Asymptotic bounded consensus tracking of double-integrator multi-agent systems with bounded-jerk target based on sampled-data without velocity measurements*

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(Received 8 September 2016; revised manuscript received 30 September 2016; published online 22 December 2016)

This paper investigates asymptotic bounded consensus tracking (ABCT) of double-integrator multi-agent systems (MASs) with an asymptotically-unbounded-acceleration and bounded-jerk target (AUABJT) available to partial agents based on sampled-data without velocity measurements. A sampled-data consensus tracking protocol (CTP) without velocity measurements is proposed to guarantee that double-integrator MASs track an AUABJT available to only partial agents. The eigenvalue analysis method together with the augmented matrix method is used to obtain the necessary and sufficient conditions for ABCT. A numerical example is provided to illustrate the effectiveness of theoretical results.

Keywords: asymptotic bounded consensus tracking, multi-agent systems, without velocity measurements, sampled-data control

PACS: 89.75.–k, 05.65.+b, 02.30.Yy **DOI:** 10.1088/1674-1056/26/1/018903

1. Introduction

Recently, cooperative control of MASs has been widely studied because of its broad applications including swarming,[1] flocking,[2,3] rendezvous,[4] formation control,[5,6] target pursuit,[7,8] and automated highway systems. As a fundamental problem in cooperative control of MASs, consensus means that all agents under properly distributed control inputs, called a consensus protocol, converge to a common value. Many consensus protocols have been proposed for single-integrator MASs.[9–15] However, many agents such as underwater robots are controlled to achieve a desired motion by their accelerations rather than their velocities in reality. Therefore, there is a need to investigate consensus of double-integrator MASs. For many existing consensus protocols for double-integrator MASs, the eventual value to be reached is a function of all agents' initial states, which is unknown in advance. This is often called the χ-consensus problem.[16] However, in some practical applications, all agents might be required to track a common moving target. This is often called the consensus tracking (CT) problem. Since the protocols about the χ-consensus cannot be directly used to solve the CT problems, proper consensus tracking protocols (CTPs) for double-integrator MASs should be proposed. Considerable efforts have been made on the CT of double-integrator MASs, where main topics include delays,[17–19] robustness,[19–21] finite-time convergence,[21–23] and containment control.[24,25]

References [17] and [25] about the CT of double-integrator MASs assumed that continuous-time CTPs are used to guarantee the CT of double-integrator MASs. However, in many cases, when a continuous-time system is equipped with digital sensors and controllers, only sampled-data can be used to synthesise the control laws. Hence, it is important to consider sampled-data-based CT of double-integrator MASs. Recently, references [26]–[32] investigated sampled-data-based CT of double-integrator MASs. Particularly, it was proved in Ref. [32] that when a communication topology is undirected and fixed and there are paths from the target to all agents, choosing an appropriate sampling period can guarantee double-integrator MASs to track an asymptotically unbounded-acceleration and bounded-jerk target (AUABJT) available to only partial agents by using the proposed sampled-data CTP. It is easy to see from the sampled-data CTP proposed in Ref. [32] that each agent needs to know its own and neighbours' positions and velocities. However, in some practical situations, to save hardware costs, an agent is often not equipped with a velocity sensor. For an agent without a velocity sensor, its velocity information cannot be measured and is thus unavailable to itself and its neighbours. In this case, the sampled-data CTP in Ref. [32] cannot be directly applied to the CT of double-integrator MASs with an AUABJT available to only partial agents based on sampled-data without velocity measurements. Therefore, a question arises that how we can devise a sampled-data CTP without velocity measurements to guarantee that double-integrator MASs track an AUABJT available to only partial agents.

*Project supported by the National Natural Science Foundation of China (Grant Nos. 61203147, 61374047, 61473138, and 61403168) and the Fundamental Research Funds for the Central Universities of China (Grant No. JUSRP51510).

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To answer the above question, we propose a sampled-data CTP without velocity measurements by using the numerical derivatives of an agent's and an AUABJT's positions to estimate their velocities. We have theoretically proved and numerically illustrated that the proposed sampled-data CTP without velocity measurements is effective to guarantee that double-integrator MASs with an AUABJT available to only partial agents achieve asymptotic bounded consensus tracking (ABCT).

2. Preliminaries and problem formulation

2.1. Algebraic graph theory and some notations

Consider a weighted undirected graph $G=(V,E,A)$, where $V=\{v_1,v_2,\ldots,v_N\}$, $E\subseteq V\times V$ with edge elements $e_{ij}=(v_i,v_j)$ and $A=[a_{ij}]$ with nonnegative adjacency elements a_{ij} are the node set, the edge set and the weighted adjacency matrix, respectively. Assume that $e_{ij}\in E\Leftrightarrow a_{ij}>0$ and $a_{ii}=0$, $\forall i\in I\triangleq\{1,2,\ldots,N\}$. The neighbour set and the degree of node v_i are denoted by $N_i=\{v_j\in V:e_{ij}\in E\}$ and $d_i=\sum\limits_{j\in N_i}a_{ij}$, respectively. $\boldsymbol{L}=\mathrm{diag}(d_1,d_2,\ldots,d_N)-\boldsymbol{A}$ is the Laplacian matrix of G. Certainly, all row sums of L are zero and thus $\boldsymbol{L}\boldsymbol{1}_N=\boldsymbol{0}_N$, where $\boldsymbol{0}_N=\underbrace{(0,0,\ldots,0)}_{N}^{\mathrm{T}}$ and $\boldsymbol{1}_N=\underbrace{(1,1,\ldots,1)}_{N}^{\mathrm{T}}$.[33] A path from node v_i to node v_j is a sequence $(v_i,v_{k_1}),(v_{k_1},v_{k_2}),\ldots,(v_{k_l},v_j)$ composed of distinct edges. A graph is said to be connected if there exists a path between any of its two distinct nodes.

Following are some notations used in this paper. O_N and I_N represent the N-dimensional zero matrix and the N-dimensional identity matrix, respectively. The 2-norm and the transpose of a vector or matrix $\boldsymbol{M}$ are denoted by $\|\boldsymbol{M}\|_2$ and $\boldsymbol{M}^{\mathrm{T}}$, respectively. The eigenvalues, the largest eigenvalue, the spectral radius and the determinant of an N-dimensional matrix $\boldsymbol{M}$ are denoted by $\lambda_i(\boldsymbol{M})$, $i\in I$, $\bar{\lambda}(\boldsymbol{M})$, $\rho(\boldsymbol{M})$, and $\det(\boldsymbol{M})$, respectively.

2.2. Consensus tracking protocols and some lemmas

Consider MASs with N agents, where each agent's dynamics is

$$\begin{cases}\dot{x}_i(t)=y_i(t),\\ \dot{y}_i(t)=u_i(t),\end{cases}\quad i\in I,\tag{1}$$

where $x_i(t)\in R$, $y_i(t)\in R$, and $u_i(t)\in R$ represent the position, the velocity, and the control input of the i-th agent, respectively. The target's dynamics to be tracked is

$$\begin{cases}\dot{x}_0(t)=y_0(t),\\ \dot{y}_0(t)=a_0(t),\end{cases}$$

where $x_0(t)\in R$, $y_0(t)\in R$, and $a_0(t)\in R$ are the position, the velocity, and the acceleration of the target, respectively.

To better introduce the issue to be considered, we provide the following definition and assumptions.

Definition 1 We say that the systems (1) achieve ABCT, if there are $C_{\mathrm{xeub}}>0$ and $C_{\mathrm{yeub}}>0$ such that[32]

$$\begin{cases}\lim_{t\to\infty}\|x_i(t)-x_0(t)\|_2\le C_{\mathrm{xeub}}<+\infty,\\ \lim_{t\to\infty}\|y_i(t)-y_0(t)\|_2\le C_{\mathrm{yeub}}<+\infty,\end{cases}\quad\forall i\in I.$$

Assumption 1 The communication topology consisting of N agents is undirected and fixed, only partial agents can directly obtain the information of the target and there are paths from the target to all agents.[32]

Assumption 2 The target's dynamics satisfies $\lim_{t\to+\infty}|a_0(t)|=+\infty$ and $|\dot{a}_0(t)|\le\xi<+\infty, t\in R$, where $\xi>0$.[32]

In Ref. [32], the authors proved that under Assumptions 1 and 2, the systems (1) using the protocol

$$\begin{aligned}u_i(t)=&\frac{y_i(kh)-y_i(kh-h)}{h}-\beta\delta_i\big[(x_i(kh)-x_0(kh))\\&+\gamma(y_i(kh)-y_0(kh))-\eta(y_i(kh-h)-y_0(kh-h))\big]\\&+\sum_{j\in N_i}a_{ij}\big[(x_j(kh)-x_i(kh))+\gamma(y_j(kh)-y_i(kh))\\&-\eta(y_j(kh-h)-y_i(kh-h))\big],\\ t\in[kh,kh+h),&\ \ k=0,1,2,\ldots;\ \ i\in I\end{aligned}\tag{2}$$

can achieve ABCT, provided the sampling period is appropriately chosen, where $h>0$ represents the sampling period, all of $\beta>0$, $\gamma>0$, and $\eta>0$ represent feedback gains, $\delta_i=1$ if the i-th agent can directly obtain the target's information and $\delta_i=0$ otherwise.

The protocol (2) requires that each agent can obtain the positions and velocities of itself, its neighbours and the target if the target is available to the agent. To deal with the case when the velocity cannot be measured, inspired by Ref. [34], we propose the protocol without velocity measurements

$$\begin{aligned}u_i(t)=&\frac{x_i(kh)-2x_i(kh-h)+x_i(kh-2h)}{h^2}\\&-\beta\delta_i\Big[(x_i(kh)-x_0(kh))\\&+\gamma\Big(\frac{x_i(kh)-x_i(kh-h)}{h}-\frac{x_0(kh)-x_0(kh-h)}{h}\Big)\\&-\eta\Big(\frac{x_i(kh-h)-x_i(kh-2h)}{h}-\frac{x_0(kh-h)-x_0(kh-2h)}{h}\Big)\Big]\\&+\sum_{j\in N_i}a_{ij}\Big[(x_j(kh)-x_i(kh))\\&+\gamma\Big(\frac{x_j(kh)-x_j(kh-h)}{h}-\frac{x_i(kh)-x_i(kh-h)}{h}\Big)\\&-\eta\Big(\frac{x_j(kh-h)-x_j(kh-2h)}{h}-\frac{x_i(kh-h)-x_i(kh-2h)}{h}\Big)\Big],\\ t\in[kh,kh+h),&\ \ k=0,1,2,\ldots;\ \ i\in I.\end{aligned}\tag{3}$$

It can be seen from the protocol (3) that each agent only uses the position information from itself, its neighbours and

the target if the target is available to it for constructing its control input. Thus, the protocol (3) is independent of velocity measurements.

To derive the main results, we provide the lemmas below.

Lemma 1 All roots of $z^4+a_3z^3+a_2z^2+a_1z+a_0=0$ with $a_0,a_1,a_2,a_3\in R$ are within the unit circle if and only if $1+a_3+a_2+a_1+a_0>0$, $1-a_3+a_2-a_1+a_0>0$, $|a_0|<1$, $\left|a_0^2-1\right|>|a_0a_3-a_1|$, and $\left|\left(a_0^2-1\right)^2-\left(a_0a_3-a_1\right)^2\right|>\left|\left(a_0^2-1\right)\left(a_0a_2-a_2\right)-\left(a_0a_3-a_1\right)\left(a_0a_1-a_3\right)\right|$ are satisfied simultaneously.[35]

Lemma 2 If there exist paths from the target to all agents, $H\triangleq\beta\,\mathrm{diag}(\delta_1,\delta_2,\ldots,\delta_N)+L$ is a positive definite matrix.[17]

Lemma 3 For a square matrix $\boldsymbol{M}$, all of its eigenvalues are within the unit circle if and only if there exists a matrix norm $\|\cdot\|_*$ such that $\|\boldsymbol{M}\|_*<1$.[33]

3. Convergence analysis

In this section, we theoretically demonstrate the effectiveness of the protocol (3). Main results are provided below.

Theorem 1 Under Assumptions 1 and 2, the systems (1) using the protocol (3) achieve ABCT if and only if

$$0<\eta h\lambda_i(H)<3,\quad \forall i\in I, \tag{4}$$

$$\begin{aligned}&\left|\,\eta\lambda_i^2(H)h^3+\left(\eta\gamma\lambda_i^2(H)-\lambda_i(H)\right)h^2\right.\\&\left.+\left(\gamma\lambda_i(H)-5\eta\lambda_i(H)\right)h+3\,\right|\\&<-\eta^2\lambda_i^2(H)h^2+2\eta\lambda_i(H)h+3,\quad\forall i\in I\end{aligned} \tag{5}$$

and

$$\begin{aligned}&\left|\left(\eta^3\lambda_i^4(H)+\eta^2\gamma\lambda_i^4(H)+2\eta\lambda_i^3(H)\right)h^4\right.\\&-\left(\eta^4\lambda_i^4(H)+6\eta^2\lambda_i^3(H)-\eta^2\gamma^2\lambda_i^4(H)\right.\\&\left.-2\eta\gamma\lambda_i^3(H)+2\lambda_i^2(H)\right)h^3\\&+\left(8\eta^3\lambda_i^3(H)+2\eta\gamma^2\lambda_i^3(H)-6\eta^2\gamma\lambda_i^3(H)\right.\\&\left.-15\eta\lambda_i^2(H)+\gamma\lambda_i^2(H)\right)h^2\\&-\left(13\eta^2\lambda_i^2(H)+12\eta\gamma\lambda_i^2(H)-\gamma^2\lambda_i^2(H)-24\lambda_i(H)\right)h\\&\left.+42\eta\lambda_i(H)-6\gamma\lambda_i(H)\,\right|\\&<-\eta^2\lambda_i^4(H)h^5+\left(-2\eta^2\gamma\lambda_i^4(H)+2\eta\lambda_i^3(H)\right)h^4\\&+\left(-\eta^2\gamma^2\lambda_i^4(H)+\eta^4\lambda_i^4(H)+10\eta^2\lambda_i^3(H)-\lambda_i^2(H)\right)h^3\\&+\left(10\eta^2\gamma\lambda_i^3(H)-2\eta\gamma^2\lambda_i^3(H)-4\eta^3\lambda_i^3(H)\right.\\&\left.-16\eta\lambda_i^2(H)+2\gamma\lambda_i^2(H)\right)h^2\\&+\left(-27\eta^2\lambda_i^2(H)+4\eta\gamma\lambda_i^2(H)-\gamma^2\lambda_i^2(H)+6\lambda_i(H)\right)h\\&+42\eta\lambda_i(H)-6\gamma\lambda_i(H),\quad\forall i\in I\end{aligned} \tag{6}$$

hold simultaneously.

Proof Employing the period sampling technology and zero-order hold circuit, we obtain the discrete-time dynamics of the systems (1)

$$\begin{cases}x_i(kh+h)=x_i(kh)+hy_i(kh)+\frac{h^2}{2}u_i(kh),\\ y_i(kh+h)=y_i(kh)+hu_i(kh),\\ k=0,1,2,\ldots;\ \ i\in I.\end{cases} \tag{7}$$

Denote

$$\begin{aligned}&\boldsymbol{x}(t)=(x_1(t),x_2(t),\ldots,x_N(t))^{\mathrm{T}},\\&\boldsymbol{y}(t)=(y_1(t),y_2(t),\ldots,y_N(t))^{\mathrm{T}},\\&\tilde{\boldsymbol{x}}(t)=\boldsymbol{x}(t)-x_0(t)1_N,\quad\tilde{\boldsymbol{y}}(t)=\boldsymbol{y}(t)-y_0(t)1_N,\\&\Gamma(t,kh)=-x_0(t)+x_0(kh)+(t-kh)y_0(kh)\\&\qquad+\frac{(t-kh)^2}{2h^2}\left(x_0(kh)-2x_0(kh-h)+x_0(kh-2h)\right),\\&\Upsilon(t,kh)=-y_0(t)+y_0(kh)+\frac{t-kh}{h^2}\left(x_0(kh)\right.\\&\qquad\left.-2x_0(kh-h)+x_0(kh-2h)\right),\\&\varphi(t,kh)=(\Gamma(t,kh)1_N^{\mathrm{T}},\Upsilon(t,kh)1_N^{\mathrm{T}},0_N^{\mathrm{T}},0_N^{\mathrm{T}})^{\mathrm{T}},\\&\chi(kh)=\left(\tilde{\boldsymbol{x}}^{\mathrm{T}}(kh),\ \tilde{\boldsymbol{y}}^{\mathrm{T}}(kh),\ \tilde{\boldsymbol{x}}^{\mathrm{T}}(kh-h),\ \tilde{\boldsymbol{x}}^{\mathrm{T}}(kh-2h)\right)^{\mathrm{T}},\end{aligned}$$

and

$$\boldsymbol{P}=\begin{pmatrix}\frac{3}{2}I_N-\frac{h(h+\gamma)}{2}H & hI_N & -I_N+\frac{(\gamma+\eta)h}{2}H & \frac{1}{2}I_N-\frac{\eta h}{2}H\\ \frac{1}{h}I_N-(h+\gamma)H & I_N & -\frac{2}{h}I_N+(\gamma+\eta)H & \frac{1}{h}I_N-\eta H\\ I_N & O_N & O_N & O_N\\ O_N & O_N & I_N & O_N\end{pmatrix}.$$

Combining Eqs. (3) and (7) and $L1_N=0_N$, we obtain the discrete-time dynamics of $\chi(kh)$

$$\chi(kh+h)=P\chi(kh)+\varphi(kh+h,kh). \tag{8}$$

On the basis of the expression of $\boldsymbol{P}$, we have

$$\begin{aligned}\det(z\boldsymbol{I}_{4N}-\boldsymbol{P})\ =\ &\prod_{i\in I}\Big[z^4+\Big(\frac{h(h+\gamma)}{2}\lambda_i(H)-\frac{5}{2}\Big)z^3\\&+\Big(\frac{h(h-\eta)}{2}\lambda_i(H)+\frac{3}{2}\Big)z^2\\&+\Big(\frac{1}{2}-\frac{\gamma h}{2}\lambda_i(H)\Big)z-\frac{1}{2}+\frac{\eta h}{2}\lambda_i(H)\Big].\end{aligned}$$

Hence, $\rho(P)<1$ if and only if all roots of

$$\begin{aligned}&z^4+\Big(\frac{h(h+\gamma)}{2}\lambda_i(H)-\frac{5}{2}\Big)z^3+\Big(\frac{h(h-\eta)}{2}\lambda_i(H)+\frac{3}{2}\Big)z^2\\&+\Big(\frac{1}{2}-\frac{\gamma h}{2}\lambda_i(H)\Big)z-\frac{1}{2}+\frac{\eta h}{2}\lambda_i(H)=0,\quad i\in I\end{aligned} \tag{9}$$

are within the unit circle. It follows from Lemma 1 that all roots of Eq. (9) are within the unit circle if and only if

$$\begin{aligned}&1+\frac{h(h+\gamma)}{2}\lambda_i(H)-\frac{5}{2}+\frac{h(h-\eta)}{2}\lambda_i(H)+\frac{3}{2}+\frac{1}{2}\\&-\frac{\gamma h}{2}\lambda_i(H)-\frac{1}{2}+\frac{\eta h}{2}\lambda_i(H)>0,\quad i\in I,\end{aligned} \tag{10}$$

$$\begin{aligned}&1-\frac{h(h+\gamma)}{2}\lambda_i(H)+\frac{5}{2}+\frac{h(h-\eta)}{2}\lambda_i(H)+\frac{3}{2}-\frac{1}{2}\\&+\frac{\gamma h}{2}\lambda_i(H)-\frac{1}{2}+\frac{\eta h}{2}\lambda_i(H)>0,\quad i\in I,\end{aligned} \tag{11}$$

$$\left|-\frac{1}{2}+\frac{\eta h}{2}\lambda_i(H)\right|<1,\quad i\in I, \tag{12}$$

$$\left|\left(-\frac{1}{2}+\frac{\eta h}{2}\lambda_i(H)\right)\left(\frac{h(h+\gamma)}{2}\lambda_i(H)-\frac{5}{2}\right)-\left(\frac{1}{2}-\frac{\gamma h}{2}\lambda_i(H)\right)\right|$$

$$< \left| \left(-\frac{1}{2} + \frac{\eta h}{2}\lambda_i(H) \right)^2 - 1 \right|, \quad i \in I \tag{13}$$

and

$$\begin{aligned} &\left| \left(\left(-\frac{1}{2} + \frac{\eta h}{2}\lambda_i(H) \right)^2 - 1 \right) \left(\left(-\frac{1}{2} + \frac{\eta h}{2}\lambda_i(H) \right) \right. \right. \\ &\times \left(\frac{h(h-\eta)}{2}\lambda_i(H) + \frac{3}{2} \right) - \left(\frac{h(h-\eta)}{2}\lambda_i(H) + \frac{3}{2} \right) \Big) \\ &- \left(\left(-\frac{1}{2} + \frac{\eta h}{2}\lambda_i(H) \right) \left(\frac{h(h+\gamma)}{2}\lambda_i(H) - \frac{5}{2} \right) \right. \\ &- \left. \left(\frac{1}{2} - \frac{\gamma h}{2}\lambda_i(H) \right) \right) \left(\left(-\frac{1}{2} + \frac{\eta h}{2}\lambda_i(H) \right) \left(\frac{1}{2} - \frac{\gamma h}{2}\lambda_i(H) \right) \right. \\ &- \left. \left. \left(\frac{h(h+\gamma)}{2}\lambda_i(H) - \frac{5}{2} \right) \right) \right| \\ &< \left| \left(\left(-\frac{1}{2} + \frac{\eta h}{2}\lambda_i(H) \right)^2 - 1 \right)^2 - \left(\left(-\frac{1}{2} + \frac{\eta h}{2}\lambda_i(H) \right) \right. \right. \\ &\times \left. \left. \left(\frac{h(h+\gamma)}{2}\lambda_i(H) - \frac{5}{2} \right) - \left(\frac{1}{2} - \frac{\gamma h}{2}\lambda_i(H) \right) \right)^2 \right|, \quad i \in I \end{aligned} \tag{14}$$

hold simultaneously. Equivalently simplifying (10) yields $h^2\lambda_i(H) > 0, i \in I$, which is naturally satisfied under Assumption 1 and $h > 0$ with the help of Lemma 2. Equivalently simplifying Eq. (11) leads to $4 > 0, i \in I$, which naturally holds. Equivalently simplifying (12) under Assumption 1 results in (4). Equivalently simplifying (13) and (14) yields (5) and (6), respectively. Therefore, all roots of (9) are within the unit circle if and only if (4)–(6) hold simultaneously. This indicates that $\rho(\boldsymbol{P}) < 1$ if and only if (4)–(6) hold simultaneously. From Lemma 3 we derive that if and only if (4)–(6) are met, there exists a matrix norm $\|\cdot\|_*$ such that

$$\|\boldsymbol{P}\|_* < 1. \tag{15}$$

Employing the matrix norms' equivalence, we derive that there are $C_2 > 0$ and $C_* > 0$ such that

$$\frac{\|\cdot\|_*}{C_2} \le \|\cdot\|_2 \le C_* \|\cdot\|_*. \tag{16}$$

Using Taylor expansion, we deduce that there are $\theta_1(k,h) \in (kh, kh+h)$, $\theta_2(k,h) \in (kh-h, kh)$, $\theta_3(k,h) \in (kh-2h, kh)$ and $\theta_4(k,h) \in (kh, kh+h)$ such that

$$\begin{aligned} x_0(kh+h) &= x_0(kh) + hy_0(kh) + \frac{h^2}{2}a_0(kh) \\ &\quad + \frac{h^3}{6}\dot{a}_0(\theta_1(k,h)), \end{aligned} \tag{17}$$

$$\begin{aligned} x_0(kh-h) &= x_0(kh) - hy_0(kh) + \frac{h^2}{2}a_0(kh) \\ &\quad - \frac{h^3}{6}\dot{a}_0(\theta_2(k,h)), \end{aligned} \tag{18}$$

$$\begin{aligned} x_0(kh-2h) &= x_0(kh) - 2hy_0(kh) + 2h^2a_0(kh) \\ &\quad - \frac{4h^3}{3}\dot{a}_0(\theta_3(k,h)), \end{aligned} \tag{19}$$

$$y_0(kh+h) = y_0(kh) + ha_0(kh) + \frac{h^2}{2}\dot{a}_0(\theta_4(k,h)). \tag{20}$$

Combining Eqs. (17)–(19) results in

$$\begin{aligned} \Gamma(kh+h,kh) &= -x_0(kh+h) + x_0(kh) + hy_0(kh) \\ &\quad + \frac{x_0(kh) - 2x_0(kh-h) + x_0(kh-2h)}{2} \\ &= -h^3\Big(\frac{1}{6}\dot{a}_0(\theta_1(k,h)) - \frac{1}{6}\dot{a}_0(\theta_2(k,h)) \\ &\quad + \frac{2}{3}\dot{a}_0(\theta_3(k,h))\Big), \end{aligned} \tag{21}$$

which together with Assumption 2 leads to

$$|\Gamma(kh+h,kh)| \le h^3\xi, \quad k = 0,1,2,\ldots \tag{22}$$

Combining Eqs. (18)–(20) results in

$$\begin{aligned} \Upsilon(kh+h,kh) &= -y_0(kh+h) + y_0(kh) \\ &\quad + \frac{x_0(kh) - 2x_0(kh-h) + x_0(kh-2h)}{h} \\ &= h^2\Big(\frac{1}{3}\dot{a}_0(\theta_2(k,h)) - \frac{4}{3}\dot{a}_0(\theta_3(k,h)) \\ &\quad + \frac{1}{2}\dot{a}_0(\theta_4(k,h))\Big), \end{aligned} \tag{23}$$

which together with Assumption 2 leads to

$$|\Upsilon(kh+h,kh)| \le \frac{13h^2\xi}{6}, \quad k = 0,1,2,\ldots \tag{24}$$

Combining (22) and (24) results in

$$\|\varphi(kh+h,kh)\|_2 \le \frac{\sqrt{N}h^2\xi\sqrt{36h^2+169}}{6}, \quad k = 0,1,2,\ldots \tag{25}$$

Considering (8), we have

$$\chi(nh) = P^n\chi(0) + \sum_{k=0}^{n-1} P^{n-k-1}\varphi(kh+h,kh). \tag{26}$$

This together with (16) and (25) leads to

$$\begin{aligned} \|\chi(nh)\|_2 &\le \|\boldsymbol{P}^n\|_2\|\chi(0)\|_2 + \left\| \sum_{k=0}^{n-1} \boldsymbol{P}^{n-k-1}\varphi(kh+h,kh) \right\|_2 \\ &\le C_*\|\chi(0)\|_2\|\boldsymbol{P}\|_*^n \\ &\quad + C_*\left\| \sum_{k=0}^{n-1} \boldsymbol{P}^{n-k-1}\varphi(kh+h,kh) \right\|_* \\ &\le C_*\|\chi(0)\|_2\|\boldsymbol{P}\|_*^n \\ &\quad + C_*\sum_{k=0}^{n-1}\left\|\boldsymbol{P}^{n-k-1}\right\|_*\|\varphi(kh+h,kh)\|_* \\ &\le C_*\|\chi(0)\|_2\|\boldsymbol{P}\|_*^n \\ &\quad + C_*C_2\sum_{k=0}^{n-1}\|\boldsymbol{P}\|_*^{n-k-1}\|\varphi(kh+h,kh)\|_2 \\ &\le C_*\|\chi(0)\|_2\|P\|_*^n \\ &\quad + \frac{C_*C_2\sqrt{N}h^2\xi\sqrt{36h^2+169}(1-\|\boldsymbol{P}\|_*^{n-1})}{6(1-\|\boldsymbol{P}\|_*)}. \end{aligned} \tag{27}$$

Combining (15) and (27) yields

$$\begin{aligned}\lim_{n\to\infty}\|\chi(nh)\|_2 &\le C_*\|\chi(0)\|_2\lim_{n\to\infty}\|\boldsymbol{P}\|_*^n\\ &\quad+\frac{C_*C_2\sqrt{N}h^2\xi\sqrt{36h^2+169}(1-\lim_{n\to\infty}\|\boldsymbol{P}\|_*^{n-1})}{6(1-\|\boldsymbol{P}\|_*)}\\ &=\frac{C_*C_2\sqrt{N}h^2\xi\sqrt{36h^2+169}}{6(1-\|\boldsymbol{P}\|_*)},\end{aligned}$$

which further leads to

$$\lim_{n\to\infty}\left\|\begin{pmatrix}\tilde{x}(nh)\\ \tilde{y}(nh)\end{pmatrix}\right\|_2\le\frac{C_*C_2\sqrt{N}h^2\xi\sqrt{36h^2+169}}{6(1-\|\boldsymbol{P}\|_*)}<+\infty. \tag{28}$$

The systems (1) under the protocol (3) become

$$\begin{cases}x_i(t)=x_i(kh)+(t-kh)y_i(kh)+\frac{(t-kh)^2}{2}u_i(kh),\\ y_i(t)=y_i(kh)+(t-kh)u_i(kh),\\ t\in[kh,kh+h),\ \ k=0,1,2,\ldots;\ \ i\in I,\end{cases} \tag{29}$$

which leads to

$$\begin{cases}\tilde{\boldsymbol{x}}(t)=\Big(I_N-\frac{(t-kh)^2}{2}H-\frac{(t-kh)^2}{2h}\gamma H+\frac{(t-kh)^2}{2h^2}I_N\Big)\tilde{x}(kh)\\ \qquad+(t-kh)I_N\tilde{y}(kh)\\ \qquad+\Big(\frac{(t-kh)^2}{2h}\gamma H+\frac{(t-kh)^2}{2h}\eta H-\frac{(t-kh)^2}{h^2}I_N\Big)\tilde{x}(kh-h)\\ \qquad+\Big(-\frac{(t-kh)^2}{2h}\eta H+\frac{(t-kh)^2}{2h^2}I_N\Big)\tilde{x}(kh-2h)+\Gamma(t,kh)1_N,\\ \tilde{\boldsymbol{y}}(t)=\Big(-(t-kh)H-\frac{t-kh}{h}\gamma H+\frac{t-kh}{h^2}I_N\Big)\tilde{x}(kh)+I_N\tilde{y}(kh)\\ \qquad+\Big(\frac{t-kh}{h}\gamma H+\frac{t-kh}{h}\eta H-2\frac{t-kh}{h^2}I_N\Big)\tilde{x}(kh-h)\\ \qquad+\Big(-\frac{t-kh}{h}\eta H+\frac{t-kh}{h^2}I_N\Big)\tilde{x}(kh-2h)+\Upsilon(t,kh)1_N\\ \qquad t\in[kh,kh+h),\ \ k=0,1,2,\ldots;\ \ i\in I.\end{cases} \tag{30}$$

With the aid of the similar analysis method in (22) and (24), we deduce that under Assumption 2

$$\begin{aligned}|\Gamma(t,kh)|&\le(t-kh)^2\Big(\frac{t-kh}{6}+\frac{h}{6}+\frac{2h}{3}\Big)\xi\\ &\le h^3\xi,\quad t\in[kh,kh+h),\quad k=0,1,2,\ldots\end{aligned} \tag{31}$$

and

$$\begin{aligned}|\Upsilon(t,kh)|&\le(t-kh)\Big(\frac{t-kh}{2}+\frac{h}{3}+\frac{4h}{3}\Big)\xi\le\frac{13h^2\xi}{6},\\ &\qquad t\in[kh,kh+h),\quad k=0,1,2,\ldots\end{aligned} \tag{32}$$

Combining (28), (30)–(32) leads to

$$\begin{cases}\lim_{t\to\infty}\|\tilde{x}(t)\|_2\le[3+h+(\gamma+\eta+\frac{h}{2})h\bar{\lambda}(H)]\\ \qquad\times\dfrac{C_*C_2\sqrt{N}h^2\xi\sqrt{36h^2+169}}{6(1-\|\boldsymbol{P}\|_*)}+\sqrt{N}h^3\xi\\ \qquad\triangleq C_{\text{xeub}}<+\infty,\\ \lim_{t\to\infty}\|\tilde{y}(t)\|_2\le[1+\frac{4}{h}+(h+2\gamma+2\eta)\bar{\lambda}(H)]\\ \qquad\times\dfrac{C_*C_2\sqrt{N}h^2\xi\sqrt{36h^2+169}}{6(1-\|\boldsymbol{P}\|_*)}+\dfrac{13\sqrt{N}h^2\xi}{6}\\ \qquad\triangleq C_{\text{yeub}}<+\infty.\end{cases}$$

According to Definition 1, the systems (1) under the protocol (3) achieve ABCT.

Remark 1 When the sampling period h is sufficiently small, (5) and (6), respectively, degenerate into

$$|(\gamma\lambda_i(H)-5\eta\lambda_i(H))h+3|<2\eta\lambda_i(H)h+3,\ \ \forall i\in I \tag{33}$$

and

$$\begin{aligned}&\big|-\big(13\eta^2\lambda_i^2(H)+12\eta\gamma\lambda_i^2(H)-\gamma^2\lambda_i^2(H)-24\lambda_i(H)\big)h\\ &+42\eta\lambda_i(H)-6\gamma\lambda_i(H)|\\ &<\big(-27\eta^2\lambda_i^2(H)+4\eta\gamma\lambda_i^2(H)-\gamma^2\lambda_i^2(H)+6\lambda_i(H)\big)h\\ &+42\eta\lambda_i(H)-6\gamma\lambda_i(H),\ \ \forall i\in I.\end{aligned} \tag{34}$$

Equivalently simplifying (33) yields

$$\gamma<7\eta, \tag{35}$$

$$(3\eta-\gamma)\lambda_i(H)h<6,\ \ \forall i\in I. \tag{36}$$

Equivalently simplifying (34) yields

$$(-20\eta^2\lambda_i(H)-8\eta\gamma\lambda_i(H)+15)h>6(\gamma-7\eta),\ \ \forall i\in I, \tag{37}$$

$$7\eta^2\lambda_i(H)+\gamma^2\lambda_i(H)+9<8\eta\gamma\lambda_i(H),\ \ \forall i\in I. \tag{38}$$

It can be verified that (35)–(38) hold simultaneously, provided h, γ and η are appropriately chosen. For example, when h is sufficiently small, $\gamma=4\eta$ and $\eta^2>\frac{1}{\lambda_i(H)}$, $\forall i\in I$, (35)–(38) hold simultaneously. On the basis of the above analysis, there are appropriate h, γ and η such that (35)–(38) hold simultaneously. Note that (4) holds when h is sufficiently small. Employing the continuous dependence of (4)–(6) on h, we infer that there are appropriate h, γ and η such that (4)–(6) hold simultaneously. Therefore, we theoretically prove that the protocol (3) is effective to guarantee that double-integrator MASs with an AUABJT available to only partial agents achieve ABCT.

4. Simulations

In this section, we provide a numerical example to demonstrate the effectiveness of the protocol (3). Consider MASs with five agents, whose communication topology is provided in Fig. 1, where the target is labelled as 0.

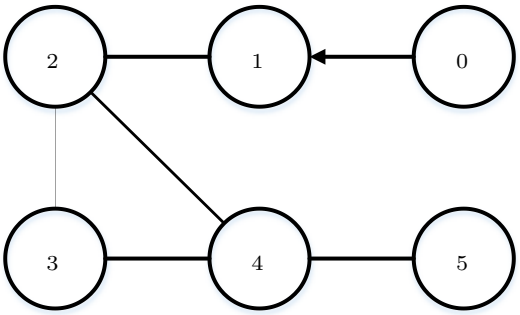


Fig. 1. Communication topology.

Without loss of generality, we assume that all weights of edges are 1. The initial states of five agents are randomly set. It can be verified that when $(\beta,\gamma,\eta,h)=(1,10,5,0.01)$, (4)–(6) hold simultaneously. Figure 2 illustrates the numerical results.

It is clear from Fig. 2 that the positions and velocities of all agents converge to the position and velocity of the target in the asymptotical and bounded sense. This means that the systems (1) under the protocol (3) achieve ABCT. This numerically illustrates the effectiveness of the protocol (3).

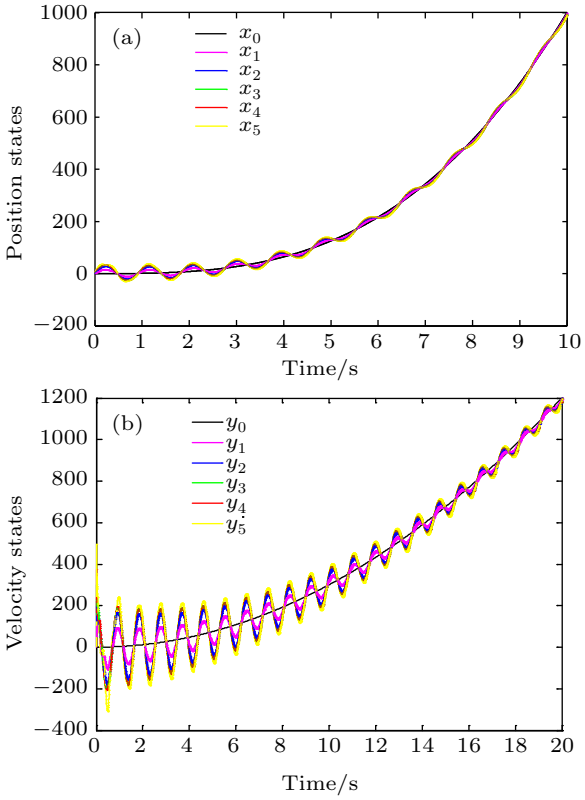


Fig. 2. (color online) Positions and velocities of systems (1) under protocol (3) with $(\beta,\gamma,\eta,h)=(1,10,5,0.01)$ and $x_0(t)=t^3$.

5. Conclusions

In this paper, we considered ABCT of double-integrator MASs with an AUABJT available to partial agents based on sampled-data without velocity measurements. We proposed a sampled-data CTP without velocity measurements and proved that the proposed protocol is effective to guarantee that double-integrator MASs with an AUABJT available to only partial agents achieve ABCT. In this paper, to overcome the difficulties caused by the unavailability of velocity information we used the numerical derivatives of an agent's and an AUABJT's positions to estimate their velocities. Recently, some distributed estimation-based CTPs have been proposed to solve CT of MASs with partial measurements.[36–39] Motivated by the methods in Refs. [36]–[39], in the future we intend to design appropriately distributed estimation-based CTPs without velocity measurements to deal with the issue considered in this paper. In this paper, we assumed the communication topology to be undirected and fixed. Considering that the communication topology might be directed and switching in some real situations, we will extend this paper's results to the case of a directed switching communication topology.

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