Moving bright solitons in a pseudo-spin polarization Bose–Einstein condensate*

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We study the moving bright solitons in the weak attractive Bose–Einstein condensate with a spin–orbit interaction. By solving the coupled nonlinear Schrödinger equation with the variational method and the imaginary time evolution method, two kinds of solitons (plane wave soliton and stripe solitons) are found in different parameter regions. It is shown that the soliton speed dominates its structure. The detuning between the Raman beam and energy states of the atoms decides the spin polarization strength of the system. The soliton dynamics is also studied for various moving speed and we find that the shape of individual components can be kept when the speed of soliton is low.

Keywords: moving bright solitons, spin–orbit interaction

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1. Introduction

The effect of spin–orbit coupling (SOC) for electrons in condensed matter system is particularly important for many condensed matter effect, such as spin Hall effect, topological insulator, spintronic devices, etc.[1–3] With the development of the laser-coupling techniques the momentum of ultracold neutral atoms can also be coupled with their pseudospin by the two-photon Raman process.[4–6] For ordinary materials Raman beams are known as changing no momentum from atoms because of very small photon recoil. However, when the temperature is low, the momentum of optical photon becomes quite large, and then Raman process can get the velocity-dependent link between the spin and movement of atoms.[7]

Since the experimental foundation of Bose–Einstein condensates (BEC) in ultracold atom gases, matter wave solitons have been the focus of various researches. According to the type of short-range interaction, these matter wave solitons can be deviated to two kinds of fundamental soltions: dark soliton with a density notch and phase slip across their density notch in repulsive interaction system, bright soliton with a density peak against a negligible background in attractive interaction system.[8–10] Associated with the dispersion and attractive nonlinearity, there also exist two important bright solitons in matter wave systems, namely stationary bright solitons and moving bright solitons. The general relation of stationary and moving matter wave soliton is not complex, one can obtain a moving bright soliton just by adding a trivial phase on a stationary solution. But this is not true for a typical SOC–BEC system because of the violation of Galilean invariance.[11–13] So it is not a general assignment to look for a moving bright soliton in a BEC with SOC.[14–16]

Compared with single component BEC, two-component solitons have more richer novel nonlinear effects in the SOC–BEC system.[17] For example, the intra- and inter-component attractive (repulsive) interactions, the detuning between the Raman beam and energy levels of the atoms, the strength of SOC and velocities of matter waves are almost all tunable. These ample nonlinear structures, such as dark–dark solitons,[18,19] dark–bright solitons,[20–22] which can be regarded as symbiotic solitons, bright–bright solitons,[23,24] and gap bright–bright solitons[25] in a spatially periodic Zeeman field had become the focus of related SOC–BEC researches.

In this paper, we attempt to systematically study moving bright solitons in a quasi one-dimensional (1D) SOC–BEC with attractive interaction. In particular, we are interested in how the speed of solitons and Raman detuning of the system change the density distribution of spinor wave functions and affect the spin polarization strength. It will be shown that the velocity of moving solitons mainly dominate the type of ground states, that is, plane wave solitons or stripe solitons and spin polarization intensity are mainly decided by Raman detuning strength.

The paper is organized as follows. In Section 2, we introduce the model equation for moving bright solitons in a SOC

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Bose system. Section 3 is devoted to computing the ground-state wave functions via numerical and variational method for zero detuning. We aim to give a direct insight of the different ground-state densities in a interval of parameters. In Section 4, the bright solitons are calculated by numerical method for nonzero detuning. In Section 5 we calculate the dynamic evolution for various parameters. Section 6 is a brief summary.

2. Model equation

We consider a quasi-one-dimensional SOC–BEC described by the following time-dependent reduced coupled nonlinear Schrödinger equation[14]

$$\begin{aligned} \mathrm{i}\frac{\partial}{\partial t}\phi_1 &= \Big[-\frac{1}{2}\frac{\partial^2}{\partial x^2}+\delta-g(|\phi_1|^2+|\phi_2|^2)\Big]\phi_1+\gamma\Big(\frac{\partial}{\partial x}+\mathrm{i}v\Big)\phi_2,\\ \mathrm{i}\frac{\partial}{\partial t}\phi_2 &= \Big[-\frac{1}{2}\frac{\partial^2}{\partial x^2}-\delta-g(|\phi_1|^2+|\phi_2|^2)\Big]\phi_2-\gamma\Big(\frac{\partial}{\partial x}+\mathrm{i}v\Big)\phi_1, \end{aligned} \tag{1}$$

where ϕ_1 and ϕ_2 are the up and down pseudospin components respectively, δ is the detuning between the Raman beam and energy states of the atoms, g is the interaction strength between atoms, γ represents the SOC strength, and v is the velocity of solitons. In fact equation (1) has a version of stationary equation as follows:

$$\begin{aligned} \mathrm{i}\frac{\partial}{\partial T}\phi_1 &= \Big[-\frac{1}{2}\frac{\partial^2}{\partial X^2}+\delta-g(|\phi_1|^2+|\phi_2|^2)\Big]\phi_1+\gamma\frac{\partial}{\partial X}\phi_2,\\ \mathrm{i}\frac{\partial}{\partial T}\phi_2 &= \Big[-\frac{1}{2}\frac{\partial^2}{\partial X^2}-\delta-g(|\phi_1|^2+|\phi_2|^2)\Big]\phi_2-\gamma\frac{\partial}{\partial X}\phi_1. \end{aligned} \tag{2}$$

Equation (1) can be introduced by Galilean transformation $X=x-vt$, $T=t$ from the stationary format Eq. (2). Using

$$\frac{\partial}{\partial t}=-v\frac{\partial}{\partial X}+\frac{\partial}{\partial T}$$

and

$$\frac{\partial^2}{\partial x^2}=\frac{\partial^2}{\partial X^2},$$

we obtain Eq. (1). For the sake of simplicity the intra- and inter-interaction are set to be equal to g.[19] The normalization wavefunction is determined via $\int_{-\infty}^{\infty}(|\phi_1|^2+|\phi_2|^2)\mathrm{d}x=1$. Equation (1) is the starting point of the numerical calculations throughout this paper.

For the SOC–BEC Eq. (1) there are analytical approximation bright soliton solutions via the variational method. The energy functional is given by

$$\begin{aligned} E=\int \mathrm{d}x\Big[&\frac{1}{2}|\partial_x\phi_1|^2+\frac{1}{2}|\partial_x\phi_2|^2+\delta|\phi_1|^2-\delta|\phi_2|^2\\ &+\gamma\phi_1^*\partial_x\phi_2-\gamma\phi_2^*\partial_x\phi_1-2\gamma\mathrm{Im}(\phi_1^*\phi_2)\\ &-\frac{g}{2}(|\phi_1|^4+|\phi_2|^4+2|\phi_1|^2|\phi_2|^2)\Big]. \end{aligned} \tag{3}$$

For moving bright solitons an appropriate variational ansatz is the following trial wavefunction:

$$\begin{aligned} \phi_1 &= a_1\left[\sin(b_1x)+\mathrm{i}c_1\cos(b_1x)\right]/\cosh(d_1x),\\ \phi_2 &= a_2\left[\sin(b_2x)+\mathrm{i}c_2\cos(b_2x)\right]/\cosh(d_2x). \end{aligned} \tag{4}$$

Under the normalization condition these parameters can be obtained by minimizing the energy function in Eq. (3).

3. Bright solitons for the case of zero detuning

In this section, we study the moving bright solitons in the absence of detuning. By solving Eq. (1) numerically, one can obtain the two bright soliton solutions for each component. It is remarked that to solve Eq. (1), we can firstly differentiate it using split-time-step Crank–Nicolson method along with the zero boundary condition,[26] and then evaluate several thousands of steps in imaginary time until the lowest energy is reached.[27] In the calculation, the problem is solved numerically on the interval of $[-10,10]$ with spatial size 0.02 and time step 0.0002. Figures 1(a1), 1(b1), and 1(c1) show the first moving bright soliton component in different SOC strengths when the speed is very low, i.e., $v=0.001$. In view of the left panel of Fig. 1, $|\phi_1|^2$ evolves from one node density distribution to many nodes one when the SOC constant γ is increased, which are typical strip solitons in 1D system.[19,25,28–32] However, the second soliton component in Fig. 1(a2) has one main peak, which is an ordinary density distribution in the absence of SOC effect, and can be called plane wave soliton.[29] But it changes to a strip soliton when SOC becomes stronger. "No-node" theorem is gradually ruled out the ground state of the Bose system. The width of the solitons decreases with the increase of the SOC strength. These results are consistent with the recent stationary bright soliton researches.[14]

In order to compare the results obtained by the imaginary-time method, we also calculate the density profiles for the SOC–BEC using the variational method by minimizing the energy in Eq. (3) according to those parameters respectively. These parameters a (c), d, and b can be regarded as the original characteristic of solitons, such as the height, width, and

numbers of nodes. It turns out that the variational curve of $|\phi_{1,2}|^2$ agrees well with the soliton densities obtained by the imaginary-time method for different γ cases (see Fig. 1). Based on the excellent agreement between the two methods, the imaginary-time method is mainly used for the following calculations.

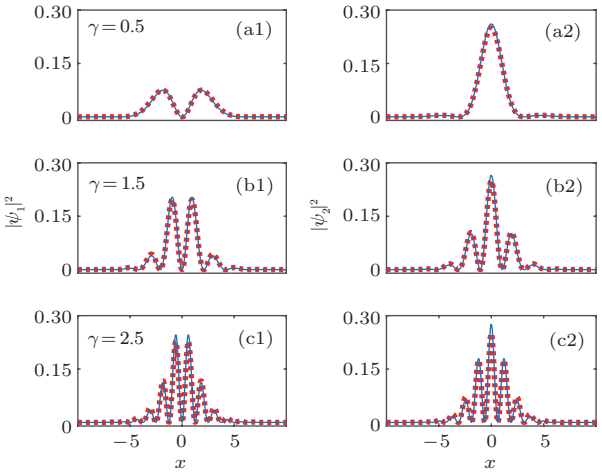


Fig. 1. (color online) [(a1), (b1), and (c1)]: The first moving bright soliton state density of an SOC–BEC. [(a2), (b2), and (c2)]: The second moving bright soliton. The solid lines are numerical results, and the dotted lines are the variational densities. The parameters for all panels (a), (b), and (c) are $g = 1$, $v = 0.001$, and $\delta = 0$. In panels (a), (b), and (c) $\gamma = 0.5$, 1.5, 2.5 respectively.

Now, let us check the solution of Eq. (1) in a larger moving speed. If the detuning δ is set to zero, we find a couple of the same bright soliton in one set of order parameters [see Fig. 2]. In this case the solution is degenerate, and the two soliton solutions share the same density mode. So, we only plot the first soliton component. It is clear that the SOC constant has less effect on the overall shape of the plain wave soliton, which is a common result in conventional BEC system.

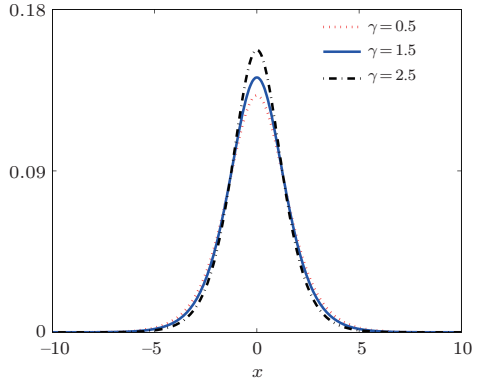


Fig. 2. (color online) The first moving bright soliton component of an SOC–BEC by the numerical method. The moving speed of soliton is $v = 3$, other parameters are the same as those in Fig. 1.

4. Bright solitons for the case of nonzero detuning

The detuning between the Raman beam and energy states of the atoms impacts SOC–BEC density distribution, which has an nonnegligible heating effect at zero detuning. Finite detuning can help to suppress the effect.[4] Different from that in nonlinear periodic system, the major characteristic feature of gap solitons in SOC–BEC admits that the number of atoms in the two components is strictly equal when the detuning is zero,[33] which can be checked by defining the pseudo spin polarization $P = \int(|\phi_1|^2 - |\phi_2|^2)\,dx$ for these bright solitons.

For slower moving speed case, $v = 0.001$, the polarization is $P = 0.2926$, 0.0023, and 0 for panels (a), (b), and (c) in Fig. 1 respectively. It shows that the strength of SOC has an effect on polarization for small moving speed even though $\delta = 0$, as shown by the polarization $P = 0.2926$ in Fig. 1(a). This is an obvious contrast with the circumstance of gap solitons in SOC BEC in optical lattice. We also find that the density distributions of the first bright soliton are different from the slower speed case [see Fig. 1(c)], and these solitons can turn into Gaussian-like density structure. This is because the effect of SOC will be coveraged by the fast speeds when the soliton speed becomes high and increase to a crucial velocity. It is like that a BECs lies in a parity–time symmetry system with complex potential trap, in which the ground state is a single Gaussian-like density peak.[34]

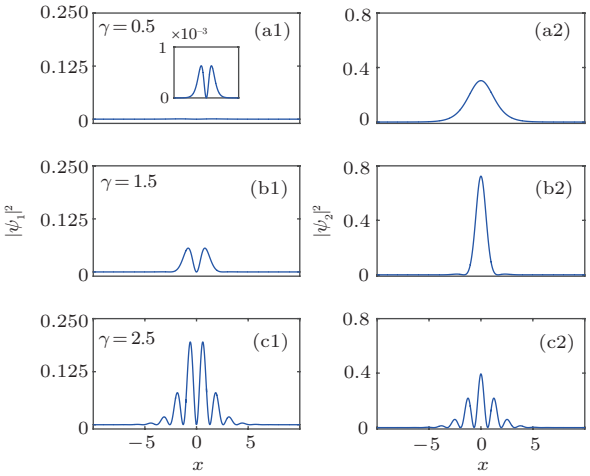


Fig. 3. (color online) The bright soliton of an SOC–BEC by the numerical method for a lower moving speed. The detuning is $\delta = 1.5$, other parameters are the same as those in Fig. 1. The inset shows an enlarged view of the curve in panel (a1).

We also investigate the dynamic process of the bright soliton with different moving speeds for the nonzero detuning case. Three typical bright solitons for smaller or bigger SOC strength is visible in Fig. 3 in the presence of finite detuning. From Figs. 1 and 3, the unbalanced distribution between the two soliton is easily seen. The number of nodes in the soliton almost has no change for different detuning, except for the case in Fig. 3(b). When a higher moving speed is present under finite detuning case, the polarization arises again in Fig. 4. With the increasing of SOC strength, the first component ϕ_1 is enhanced gradually and the polarization tendency also decreases in Fig. 4. Gaussian-like density peaks are seem to Fig. 2. These results show that the speed of solitons dominates their types, plane wave soliton or stripe solitons, and spin polarization strength is mainly decided by the detuning.

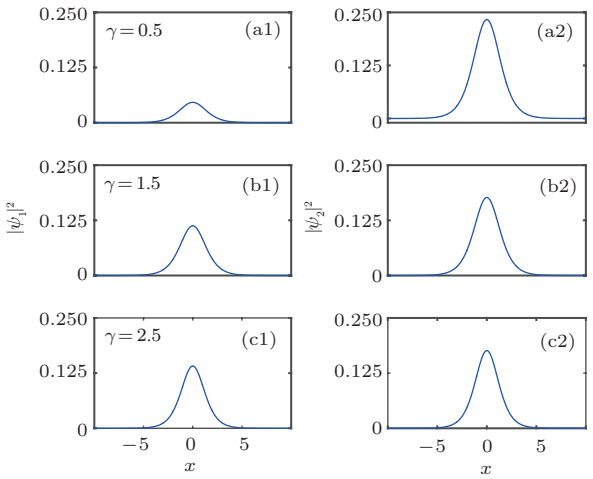


Fig. 4. (color online) The bright soliton of an SOC–BEC by the numerical method for a faster moving speed. The detuning is $\delta = 1.5$, other parameters are the same as those in Fig. 2.

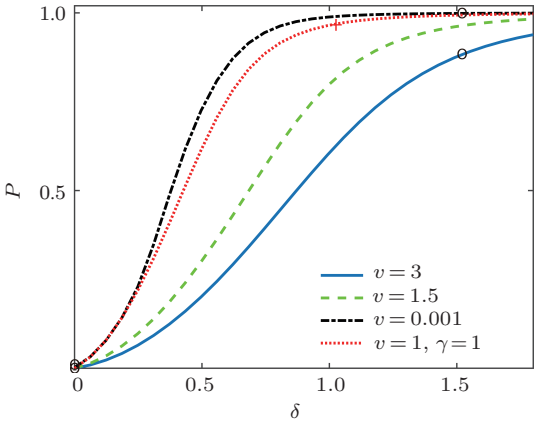


Fig. 5. (color online) The relation between polarization and detuning for different velocities of solitons. The open circles correspond to the bright soliton in Figs. 1–4 for $\gamma = 1.5$. The mark "+", corresponding to $\gamma = 1$, is what used in Fig. 4(b) in Ref. [14] to discuss the moving bright solitons.

In fact, the moving speed of bright soliton may also change the polarization amplitude. As is shown in Fig. 5, it is easy to see the differences in the relation between the polarization and detuning for various moving speeds. For the same polarization, the velocities of solitons are bigger for stronger detuning between the Raman beam and energy states of the atoms. While for the same detuning, the polarization is stronger for relative weaker moving speed. The open circle correspond the bright solitons in Figs. 1–4 for the same spin–orbit coupling strength $\gamma = 1.5$. The mark "+", corresponding to $\gamma = 1$, is what used in Fig. 4(b) in Ref. [14] to discuss the moving bright solitons.

5. The dynamics of bright solitons for different velocities

When we study the dynamics of a soliton in SOC–BEC, the dynamics property is decided by the following form

$$\Phi_1(x,t) = \phi_1(x - vt)\exp(\mathrm{i}vx),$$
$$\Phi_2(x,t) = \phi_2(x - vt)\exp(\mathrm{i}vx), \qquad (5)$$

where the wave functions $\phi_{1,2}$ are the solutions of Eq. (1) by imaginary time evolution method. Substituting Eq. (5) into Eq. (1) and setting initial state $t = 0$, we can obtain the effect of different moving speeds on the dynamic property of solitons. Comparing Eq. (1) with Eq. (2), the speed item in Eq. (1) indicates that the Hamiltonian of SOC–BEC has violated Galilean-invariant. The dynamic properties are significantly different from those in the absence of spin–orbit coupling system,[35] where the shape, height, and peaks of solitons can be held in different velocities. In the literature,[36] a little amplified bright or dark solitons can slightly change their shapes according to different velocities of solitons by the lack of Galilean invariance.

We numerically solve the coupled Eqs. (1) using split-time-step Crank–Nicolson method in real time. The results are shown in Fig. 6, in which the density and the phase of the bright solitons is denoted in panels (a) and (b), respectively. The moving speed for panels (a1) and (a2) is a smaller velocity $v = 0.001$, and for panels (a3) and (a4) is a bigger speed $v = 3$. In panels (a1) and (a3) we plot the first bright solitons component and the second bright solitons component is plotted in panels (a2) and (a4). We see clearly that the moving multi-peaks solitons maintain their shapes and heights in the parameters region, which shows no spin mixing dynamics in the parameters region. While the density of one peak solitons propagates with changing slightly their shapes.

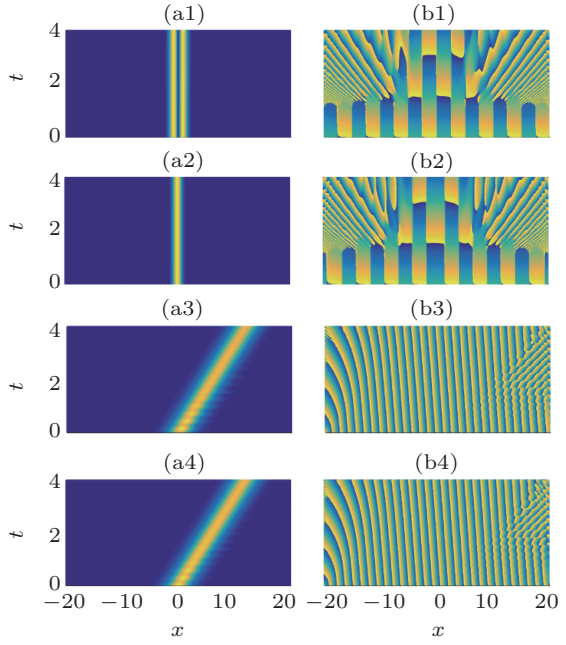


Fig. 6. (color online) Dynamical evolution of moving bright solitons for different velocities. The detuning is $\delta = 1.5$, the SOC is $\gamma = 1.5$, and interaction strength is $g = 1$. Panels (a) and (b) represent the density and the phase of the bright solitons, respectively. The parameter for panels (a1) and (a2) is $v = 0.001$, and that for panels (a3) and (a4) is $v = 3$.

Especially, one-peak soliton evolves (Figs. 6(a3) and 6(a4) with a little ripple in the initial time interval and then forms a stable propagation state. This can be understood in the following manner. When the speed of soliton is bigger in

the initial time, the SOC effect cannot be balanced. So the ripple appears, and then disappears when the balance between the speed and SOC is attained. Due to the lack of the Galilean invariance in the SOC–BEC system, it needs time to balance the anisotropy to reach a same velocity. Figure 6(b) capures many features of the phase of moving bright solitons. The phase of the individual component in one set of parameters is almost the same. It is easily seen than the difference of the phases exists between Figs. 6(b1), 6(b2) and Figs. 6(b3), 6(b4). For a smaller velocity the phase can reverse during the dynamic evolution, see Figs. 6(b1) and 6(b2), while the phase almost keeps the same shape in Figs. 6(b3) and 6(b4). We also check the dynamic of more than two peaks of solitons, and they can also keep their shapes and heights during the propagation so long as the velocity of soliton is low.

6. Summary

In summary, we have investigated the moving bright soliton properties of attractive interacting bosons in one-dimensional SOC–BEC system. The main focus is to consider the influence of the detuning and the moving velocity of the two-component solitons. Our numerical results verify that the speed of solitons dominate their structure and spin polarization strength is mainly decided by the detuning. Two types of solitons (plane wave soliton and stripe soliton) are found in lower and higher moving speed regions. The dynamics of solitons show that the shape of individual components can be kept when the speeds of solitons are small enough. This property is not tied to the number of peaks of solitons.

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