Quantum coherence preservation of atom with a classical driving field under non-Markovian environment*

De-Ying Gao(高德营)[1,2], Qiang Gao(高强)[1], and Yun-Jie Xia(夏云杰)[1,†]

[1] *Shandong Provincial Key Laboratory of Laser Polarization and Information Technology, Department of Physics, Qufu Normal University, Qufu* 273165, *China*

[2] *College of Dong Chang, Liaocheng University, Liaocheng, Shandong* 252000, *China*

(Received 5 May 2017; revised manuscript received 25 July 2017; published online 29 September 2017)

The exact dynamics of an open quantum system consisting of one qubit driven by a classical driving field is investigated. Our attention is focused on the influences of single- and two-photon excitations on the dynamics of quantum coherence and quantum entanglement. It is shown that the atomic coherence can be improved or even maintained by the classical driving field, the non-Markovian effect, and the atom-reservoir detuning. The interconversion between the atomic coherence and the atom-reservoir entanglement exists and can be controlled by the appropriate conditions. The conservation of coherence for different partitions is explored, and the dynamics of a system with two-photon excitations is different from the case of single- photon excitation.

Keywords: quantum coherence, non-Markovian process, classical driving field

PACS: 03.67.–a, 03.65.–w, 03.65.Yz **DOI:** 10.1088/1674-1056/26/11/110303

1. Introduction

Quantum coherence and quantum entanglement are the basic concepts of quantum physics, which are important features of quantum physics. However, compared with the entanglement in quantum information processing,[1–6] quantum coherence has been studied little. Quantum coherence is caused by the superposition of quantum states, which means that the wave of the quantum state is divided into two parts, and the interference occurs between the two waves, and the quantum state is formed from the superposition of the two waves. It is generally believed that quantum coherence is a necessary precondition for various nonclassical effects such as quantum phase transition,[7,8] quantum algorithms,[9] quantum interference,[10,11] and quantum thermodynamics.[12–16] Although quantum coherence is important, the measure theory of quantum coherence has not been developed until only very recently and some necessary constraints have been put forward to ensure that quantum coherence is a physical resource.[17] Subsequently, lots of methods of measuring the quantum coherence have been proposed, such as the l_1 norm and relative entropy of coherence,[17] skew information,[18] etc. For the dynamics of quantum coherence in an open quantum system, one of the most interesting phenomena is its freezing in some special initial states and dynamic conditions,[19,20] which means that the coherence will remain exactly constant (frozen) during the whole evolution. Another interesting phenomenon is coherence trapping,[21,22] and it is different from the coherence freezing, which means that the stationary state has a nonzero coherence at $t \to \infty$, and it can only occur in the presence of non-Markovian dynamics.

As is well known, the coherence is a prerequisite to the existence of quantum entanglement and some nonclassical effects, such as squeezing and spin squeezing, and can be converted into entanglement by a beam splitter.[23–25] Recently, any nonzero amount of coherence in a system can be converted into entanglement between the system and an initially incoherent ancilla by incoherent operation, which means that quantum coherence and quantum entanglement are equivalent in quantity.[26] Another study has presented the relation between quantum coherence and quantum discord in a multipartite system; it is proved that the quantum coherence is equivalent to the quantum discord.[27]

Under the non-Markovian environment, quantum entanglement and quantum discord have been studied more than quantum coherence,[28] in addition, the influence of classical driving field on quantum coherence has not been studied as far as we know. In the paper, we investigate the interaction between a two-level atom and a non-Markovian reservoir with single- and two-photon excitations. The influences of the classical driving field and the non-Markovian effect on the atomic coherence, the atom-reservoir entanglement and the reservoir coherence are explored. Besides, the interconversion between quantum coherence and quantum entanglement and the conservation of coherence for different partitions of two identical single-body systems are also explored. The dynamics of a system with two-photon excitations is discussed, and its dynamics is different from the case of single-photon excitation. The rest of this paper is organized as follows. In Section 2, we

*Project supported by the National Natural Science Foundation of China (Grant Nos. 61675115, 11204156, 11574178, 11304179, and 11647172) and the Science and Technology Plan Projects of Shandong University, China (Grant No. J16LJ52).

†Corresponding author. E-mail: yjxia@mail.qfnu.edu.cn

© 2017 Chinese Physical Society and IOP Publishing Ltd

present the theoretical model of the system with single- and two-photon excitations. Based on the analysis of the model, the expressions for quantum coherence and quantum entanglement are obtained. In Section 3, the effects of single- and two-photon excitations on coherence and entanglement are explored. Finally, the conclusion is drawn from the present studies in Section 4.

2. Theoretical model

The system consists of a two-level atom locally interacting with one zero-temperature bosonic reservoir, and the atom with frequency ω_0 is driven by a classical driving field with frequency $\omega_{\rm L}$ additionally. The Hamiltonian of the total system is described by (in units of $\hbar = 1$)[29]

$$\begin{aligned} H &= \frac{1}{2}\omega_0\sigma_z + \sum_k \omega_k b_k^+ b_k + \sum_k g_k(b_k\sigma_+ + b_k^+\sigma_-) \\ &\quad + \Omega(\mathrm{e}^{-\mathrm{i}\omega_{\rm L}t}\sigma_+ + \mathrm{e}^{\mathrm{i}\omega_{\rm L}t}\sigma_-), \end{aligned} \tag{1}$$

where $b_k^+(b_k)$ is the creation(annihilation) operator of the kth mode with frequency ω_k of the reservoir. In addition, g_k is the coupling constant of the atom with the kth mode of reservoir. $\sigma_+ = |e\rangle\langle g|$, $\sigma_- = |g\rangle\langle e|$, and $\sigma_z = |e\rangle\langle e| - |g\rangle\langle g|$ denote the raising, lowering and inversion operators of the atom, respectively, while Ω refers to the Rabi frequency of the driving field.

The unitary transformation of the Hamiltonian of the system does not change the eigenvalues of the system. Under a unitary transformation $U_1 = \mathrm{e}^{-\mathrm{i}\omega_{\rm L}\sigma_z t/2}$, the Hamiltonian of the system is transformed into H_1 as

$$\begin{aligned} H_1 &= \mathrm{i}\hbar\frac{\mathrm{d}U_1^+}{\mathrm{d}t}U_1 + U_1^+ H U_1 \\ &= \frac{\Delta}{2}\sigma_Z + \Omega(\sigma_+ + \sigma_-) + \sum_k \omega_k b_k^+ b_k \\ &\quad + \sum_k g_k(\mathrm{e}^{\mathrm{i}\omega_{\rm L}t}\sigma_+ b_k + \mathrm{e}^{-\mathrm{i}\omega_{\rm L}t}\sigma_- b_k^+) \\ &= H_1^{(1)} + H_1^{(2)}, \end{aligned} \tag{2}$$

with

$$\begin{aligned} H_1^{(1)} &= \frac{\Delta}{2}\sigma_Z + \Omega(\sigma_+ + \sigma_-), \\ H_1^{(2)} &= \sum_k \omega_k b_k^+ b_k + \sum_k g_k(\mathrm{e}^{\mathrm{i}\omega_{\rm L}t}\sigma_+ b_k + \mathrm{e}^{-\mathrm{i}\omega_{\rm L}t}\sigma_- b_k^+), \end{aligned}$$

where $\Delta = \omega_0 - \omega_{\rm L}$ denoting the frequency detuning of atom and the classical driving field. Using the method similar to that used in Ref. [30], diagonalizing the Hamiltonian $H_1^{(1)}$, $H_1^{(1)} = \Lambda S_Z/2$, with $\Lambda = \sqrt{\Delta^2 + 4\Omega^2}$, $S_Z = |+\rangle\langle +| - |-\rangle\langle -|$, S_Z being the inversion operator of the atom in the presence of the dressed-states, and $|+\rangle = \cos(\theta/2)|e\rangle + \sin(\theta/2)|g\rangle$ and $|-\rangle = \cos(\theta/2)|g\rangle - \sin(\theta/2)|e\rangle$ being the excited and ground states of the atom in the presence of the dressed-states with $\theta = \arg\tan(2\Omega/\Delta)$.

For $H_1^{(2)}$, neglecting the terms which do not conserve energies (rotating wave approximation), we have $H_1^{(2)} = \sum_k \omega_k b_k^+ b_k + \cos^2(\theta/2)\sum_k (g_k \mathrm{e}^{\mathrm{i}\omega_{\rm L}t} b_k S_+ + \mathrm{h.c.})$, with $S_+ = |+\rangle\langle -|$ and $S_- = |-\rangle\langle +|$ being the raising and lowering operators of the atom in the presence of the dressed-states.

The Hamiltonian is

$$H = \frac{\Lambda}{2}S_Z + \sum_k \omega_k b_k^+ b_k + \cos^2\frac{\theta}{2}\sum_k (g_k \mathrm{e}^{\mathrm{i}\omega_{\rm L}t} b_k S_+ + \mathrm{h.c.}), \tag{3}$$

in the interaction representation, and the Hamiltonian can be rewritten as

$$H_{\rm I}' = \cos^2\frac{\theta}{2}\sum_k (g_k b_k S_+ \mathrm{e}^{-\mathrm{i}(\omega_k - \omega_{\rm L} - \Lambda)t} + \mathrm{h.c.}). \tag{4}$$

2.1. Single-photon excitation in the whole system

Considering the single-photon excitation in the whole system, in the presence of the dressed-states, the initial state of the system is

$$\begin{aligned} |\psi(0)\rangle &= C_0|-\rangle_{\rm S}|0\rangle_{\rm R} + C_1(0)|+\rangle_{\rm S}|0\rangle_{\rm R} \\ &\quad + \sum_k C_k(0)|-\rangle_{\rm S}|1_k\rangle_{\rm R}, \end{aligned} \tag{5}$$

where $|0\rangle_{\rm R}$ is the vacuum state of the reservoir, and $|1_k\rangle_{\rm R}$ indicates that there exists a photon in the kth mode of the reservoir.

At the time t, the state of the system is

$$\begin{aligned} |\psi(t)\rangle &= C_0|-\rangle_{\rm S}|0\rangle_{\rm R} + C_1(t)|+\rangle_{\rm S}|0\rangle_{\rm R} \\ &\quad + \sum_k C_k(t)|-\rangle_{\rm S}|1_k\rangle_{\rm R}, \end{aligned} \tag{6}$$

for $H_{\rm I}'|-\rangle_{\rm S}|0\rangle_{\rm R} = 0$, so C_0 is a constant, the evolution of $C_1(t)$ and $C_k(t)$ can be obtained by the Schrodinger equation as follows:

$$\dot{C}_1(t) = -\mathrm{i}\cos^2\frac{\theta}{2}\sum_k g_k \mathrm{e}^{-\mathrm{i}(\omega_k - \omega_{\rm L} - \Lambda)t} C_k(t), \tag{7}$$

$$\dot{C}_k(t) = -\mathrm{i}\cos^2\frac{\theta}{2} g_k^* \mathrm{e}^{\mathrm{i}(\omega_k - \omega_{\rm L} - \Lambda)t} C_1(t). \tag{8}$$

Assuming $C_k(0) = 0$, through integrating $\dot{C}_k(t)$ of Eq. (8), then substitute the resulting expression into Eq. (7), we can obtain

$$\dot{C}_1(t) = -\cos^4\frac{\theta}{2}\int_0^t \mathrm{d}t_1 f(t - t_1)C_1(t_1), \tag{9}$$

where $f(t - t_1) = \int \mathrm{d}\omega J(\omega)\mathrm{e}^{\mathrm{i}(\Lambda + \omega_{\rm L} - \omega)(t - t_1)}$, which is a correlation function of the environment, it is obvious that the $f(t - t_1)$ is determined by the reservoir spectral density $J(\omega)$, and the spectral distribution of the Lorentzian form[31] is considered as

$$J(\omega) = \frac{1}{2\pi}\frac{\gamma_0\lambda^2}{(\omega_0 - \omega - \delta)^2 + \lambda^2}. \tag{10}$$

In Eq. (10), δ is the detuning between the atom transition frequency and center frequency of the structured spectrum, the

parameter λ defines the spectral width of the coupling and is related to the reservoir correlation time $T_{\rm E}$ by the relation $T_{\rm E} \approx \lambda^{-1}$, and the other parameter γ_0 is the decay of the excited atomic state in the Markovian limit of flat spectrum and is connected to the system relaxation time $T_{\rm S}$ by the relation $\gamma_0 \approx T_{\rm S}^{-1}$. Generally, the Markovian regime corresponds to the $\lambda > 2\gamma_0$, irreversible decay of the behavior of the system occurs, and in the non-Markovian regime there is $\lambda < 2\gamma_0$, the behavior of the system displays non-Markovian dynamics.[32]

According to the above $J(\omega)$, we can obtain the correlation function

$$f(t-t_1) = \frac{\gamma_0\lambda}{2}\,\mathrm{e}^{-(\lambda+\mathrm{i}\Delta-\mathrm{i}\Lambda-\mathrm{i}\delta)(t-t_1)}, \tag{11}$$

then the solution of Eq. (9) can be obtained as follows:

$$C_1(t) = C_1(0)q(t), \tag{12}$$

with

$$q(t) = \mathrm{e}^{\frac{-at}{2}}\left(\cosh\left(\frac{bt}{4}\right) + \frac{2a}{b}\sinh\left(\frac{bt}{4}\right)\right),$$
$$a = \lambda + \mathrm{i}\Delta - \mathrm{i}\delta - \mathrm{i}\Lambda,$$

and

$$b = \sqrt{4a^2 - 2\gamma_0\lambda(1+\cos\theta)^2}.$$

Suppose that the initial state of the system is

$$|\phi(0)\rangle = \left(\cos\frac{\eta}{2}|e\rangle + \sin\frac{\eta}{2}\,\mathrm{e}^{-\mathrm{i}\varphi}|g\rangle\right)_{\rm S} \otimes |0\rangle_{\rm R},$$

based on the previous analysis, then the quantum state at the time t will be

$$|\phi(t)\rangle = C_0|-\rangle|0\rangle_{\rm R} + C_1(t)|+\rangle|0\rangle_{\rm R} + D(t)|-\rangle|\tilde{1}_k\rangle_{\rm R}, \tag{13}$$

with

$$|\tilde{1}_k\rangle = \frac{\sum_k C_k|1_k\rangle_{\rm R}}{D(t)},\quad C_0 = -\cos\frac{\eta}{2}\sin\frac{\theta}{2} + \sin\frac{\eta}{2}\cos\frac{\theta}{2}\,\mathrm{e}^{-\mathrm{i}\varphi},$$
$$C_1(t) = C_1(0)q(t),\quad D(t) = \sqrt{1-|C_0|^2-|C_1(t)|^2},$$

and

$$C_1(0) = \cos\frac{\eta}{2}\cos\frac{\theta}{2} + \sin\frac{\eta}{2}\sin\frac{\theta}{2}\,\mathrm{e}^{-\mathrm{i}\varphi}.$$

Considering the atom and the bosonic reservoir as a bipartite system, in the basis $(|+,\tilde{1}_k\rangle, |+,0\rangle, |-,\tilde{1}_k\rangle, |-,0\rangle)$, the density matrix of the system is

$$\begin{aligned}\rho &= |\phi(t)\rangle\langle\phi(t)| \\ &= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \rho_{22} & \rho_{23} & \rho_{24} \\ 0 & \rho_{32} & \rho_{33} & \rho_{34} \\ 0 & \rho_{42} & \rho_{43} & \rho_{44} \end{pmatrix} \\ &= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & |C_1(t)|^2 & D(t)C_1(t) & C_1(t)C_0^* \\ 0 & D(t)C_1^*(t) & D(t)^2 & D(t)C_0^* \\ 0 & C_1^*(t)C_0 & D(t)C_0 & |C_0|^2 \end{pmatrix},\end{aligned} \tag{14}$$

tracing over the reservoir degree of freedom, the reduced density matrix of the atom in the basis $(|+\rangle, |-\rangle)$ is obtained as follows:

$$\rho_a(t) = \mathrm{Tr}_{\rm R}(\rho) = \begin{pmatrix} |C_1(t)|^2 & C_1(t)C_0^* \\ C_0C_1^*(t) & |C_0|^2 + D(t)^2 \end{pmatrix}, \tag{15}$$

in the basis $(|e\rangle, |g\rangle)$, the reduced density matrix of the atom

$$\rho_a = \begin{pmatrix} \rho_{ee} & \rho_{eg} \\ \rho_{ge} & \rho_{gg} \end{pmatrix},$$

with matrix elements being

$$\begin{aligned}\rho_{ee} &= \sin^2\frac{\theta}{2} - C_0C_1^*(t)\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\ &\quad - C_0^*C_1(t)\sin\frac{\theta}{2}\cos\frac{\theta}{2} + |C_1(t)|^2\cos\theta, \\ \rho_{eg} &= \rho_{ge}^* = -\sin\frac{\theta}{2}\cos\frac{\theta}{2} - C_0C_1^*(t)\sin^2\frac{\theta}{2} \\ &\quad + C_0^*C_1(t)\cos^2\frac{\theta}{2} + |C_1(t)|^2\sin\theta, \\ \rho_{gg} &= \cos^2\frac{\theta}{2} + C_0C_1^*(t)\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\ &\quad + C_0^*C_1(t)\sin\frac{\theta}{2}\cos\frac{\theta}{2} - |C_1(t)|^2\cos\theta.\end{aligned} \tag{16}$$

Using a similar method to the above, we can obtain the reduced density matrix of the bosonic reservoir

$$\rho_{\rm R} = \begin{pmatrix} \rho_{00} & \rho_{0\tilde{1}_k} \\ \rho_{\tilde{1}_k0} & \rho_{\tilde{1}_k\tilde{1}_k} \end{pmatrix},$$

in the basis $(|0\rangle, |\tilde{1}_k\rangle)$,

$$\begin{aligned}&\rho_{00} = |C_0|^2 + |C_1(t)|^2, \\ &\rho_{0\tilde{1}_k} = \rho_{\tilde{1}_k0}^* = C_0\sqrt{1-|C_0|^2-|C_1(t)|^2}, \\ &\rho_{\tilde{1}_k\tilde{1}_k} = 1-|C_0|^2-|C_1(t)|^2.\end{aligned} \tag{17}$$

Our aim is to obtain the dynamics of quantum coherence for the atom and the bosonic reservoir, so we adopt the l_1 norm of coherence

$$C_{l_1}(\rho) = \sum_{i,j,i\neq j}|\rho_{ij}|. \tag{18}$$

The quantum coherence of the atom and the bosonic reservoir can be easily obtained according to Eqs. (16)–(18).

For the atom

$$\begin{aligned}C_{l_1}(\rho) &= \Big|2C_0^*C_1(t)\cos^2\frac{\theta}{2} + (2|C_1(t)|^2 - 1)\sin\theta \\ &\quad - 2C_0C_1^*(t)\sin^2\frac{\theta}{2}\Big|,\end{aligned} \tag{19}$$

and for the bosonic reservoir

$$C_{l_1}(\rho) = 2|C_0\sqrt{1-|C_0|^2-|C_1(t)|^2}|. \quad (20)$$

In order to study the entanglement evolution between the two-level atom and the bosonic reservoir, a feasible method is to calculate the linear entropy of the two-level atom[33]

$$S = 1-\mathrm{Tr}(\rho_a(t)^2) = 2|C_1(t)|^2(1-|C_0|^2-|C_1(t)|^2). \quad (21)$$

2.2. Two-photon excitation in the whole system

Considering two-photon excitation in the whole system, the initial state of the system is

$$|\psi(0)\rangle = \left(\cos\frac{\eta}{2}|e\rangle + \sin\frac{\eta}{2}\mathrm{e}^{-\mathrm{i}\varphi}|g\rangle\right)_{\mathrm{S}} \otimes |1\rangle_{\mathrm{R}}, \quad (22)$$

at time t, the state of the quantum system is

$$\begin{aligned}|\psi(t)\rangle =\ & b_1(t)|-\rangle_{\mathrm{S}}|1\rangle_{\mathrm{R}} + b_2(t)|+\rangle_{\mathrm{S}}|0\rangle_{\mathrm{R}} + b_3(t)|+\rangle_{\mathrm{S}}|1\rangle_{\mathrm{R}} \\ & + b_4(t)|-\rangle_{\mathrm{S}}|2\rangle_{\mathrm{R}}, \end{aligned} \quad (23)$$

with

$$\begin{aligned} b_1(0) &= -\cos\frac{\eta}{2}\sin\frac{\theta}{2} + \sin\frac{\eta}{2}\cos\frac{\theta}{2}\mathrm{e}^{-\mathrm{i}\varphi}, \\ b_2(0) &= b_4(0) = 0, \\ b_3(0) &= \cos\frac{\eta}{2}\cos\frac{\theta}{2} + \sin\frac{\eta}{2}\sin\frac{\theta}{2}\mathrm{e}^{-\mathrm{i}\varphi}. \end{aligned}$$

In order to find the dynamics of the system with two-photon excitation, we solve the master equation by using the pseudomode approach.[34,35] The exact dynamics of the atom interacting with a Lorentzian structured reservoir is contained in the following pseudomode master equation:

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -\mathrm{i}[H,\rho] - \frac{\Gamma}{2}\left[b^+b\rho - 2b\rho b^+ + \rho b^+b\right],$$

where

$$H = \Lambda S_+S_- + \omega_{\mathrm{c}}b^+b + \cos^2\frac{\theta}{2}\Omega_0(S_+b + S_-b^+), \quad (24)$$

where ρ is the density operator for the atom and the pseudomode of the structured reservoir, b^+ and b are the creation and annihilation operators of the pseudomode of the structured reservoir; constants ω_{c} and Γ are, respectively, the oscillation frequency and the decay rate of the pseudomode and they depend on the position of the pole $z = \omega_{\mathrm{c}} - \mathrm{i}\Gamma/2$. The atom interacts coherently with the pseudomode (the strength of the coupling is Ω_0). In order to find the dynamics of the system, we solve the master equation (Eq. (24)). Then, tracing out the pseudomode degree of freedom, we obtain the reduced density matrix of the atomic system as follows:

$$\rho_a = \begin{pmatrix} \rho_{ee} & \rho_{eg} \\ \rho_{ge} & \rho_{gg} \end{pmatrix},$$

with matrix elements

$$\begin{aligned} \rho_{ee} =\ & (b_2b_2^* + b_3b_3^*)\cos^2\frac{\eta}{2} + (b_1b_1^* + b_4b_4^*)\sin^2\frac{\eta}{2} \\ & - b_3b_1^*\cos\frac{\eta}{2}\sin\frac{\eta}{2} - b_1b_3^*\cos\frac{\eta}{2}\sin\frac{\eta}{2}, \\ \rho_{eg} =\ & (b_2b_2^* + b_3b_3^*)\cos\frac{\eta}{2}\sin\frac{\eta}{2} - (b_1b_1^* + b_4b_4^*)\cos\frac{\eta}{2}\sin\frac{\eta}{2} \\ & + b_3b_1^*\cos^2\frac{\eta}{2} - b_1b_3^*\sin^2\frac{\eta}{2}, \\ \rho_{ge} =\ & \rho_{eg}^*, \\ \rho_{gg} =\ & (b_2b_2^* + b_3b_3^*)\sin^2\frac{\eta}{2} + (b_1b_1^* + b_4b_4^*)\cos^2\frac{\eta}{2} \\ & + b_3b_1^*\cos\frac{\eta}{2}\sin\frac{\eta}{2} + b_1b_3^*\cos\frac{\eta}{2}\sin\frac{\eta}{2}. \end{aligned}$$

By using a similar method to the above, we can obtain the reduced density matrix of the bosonic reservoir

$$\rho_{\mathrm{R}} = \begin{pmatrix} \rho_{00} & \rho_{01} & \rho_{02} \\ \rho_{10} & \rho_{11} & \rho_{12} \\ \rho_{20} & \rho_{21} & \rho_{22} \end{pmatrix},$$

in the basis $(|0\rangle, |\tilde{1}_k\rangle, |\tilde{2}_k\rangle)$.

$$\begin{aligned} &\rho_{00} = |b_2(t)|^2, \ \ \rho_{01} = \rho_{10}^* = b_2b_3^*, \\ &\rho_{11} = b_3b_3^* + b_1b_1^*, \ \ \rho_{12} = \rho_{21}^* = b_1b_4^*, \\ &\rho_{22} = b_4b_4^*, \ \ \rho_{02} = \rho_{20} = 0. \end{aligned}$$

The l_1 norm of coherence for the atom,

$$\begin{aligned} C_{l_1}(\rho) =\ & \Big|(b_2b_2^* + b_3b_3^*)\cos\frac{\eta}{2}\sin\frac{\eta}{2} \\ & - (b_1b_1^* + b_4b_4^*)\cos\frac{\eta}{2}\sin\frac{\eta}{2} \\ & + b_3b_1^*\cos^2\frac{\eta}{2} - b_1b_3^*\sin^2\frac{\eta}{2}\Big|, \end{aligned} \quad (25)$$

and for the bosonic reservoir,

$$C_{l_1}(\rho) = 2|b_1b_4^*| + 2|b_2b_3^*|. \quad (26)$$

By using the same method as the above, the linear entropy of the two-level atom is

$$\begin{aligned} S =\ & 1 - \{(|b_2|^2 + |b_3|^2)^2 + (|b_1|^2 + |b_4|^2)^2 \\ & + 2|b_1|^2|b_3|^2\}. \end{aligned} \quad (27)$$

3. Discussion and results

3.1. Effects of single-photon excitation on coherence/entanglement

Firstly, we discuss the dynamics of the atomic coherence under a classical driving field, the detuning and the non-Markovian effects.

In Fig. 1, the atomic coherence is plotted as a function of dimensionless quantity $\gamma_0 t$ with $\Delta = 0$, $\delta = 0$ for different driving field strengths with the non-Markovian regime $\lambda = 0.2\gamma_0$. It is noted that without the classical driving field, the atomic coherence is damped, and finally vanishes; with the increase of the driving field strength, the atomic coherence decays slowly and even stays unchanged, that is to say, adjusting the driving field can preserve the atomic coherence.

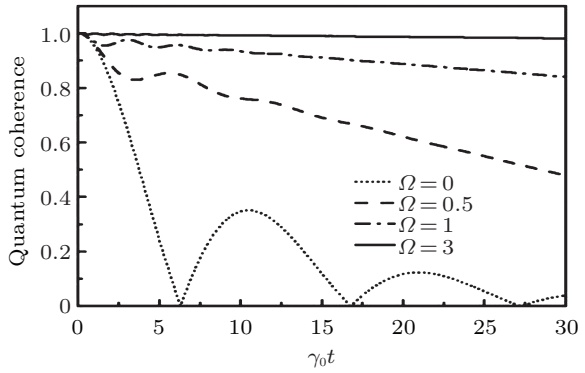


Fig. 1. Plots of quantum coherence versus dimensionless quantity $\gamma_0 t$ in the non-Markovian regime $\lambda = 0.2\gamma_0$ with $\Delta = 0$, $\delta = 0$ for driving field strengths of $\Omega = 0, 0.5, 1, 3$.

Figure 2(a) shows the plot of atomic coherence versus dimensionless quantity $\gamma_0 t$, and detuning Δ with $\delta = 0$, $\Omega = 2$ in the non-Markovian regime $\lambda = 0.2\gamma_0$. From the figure we find that the atomic coherence decreases with the increase of the atom-driving field detuning, and this phenomenon can be understood as follows: as the detuning increases, the coupling between the driving field and atom weakens, meanwhile, the effective coupling between atoms and the reservoirs increases, so the atomic coherence damped oscillations occur as displayed in Fig. 2(a).

From Fig. 2(b), we can find that the atomic coherence increases with the increase of the detuning between the atom frequency and the center frequency of the reservoirs; this is because the decay of the atom is suppressed with the increasing of the detuning. As a result, the atomic coherence can be protected. In addition, we can infer that the spectral width λ also affects the atomic coherence. The spectral λ is related to the degree of the non-Markovian effect, and the value of λ/γ_0 is smaller and the non-Markovian effect is stronger, so the atomic coherence becomes larger with the spectral width decreasing.

Secondly, we discuss the interconversion of the atomic coherence, the atom-reservoir entanglement and the reservoir coherence.

Without the driving fields, the atom-reservoir entanglement increases from zero to a certain value and then decreases to zero, next, the above evolution behavior is repeated and finally vanishes, the atomic coherence always follows the entanglement as displayed in Fig. 3(a). Besides, the reservoir coherence increases from zero and reaches a maximum at $\gamma_0 t = 6.3$, meanwhile, the atomic coherence and the atom-reservoir entanglement decrease to zero, and the dash-dotted line as displayed in Fig. 3(a) can clearly show the above relation. This phenomenon indicates that the atomic coherence and the atom-reservoir entanglement are converted into the reservoir coherence.

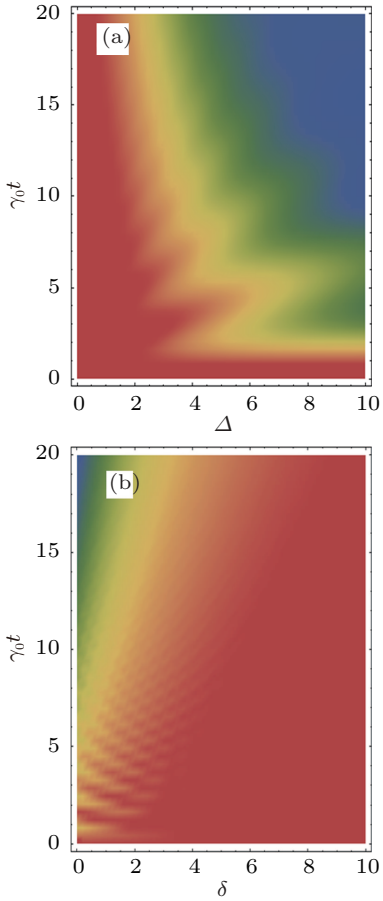


Fig. 2. (color online) Plots of the atomic coherence versus dimensionless quantity $\gamma_0 t$, and versus (a) detuing Δ with $\delta = 0$, $\Omega = 2$, (b) detuning δ with $\Delta = 0$, $\Omega = 2$ in the non-Markovian regime $\lambda = 0.2\gamma_0$.

In the cases of $\Omega = 0.5$ and $\Omega = 1$, the atomic coherence decreases and the atom-reservoir entanglement increases. For the case of $\Omega = 3$, the atomic coherence decreases very slowly, and the entanglement of atom-reservoir also increases very slowly as displayed in Figs. 3(b)–3(d). With the increase of the driving field strength, the atomic coherence becomes larger, so the interconversion from the atomic coherence to the atom-reservoir entanglement takes place, and the reservoir coherence becomes smaller.

From Fig. 4, we can find that with the increase of detuning Δ, the atomic coherence decreases, but the atom-reservoir entanglement and the reservoir coherence increase. As the detuning increases, the effective coupling between the atom and the reservoir increases, and the information which flows into the environment returns to the system, so we can see the oscillation of the atomic coherence and the entanglement of atom-reservoir in the case of detuning $\Delta = 3$. That is to say, the interconversion between the atomic coherence and the atom-reservoirs entanglement can be improved by the atom-field detuning.

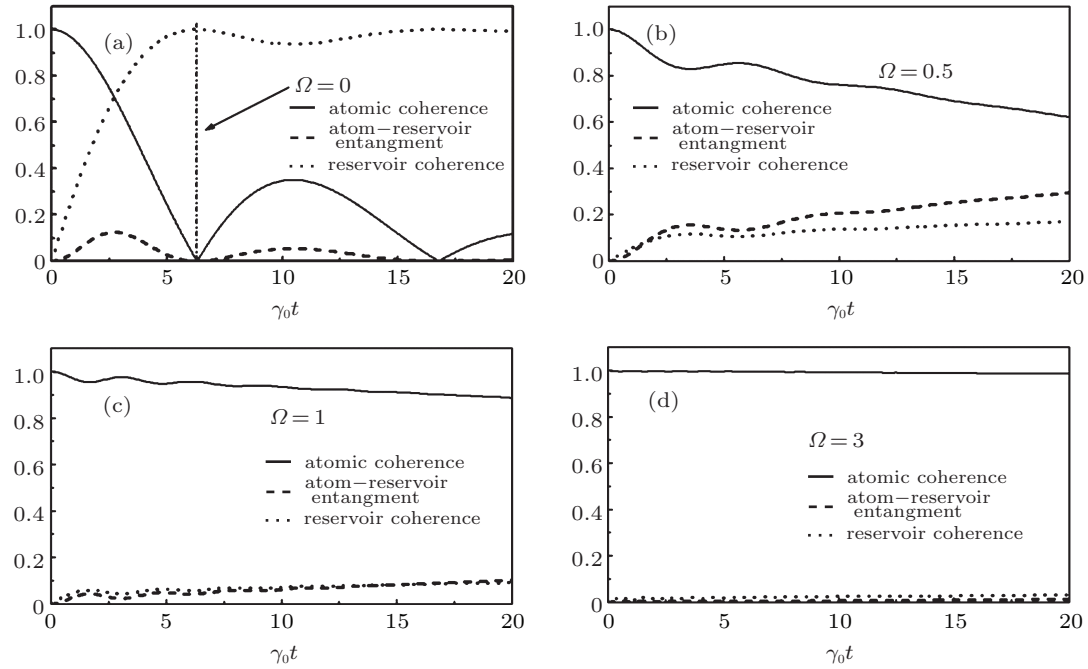


Fig. 3. Atomic coherence (solid line), the atom-reservoir entanglement (dash line), the reservoir coherence (dotted line) as a function of $\gamma_0 t$ with $\Delta = 0$, $\delta = 0$ for different driving field strengths in the non-Markovian regime $\lambda = 0.2\gamma_0$. (a) $\Omega = 0$, (b) $\Omega = 0.5$, (c) $\Omega = 1$, and (d) $\Omega = 3$.

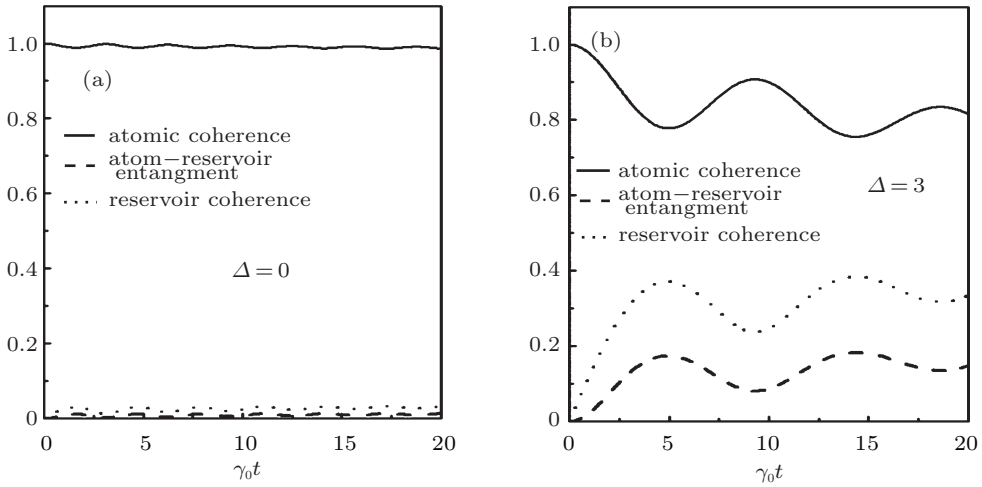


Fig. 4. Atomic coherence (solid line), the atom-reservoir entanglement (dash line), the reservoir coherence (dotted line) as a function of $\gamma_0 t$ with $\delta = 0$, $\Omega = 1$ in the non-Markovian regime $\lambda = 0.05\gamma_0$. (a) $\Delta = 0$ and (b) $\Delta = 3$.

In Fig. 5, we can also see results similar to those in Figs. 3(b)–3(d), in other words, the influences of the detuning δ on the atomic coherence and the atom-reservoir entanglement are similar to those of driving strength Ω. In addition, it is noticed that the increase of the detuning δ can speed up the conversion as displayed in Fig. 5.

Now, we come to study the joint evolutions of two identical single-body systems with the initial density matrix $\rho(0) = |\varphi(0)\rangle\langle\varphi(0)| \otimes |\tilde{0}\rangle|\tilde{0}\rangle\langle\tilde{0}|\langle\tilde{0}|$, with $|\varphi(0)\rangle = \cos\alpha|--\rangle + \sin\alpha|++\rangle$. According to Eqs. (5) and (6), the density matrix of the system at time t can be obtained as follows:

$$\begin{aligned}\rho(t) \;=\; & \sin^2\alpha|\varphi\rangle_1|\varphi\rangle_{21}\langle\varphi|_2\langle\varphi| + \cos^2\alpha(|--\tilde{0}\tilde{0}\rangle\langle--\tilde{0}\tilde{0}|) \\ & + \sin\alpha\cos\alpha|\varphi\rangle_1|\varphi\rangle_{21}\langle--|_2\langle\tilde{0}\tilde{0}| \\ & + \sin\alpha\cos\alpha|--\rangle_1|\tilde{0}\tilde{0}\rangle_{21}\langle\varphi|_2\langle\varphi|,\end{aligned}$$

where $|\varphi\rangle_i = q(t)|+\rangle_{Si}|\bar{0}\rangle_{Ri} + \sqrt{1-q(t)^2}|-\rangle_{Si}|\bar{1}_k\rangle_{Ri}(i = 1,2)$ represents the ith single-body evolution, by using the l_1 norm of coherence, we can write down quantum coherence for partitions of $q_1 \otimes q_2$, $r_1 \otimes r_2$, $q_1 \otimes r_2$, $q_2 \otimes r_1$, $q_1 \otimes r_1$, and $q_2 \otimes r_2$, respectively,

$$\begin{aligned}& C_{q_1q_2} = 2|\sin\alpha\cos\alpha q(t)^2|, \\ & C_{r_1r_2} = 2|\sin\alpha\cos\alpha(1-q(t)^2)|, \\ & C_{q_1r_2} = 2|\sin\alpha\cos\alpha q(t)\sqrt{1-q(t)^2}|, \;\; C_{q_2r_1} = C_{q_1r_2}, \\ & C_{q_1r_1} = 2|\sin^2\alpha q(t)\sqrt{1-q(t)^2}|, \;\; C_{q_2r_2} = C_{q_1r_1},\end{aligned}$$

where $q_i(r_i)$ represents the ith atom (reservoir).

Taking the symmetry of the composite system into account, we analyze the bipartite coherence of the six partitions of $q_1 \otimes q_2$, $r_1 \otimes r_2$, $q_1 \otimes r_2$, $q_2 \otimes r_1$, $q_1 \otimes r_1$ and $q_2 \otimes r_2$ in

the non-Markovian limit as shown in Fig. 6(a). In the initial period of time, due to the atom-reservoir coupling, $C_{q_1q_2}$ decreases from the original value to zero, and then increases due to the memory effect of the non-Markovian reservoirs. $C_{q_1r_1}$ first increases from zero to a maximum, and then decreases from the maximum. Meanwhile, $C_{q_1r_2}(C_{q_2r_1})$ and $C_{q_1r_1}(C_{q_2r_2})$ increase from zero to a maximum value and then decrease from the maximum to zero. The above effects indicate that the coherence of the qubits does not disappear, but has been transferred to other partitions, e.g., $q_1 \otimes q_2$, $r_1 \otimes r_2$, $q_1 \otimes r_2$, $q_2 \otimes r_1$, $q_1 \otimes r_1$, $q_2 \otimes r_2$, etc. Furthermore, through the further analysis in our model, $C^{+}_{q_1q_2}C_{r_1r_2}$ is conserved and its value is equivalent to the initial value of $C_{q_1q_2}$, which means that there is a perfect compensation of $C_{r_1r_2}$ for $C_{q_1q_2}$. Interestingly we find that the $Q(t) = C_{q_1q_2} + C_{r_1r_2} + 2C_{q_1r_1}|\cot\alpha| - 2C_{q_1r_2}$ is also conserved, and it means that the atomic coherence is transferred among all the bipartite partitions of the whole system but an identity (the above equation) is required to be satisfied at any time. Besides, $C_{q_1q_2} + C_{r_1r_2}$ and $Q(t) = C_{q_1q_2} + C_{r_1r_2} + 2C_{q_1r_1}|\cot\alpha| - 2C_{q_1r_2}$ are also conserved in the Markovian limit as shown in Fig. 6(b).

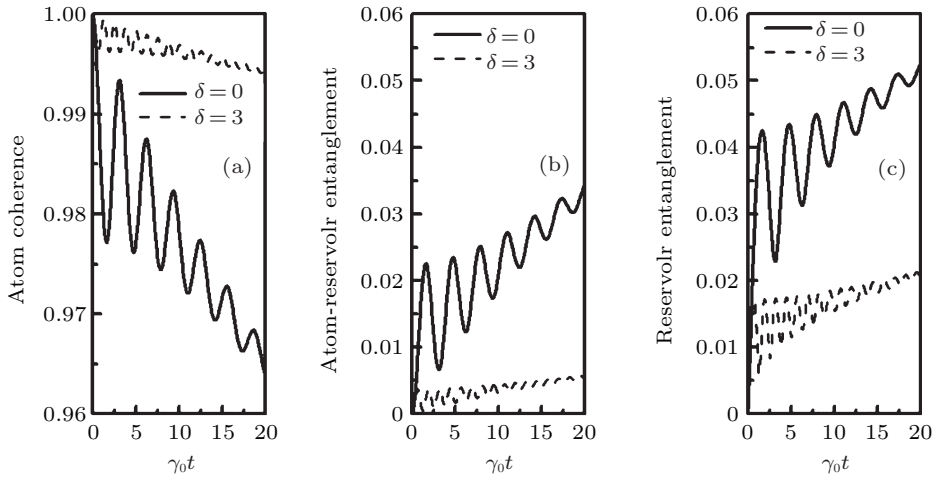


Fig. 5. (a) Atomic coherence, (b) atom-reservoir entanglement, (c) reservoir coherence as a function of $\gamma_0 t$ with $\Delta = 0$, $\Omega = 1$, $\lambda = 0.1\gamma_0$.

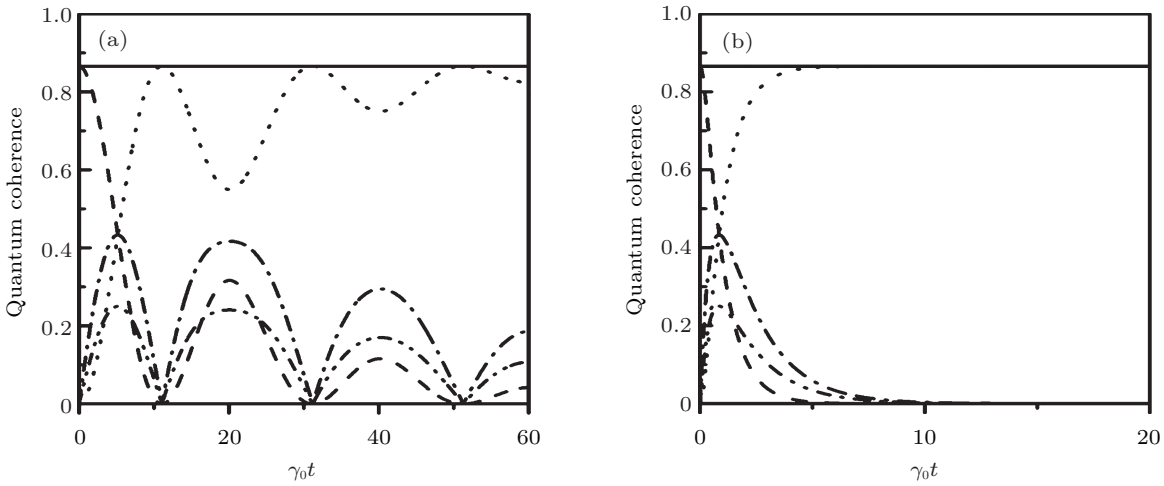


Fig. 6. Quantum coherences for partitions $C_{q_1q_2}$ (dash line), $C_{r_1r_2}$ (dot line), $C_{q_1r_2} = C_{q_2r_1}$ (dash-dotted line), $C_{q_1r_1} = C_{q_2r_2}$ (dash-dot-dotted line) as a function of $\gamma_0 t$ with $\Omega = 0$, $\alpha = \pi/6$. The solid line denotes the coherence invariant of $C_{q_1q_2} + C_{r_1r_2}$ for (a) $\lambda = 0.05\gamma_0$ and (b) $\lambda = 5\gamma_0$.

3.2. Effects of two-photon excitations on the coherence/entanglement

Comparing Fig. 7 with Fig. 3, we can find that the decay of the atomic coherence is faster with the two-photon excitation in the whole system. To our surprise, when there is one excitation in the system, the atomic coherence eventually transfers into the reservoir coherence, and the atomic coherence and the atom-reservoir entanglement vanish as displayed in Fig. 3(a). However, the atomic coherence transfers into the atom-reservoir entanglement in the end with two-photon excitation in the system, and the atomic coherence and the reservoir coherence vanish as shown in Fig. 7. These results demonstrate that the dynamics of a system with two-photon excitations is quite different from the case of single-photon excitation.

With single-photon excitation in the system, the atom has

two energy levels, i.e., $|e\rangle$ and $|g\rangle$, and the reservoir also has two energy levels, i.e., $|\tilde{0}\rangle$, $|\tilde{1}\rangle$, the coherence transfers between the two energy levels via atom–reservoir interaction. While for the two or more excitations in the system, the atom has two energy levels, and the reservoir is a multilevel system. The difference in energy level leads to the above phenomenon.

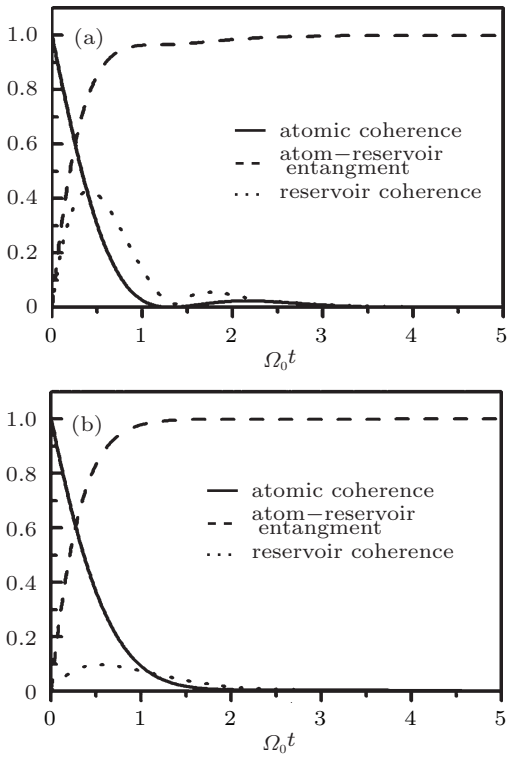


Fig. 7. Atomic coherence (solid line), the atom-reservoir entanglement (dash line), the reservoir coherence (dotted line) as a function of $\Omega_0 t$ with $\omega_c = \Lambda$ in the non-Markovian regime $\Gamma = 1.6\Omega_0$, for (a) $\Omega = 0$ and (b) $\Omega = 0.5$.

4. Conclusions

In the paper, we investigate the interaction between a two-level atom and a non-Markovian reservoir under a classical driving field. The influences of the classical driving fields and non-Markovian effects on atomic coherence and atom-reservoir entanglement are explored. It is shown that the atomic coherence can be improved or even maintained by the classical driving field, non-Markovian effect and the atom-reservoir detuning, however, the increase of the atom-field detuning weakens the atomic coherence. The interconversion between the atom coherence and the atom-reservoir entanglement exists and can be improved by the atom-driving field detuning, and the increase of the atom-reservoirs detuning can speed up the conversion. Besides, $C_{q_1q_2} + C_{r_1r_2}$ and $Q(t) = C_{q_1q_2} + C_{r_1r_2} + 2C_{q_1r_1}|\cot\alpha| - 2C_{q_1r_2}$ are conserved in the non-Markovian limit or in the Markovian limit, and the dynamic of a system with two-photon excitation is quite different from the case of one-photon excitation.

In the end, with the development of the control technology for spectral density of the reservoir and the system-environment,[36,37] our results provide an effective solution to maintain the coherence of the system. In addition, by the interconversion between the coherence and the entanglement, we have an in-depth understanding of the relationship between quantum coherence and quantum entanglement. By the dynamics of a system with single- or two-photon excitations, we understand the effects of reservoir excitations on the coherence/entanglement of the atom-reservoir system deeply.

References

[1] Bennett C H, Brassard G, Crepeau C, Jozsa R, Peres A and Wootters W K 1993 *Phys. Rev. Lett.* **70** 1895
[2] Zukowshi M, Zeilinger A, Horne M A and Ekert A K 1993 *Phys. Rev. Lett.* **71** 4287
[3] Bennett C H and Wiesner S J 1992 *Phys. Rev. Lett.* **69** 2881
[4] Ekert A K 1991 *Phys. Rev. Lett.* **67** 661
[5] Barenco A, Deutsch D, Ekert A and Jozsa R 1995 *Phys. Rev. Lett.* **74** 4083
[6] Nielsen M A and Chuang I L 2000 *Quantum Computation and Quantun Information* (Cambridge: Cambridge University press)
[7] Karpat G, Çakmak B and Fanchini F F 2014 *Phys. Rev. B* **90** 104431
[8] Çkmak B, Karpat G and Fanchini F F 2015 *Entropy* **17** 790
[9] Hillery M 2016 *Phys. Rev. A* **93** 012111
[10] Kai v P, Rudnicki Ł and Mintert F 2015 *Phys. Rev. A* **92** 052114
[11] Bera M N, Qureshi T, Siddiqui M A and Pati A K 2015 *Phys. Rev. A* **92** 012118
[12] Ford L H 1978 *Proc. R. Soc. A* **364** 227
[13] Correa L A, Palao J P, Alonso D and Adesso G 2014 *Sci. Rep.* **4** 3949
[14] Roßnagel J, Abah O, Schmidtkaler F, Singer K and Lutz E 2014 *Phys. Rev. Lett.* **112** 030602
[15] Narasimhachar V and Gour G 2015 *Nat. Commun.* **6** 7689
[16] Lostaglio L, Jennings D and Rudolph T 2015 *Nat. Commun.* **6** 6383
[17] Baumgratz T, Cramer M and Plenio M B 2014 *Phys. Rev. Lett.* **113** 140401
[18] Girolami D 2014 *Phys. Rev. Lett.* **113** 170401
[19] Bromley T R, Cianciaruso M and Adesso G 2015 *Phys. Rev. Lett.* **114** 210401
[20] Yang L W and Xia Y J 2016 *Chin. Phys. B* **25** 110303
[21] Carole A C, Brebner G, Haikka P and Maniscalco S 2014 *Phys. Rev. A* **89** 024101
[22] Zhang Y J, Han W, Xia Y J, Yu Y M and Fan H 2015 *Sci. Rep.* **5** 13359
[23] Kim M S, Son W, Bužek V and Knight P L 2002 *Phys. Rev. A* **65** 032323
[24] Wang X B 2002 *Phys. Rev. A* **66** 024303
[25] Asbóth J K, Calsamiglia J and Ritsch H 2005 *Phys. Rev. Lett.* **94** 173602
[26] Streltsov L, Singh U, Dhar H S, Bera M N and Adesso G 2015 *Phys. Rev. Lett.* **115** 020403
[27] Yao Y, Xiao X, Ge L and Sun C P 2015 *Phys. Rev. A* **92** 022112
[28] Zou H M and Fang M F 2016 *Chin. Phys. B* **25** 090302
[29] Ge G Q, Luo X L, Wu Y and Li Z G 1996 *Phys. Rev. A* **54** 1604
[30] Liu Y X, Sun C P and Nori F 2006 *Phys. Rev. A* **74** 052321
[31] Breuer H P and Petruccione F 2002 *The Theory of Open Quantum Systems* (Oxford: Oxford University Press) p. 472
[32] Dalton B J, Barnett S M and Garrraway B M 2001 *Phys. Rev. A***64** 053813
[33] Berry D W and Sanders B C 2003 *J. Phys. A General* **36** 12255
[34] Garraway B M 1997 *Phys. Rev. A* **55** 2290
[35] Zhang Y J, Zou X B, Xia Y J and Guo G C 2010 *Phys. Rev. A* **82** 022108
[36] Myatt C J, King B E, Turchette Q A, Sackett C A, Kielpinski D, Itano W M, Monroe C and Wineland D J 2000 *Nature* **403** 269
[37] Diehl S, Micheli A, Kantian A, Kraus B, Büchler H P and Zoller P 2008 *Nat. Phys.* **4** 878