhibits rotational symmetry. The projectional vectors of local spin orientations do not point to the center of the system in the chain subjected to the DSOC, and consequently, the cylindrical symmetry of the projectional vectors of the local spin orientations is lost.[11]

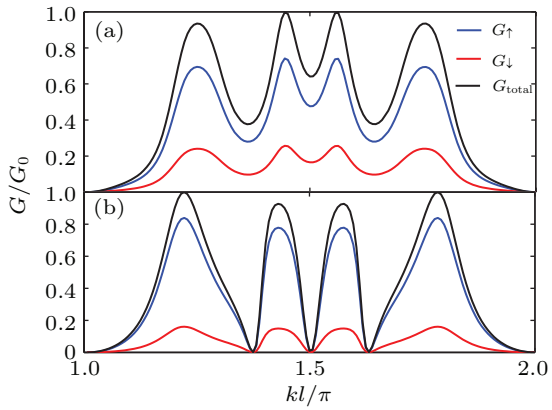


Fig. 4. (color online) Spin conductance as a function of kl (a) when a chain of hexagons is connected at the nodes ($m = 3$), (b) when a chain of squares is connected along segments ($m = 3$). The strengths of the RSOC and the DSOC have the same value $\alpha_R = \alpha_D = 0.02\pi$. Note that the total conductance for a chain without SOC overlaps the lines for $G_{\rm total}$.

Here, we should point out that the total conductance for a chain without SOC overlaps the lines for $G_{\rm total}$. When the two types of SOC have the same value, we can see that the total conductance for a chain with the same RSOC and DSOC is the same as the total conductance for a chain without SOC no matter which kind of connection type is assumed for the polygonal rings. For $\alpha_R = \alpha_D$, the SOC shifts down the entire spectrum of a chain by a constant value.[29] The SOC cannot affect the charge density for the Hamiltonian eigenstates. This is a result of the intrinsic symmetry of the Hamiltonian.[30] Unlike the conductance for a chain without SOC, the RSOC and the DSOC can each separately induce spin polarization in the conductance through a chain of polygonal rings. However, the total conductance for a chain of polygonal rings cannot be regulated by the SOC when the two types of SOC have the same value. Using this property, the strength of the DSOC can be measured using the known strength of the RSOC, which can be controlled accurately by an electric field.

Next, we concentrate on the total conductance in a chain of polygonal rings subject to both RSOC and DSOC when the two types of SOC have arbitrary values. The total conductance through a chain when hexagonal rings are connected at nodes is shown in Fig. 5. From Fig. 5, we can see that the pattern of the conductance is symmetric with respect to the lines $\alpha_R = 0$, which is different from the conductance of a polygon with RSOC and DSOC. The reason is that the effective magnetic field induced by the DSOC is complicated.[32] The z-rotational symmetry of local spin orientations is broken, and it is difficult to visualize the spin vectors intuitively. Therefore, the influence of the DSOC on the phase of the quantum interference in a chain of polygonal rings is complex. Then, the conductance of a chain with both the RSOC and the DSOC is different from the conductance of a single polygonal ring with both the RSOC and the DSOC.

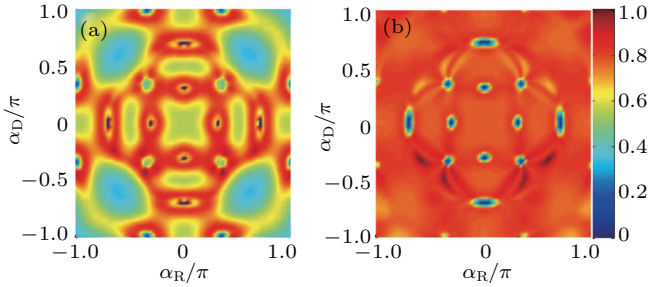


Fig. 5. (color online) When $kl = 2.5\pi$, total conductance as a function of the RSOC and the DSOC when a chain of hexagons is connected at nodes when (a) $m = 3$ and (b) $m = 4$.

As a realistic model that illustrates this discussion, we note that a spin interferometer based on square loops consisting of $In_{0.52}Al_{0.48}As/In_{0.53}Ga_{0.47}As/In_{0.52}Al_{0.48}As$ quantum wells with RSOC has been designed by Koga *et al.*[9] In addition, Meier *et al.*[10] have presented an experiment based on GaAs/InGaAs quantum wells for demonstrating spin resonance induced by oscillating spin–orbit fields such as the DSOC or RSOC. We would therefore like to stress that the use of the effects of DSOC on electron transport in regular polygonal quantum rings is within the reach of today's technology.

4. Summary

In this paper, using a standard transfer matrix method, we have investigated the spin transport properties of a chain of polygonal quantum rings which are subjected to DSOC. We

considered two cases: polygonal rings connected at nodes and polygonal rings connected along polygon segments. When only the DSOC is considered, spin quantum interference appears in both cases. In distinction from the conductance of a single polygonal ring, the conductance of a multi-polygon chain is dependent on the shape of the polygons. Due to the competition between spin precession and quantum interference, the spin conductance for the two cases shows different spectrums. When both the DSOC and the RSOC are considered in a chain of polygonal rings, we find that when the two types of SOC have the same value, the total conductance of a chain is the same as the conductance without SOC of either type. This property can be used to measure the strength of the DSOC using the strength of the RSOC as a reference. The case in which the two types of SOC have arbitrary values has also been discussed.

References

[1] Žutić I, Fabian J and Das Sarma S 2004 *Rev. Mod. Phys.* **76** 323
[2] Prinz G A 1998 *Science* **282** 1660
[3] Datta S and Das B 1990 *Appl. Phys. Lett.* **56** 665
[4] Vlaminck V and Bailleul M 2008 *Science* **322** 410
[5] Ando K, Takahashi S, Harii K, Sasage K, Ieda J, Maekawa S and Saitoh E 2008 *Phys. Rev. Lett.* **101** 036601
[6] Rashba E I 1960 *Sov. Phys. Solid State* **2** 1109
[7] Dresselhaus G 1955 *Phys. Rev.* **100** 580
[8] Bercioux D, Governale M, Cataudella V and Ramaglia V M 2004 *Phys. Rev. Lett.* **93** 056802
[9] Koga T, Sekine Y and Nitta J 2006 *Phys. Rev. B* **74** 041302(R)
[10] Meier L, Salis G, Shorubalko I, Gini E, Schön S and Ensslin K 2007 *Nat. Phys.* **3** 650
[11] Wang M and Chang K 2008 *Phys. Rev. B* **77** 125330
[12] Tang H Z, Zhai L X and Liu J J 2013 *J. Appl. Phys.* **114** 023702
[13] Tang H Z, Zhai L X, Shen M and Liu J J 2015 *Chin. Phys. B* **24** 030303
[14] Tang H Z, Zhai L X, Shen M and Liu J J 2014 *Phys. Lett. A* **378** 2790
[15] van Veenhuizen M J, Koga T and Nitta J 2006 *Phys. Rev. B* **73** 235315
[16] Molnár B, Vasilopoulos P and Peeters F M 2005 *Phys. Rev. B* **72** 075330
[17] Bercioux D, Governale M, Cataudella V and Ramaglia V M 2005 *Phys. Rev. B* **72** 075305
[18] Wang H X, Xiong S J and Evangeloub S N 2006 *Phys. Lett. A* **356** 376
[19] Wang H X and Xiong S J 2006 *Phys. Rev. B* **73** 045321
[20] Fu H H and Yao K L 2012 *Appl. Phys. Lett.* **100** 013502
[21] Sil S, Maiti S K and Chakrabarti A 2012 *J. Appl. Phys.* **112** 024321
[22] Xia J B 1992 *Phys. Rev. B* **45** 3593
[23] Mireles F and Kirczenow G 2001 *Phys. Rev. B* **64** 024426
[24] Büttiker M, Imry Y, Landauer R and Pinhas S 1985 *Phys. Rev. B* **31** 6207
[25] Landauer R 1970 *Philos. Mag.* **21** 863
[26] Aeberhard U, Wakabayashi K and Sigrist M 2005 *Phys. Rev. B* **72** 075328
[27] Zhai L X, Wang Y and Liu J J 2014 *J. Appl. Phys.* **116** 203703
[28] Li J B, Zhang Z Q and Liu Y Y 1997 *Phys. Rev. B* **55** 5337
[29] Nowak M P and Szafranc B 2009 *Phys. Rev. B* **80** 195319
[30] Duckheim M, Loss D, Scheid M, Richter K and Adagideli İ 2010 *Phys. Rev. B* **81** 085303
[31] Sun Q F, Wang J and Guo H 2005 *Phys. Rev. B* **71** 165310
[32] Sheng J S and Chang K 2006 *Phys. Rev. B* **74** 235315