A hybrid strategy to control uncertain nonlinear chaotic system*

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In this paper, a new method, based on firefly algorithm (FA) and extreme learning machine (ELM), is proposed to control chaos in nonlinear system. ELM is an efficient predicted and classified tool, and can match and fit nonlinear systems efficiently. Hence, mathematical model of uncertain nonlinear system is obtained indirectly. For higher fitting accuracy, a novel swarm intelligence algorithm FA is drawn in our proposed way. The main advantage is that our proposed method can remove the limitation that mathematical model must be known clearly and can be applied to unknown nonlinear chaotic system.

Keywords: chaos, firefly algorithm, extreme learning machine

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1. Introduction

When chaos was proposed and introduced by Edward Norton Lorenz in 1963,[1] for the first time chaos and chaotic system obtained attention. Edward gives metaphor that the wings waving of a butterfly in amazon rainforest may cause a terrible tornado in Texas, which indicates the irregularity, unpredictability and sensibility to initial value of chaotic system.[2] Chaotic phenomenon is not expected to exist in majority of systems, especially power grid and electronic circuit etc.[3] The efficient control of chaos in chaotic systems has been an attractive research area for few decades.

For nonlinear chaotic system, the essence of chaos control is to restrain chaos and trace target signals. Generally, the main way to control chaos is by adding controller into chaotic system and constructing the controller is the key to achieve chaos control. In recently years, the design of controller for chaotic systems has made many significant progresses and many controllers were proposed by researchers. For control of chaos, Heng-Hui Chen *et al.* built a nonlinear feedback controller by exact mathematical cancellation of nonvanishing perturbation terms.[4] In order to control chaos and trace target signals in permanent-magnet synchronous motor (PMSM), Jian Hu *et al.* added three outputs to system and proposed an adaptive robust nonlinear feedback controller.[5] The biggest advantage with this method is that it is available for systems with unknown parameters. Similarly, Chunlai Li proposed an adaptive controller by a unified mathematical expression to solve projective synchronization in chaotic or hyperchaotic. This adaptive controller is available for different conditions, including hyperchaotic condition, chaotic condition, periodic target signal or fixed value, even unknown parameter and disturbance conditions.[6] For control of chaos in systems, another widely applied method is sliding mode control, which is mainly due to its insensitivity to parameter variations and complete rejection of disturbances. Liu Shuang *et al.* designed a sliding mode controller using a chain network.[7] Chunlai Li *et al.* proposed a three dimensional chaotic system and then used an adaptive sliding mode control to achieve synchronization in fractional-order chaotic system.[8] Its excellent performances also were verified by numerical simulations. Unlike frequently switching the controller of system in Ref. [7], synchronization and trace can be achieved easily by only adjusting the system parameter and the controller do not need to be adjusted. In the methods listed above, although the synchronization of chaos can be solved and target signal can be traced, they can work only if the relations between variables are known, or even the mathematical models of chaotic system are known clearly. As is well known, majority of chaotic systems are uncertain in actual industry and the mathematical models cannot be described clearly. As a result, for the above issues, radial basis function neural network (RBFNN) is a very useful tool for unknown nonlinear systems. For ferroresonance overvoltage of neutral grounded power system, Liu Fan *et al.* proposed a maximum-entropy learning algorithm based on RBFNN to control the chaotic system.[9] Chun-Fei Hsu *et al.* combined RBFNN with Fuzzy Inference System (FIS) for inverted pendulum system.[10] However, as traditional neural network, it is more suitable for continuous chaotic system. Besides, some other factors, such as slow convergence speed, can also limit

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its effectiveness.

In this paper, a novel method based on extreme learning machine and firefly algorithm is introduced in detail. For uncertain nonlinear chaotic system, it can be fitted satisfactorily, avoiding the drawback that mathematical model needs to be known clearly. Moreover, firefly algorithm is drawn into our proposed method in order to guarantee fast convergence speed. Further, various chaotic systems, target signals and disturbance conditions are tested to prove the effectiveness of proposed method.

The rest of the paper is organized as follows. In Section 2, the proposed control strategy based on FA and ELM is explained in detail. Different chaotic systems, different target signals and different disturbance conditions are tested in Section 3. Conclusion is given in Section 4, finally.

2. The proposed control strategy based on FA and ELM

2.1. Proposed control strategy

For nonlinear chaotic system with single input and single output, it can be described by

$$\chi(t+1) = f(\chi(t), \omega), \tag{1}$$

where $\chi \in \mathbb{R}_n$ is system state variable, ω is system parameter and f is system function.

As is showed in Fig. 1, the modified system is

$$\chi(t+1) = f(\chi(t), \omega) + \Gamma(t), \tag{2}$$

where Γ is control section. So the control section can be defined by

$$\Gamma(t) = -F(\chi(t), \omega) + \chi_{\mathrm{r}}(t+1) + \alpha(\chi(t) - \chi_{\mathrm{r}}(t)), \tag{3}$$

where $F(\chi(t), \omega)$, $\chi(t)$, ω, and χ_{r} are fitting function, state variable, state parameter, reference signal, and convergence parameter of nonlinear system, respectively.

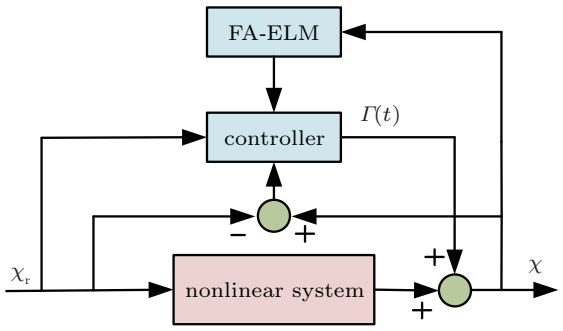


Fig. 1. (color online) The modified nonlinear system.

Although mathematical model of chaotic system is not known clearly, its data can be obtained easily and fitted by ELM, which is

$$\widehat{\chi}(t+1) = F(\chi(t), \omega). \tag{4}$$

So the residual is

$$e(t) = \chi(t) - \chi_{\mathrm{r}}(t). \tag{5}$$

Then

$$\begin{aligned} e(t+1) &= \chi(t+1) - \chi_{\mathrm{r}}(t+1) \\ &= f(\chi(k), \omega) - F(\chi(k), \omega) + \chi_{\mathrm{r}}(t+1) \\ &\quad + \alpha(\chi(k) - \chi_{\mathrm{r}}(k)) - \chi_{\mathrm{r}}(t+1). \end{aligned} \tag{6}$$

Assuming that fitting function can match nonlinear system with high accuracy, the above equation can be rewritten as

$$e(t+1) = \alpha(\chi(t) - \chi_{\mathrm{r}}(t)) = \alpha e(t). \tag{7}$$

The above equation indicates the relation of residual on time t and $t+1$. So if $|\alpha| < 1$, residual decreases with time and system is convergent and stable, that is, the modified nonlinear system gradually becomes stable and reference signal can be traced effectively. Similarly, this proposed way also can be applied in nonlinear chaotic system with multiple input multiple output.

2.2. Proposed control algorithm

As is introduced above, the modified nonlinear system functions well and chaos is restrained effectively only if fitting function matches nonlinear system with high accuracy. In order to solve this issue, a strategy based on ELM and FA is proposed. Nonlinear chaotic system is fitted by ELM and ELM is optimized by FA with optimal parameters and high fitting accuracy.

ELM is a very useful and efficient tool to classify and predict. Originating from neural network (NN), since it was proposed in 2005,[11] it has received much attention and been applied to various areas, such as image processing[12,13] and fault classification,[14–16] etc. In this study, ELM is utilized to fit nonlinear chaos system. Its expressions is[17]

$$HV = Y. \tag{8}$$

H, V, and Y are

$$H = \begin{bmatrix} g(\mu_1, \nu_1, x_1) g(\mu_2, \nu_2, x_1) \ldots g(\mu_N, \nu_N, x_1) \\ \vdots \\ g(\mu_1, \nu_1, x_N) g(\mu_2, \nu_2, x_N) \ldots g(\mu_N, \nu_N, x_N) \end{bmatrix}_{N \times N},$$

$$V = \begin{bmatrix} \nu_1^T \\ \vdots \\ \nu_N^T \end{bmatrix}_{N \times m}, \quad Y = \begin{bmatrix} y_1^T \\ \vdots \\ y_N^T \end{bmatrix}_{N \times m},$$

where x_i $(i = 1, 2, 3, \ldots, N)$ is input data, y_i $(i = 1, 2, 3, \ldots, N)$ is output data. $g(x)$ is hidden layer activation function, μ and ν are the weight coefficient and bias between input layer and

hidden layer. When μ and ν are chosen, y is determined accordingly.

There is also drawback associated with this method. The random way to define weight coefficient and bias in traditional mode limits its performance. Taking the blemish into consideration, ELM is improved by swarm intelligence algorithm to improve convergence rate and fitting precision.

Originating from swarm intelligence, FA is an outstanding intelligence algorithm proposed by Yang in 2008.[17] The basic equations of FA are

$$x_i(t+1) = x_i(t) + \beta(x_j(t) - x_i(t)) + \alpha\left(\text{rand} - \frac{1}{2}\right), \quad (9)$$

$$I = I_0 \times \mathrm{e}^{-\gamma r_{ij}}, \beta = \beta_0 \times \mathrm{e}^{-\gamma r_{ij}^2}, \quad (10)$$

where I and β are brightness and attraction of firefly, $x_i(t+1)$ is location of the i-th firefly in $(t+1)$-th iteration. So the structure of hybrid algorithm is showed in Fig. 2.

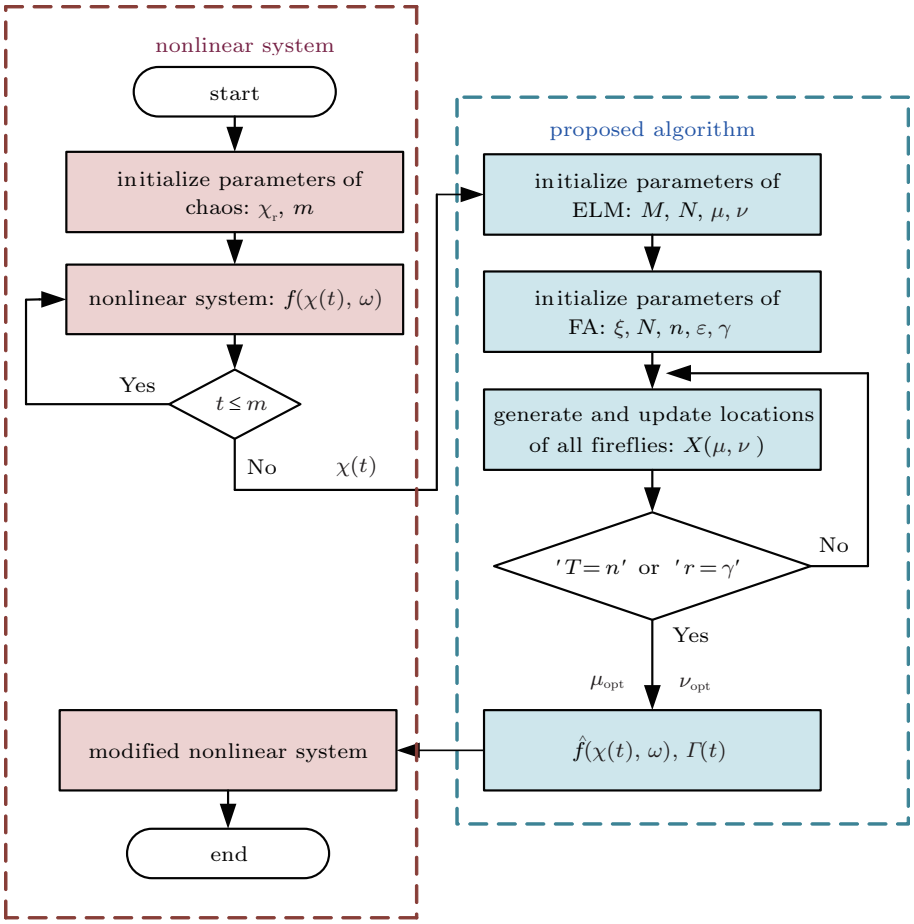


Fig. 2. (color online) The structure of hybrid algorithm. ξ, D, n, ε, and Υ denote the number of fireflies, dimension, the maximum iterations, the random factor, and reference accuracy in FA. Similarly, T and r are current iterations and accuracy.

As is showed in Fig. 2, some related parameters are: χ_r is reference signal, m is data of chaotic system used to train and test ELM, t is the current number of chaotic data. M, N, μ, and ν denote the number of input data, the number of hidden neurons, the weight, and bias between input layer and hidden layer in ELM, respectively.

From the descriptions above, the processes to control nonlinear chaotic system in this study are followed.

Initialization

Step 1 Initialize reference signal χ_r and number of sample data m in nonlinear chaotic system.

Step 2 Obtain training data and testing data $\chi_i, i = 1, 2, 3, \ldots, m$.

Step 3 Initialize parameters in FA: the number of fireflies ξ, the maximum iterations n, the random factor ε and reference accuracy Υ.

Step 4 Define number of neurons N in hidden layer and construct ELM fitting model.

Search process

Step 5 Define the dimension D of firefly using the optimization object "weight matrix μ and bias ν"

$$D = N \times \delta_i + N, \quad (11)$$

where δ_i and N are the number of neuros in input layer and hidden layer, respectively.

Step 6 Update locations of fireflies, brightness and attraction using Eqs. (9) and (10).

Step 7 Calculate output training samples $\hat{y}_i$ by Eq. (8).

Step 8 Calculate mean squared error (MSE)

$$\mathrm{RESM} = \sqrt{\frac{1}{m}\sum_{i=1}^{m}(\hat{y}_i - y_i)^2}, \quad i = (1,2,\ldots,m). \tag{12}$$

Convergence

Step 9 Update the global optimal individual.

Step 10 Judge whether the current iteration T is equal to maximum iteration n and the current accuracy r is less than Γ or not. If not, $T = T+1$ and back to Step 6.

Step 11 Judge whether N is less than the maximum searching range in the hidden layer neurons. If not, initialize iterations, $N = N+1$ and back to Step 4.

Step 12 Output optimal weight μ_{opt}, optimal bias v_{opt} and fitting function F with optimal ELM.

Step 13 Test nonlinear chaotic system with controller.

3. The simulations and discussions

In this section, performances of our proposed method are evaluated under various reference signals, chaotic systems, and disturbance conditions.

3.1. Different reference signals

Sinusodial chaotic map can be defined by

$$\chi(t+1) = \sin(\pi\chi(t)), \quad x_n \in (0,1). \tag{13}$$

So the fitting function with FA-ELM is

$$\widehat{\chi}(t+1) = F(\chi(t)). \tag{14}$$

The controller can be written as

$$\Gamma(t) = -F(\chi(t)) + \chi_{\mathrm{r}}(t+1) + \alpha(\chi(t) - \chi_{\mathrm{r}}(t)). \tag{15}$$

Finally, the modified nonlinear system is

$$\chi(t+1) = \sin(\pi\chi(t)) + \Gamma(t). \tag{16}$$

In modified nonlinear system, $\alpha = 0.01$ and $m = 500$, which means that the controller is added in system when $t = 500$. The entire data points 300 are used to train ELM and the rest are used to test fitting function. The size of fireflies ξ, the maximum iterations n, the random factor ε, and reference accuracy Υ are set 10, 100, 0.5, and 10^{-5} respectively in FA. The number of hidden neurons is set to be 10. The reference signals include

(i) $\chi_{\mathrm{r1}} = 0.5$,

(ii) $\chi_{\mathrm{r2}} = 0.5 + 0.1\sin(t * \pi/100)$,

(iii) $\chi_{\mathrm{r3}} = 0.1 * \mathrm{square}(t * \pi/40) + 0.6$,

(iv) $\chi_{\mathrm{r4}} = 0.1 * \mathrm{sawtooth}(t * \pi/50) + 0.5$.

All fitness value curves and outputs of system with different reference signals are shown clearly in Fig. 3. It is clear that chaos can be controlled and different reference signals are traced effectively.

3.2. Different chaotic systems

In this section, different chaotic systems are used to test adaptation of our proposed way.

System 1: Logistic map

The famous Logistic map, depicted in Eq. (17), was introduced by Robert in 1976, which can generates chaotic sequences in (0,1) with $\varpi \in (0,4]$.[18]

$$\chi(t+1) = \varpi\chi(t)(1-\chi(t)). \tag{17}$$

System 2: Chebyshev map

Chebyshev map is initially used in machine learning and image recognition and can generate chaotic phenomenon in $(-1, 1)$. This map is expressed as

$$\chi(t+1) = \cos(\varpi\cos^{-1}\chi(t)), \varpi > 0. \tag{18}$$

System 3: Tent map

Similarly, tent map can generate chaotic phenomenon in (0,1) and its formula is

$$\begin{cases} \chi(t+1) = \dfrac{\chi(t)}{0.7}, & \chi(t) < 0.7, \\ \chi(t+1) = \dfrac{10}{3}\chi(t)(1-\chi(t)), & \text{else}. \end{cases} \tag{19}$$

System 4: Cubic map

Cubic map is also a classical chaotic map in cryptography and its formula is

$$\chi(t+1) = \varpi\chi(t)(1-\chi(t)^2), \chi(t) \in (0,1). \tag{20}$$

Typical behaviors of four chaotic maps with 500 iterations are demonstrated in Fig. 4. In Logistic map, the initial value is 0.2 and $\varpi = 4$, so its behavior presents chaos in Fig. 4(a). Figure 4(b) is the result of chebyshev map with $\varpi = 5$. For divided-interval, tent chaotic map, which is a classical chaos, is a bit different from the others. Figure 4(d) is the result of cubic map with $\varpi = 2.59$.

For same sawtooth signal, figure 5 shows output of four chaotic maps. As is illustrated, low spikes appear in Figs. 5(d) and 5(f), demonstrating that the fitting precisions of chebyshev map and tent map are lower than the others. This result is a little different from our expectation, but it does not compromise the correctness of our approach.

3.3. Parameter perturbation

For the nonlinear systems in industry, parameter perturbation may appear in complex conditions, which increases barriers to effectively control chaos. In this subsection, logistic map is tested to prove performance of our proposed controller in parameter perturbation.

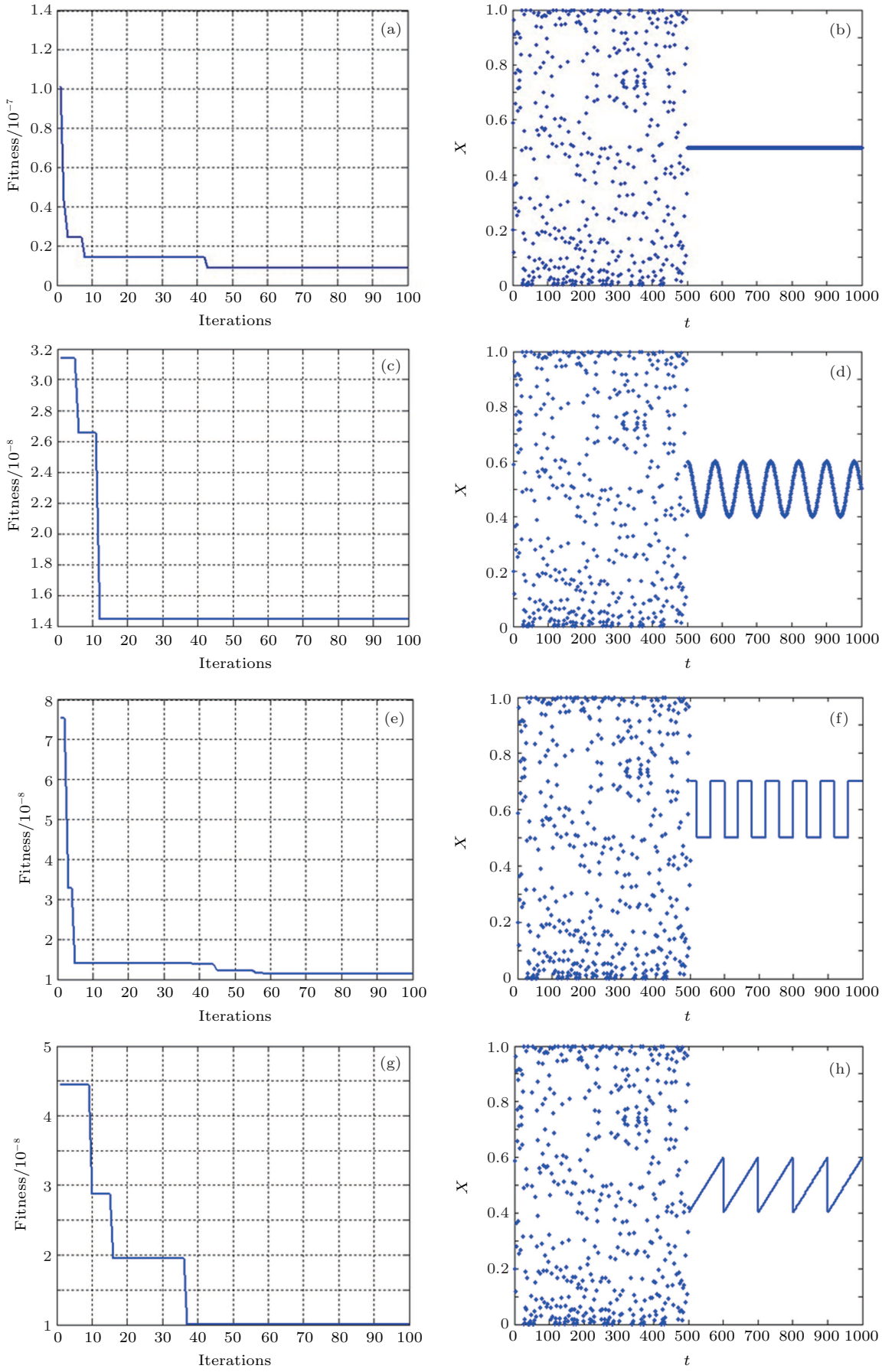


Fig. 3. (color online) Fitness value curves and output of system with different reference signals: Panels (a) and (b) are fitness value curve and output of Sinusodial chaotic system with constant signal. Panels (c) and (d) are fitness value curve and output of sinusodial chaotic system with sinusoidal signal. Panels (e) and (f) are fitness value curve and output of sinusodial chaotic system with square signal. Panels (g) and (h) are fitness value curve and output of sinusodial chaotic system with sawtooth signal.

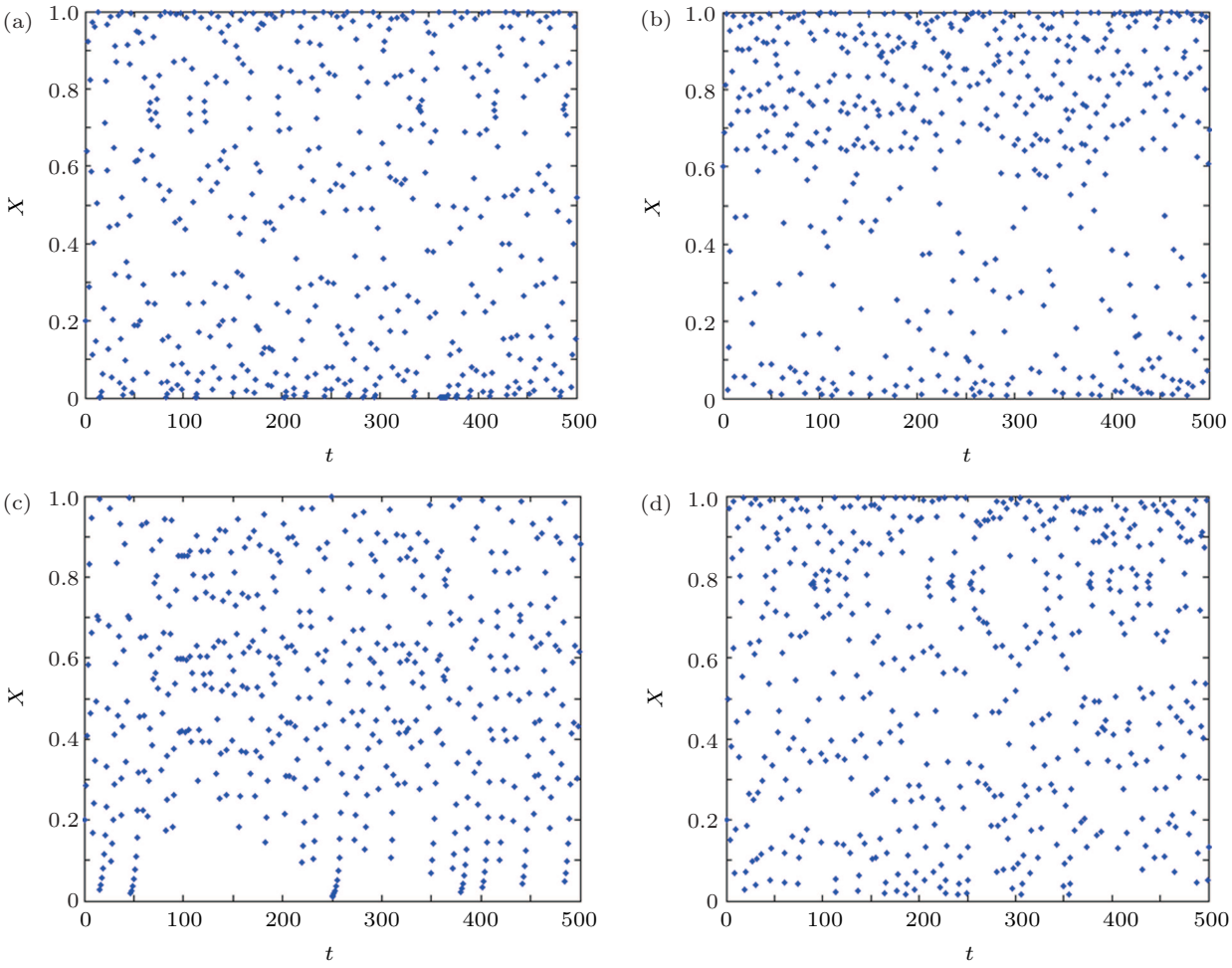


Fig. 4. (color online) Different chaotic phenomena for four chaotic maps: Panel (a) is Logistic map, panel (b) is Chebyshev map, panel (c) is Tent map, and panel (d) is Cubic map, respectively.

According to Eq. (17), assume the nonlinear is

$$\chi(t+1) = (\varpi + \Delta\varpi)\chi(t)(1-\chi(t)), \tag{21}$$

where $\Delta\varpi$ is the parameter perturbation in the system.

Similarly, the fitting function with FA-ELM is

$$\widehat{\chi}(t+1) = F(\chi(t)). \tag{22}$$

The controller can be written as

$$\Gamma(t) = -F(\chi(t)) + \chi_{\mathrm{r}}(t+1) + \alpha(\chi(t) - \chi_{\mathrm{r}}(t)). \tag{23}$$

Finally, the modified nonlinear system is

$$\chi(t+1) = (\varpi + \Delta\varpi)\chi(t)(1-\chi(t)) + \Gamma(t). \tag{24}$$

Assume parameter of nonlinear system changes in a range, that is, the parameter perturbation $\Delta\varpi = 0.01$. In this controller, the reference signals are set with $\chi_{\mathrm{r3}} = 0.1 * \mathrm{square}(t * \pi/40) + 0.6$ and $\chi_{\mathrm{r4}} = 0.1 * \mathrm{sawtooth}(t * \pi/50) + 0.5$. Other parameters are same to the ones in Subsection 3.2. The reference accuracy is set to be 10^{-5} for two signals. As is shown in Fig. 6, two signals are still traced precisely with our proposed controller.

3.4. Noise perturbation

As is known, noise cannot be avoided in process to control system, especially sampling data from it.[19] So, various ways are proposed by researchers to remove noise, such as wavelet transform (WT). In this subsection, noise perturbation is added into nonlinear system to verify our modified control scheme.

Similarly, assume the nonlinear system is Logistic chaotic map, that is

$$\chi(t+1) = \varpi\chi(t)(1-\chi(t)). \tag{25}$$

Then the fitting function by FA-ELM is

$$\widehat{\chi}(t+1) = F(\chi(t)). \tag{26}$$

So the controller is

$$\Gamma(t) = -F(\chi(t)) + \chi_{\mathrm{r}}(t+1) + \alpha(\chi(t) - \chi_{\mathrm{r}}(t)). \tag{27}$$

Finally, the modified nonlinear system is

$$\chi(t+1) = \varpi\chi(t)(1-\chi(t)) + \Gamma(t). \tag{28}$$

It is assumed that the sample data in nonlinear system are mixed by random noise, whose amplitude is 0.01 and

FA-ELM is trained by them. Other parameters are the same as given in Subsection 3.2. The reference signal includes $\chi_{r3} = 0.1 * \text{square}(t * \pi/40) + 0.6$ and $\chi_{r4} = 0.1 * \text{sawtooth}(t * \pi/50) + 0.5$. The reference accuracy Υ is set to be 10^{-5} for two target signals. The convergence parameter α is set to be 0.01. Figure 7 shows outputs of the Logistic chaotic system with the two target signals. It is clear that, even when noise perturbation exists in nonlinear system, can the performance to control chaos and trace reference signals still meet our satisfaction.

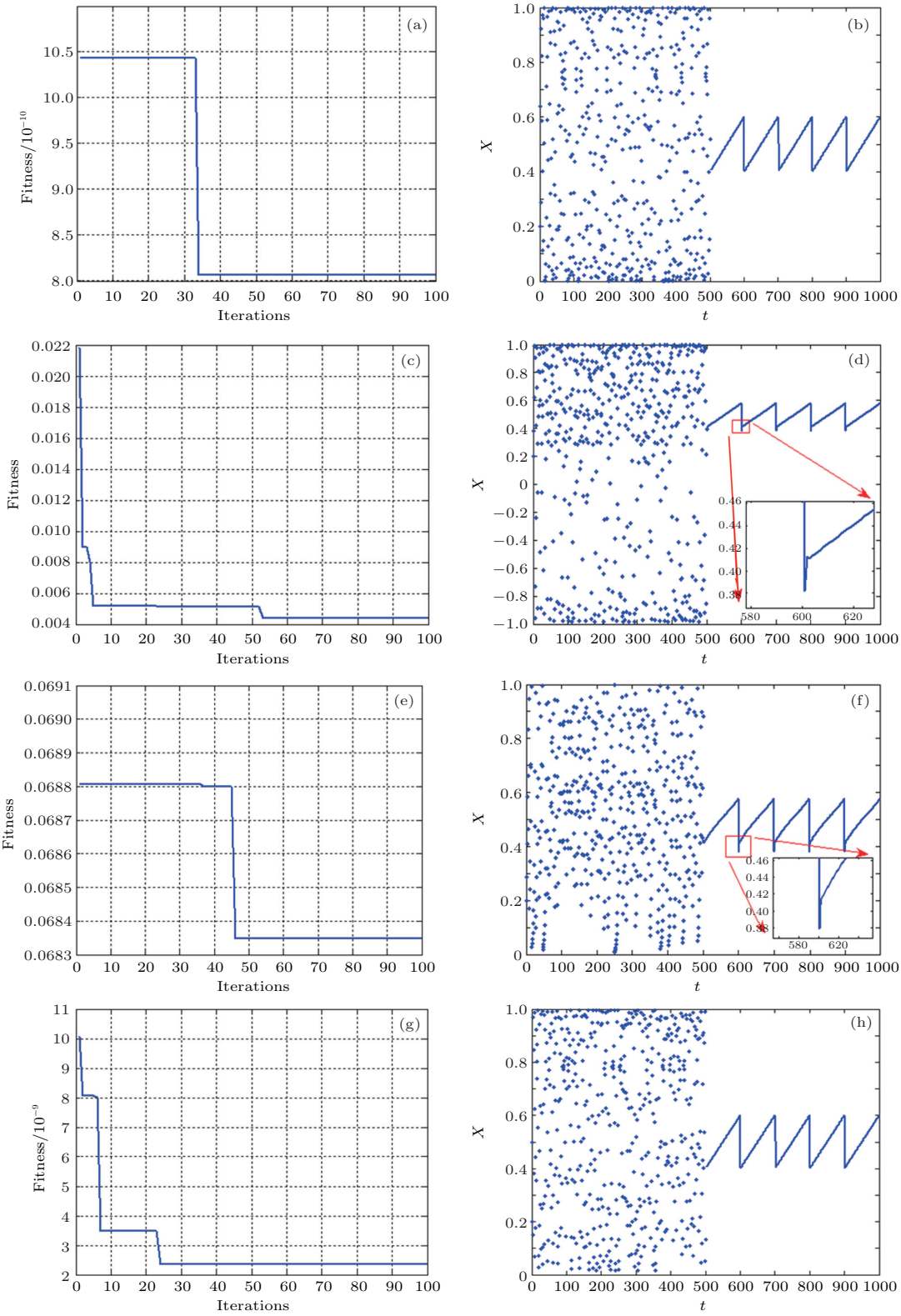


Fig. 5. (color online) Fitness value curves and output of system for four chaotic maps with sawtooth signal. Panels (a) and (b) are fitness value curve and output of Logistic chaotic system with sawtooth signal. Panels (c) and (d) are fitness value curve and output of nonlinear Chebyshev chaotic system with sawtooth signal. Panels (d) and (f) are fitness value curve and output of nonlinear Tent chaotic system with sawtooth signal. Panels (g) and (h) are fitness value curve and output of nonlinear Cubic chaotic system with sawtooth signal, respectively.

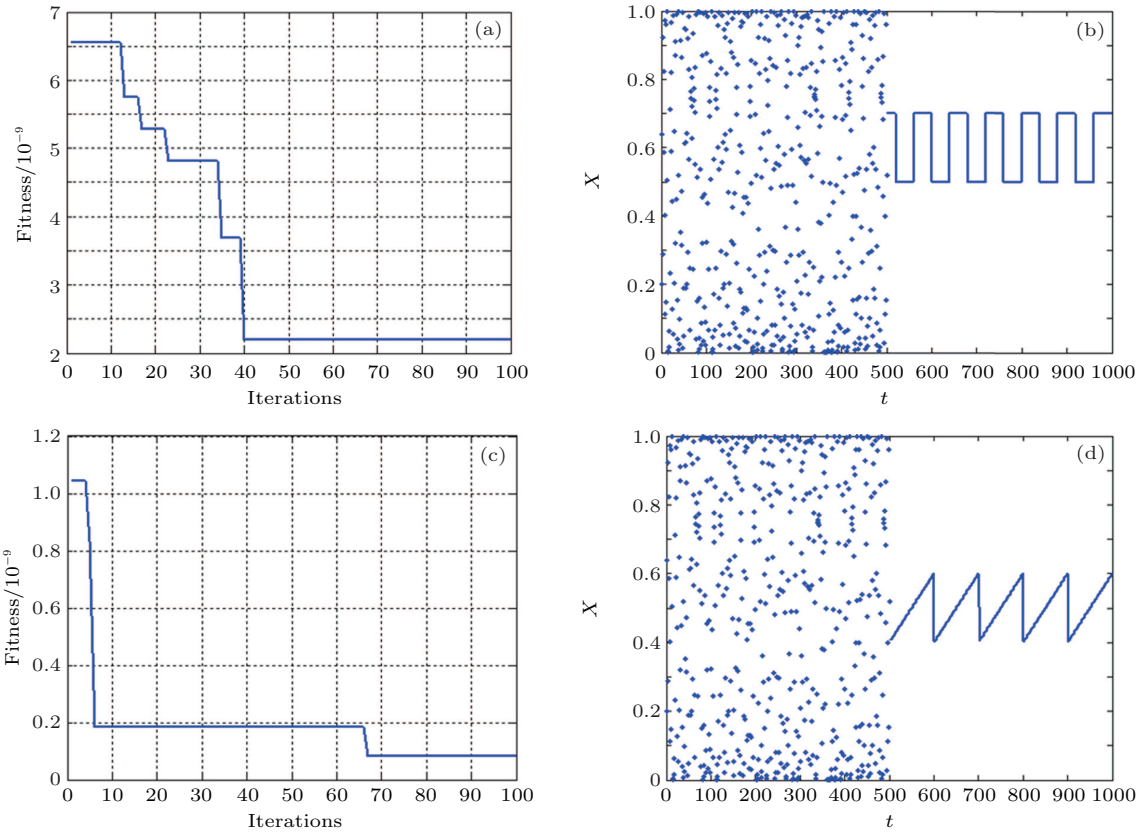


Fig. 6. (color online) Fitness value curves and output of system with parameter perturbation. Panels (a) and (b) are fitness value curve and output of Logistic chaotic system with square signal in parameter perturbation. Panels (c) and (d) are fitness value curve and output of Logistic chaotic system with sawtooth signal in parameter perturbation.

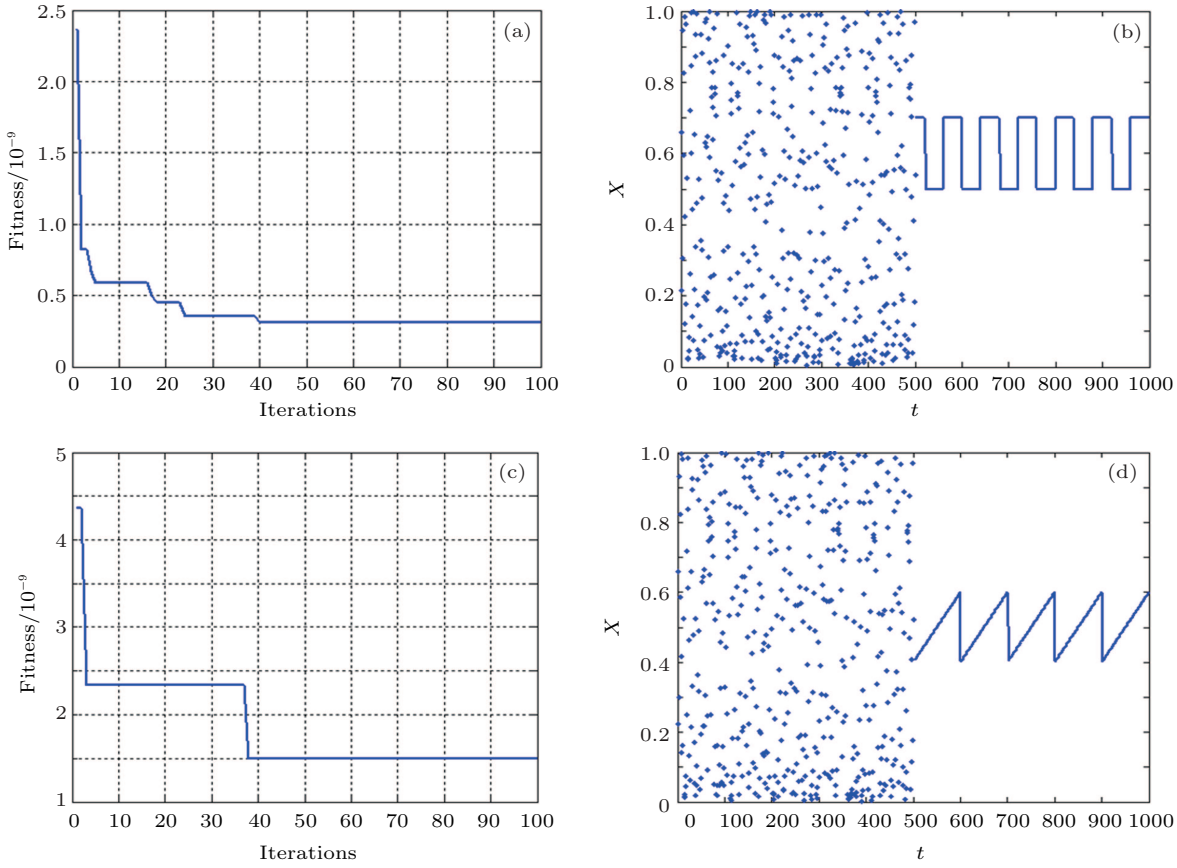


Fig. 7. (color online) Fitness value curves and output of system with noise perturbation. Panels (a) and (b) are fitness value curve and output of Logistic chaotic system with square signal in noise perturbation. Panels (c) and (d) are fitness value curve and output of Logistic chaotic system with sawtooth signal in noise perturbation.

4. Conclusion

In this paper, a novel method to control chaotic system based on FA-ELM is introduced. The proposed method can be applied in uncertain nonlinear system and eliminate the drawback associated with the traditional method. In our research, various chaotic systems, reference signals, and disturbance conditions are evaluated by controller of FA-ELM. From the results, we are able to successfully verify the correctness and effectiveness of our proposed strategy.

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