Quadratic interaction effect on the dark energy density in the universe

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(Received 6 June 2017; revised manuscript received 8 July 2017; published online 20 August 2017)

In this study, we deal with the holographic model of interacting dark components of dark energy and dark matter quadratic case of the equation of state parameter (EoS). The effective equations of states for the interacting holographic energy density are derived and the results are analyzed and compared with the solution of the linear form in the literature. The result of our work shows that the value of interaction term between dark components affects the fixed points at far future in the DE-dominated universe in the case of quadratic EoS parameter; it is a different result from the linear case in the theoretical results in the literature, and as the Quintom scenario the equations of state had coincidence at the cosmological constant boundary of -1 from above to below.

Keywords: dark energy, dark matter

PACS: 95.36.+x, 98.80.–k, 95.35.+d **DOI:** 10.1088/1674-1056/26/10/109501

1. Introduction

Recent Supernova Type Ia observations (SNIa)[1,2] have showed that our Universe appears to be accelerating and the universe was expanding more slowly than it is today. They also suggest that there is a form of energy which is unknown and is shelving to accelerated expansion of the universe by energy contribution $\Omega_{\rm DE} \simeq 0.75$. This draws attention in the astronomy and particle physics research. Besides, cosmic microwave background (CMB) observations[3] provide evidence of the Universe to be expanding at an increasing rate. Cosmological observations suggest that a large part of the Universe arises from dark components which are named dark energy (DE) and dark matter (DM).[4–6] There are a number of dark energy candidates such as the cosmological constant model and scalar fields model including quintessence scenario, k-essence model, Chaplygin gas, holographic dark energy (HDE) models, and phantom dark energy model.[7] Also two famous cosmological constant problems survive that researchers try to solve: the fine tuning and coincidence problems. In the observational cosmology, the interaction between dark components giving EoS parameter of DE $\omega_{\rm DE} = p_{\rm DE}/\rho_{\rm DE}$ plays an important role in solving the mentioned problems where $p_{\rm DE}$ is the pressure and $\rho_{\rm DE}$ is the energy density. According to literature, in order to derive accelerated expansion of the Universe the EoS parameter of DE must satisfy $\omega_{\rm DE} < -0.33$ condition.[8] The EoS of the positive cosmological constant represented by Λ is the simplest candidate of the DE which has value -1, the value for quintessence model is $-1 < \omega_{\rm DE} < -0.33$, for k-essence model is $\omega_{\rm DE} < -1$ or $\omega_{\rm DE} > -1$. In the Chaplygin gas and HDE model $\omega_{\rm DE} > -1$, and not crossing -1. In the phantom case $\omega_{\rm DE}$ is smaller than -1. Most cosmological models assume that dark components only interact gravitationally.[9] In the recent works the parameter of EoS evolves across -1, the cosmological constant boundary of this phenomenon is called the quintom scenario, that was first pointed out in a pioneer work[10] and was later comprehensively reviewed in Ref. [11]. In cosmological models the idea that DM and DE are not evolving separately but interact with each other was introduced first to justify the currently small value of the cosmological constant but they were found to be very useful to alleviate the problem of coincidence namely: Why are DE and DM densities of precisely the same order today? discussed in recent works.[12–14] Since DE and DM dominate the energy content of the universe today, it is equally reasonable to assume that these dark components could interact among themselves and with other components. The mutual interactions between DE and DM can be suggested as a possible scenario of the Universe evolution. On the other hand, the interacting dark components are provided a new direction to understand the nature of DE. The particle physics can be explained by an effective field theory with a (short distance) UV cut-off that is less than the Planck mass given by $M_{\rm p}$. For an effective quantum field theory in a box of size L with UV cut-off Λ the entropy S scales extensively, $S \sim L^3\Lambda^3$. According to Bekenstein[15] the peculiar thermodynamics of black holes, which have the maximum entropy in a box of volume is L^3, and they behave non-extensively, therefore Bekenstein entropy bound is called Bekenstein–Hawking bound. The recent works[16] show that $L^3\Lambda^3 \leq S_{\rm BH} \equiv \pi L^2 M_{\rm p}^2$ where $S_{\rm BH}$ called by the entropy of a black hole with radius L acting as an IR cutoff, however it cannot be chosen inde-

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pendently of the UV cut-off, and scales as Λ^3. It means that UV cut-off is connected to the IR cut-off (long distance) and the density of zero point energy is proportional to UV cut-off Λ. The whole system total energy having size L may not exceed the mass of black hole with the same size. It means that the black hole entropy is directly related to the area of the event horizon. The holographic principle[17] was originated by black hole thermodynamics. It states how quantum theory and gravity interact to construct the reality that we are in and it suggests that the Universe can be seen as two-dimensional (2D) information on the event horizon and can be thought of as encoded on a lower-dimensional boundary to the region, therefore IR cut-off related to the DE is the size of the event horizon. HDE[18] arises from a theoretical attempt of applying the holographic principle to the DE problem; it has been introduced recently.[19] In this model the future event horizon is chosen as the characteristic length scale L and the DE density given by $\rho_{\rm DE} = 3c^2M_{\rm p}^2L^{-2}$, where c is a free dimensionless parameter. Many research studies try to solve the mentioned cosmological problems with working different interacting models. Recent observations of clusters of galaxies indicated that the density of matter is less than critical density. Observations, especially CMB signs, specify that the Universe has flat form and the Universe is dominated by DE with negative pressure driving to accelerate expansion of the Universe. Research where the interacting DE models are derived with linear form of EoS parameter, concluded that the mutual interactions of DE and DM may play a crucial role in the dynamics of the Universe by alleviating the mentioned cosmological problems.[20–24]

2. Interacting model

Generally it is assumed that interaction between DM and DE can be defined as

$$\begin{aligned}\dot{\rho}_{\rm DE} + 3H(\rho_{\rm DE} + p_{\rm DE}) &= -Q,\\ \dot{\rho}_{\rm DM} + 3H(\rho_{\rm DM} + p_{\rm DM}) &= Q\end{aligned} \tag{1}$$

with the total energy density $\rho_{\rm tot} = \rho_{\rm DE} + \rho_{\rm DM}$ where $\rho_{\rm DM}$ and $\rho_{\rm DE}$ are energy densities of DM and DE respectively. $p_{\rm DM}$ and $p_{\rm DM}$ are also the pressure of DM and DE, respectively. Q represents the overall conservation of the energy–momentum tensor indicating a transfer from DE component to DM component and *vice versa*. In the literature there are non-zero matter components of $\rho_{\rm DM}$[24] but in this study pressure is taken as zero for the matter component ($p_{\rm DM} = 0$) because we know that the normal dust matter is pressureless. In this study $p_{\rm DM}$ is considered as a specific case of a general quadratic EoS of the form[25]

$$p = \sum_{n=0}^{N} A_n p^n = p_0 + A_1\rho + A_2\rho^2 + \cdots. \tag{2}$$

We investigate a quadratic EoS parameter focusing on the quadratic case and ignore higher order terms. In high-energy regime p_0 can be chosen as zero. The parameters A_1 and A_2 set the characteristic energy scale. They can be chosen to be the same. For the standard linear EoS parameter for a cosmological fluid, p_0 and A_2 set to be zero, but now we are interested in the $A_1 = A_2$ condition with $\omega_{\rm DE}$ value. Rewrite Eq. (2) as

$$p_{\rm DE} = \sum_{n=0}^{N} \omega_{\rm DE}\rho_{\rm DE}^n = \omega_{\rm DE}(\rho_{\rm DE} + \rho_{\rm DE}^2 + \cdots). \tag{3}$$

This is decaying of the HDE into cold dark matter (CDM). We assume a form of interaction $Q = \Gamma\rho_{\rm DE}$ with the decay rate Γ. Equation (1) can be reorganized by using Eq. (3)

$$\begin{aligned}\dot{\rho}_{\rm DE} + 3H(1 + \omega_{\rm DE}^{\rm eff})\rho_{\rm DE} &= 0,\\ \dot{\rho}_{\rm DM} + 3H(1 + \omega_{\rm DM}^{\rm eff})\rho_{\rm DM} &= 0,\end{aligned} \tag{4}$$

where the effective equations of states are defined as follows:

$$\begin{aligned}\omega_{\rm DE}^{\rm eff} &= \omega_{\rm DE}(1 + \rho_{\rm DE} + \rho_{\rm DE}^2 + \cdots) + \Gamma/3H,\\ \omega_{\rm DM}^{\rm eff} &= -\Gamma/u3H,\end{aligned} \tag{5}$$

in this equation the ratio of the two energy densities is given by $u = \rho_{\rm DM}/\rho_{\rm DE}$. The homogenous and isotropic universe can be easily described by Friedmann–Robertson–Walker (FRW) metric[26]

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 ds^2 d\Omega^2\right). \tag{6}$$

where k refers to the curvature of the space with $k = -1, 0, 1$, the curvature of space depending on whether the shape of the Universe is an open 3-hyperboloid, flat (i.e., Euclidean space) or a closed 3-sphere, respectively, and a is the cosmic scale factor depending on time and is a measure of the size of the Universe representing relative expansion of the Universe. $x = 0$ when $a = 1$ since $x = \ln(a)$. In order to combine the energy density with the curvature of the Universe, we employ the first Friedman equation given by

$$H^2 = \frac{1}{3M_{\rm p}^2}[\rho_{\rm tot}] - \frac{kc^2}{a^2}, \tag{7}$$

where $\rho_{\rm tot} = \rho_{\rm DE} + \rho_{\rm DM}$, c is velocity of the light, $c = 1$ is known for the flat universe. $M_{\rm p}^{-2} = 8\pi G$ is the reduced Planck mass and $H = \dot{a}/a$ is the Hubble parameter. The geometry of the Universe is defined by the density parameter that is expressed in terms of the ratio of the Universe density to the critical density. This critical density is also called closure density which is a remarkably small number for our Universe and it is described by $\rho_{\rm cr}$. Ω_0 is defined as the average matter Universe density divided by a critical value of that density ($\Omega_0 = \rho_0/\rho_{\rm cr}$). This selects also one of the three possible geometries depending on whether Ω_0 is equal to, less than, or greater than 1 and it governs whether the curvature

is negative ($\Omega_0 < 1$), positive ($\Omega_0 > 1$), flat ($\Omega_0 = 1$). Today, the research shows that the Universe is nearly flat, therefore the average density is nearly close to the critical density ($\rho_{\rm cr} = 10^{-26}$ kg/m^3). Define the dimensionless density parameters for DE, DM, and the curvature relative to critical density parameter;

$$\Omega_{\rm DM} = \frac{\rho_{\rm DM}}{\rho_{\rm cr}}, \quad \Omega_{\rm DE} = \frac{\rho_{\rm DE}}{\rho_{\rm cr}}, \quad \Omega_k = \frac{kc^2}{a^2H^2}, \tag{8}$$

where $\rho_{\rm cr} = 3M_{\rm p}^2H^2$ is the critical energy density and the First–Friedman equation can be rewritten by using Eqs. (7) and (8)

$$\Omega_{\rm DM} + \Omega_{\rm DE} = 1 + \Omega_k, \tag{9}$$

which is for the holographic condition involving the future event horizon and then expresses the density ratio in terms of Eq. (8) by using Eq. (9)

$$u = \frac{\Omega_{\rm DM}}{\Omega_{\rm DE}} = \frac{1+\Omega_k-\Omega_{\rm DE}}{\Omega_{\rm DE}},$$
$$r_k = \frac{\Omega_k}{\Omega_{\rm DE}}, \tag{10}$$

where $u = \rho_{\rm DM}/\rho_{\rm DE}$ which should be a solution for the coincidence problem. Its derivative can be rewritten by using Eq. (10)

$$\dot{u} = 3Hu\left[\omega_{\rm DE}(1+\rho_{\rm DE}) + \frac{\Gamma}{3H}\left(\frac{1+u}{u}\right)\right], \tag{11}$$

$$\dot{u} = 3Hu\left[\omega_{\rm DE}^{\rm eff} - \omega_{\rm DM}^{\rm eff}\right], \tag{12}$$

which means that the ratio of the densities depends on the defined form of the interaction. In the evolution $\omega_{\rm DE}^{\rm eff} = \omega_{\rm DM}^{\rm eff}$ when $\dot{u}$ is equal to 0, which means that the ratio of the energy densities provides constant value for the DE and DM. $\omega_{\rm DE}^{\rm eff} > \omega_{\rm DM}^{\rm eff}$ means that the dark energy dominates the system and it is not vanishing. If $\omega_{\rm DE}^{\rm eff} < \omega_{\rm DM}^{\rm eff}$ the vanishing dark energy density is being apparent. The IR cut-off of the Universe in the case of the holographic model with the HDE density is defined in terms of L

$$\rho_{\rm DE} = 3c^2M_{\rm p}^2L^{-2}, \tag{13}$$

where L is IR cut-off which can be identified by the radius of the future event horizon r defined as follows:

$$L = ar(t), \tag{14}$$

while referring holographic model, when DE-dominated universe model is assumed, the future event horizon has a tendency to a constant value with the order H^{-1}, therefore equation (13) can be rewritten by $L = 1/H$. In this study we have chosen the radius of $r(t)$ as follows:

$$\int_0^{r(t)} \frac{{\rm d}r}{\sqrt{1-kr^2}} = \int_t^{\infty} \frac{{\rm d}t}{a} = \frac{R_h}{a}, \tag{15}$$

where the event horizon is given by R_h and the radial coordinate is given by

$$r(t) = \frac{1}{k}\sin y, \qquad y = \sqrt{k}R_h/a, \tag{16}$$

using these definitions $\Omega_{\rm DE}$ and $\rho_{\rm cr}$, we can easily obtain

$$HL = \frac{c}{\Omega_{\rm DE}}, \tag{17}$$

using Eqs. (14)–(17) we gain

$$\dot{L} = HL - a\dot{r} \tag{18}$$

and then using the definition Ω_k and Eq. (18) we obtain

$$\dot{L} = \frac{c}{\sqrt{\Omega_{\rm DE}}} - \cos y, \tag{19}$$

where $\cos y = \cos y$ for $k = 1$, $\cos y = y$ for $k = 0$, and $\cos y = \cosh y$ for $k = -1$ with $y = \sqrt{k}R_{\rm FH}/a$. If equation (13) is used $\cos y$ can be rewritten as $\cos y = y = \sqrt{1-\Omega_k/\Omega_{\rm DE}}$. Taking again the holographic energy density definition $\rho_{\rm DE}$ by using Eq. (4) one finds the holographic energy equation of the state by using Eq. (13) takes the following form

$$\omega_{\rm DE} = \frac{1}{\gamma} - \left(-1 - \frac{2\cos y\sqrt{\Omega_{\rm DE}}}{c}\right) - \frac{\Gamma}{\gamma}, \tag{20}$$

where $\gamma = 3(1+3M_{\rm p}^2H^2\Omega_{\rm DE})$. In this work, the decay rate can be chosen by $\Gamma = 3b^2(1+u)^nH$ (n is a constant) where b is the coupling constant and $1 \geq n$. Substituting this relation into Eq. (5) with the chosen decay rate we find the effective state equation

$$\omega_{\rm DE}^{\rm eff} = -\frac{1}{3} - \frac{2}{3}\frac{\sqrt{\Omega_{\rm DE}}}{c}\cos y,$$
$$\omega_{\rm DM}^{\rm eff} = \frac{-b^2(1+\Omega_k)^n}{\Omega_{\rm DE}^{n-1}(1-\Omega_{\rm DE}+\Omega_k)}. \tag{21}$$

Now we derive here two dimensionless coupled equations with solutions determining the effective equations of state. $\Omega_{\rm DE}$ is coming from the derivative of u given by $\Omega_{\rm DM}/\Omega_{\rm DE}$ and Ω_k is coming from the derivative of the r_k given by $\Omega_k/\Omega_{\rm DE}$. The corresponding coupled equations are derived by using Eqs. (11), (12), and (21)

$$\frac{{\rm d}\Omega_{\rm DE}}{{\rm d}x} = -3\Omega_{\rm DE}\eta + \Omega_k\Omega_{\rm DE}(1+3\omega_{\rm DE}^{\rm eff}),$$
$$\frac{zd\Omega_k}{{\rm d}x} = -3\Omega_k\eta + \Omega_k(1+\Omega_k)(1+3\omega_{\rm DE}^{\rm eff}), \tag{22}$$

where $x = \ln(a/a_0)$ and $\eta = (1-\Omega_{\rm DE}+\Omega_k)[\omega_{\rm DE}^{\rm eff} - \omega_{\rm DM}^{\rm eff}]$. When we compare our results with literature it can be seen that our model is in accordance with the literature.

3. Numerical results

In this study we have obtained the state equation for the interacting HDE density by using quadratic form of the EoS parameter. The numerical results are analyzed for the cases of $k = 0, 1, -1$ the curvature of space flat, closed, and open forms of the universe, respectively. In this study we have used the effective equations of state (ω_{DE}^{eff}, ω_{DM}^{eff}). To analyze the results of our numerical solutions we represent the behavior of the energy densities and effective equations by plotting versus x values in the case of quadratic form of the EoS parameter.

In the literature the effective equation of the state ω_{DM}^{eff} for $n > 1$ diverges for small Ω_{DE} for choices of the holographic condition that could be consistent with observations, therefore in literature the parameters were assigned as $c = 1$, $n = 1$, and $b^2 = 0.1$ and 0.2 with the initial conditions $\dot{\omega}_{DE} > 0$ at $x = 0$, $\omega_{DE}(0) = 0.72$, $\omega_k(0) = 0$ for $k = 0$, $\omega_k(0) = 0.08$ for $k = 1$, $\omega_k(0) = -0.01$ for $k = -1$.

First of all the evolution behavior of the effective equation of state ω_{DE}^{eff} (blue) and ω_{DM}^{eff} (red) in Eqs. (21) and the evolution of Ω_{DE} and Ω_k with respect to x is qualitatively determined in Eqs. (22) are plotted in Figs. 1, 2, and 3 to analyze the numerical results and compare them with previous literature.

In Fig. 1 the density parameter and effective equations are plotted with the initial parameters $b^2 = 0.2$, for an interaction of $n = 1$ with the initial conditions $\Omega_{DE}(0) = 0.72$, $\Omega_k(0) = 0.08$, where the positive curvature constant $k = 1$. In this case the density is so high that the gravitational attraction can stop the expansion and collapse backward named "big crunch". This kind of universe is described as being a closed universe, or a gravitationally bound universe with a small positive curvature ($\Omega_k = 0.02$). As shown in Fig. 1(a) the two effective equations of state start different points. In far future two effective equations take the same value ($\omega_{DE}^{eff} = \omega_{DM}^{eff}$) between 0 and -1, which means that there is a stable point in the far future. It is clear in Eq. (12) when r is equal to zero $\omega_{DE}^{eff} = \omega_{DM}^{eff}$ meaning that the Universe may be re-collapsed in the future time. From Eq. (9) the density of matter exceeds the critical value (0.28) when the curvature term goes to zero at the far future it is in accordance with Ref. [20]. In Fig. 1(b) it is seen that Ω_k has maximum positive curvature with value 0.04 and Ω_k goes to zero for the past and far future and the density parameter of Ω_{DE} is a monotonically increasing function of x which goes to 0.77 at far future, therefore $\Omega_{DM} = 0, 27$. The research shows that the observed best fit of the closed model of the Universe has $\Omega_{DM} = 0.415$, $\Omega_{DE} = 0.630$.[30]

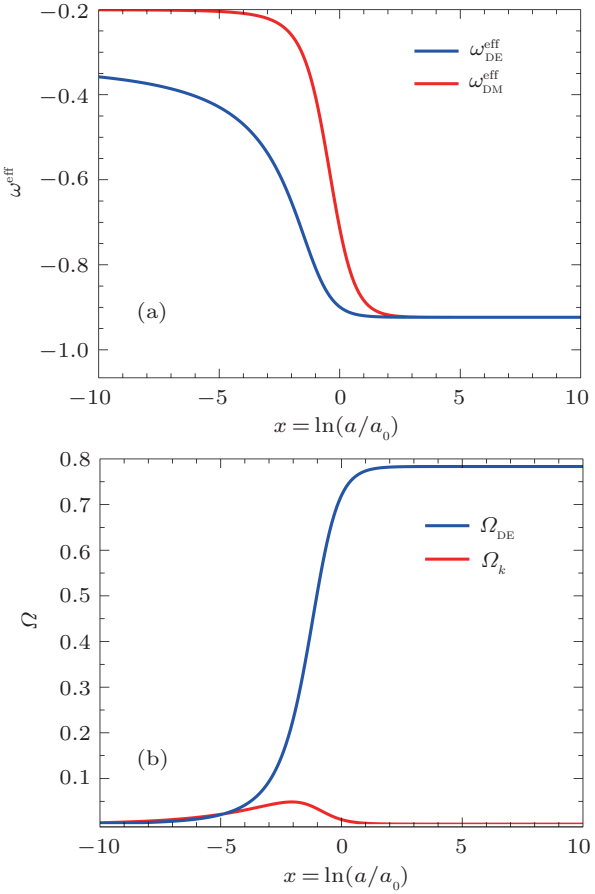


Fig. 1. (color online) (a) The effective equations of state ω_{DE}^{eff} (blue) and ω_{DM}^{eff} (red) versus x where $x = ln(a/a_0)$ and (b) the density parameters of Ω_{DE} (blue) and Ω_k (red) versus x are represented for the non-flat universe (closed case) $b^2 = 0.2$, for an interaction of $n = 1$ with the initial conditions $\Omega_{DE}(0) = 0.72$, $\Omega_k(0) = 0.08$ where $k = 1$.

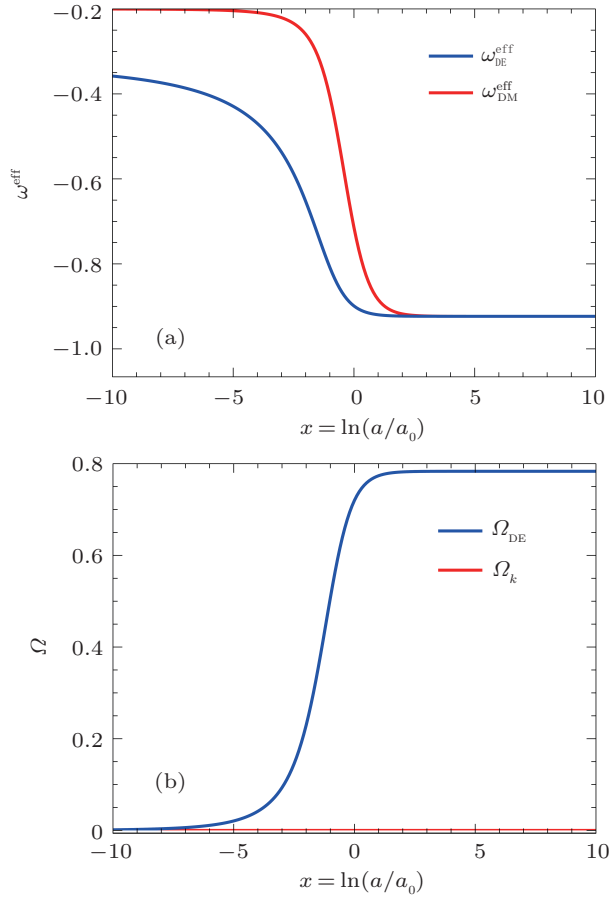


Fig. 2. (color online) (a) The effective equations of state ω_{DE}^{eff} (blue) and ω_{DM}^{eff} (red) versus x where $x = ln(a/a_0)$ and (b) the density parameters of Ω_{DE} (blue) and Ω_k (red) versus x are represented for the flat universe $b^2 = 0.2$, for an interaction of $n = 1$ with the initial conditions $\Omega_{DE}(0) = 0.72$, $\Omega_k(0) = 0.0$ where $k = 0$.

In Fig. 2 the density parameter and effective equations are plotted with parameters $b^2 = 0.2$, for an interaction of $n = 1$ with the initial conditions $\Omega_{DE}(0) = 0.72$, $\Omega_k(0) = 0.0$ where the curvature constant $k = 0$. In theory a space with zero curvature is called a flat space which is non-compact and extends infinitely far in any direction and in this case the density value is equal nearly to the critical value, so the Universe may expand forever. As shown in Fig. 2(a) the two effective equations of state start also from different points. In the far future the two effective equations approach their limiting (equal) value ($\omega_{DE}^{eff} = \omega_{DM}^{eff}$) which means that there is a stable point in the far future. The two effective equations coincide at -0.98 fixed point. In Fig. 2(b) Ω_k has zero value for the past and far future. The density parameter Ω_{DE} has monotonically increasing behavior like non-flat universe 0.78 at the far future; it is compatible with observations.[27]

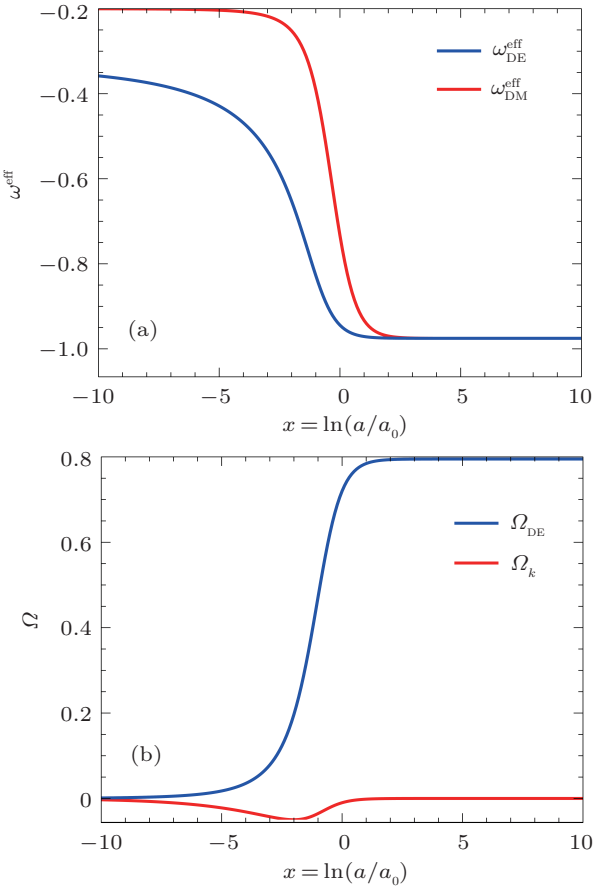


Fig. 3. (color online) (a) The effective equations of state ω_{DE}^{eff} (blue) and ω_{DM}^{eff} (red) versus x where $x = \ln(a/a_0)$ and (b) the density parameters of Ω_{DE} (blue) and Ω_k (red) versus x are represented for the non-flat universe (open case) $b^2 = 0.2$, for an interaction of $n = 1$ with the initial conditions $\Omega_{DE}(0) = 0.72$, $\Omega_k(0) = 0.01$ where $k = -1$.

In Fig. 3 the density parameter and effective equations are also plotted with the same parameters except the initial condition $\Omega_k(0) = -0.01$ where the Universe has open form with negative curvature, space has infinite volume. This space–time also has space expanding forever in time. In Fig. 3(a) in the far future the two effective equations are also coincident at a fixed point ($\omega_{DE}^{eff} = \omega_{DM}^{eff}$), and take the value between 0 and -1. In Fig. 3(b) Ω_k has negative value with tending zero value for the past and far future. The recent research shows that, in the case a non-flat universe, the models with small negative value of Ω_k are a better fit than ΛCDM model.[30] The density parameter Ω_{DE} is also monotonically increasing.

In Fig. 4 the equation of states and density parameters are plotted with different interaction parameters to see the effect on the nature of the holographic interaction. In the case of $b^2 = 0$ non-interaction we see $\Omega_{DE}(x) \to 1$ as x value increases. When we have chosen the interaction parameter value to be 0.1 as shown in Fig. 4(a) in the far future two effective equations are also coincident at a fixed point, and take the value between 0 and -1, however it is a bigger value than the $b^2 = 0.2$ case. In Fig. 4(b) $\Omega_{DE}(x)$ is also a monotonically increasing function of x and reaches 0.90 value at the far future.

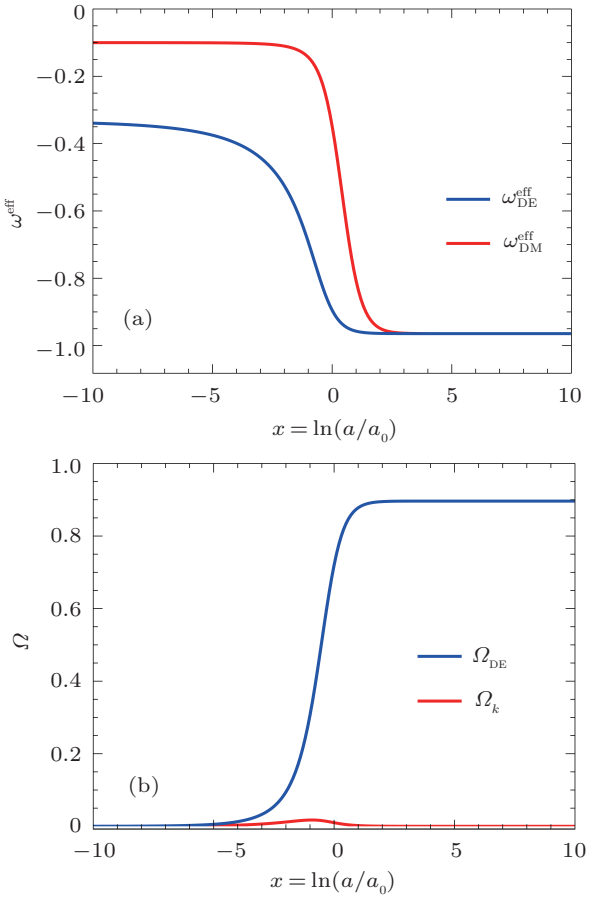


Fig. 4. (color online) (a) The evolution behavior of the effective equations of state ω_{DE}^{eff} (blue) and ω_{DM}^{eff} (red) versus x where $x = \ln(a/a_0)$ and (b) the evolution behaviors of the density parameters of Ω_{DE} (blue) and Ω_k (red) versus x are represented for the non-flat universe (open case) in the case of quadratic EoS with the parameters $b^2 = 0.1$, for an interaction of $n = 1$ with the initial conditions $\Omega_{DE}(0) = 0.72$, $\Omega_k(0) = 0.08$ for $k = 1$.

4. Conclusions

In cosmology, driving to an accelerated universe, a quadratic EoS parameter has been shown to be concerned with the expounding of DE and DM,[28,29] it is also appearing in the

energy density evolution equations and may be liable for the annihilation of anisotropy and inhomogeneity. In contrast, for the standard general relativistic cosmology case, a linear EoS parameter represents the high isotropy of the Universe. In this study, we have chosen the DE-dominated non-flat universe and three parameters b^2 (interaction strength), c, n are taken in the observational ranges $\Omega_{\rm DE}$, $\Omega_{\rm DM}$, Ω_k. First of all we have extended the interacting HDE model to the non-flat model of the Universe with quadratic EoS parameter which represents the state of the composition of two comparable components that arise from decaying of HDE into CDM. Using HDE, we have obtained the equation of state for interacting HDE density enveloped by L universe. The density parameters of $\Omega_{\rm DE}$ and Ω_k have calculated numerically for the quadratic EoS parameter, and then the effective equations $\omega_{\rm DE}^{\rm eff}$ and $\omega_{\rm DM}^{\rm eff}$ are calculated. The behavior of the density parameters and effective equations are analyzed. As can be seen from the numerical results mentioned above the effect of the non-flat universe is trivial, because from Figs. 1, 2, and 3(a) it can be seen easily that $\Omega_k \to 0$ for the past and future, and this means that for the non-flat universe with value $k \neq 0$, the phantom regime could not be caused for which $\Omega_{\rm DE}^{\rm eff} < -1$, even if one comprises an interaction between HDE and DM where $\Omega_{\rm DE}^{\rm eff} > -1$. For the non-interacting case of $b^2 = 0$ we see that $\Omega_{\rm DE}(x) \to 1$ as x increases. As shown in Figs. 1, 2, 3, and 4(b), $\Omega_{\rm DE}(x)$ is a droningly increasing function of x with the interacting cases of $b^2 = 0.1$ and $b^2 = 0.2$. The DM density is affected by small changes of the interaction term, but the DE density is not. This means that the interaction term affects strongly the effect on the nature of the holographic interaction in the DE-dominated universe in the case of the quadratic EoS parameter, which is different from the linear case. These figures show also that the quadratic effective EoS of the two equation states of $\omega_{\rm DE}^{\rm eff}$ and $\omega_{\rm DM}^{\rm eff}$ will take negative values which are greater than -1 during the whole evolution of the Universe. This means that the holographic energy density preserves its nature for the future even if the interaction term is changed and as the Quintom scenario the equations of state had coincidence at the cosmological constant boundary of -1 from above to below. As can be seen in Fig. 4 a small change is important when it comes to quadratic additions to EoS parameter. This refers that the interaction term affects the fixed points at the far future in the DE-dominated non-flat universe. The effective equation of matter component is also affected by small changes in the interaction parameter; however the effective equation of DE is not affected strongly. When it is compared with the recent research, it is concluded that use of quadratic EoS parameter leads to the overriding of the term of the interactions between DM and DE, therefore the effective equation of state parameter fails to evolve from the regime greater than -1 to the regime less than -1 or the opposite way. In this regard, this can lead to the conflict between the theoretical and observational expectations.

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