Quantum statistical properties of photon-added spin coherent states

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The photon-added spin coherent state as a new kind of coherent state has been defined by iterated actions of the proper raising operator on the ordinary spin coherent state. In this paper, the quantum statistical properties of photon-added spin coherent states such as photon number distribution, second-order correlation function and Wigner function are studied. It is found that the Wigner function shows the negativity in some regions and the second-order correlation function is less than unity. Therefore, the photon-added spin coherent state is a nonclassical state.

Keywords: photon-added spin coherent states, photon number distribution, Wigner function

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1. Introduction

One of the most widely used and interesting quantum states in the field of quantum optics is the coherent state. The standard coherent states are related to the Heisenberg–Weyl group.[1] The generalized coherent states related to any arbitrary Lie group have been introduced and investigated by Perelomov.[2] Along these generalizations, the spin states analogous to the standard coherent states have been introduced by Radcliffe.[3] These states are related to the SU(2) group and have been receiving attention widely.[4–11] On the other hand, photon-added coherent states have been introduced by Agarwal and Tara and generated experimentally by Zavatta *et al.*[12,13] These states have received considerable attention and some generalizations on them take place.[14–16] Recently, analogous to photon added coherent states, the photon added spin coherent states have been introduced by Berrada.[17] Since nonclassical states attracted a great deal of interest in quantum computation and quantum information technology,[18–22] in the present paper, the nonclassical properties and statistical properties of these states are investigated.

The paper is organized as follows. In Section 2, the photon-added coherent states and their properties are reviewed, briefly. Next, some statistical properties of these states such as photon number distribution, second-order correlation function, and Wigner function are studied in Section 3. Finally, the paper is concluded in Section 4.

2. Photon-added spin coherent states

The standard spin coherent states have been introduced by acting spin raising operators J_+ on the ground state. The spin raising (J_+) and lowering operator (J_-) obey the $[J_+, J_-] = J_z$ commutation relation. The spin coherent states are defined as the complex rotation of the ground state by complex amplitude z and in the number state bases are given by[17]

$$|z,j\rangle = \frac{1}{(1+|z|^2)^j}\sum_{n=0}^{2j}\left[\frac{(2j)!}{n!(2j-n)!}\right]^{1/2} z^n|n\rangle, \tag{1}$$

where the number state $|n\rangle$ is related to the eigenstate of the J_z operator, i.e., $|j,m\rangle$ by $n = j+m$. Here, j is a positive and half-integer parameter and $m = j, j-1, \ldots, -j$. These states satisfy the minimal conditions to be a coherent state.

Analogous to ordinary photon-added coherent states, the photon-added spin coherent states have been introduced by Berrada by iterated actions (k times) of J_+ on the spin coherent state. The explicit form of photon-added spin coherent states is as follows:

$$|z,j\rangle_k = \mathcal{N}_k(|z|^2)\sum_{n=0}^{2j-k}\left[\frac{(2j)!(n+k)!}{(n!)^2(2j-n-k)!}\right]^{1/2} z^n\,|n+k\rangle, \tag{2}$$

where the normalization constant $\mathcal{N}_k(|z|^2)$ is given by

$$\mathcal{N}_k(|z|^2) = \left[\sum_{n=0}^{2j-k}\frac{(2j)!(n+k)!}{(n!)^2(2j-n-k)!}|z|^{2n}\right]^{-1/2}. \tag{3}$$

Berrada showed that these states satisfy the main axioms of Gazeau–Klauder coherent states.

3. Statistical properties

In this section, some of the statistical properties of photon-added spin coherent states are investigated. For this purpose, the photon number distribution, the second-order correlation function, and the Wigner function of photon-added spin coherent states are discussed.

3.1. Photon number distribution

The photon number distribution of a photon-added spin coherent state is given by

$$p(n) = |\langle n|z,j\rangle_k|^2$$

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$$= \mathcal{N}_k^2 \frac{(2j)!n!|z|^{2(n-k)}}{(n-k)!(2j-n)!}. \quad (4)$$

Photon number distributions of the photon-added spin coherent states have been plotted for several values of j and k in Fig. 1. Clearly, it is observed that by increasing the parameter j or k, the peak moves to higher values of photon number. The figure shows a wider photon number distribution for larger and smaller values of j and k, respectively.

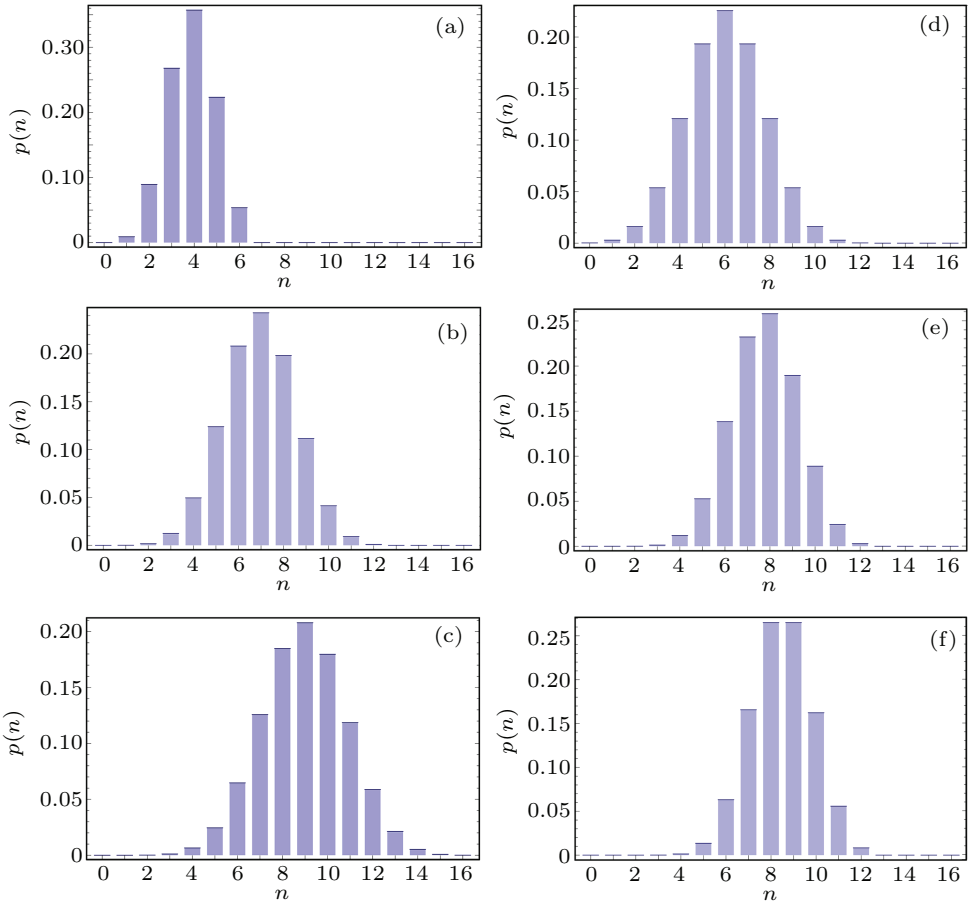


Fig. 1. (color online) Photon number distributions of photon-added spin coherent states with $|z| = 1$ for (a) $j = 3$, $k = 1$, (b) $j = 6$, $k = 1$, (c) $j = 8$, $k = 1$, (d) $j = 6$, $k = 0$, (e) $j = 6$, $k = 2$, and (f) $j = 6$, $k = 3$.

3.2. Second-order correlation function

Quantum statistics of a state can be characterized by the second-order correlation function which is obtained by[23]

$$g^{(2)}(0) = \frac{\langle a^{\dagger 2} a^2 \rangle}{\langle a^\dagger a \rangle^2}. \quad (5)$$

If $g^{(2)}(0) \geq 1$, the state shows classical behavior (bunching) and if $g^{(2)}(0) < 1$, the state represents nonclassical behavior (antibunching). The mean photon (quanta) number and $\langle a^{\dagger 2} a^2 \rangle$ for photon-added spin coherent states can be found as follows:

$$\langle a^\dagger a \rangle = \mathcal{N}_k^2 \sum_{n=0}^{2j-k} \frac{(2j)!(n+k)!(n+k)}{(n!)^2(2j-n-k)!}|z|^{2n}, \quad (6)$$

$$\langle a^{\dagger 2} a^2 \rangle = \langle n(n-1) \rangle = \mathcal{N}_k^2 z^2 \sum_{n=0}^{2j-k} \frac{(2j)!(n+k)!(n+k)(n+k-1)}{(n!)^2(2j-n-k)!}|z|^{2n}. \quad (7)$$

The second-order correlation function versus the amplitude $|z|$ for various values of photon added number k and spin number j has been plotted in Fig. 2. From the figure it is seen that, $g^{(2)}(0) < 1$ for all values of $|z|$, k and j, which corresponds to the nonclassical behavior. Interestingly, the second-order correlation function for high values of $|z|$, tends to a fixed value ($\simeq 0.83$ for $j = 3$). This fixed value depends on the spin number j, but it is less than one for all values of j.

3.3. Wigner function

The Wigner function is one of the main tools to investigate the behavior of a quantum state and its nonclassicality.[24] Partial negativity of the Wigner function indicates the nonclassical feature of the quantum state. The Wigner function of a single-mode system in the coherent state representation β can be expressed as[25]

$$W(\alpha) = \frac{2}{\pi^2} e^{2|\alpha|^2} \int \langle -\beta|\rho|\beta\rangle e^{-2(\beta\alpha^* - \beta^*\alpha)} d^2\beta, \quad (8)$$

where $\alpha = (q + \mathrm{i}p)/\sqrt{2}$ and $\rho = |z, j\rangle_{kk}\langle z, j|$ is the density operator for photon-added spin coherent states. Substituting Eq. (2) into Eq. (8) and recalling the integral equation, we obtain

$$\int f(\xi) \exp\left[\xi x + \xi^* y - z|\xi|^2\right] \mathrm{d}^2\xi = \frac{\pi}{z} \exp\left[\frac{xy}{z}\right] f(y/z), \quad \mathrm{Re}(z) > 0, \tag{9}$$

and the Wigner function of photon-added spin coherent states is given by

$$W(\alpha) = \frac{2}{\pi} \mathrm{e}^{-2|\alpha|^2} \mathcal{N}_k^2 \sum_{n=0}^{2j-k} \sum_{m=0}^{2j-k} \times \left[\frac{(2j)!}{n!(2j-n-k)!}\right]^{1/2} \left[\frac{(2j)!}{m!(2j-m-k)!}\right]^{1/2} \times z^n z^{*m} (-2\alpha^*)^n (2\alpha)^m. \tag{10}$$

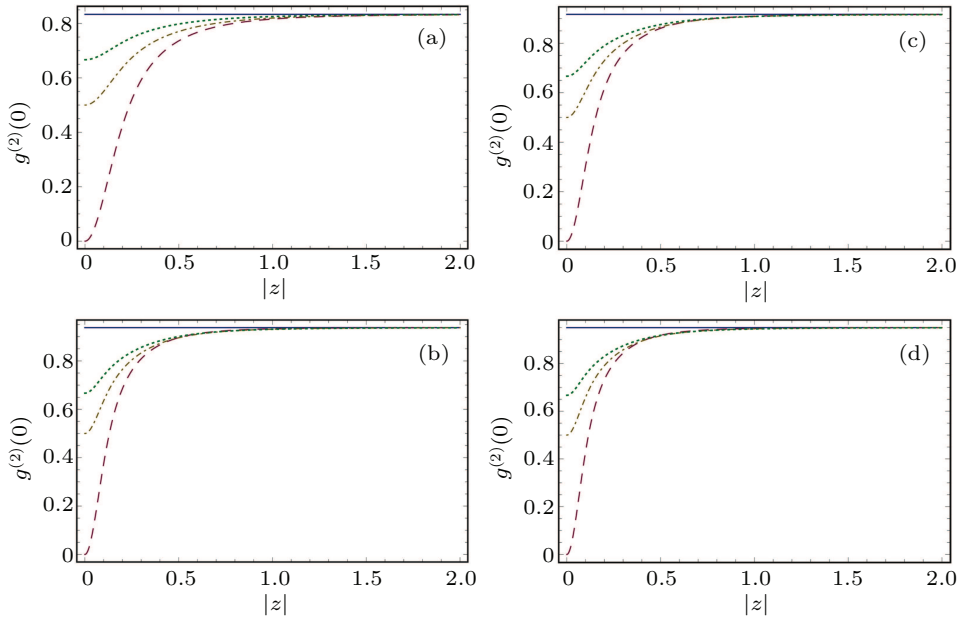


Fig. 2. (color online) Plot of the second-order correlation function of photon-added spin coherent states versus $|z|$ with for $k = 0$ (solid line), $k = 1$ (dashed line), $k = 2$ (dashed-dotted line), and $k = 3$ (dotted line) for (a) $j = 3$, (b) $j = 6$, (c) $j = 8$, and (d) $j = 10$.

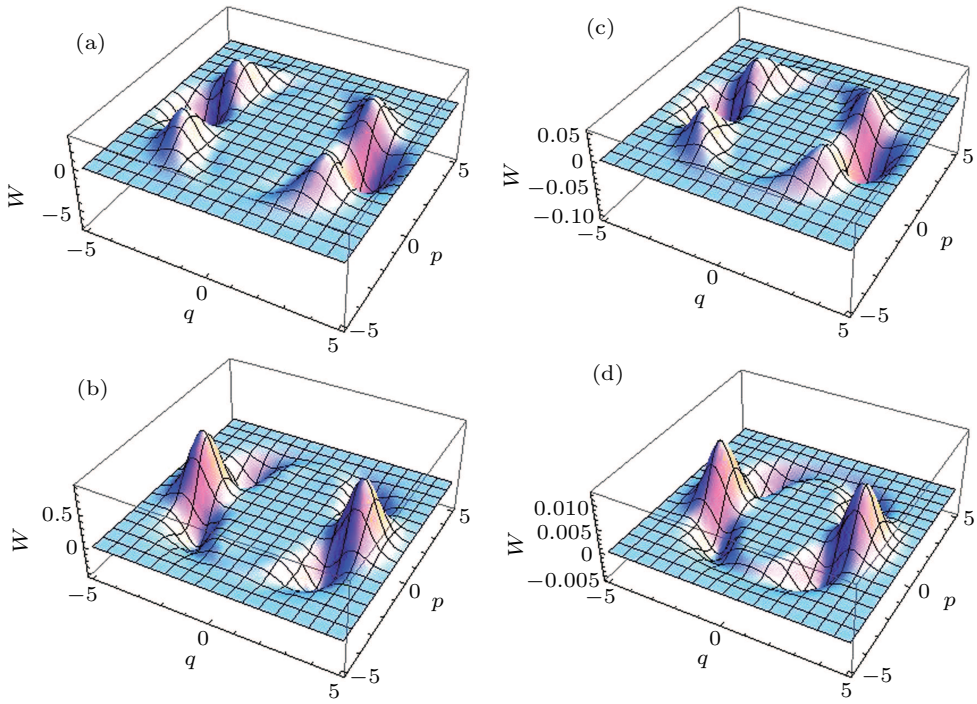


Fig. 3. (color online) Plot of the Wigner function of photon-added spin coherent states as a function of q and p with $j = 6$ and $|z| = 1$ for (a) $k = 0$, (b) $k = 1$, (c) $k = 2$, and (d) $k = 3$.

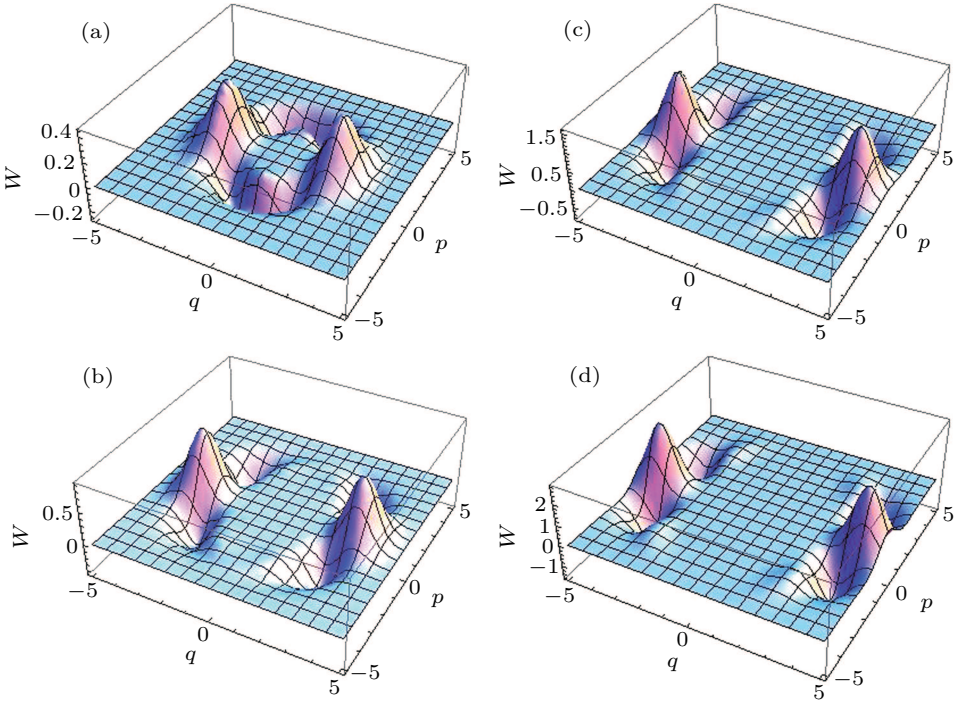


Fig. 4. (color online) Plot of the Wigner function of photon-added spin coherent states as a function of q and p with $k = 1$ and $|z| = 1$ for (a) $j = 3$, (b) $j = 6$, (c) $j = 8$, and (d) $j = 10$.

Figure 3 shows the Wigner function as a function of q and p for various values of the photon added number k, keeping j and $|z|$ fixed at 6 and 1, respectively. It is clear that the Wigner function is sensitive to the photon added number k. Obviously, the Wigner function is negative in some regions of the q–p plane for all values of k, indicating the nonclassicality behavior of photon-added spin coherent states. The Wigner function for various values of spin number j has been plotted in Fig. 4, keeping both k and $|z|$ fixed at 1. Similar to Fig. 3, the Wigner function is negative in some regions of the q–p plane for all values of j. From the figure, we find that upward and downward peaks occur at larger values of q as j increases.

4. Summary

In summary, we study the statistical properties of the photon-added spin coherent states through the photon number distribution, second-order correlation function, and Wigner function. It is found that the photon added number k and spin number j affect these properties. In addition, the results show that the Wigner function is negative in some regions and the second-order correlation function represents antibunching behavior and so, the photon-added coherent state can be considered as a nonclassical state.

In general, the photon-added coherent state is an intermediate state between a (classical) coherent state and a (full quantum) Fock state. Here, adding photon (quanta) to the spin coherent states is a proper way to construct the photon-added spin coherent states and these states show more nonclassicality. This opportunity helps us to manipulate a nonclassical state with more control on its nonclassicality.

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