Second-order temporal interference of two independent light beams at an asymmetrical beam splitter*

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The second-order temporal interference of classical and nonclassical light at an asymmetrical beam splitter is discussed based on two-photon interference in Feynman's path integral theory. The visibility of the second-order interference pattern is determined by the properties of the superposed light beams, the ratio between the intensities of these two light beams, and the reflectivity of the asymmetrical beam splitter. Some requirements about the asymmetrical beam splitter have to be satisfied in order to ensure that the visibility of the second-order interference pattern of nonclassical light beams exceeds the classical limit. The visibility of the second-order interference pattern of photons emitted by two independent single-photon sources is independent of the ratio between the intensities. These conclusions are important for the researches and applications in quantum optics and quantum information when an asymmetrical beam splitter is employed.

Keywords: two-photon interference, Feynman's path integral theory, asymmetrical beam splitter

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1. Introduction

The beam splitter (BS) is a simple yet important element in classical optics,[1] quantum optics,[2] and quantum information.[3] The symmetrical BS was assumed in most existing studies in order to simplify the calculations.[2,3] However, the asymmetrical BS is more general than the symmetrical BS, since it is difficult to produce a symmetrical BS in practice. Further more, the asymmetrical BS has important applications in quantum cryptography,[4] multiphoton de Broglie wavelength measurement,[5–8] filtering out photonic Fock states,[9] and other interesting applications.[10–16] It will be helpful to understand how different the interference patterns are for symmetrical and asymmetrical beam splitters. There have been studies about the properties of both symmetrical and asymmetrical beam splitters in quantum theory.[17–19] However, the systematical study about the second-order interference of two independent light beams at an asymmetrical BS is still missing. In this paper, we will employ the two-photon interference theory to study this topic and show how the visibility of the second-order interference pattern is influenced by the superposed light beams and the asymmetrical BS.

Although both classical and quantum theories can be employed to calculate the second-order interference of the classical light, only quantum theory is valid when the nonclassical light is employed.[20–22] We have employed two-photon interference theory to discuss the second-order interference of light at a symmetrical BS,[23–27] which is helpful to understand the physics behind the mathematical calculations. The same method will be employed to calculate the second-order interference of two independent light beams at an asymmetrical BS.

The following parts are organized as follows. In Section 2, we will calculate the second-order interference of different types of light beams superposed at an asymmetrical BS based on the two-photon interference in Feynman's path integral theory. The discussion and conclusions are in Sections 3 and 4, respectively.

2. Theory

The scheme for the second-order interference of two independent light beams at an asymmetrical BS is shown in Fig. 1. Here, S_a and S_b are two independent light sources, which can emit classical or nonclassical light, ABS is an asymmetrical beam splitter, D_1 and D_2 are two single-photon detectors, and CC is a two-photon coincidence counting detection system. The optical distances via ABS between S_a and D_1, S_a and D_2, S_b and D_1, S_b and D_2 are all assumed to be equal.

There are three different ways to trigger a two-photon coincidence count in Fig. 1. The first way is that these two photons are both emitted by S_a. The second way is that these two photons are both emitted by S_b. The third way is that these two photons are emitted by S_a and S_b, respectively. The intensities of the light beams emitted by S_a and S_b are I_a and I_b, respectively. The reflectivity and transmittivity of ABS are R and T,

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respectively. The sum of R and T is 1 for a lossless BS.[17–19] The probability for the photon detected by D_1 coming from S_a is

$$P_{1a} = \frac{I_a T}{I_a T + I_b R}, \tag{1}$$

and P_{1b}, the probability for the photon detected by D_1 coming from S_b, is $1 - P_{1a}$. The probability for the photon detected by D_2 coming from S_a is

$$P_{2a} = \frac{I_a R}{I_a R + I_b T}, \tag{2}$$

and P_{2b}, the probability for the photon detected by D_2 coming from S_b, is $1 - P_{2a}$. With the preparation above, we can calculate the probability for the three different ways to trigger a two-photon coincidence count in Fig. 1. The probability for these two photons coming from S_a is $P_{1a}P_{2a}$. The probability for these two photons coming from S_b is $P_{1b}P_{2b}$. The probability for these two photons coming from S_a and S_b, respectively, is $P_{1a}P_{2b} + P_{1b}P_{2a}$. The sum of these three probabilities equals to 1.

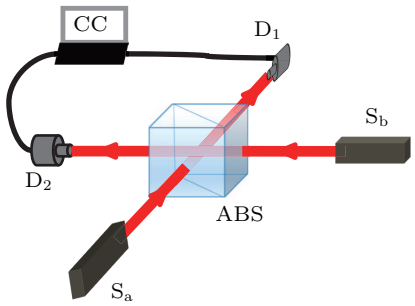


Fig. 1. (color online) The scheme for the second-order interference of two independent light beams at an asymmetrical BS. Here, S_a and S_b are two independent light sources, ABS is an asymmetrical beam splitter, D_1 and D_2 are two single-photon detectors, and CC is a two-photon coincidence counting detection system.

For simplicity, we assume S_a and S_b are two point sources. The Feynman's photon propagator for a point light source is[28]

$$K_{\alpha\beta} = \frac{\exp[-\mathrm{i}(\boldsymbol{k}_{\alpha\beta}\cdot\boldsymbol{r}_{\alpha\beta} - 2\pi\nu_\alpha t_\beta)]}{r_{\alpha\beta}}, \tag{3}$$

which is the same as the Green function for a point light source in classical optics,[1] $\boldsymbol{k}_{\alpha\beta}$ and $\boldsymbol{r}_{\alpha\beta}$ are the wave and position vectors of the photon emitted by S_α and detected at D_β, respectively, $r_{\alpha\beta} = |\boldsymbol{r}_{\alpha\beta}|$ is the distance between S_α and D_β, and ν_α and t_β are the frequency and time for the photon that is emitted by S_α and detected at D_β, respectively ($\alpha =$ a and b, $\beta = 1$ and 2).

2.1. The second-order interference of laser and thermal light

We will first calculate the second-order interference of laser and thermal light in Fig. 1. S_a is assumed to be the thermal light source and S_b is assumed to be the single-mode laser light source. If the photons emitted by these two sources are indistinguishable, these three different ways to trigger a two-photon coincidence count are indistinguishable. Based on the superposition principle in Feynman's path integral theory,[29] the second-order coherence function in Fig. 1 is[24–26]

$$\begin{aligned} & G^{(2)}_{\mathrm{lt}}(\boldsymbol{r}_1, t_1; \boldsymbol{r}_2, t_2) \\ = {} & \langle |\sqrt{P_{1a}P_{2a}}\Big(\frac{1}{\sqrt{2}}\mathrm{e}^{\mathrm{i}\varphi_a}K_{a1}\,\mathrm{e}^{\mathrm{i}(\varphi'_a+\frac{\pi}{2})}K_{a2} \\ & + \frac{1}{\sqrt{2}}\mathrm{e}^{\mathrm{i}(\varphi_a+\frac{\pi}{2})}K_{a2}\,\mathrm{e}^{\mathrm{i}\varphi'_a}K_{a1}\Big) \\ & + \sqrt{P_{1b}P_{2b}}\,\mathrm{e}^{\mathrm{i}(\varphi_b+\frac{\pi}{2})}K_{b1}\,\mathrm{e}^{\mathrm{i}\varphi'_b}K_{b2} \\ & + \sqrt{P_{1a}P_{2b}}\,\mathrm{e}^{\mathrm{i}\varphi''_a}K_{a1}\,\mathrm{e}^{\mathrm{i}\varphi''_b}K_{b2} \\ & + \sqrt{P_{1b}P_{2a}}\,\mathrm{e}^{\mathrm{i}(\varphi''_b+\frac{\pi}{2})}K_{b1}\,\mathrm{e}^{\mathrm{i}(\varphi''_a+\frac{\pi}{2})}K_{a2}|^2\rangle, \end{aligned} \tag{4}$$

where $\langle\cdot\rangle$ is the ensemble average by taking all the possible phases into consideration, φ_α is the phase of the photon emitted by S_α ($\alpha =$ a and b), The extra phase $\pi/2$ is due to that the photon reflected by a BS will gain an extra phase comparing to the transmitted one. The first two terms on the righthand side of Eq. (4) correspond to that the two detected photons are both emitted by the thermal light source S_a. There are two indistinguishable alternatives for the two photons in thermal light to trigger a two-photon coincidence count.[30] The third term on the righthand side of Eq. (4) corresponds to that the two detected photons are both emitted by the laser light source S_b. There is only one alternative.[24–26] The last two terms on the righthand side of Eq. (4) correspond to that the two detected photons are emitted by S_a and S_b, respectively.

The phases of photons in the thermal light are random and the phases of photons in the single-mode laser light are identical within the coherence time.[31] Taking these phase relations into consideration, equation (4) can be simplified as

$$\begin{aligned} & G^{(2)}_{\mathrm{lt}}(\boldsymbol{r}_1, t_1; \boldsymbol{r}_2, t_2) \\ = {} & \frac{P_{1a}P_{2a}}{2}\langle |K_{a1}K_{a2} + K_{a2}K_{a1}|^2\rangle \\ & + P_{1b}P_{2b}\langle |K_{b1}K_{b2}|^2\rangle \\ & + \langle |\sqrt{P_{1a}P_{2b}}K_{a1}K_{b2} - \sqrt{P_{1b}P_{2a}}K_{b1}K_{a2}|^2\rangle. \end{aligned} \tag{5}$$

Substituting Eq. (3) into Eq. (5) and with the same method as the one in Refs. [24]–[26] it is straightforward to have a one-dimension second-order temporal coherence function as

$$\begin{aligned} & G^{(2)}_{\mathrm{lt}}(t_1 - t_2) \\ \propto {} & 1 + P_{1a}P_{2a} - 2\sqrt{P_{1a}P_{1b}P_{2a}P_{2b}}\cos[2\pi\Delta\nu(t_1 - t_2)], \end{aligned} \tag{6}$$

where a quasi-monochromatic assumption is employed.[2] These two detectors are assumed to be at symmetrical positions in order to concentrate on the temporal interference pattern. The $\Delta\nu$ is the difference between the mean frequencies of the light beams emitted by S_a and S_b, which equals to $|\nu_a - \nu_b|$.

The definition of the visibility of the second-order interference pattern is

$$V = \frac{G^{(2)}_{\max} - G^{(2)}_{\min}}{G^{(2)}_{\max} + G^{(2)}_{\min}}, \tag{7}$$

where $G^{(2)}_{\max}$ and $G^{(2)}_{\min}$ are the maximal and minimal values of the second-order coherence function, respectively. The visibility of the second-order interference pattern of the laser and thermal light in Fig. 1 is

$$V_{\rm lt} = \frac{2\sqrt{P_{1a}P_{1b}P_{2a}P_{2b}}}{1 + P_{1a}P_{2a}}. \tag{8}$$

Substituting Eqs. (1) and (2) into Eq. (8), the visibility equals to

$$V_{\rm lt} = \frac{2xR(1-R)}{(x+R-xR)(1-R+xR)+x^2R(1-R)}, \tag{9}$$

where x equals to $I_{\rm a}/I_{\rm b}$. The visibility of the second-order interference pattern of laser and thermal light depends on the ratio between the intensities of these two light beams and the reflectivity of the asymmetrical BS. Figure 2 shows how the visibility changes when the ratio x and reflectivity R vary. When x equals to $\sqrt{2}/2$ and R equals to 0.5, the visibility reaches its maximal value, $1/(\sqrt{2}+1)$, which is less than the limit of the visibility of the second-order interference pattern of classical light beams in Fig. 1.[32]

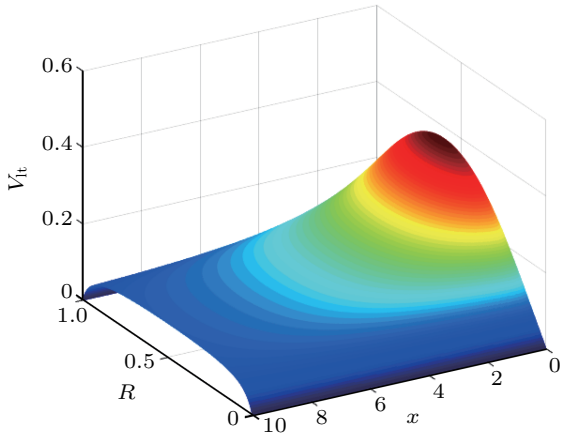


Fig. 2. (color online) Visibility of the second-order interference pattern of laser and thermal light. Here, $V_{\rm lt}$: visibility; x: ratio between $I_{\rm a}$ and $I_{\rm b}$; R: reflectivity.

The visibility in Fig. 2 is symmetrical about $R = 0.5$, which can be clearly seen in Fig. 3. The lines of a–f correspond to $x = 0.1$, 0.5, 0.71, 2, 5, and 10, respectively. For a fixed x, the visibility reaches its maximum when R equals to 0.5 and decreases when R deviates from 0.5.

Figure 4 shows how the visibility varies as x changes for different reflectivity. The visibility increases as x increases from 0 to $\sqrt{2}/2$ and gets its maximum when x equals to $\sqrt{2}/2$, and then the visibility decreases as x increases from $\sqrt{2}/2$. We only draw the situations when x is not greater than 10 in Fig. 4. It is easy to predict that the visibility will continue to decrease when x increases from 10.

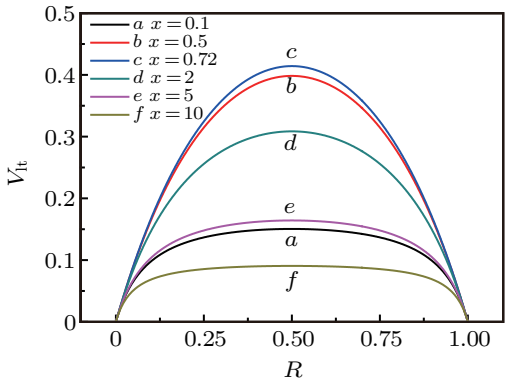


Fig. 3. (color online) Visibility versus the reflectivity for different ratios. The lines of a–f correspond to $x = 0.1$, 0.5, 0.71, 2, 5, and 10, respectively.

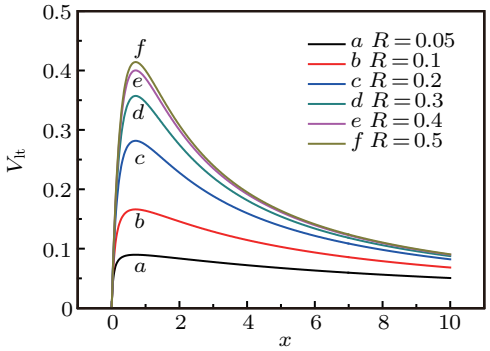


Fig. 4. (color online) Visibility versus the ratio for different reflectivities. The lines of a–f correspond to $x = 0.05$, 0.1, 0.2, 0.3, 0.4, and 0.5, respectively.

2.2. The second-order interference of laser and laser light

With the same method mentioned above, it is straightforward to calculate the second-order interference of other types of light beams. The second-order coherence function of two independent single-mode laser light beams superposed at an asymmetrical BS in Fig. 1 is

$$\begin{aligned} & G^{(2)}_{\rm ll}(\boldsymbol{r}_1, t_1; \boldsymbol{r}_2, t_2) \\ = {} & \langle | \sqrt{P_{1a}P_{2a}}\, {\rm e}^{{\rm i}\varphi_{\rm a}} K_{\rm a1}\, {\rm e}^{{\rm i}(\varphi'_{\rm a}+\frac{\pi}{2})} K_{\rm a2} \\ & + \sqrt{P_{1b}P_{2b}}\, {\rm e}^{{\rm i}(\varphi_{\rm b}+\frac{\pi}{2})} K_{\rm b1}\, {\rm e}^{{\rm i}\varphi'_{\rm b}} K_{\rm b2} \\ & + \sqrt{P_{1a}P_{2b}}\, {\rm e}^{{\rm i}\varphi''_{\rm a}} K_{\rm a1}\, {\rm e}^{{\rm i}\varphi''_{\rm b}} K_{\rm b2} \\ & + \sqrt{P_{1b}P_{2a}}\, {\rm e}^{{\rm i}(\varphi''_{\rm b}+\frac{\pi}{2})} K_{\rm b1}\, {\rm e}^{{\rm i}(\varphi''_{\rm a}+\frac{\pi}{2})} K_{\rm a2} |^2 \rangle, \end{aligned} \tag{10}$$

where the meanings of the symbols are similar to the ones in Eq. (4). The first term on the righthand side of Eq. (10) corresponds to that the two detected photons are both emitted by S_a. The second term on the righthand side of Eq. (10) corresponds to that the two detected photons are both emitted by S_b. The last two terms on the righthand side of Eq. (10) correspond

to that the two detected photons are emitted by S_a and S_b, respectively. Since these two laser light beams are independent, equation (10) can be simplified as[23]

$$\begin{aligned} & G_{\rm ll}^{(2)}(\boldsymbol{r}_1,t_1;\boldsymbol{r}_2,t_2) \\ = \; & P_{1a}P_{2a}\langle|K_{a1}K_{a2}|^2\rangle \\ & + P_{1b}P_{2b}\langle|K_{b1}K_{b2}|^2\rangle \\ & + \langle|\sqrt{P_{1a}P_{2b}}K_{a1}K_{b2} - \sqrt{P_{1b}P_{2a}}K_{b1}K_{a2}|^2\rangle. \end{aligned} \tag{11}$$

With the same method mentioned above,[24–26] the one-dimension second-order temporal coherence function of two independent single-mode laser light beams at an asymmetrical BS in Fig. 1 is

$$\begin{aligned} & G_{\rm ll}^{(2)}(t_1-t_2) \\ \propto \; & 1-2\sqrt{P_{1a}P_{1b}P_{2a}P_{2b}}\cos[2\pi\Delta\nu(t_1-t_2)], \end{aligned} \tag{12}$$

where all the approximations as the ones in Eq. (6) have been assumed to obtain Eq. (10). The visibility of the second-order interference pattern of these two independent laser light beams is

$$V_{\rm ll} = \frac{2xR(1-R)}{(x+R-xR)(1-R+xR)}. \tag{13}$$

Figure 5 shows how the visibility of the second-order interference pattern changes with the ratio x and the reflectivity R, which is similar to the ones in Fig. 2. The visibility in these two cases is symmetrical about $R=0.5$. The visibility first increases as x increases and then decreases as x continues to increase. However, there are some differences between these two situations. One difference is that the maximal visibility in Fig. 5 is 0.5, while it is $1/(\sqrt{2}+1)$ in Fig. 2. Another difference is that the visibility of the second-order interference pattern of these two independent single-mode laser light beams reaches its maximum when x equals to 1 and R equals to 0.5, while the visibility reaches its maximum when x equals to $\sqrt{2}/2$ and R equals to 0.5 for the second-order interference of laser and thermal light.

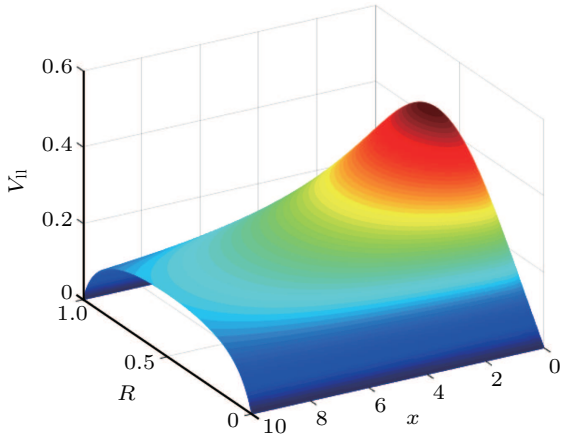


Fig. 5. (color online) Visibility of the second-order interference pattern of laser and laser light.

2.3. The second-order interference of thermal and thermal light

The second-order coherence function of two independent thermal light beams at an asymmetrical BS in Fig. 1 is

$$\begin{aligned} & G_{\rm tt}^{(2)}(\boldsymbol{r}_1,t_1;\boldsymbol{r}_2,t_2) \\ = \; & \langle|\sqrt{P_{1a}P_{2a}}[\frac{1}{\sqrt{2}}e^{i\varphi_a}K_{a1}e^{i(\varphi'_a+\frac{\pi}{2})}K_{a2} \\ & + \frac{1}{\sqrt{2}}e^{i(\varphi_a+\frac{\pi}{2})}K_{a2}e^{i\varphi'_a}K_{a1}] \\ & + \sqrt{P_{1b}P_{2b}}[\frac{1}{\sqrt{2}}e^{i(\varphi_b+\frac{\pi}{2})}K_{b1}e^{i\varphi'_b}K_{b2} \\ & + \frac{1}{\sqrt{2}}e^{i\varphi_b}K_{b2}e^{i(\varphi'_b+\frac{\pi}{2})}K_{b1}] \\ & + \sqrt{P_{1a}P_{2b}}e^{i\varphi''_a}K_{a1}e^{i\varphi''_b}K_{b2} \\ & + \sqrt{P_{1b}P_{2a}}e^{i(\varphi''_b+\frac{\pi}{2})}K_{b1}e^{i(\varphi''_a+\frac{\pi}{2})}K_{a2}|^2\rangle, \end{aligned} \tag{14}$$

where the meanings of the symbols are the same as those above. The visibility of the second-order interference pattern of these two independent thermal light beams is

$$V_{\rm tt} = \frac{2xR(1-R)}{(x+R-xR)(1-R+xR)+x^2R(1-R)+R(1-R)}. \tag{15}$$

Figure 6 shows how the visibility changes with the ratio x and reflectivity R. The maximal visibility is $1/3$, which is achieved when x equals to 1 and R equals to 0.5.

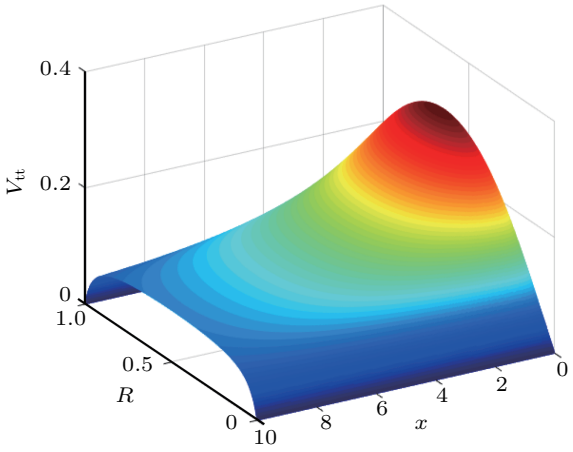


Fig. 6. (color online) Visibility of the second-order interference pattern of two independent thermal light beams.

2.4. The second-order interference of photons emitted by two independent single-photon sources

We have calculated the second-order interference of thermal and laser light beams at an asymmetrical BS based on two-photon interference theory. The calculation for the second-order interference of nonclassical light at an asymmetrical BS is similar to that of the classical light. The second-order coherence function of photons emitted by two independent single-photon sources in Fig. 1 is

$$G_{\rm ss}^{(2)}(\boldsymbol{r}_1,t_1;\boldsymbol{r}_2,t_2)$$

$$= \langle | \sqrt{P_{1a}P_{2b}}\, e^{i\varphi_a} K_{a1}\, e^{i\varphi_b} K_{b2} + \sqrt{P_{1b}P_{2a}}\, e^{i(\varphi_b+\frac{\pi}{2})} K_{b1}\, e^{i(\varphi_a+\frac{\pi}{2})} K_{a2} |^2 \rangle. \quad (16)$$

The reason why there are only two terms on the righthand side of Eq. (16) is that the single-photon source can only emit one photon at a time. The probability for the two detected photons coming from the same source is zero. The only possible way to trigger a two-photon coincidence count in Fig. 1 is the two detected photons coming from S_a and S_b, respectively. The visibility of the second-order interference pattern of photons emitted by two independent single-photon sources in Fig. 1 is

$$V_{ss} = \frac{2R(1-R)}{1-2R+2R^2}. \quad (17)$$

Figure 7 shows the visibility of the second-order interference pattern of photons emitted by two independent single-photon sources. The maximal visibility is 1, which exceeds the limit for the visibility of the second-order interference pattern with two independent classical light beams.[32] The visibility is independent of the ratio x and only determined by the reflectivity. Since it does not matter what the intensity of the light emitted by the single-photon source is, the two-photon coincidence counting rate is determined by the single-photon source whose emission rate is less.

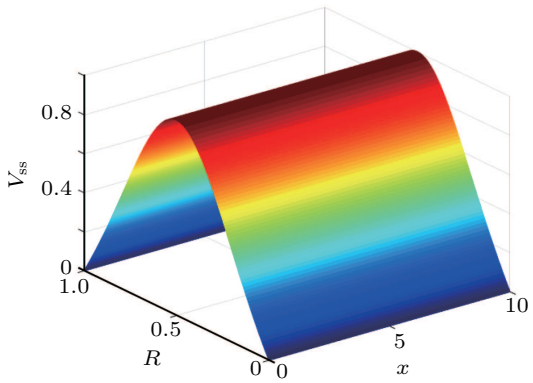


Fig. 7. (color online) Visibility of the second-order interference pattern of photons emitted by two independent single-photon sources.

2.5. The second-order interference of photons emitted by a single-photon source and a single-mode laser

If the photons emitted by the single-photon source and the single-mode laser are indistinguishable, the second-order coherence function of the photons emitted by these two sources in Fig. 1 is

$$G^{(2)}_{sl}(\boldsymbol{r}_1,t_1;\boldsymbol{r}_2,t_2) = \langle | \sqrt{P_{1b}P_{2b}}\, e^{i(\varphi_b+\frac{\pi}{2})} K_{b1}\, e^{i\varphi'_b} K_{b2} + \sqrt{P_{1a}P_{2b}}\, e^{i\varphi''_a} K_{a1}\, e^{i\varphi''_b} K_{b2} + \sqrt{P_{1b}P_{2a}}\, e^{i(\varphi''_b+\frac{\pi}{2})} K_{b1}\, e^{i(\varphi''_a+\frac{\pi}{2})} K_{a2} |^2 \rangle, \quad (18)$$

where S_a is assumed to be the single-photon source and S_b is assumed to be the single-mode laser. The probability for the two detected photons coming from the single-photon source (S_a) is zero. The visibility of the second-order interference pattern is

$$V_{sl} = \frac{2xR(1-R)}{x(1-2R+2R^2)+R(1-R)}. \quad (19)$$

Figure 8 shows the visibility of the second-order interference pattern of photons emitted by the single-photon source and the single-mode laser. Here, x is the ratio between the intensities of the light emitted by the single-photon source and the single-mode laser. For a fixed reflectivity, the visibility approaches the value in Fig. 7 as x approaches infinity. Since x cannot be infinity in a real experiment, the visibility of the second-order interference pattern of photons emitted by the single-photon source and the single-mode laser cannot reach 1. The visibility in Fig. 8 can exceed 0.5, which is the limit for the classical light.[32] The condition for V_{sl} to exceed 0.5 is different for different values of x and R. When R is not in the interval of $((\sqrt{3}-1)/2\sqrt{3}, (\sqrt{3}+1)/2\sqrt{3})$, it is impossible for V_{sl} to exceed 0.5 no matter what the value of the ratio might be. When R is in the interval of $((\sqrt{3}-1)/2\sqrt{3}, (\sqrt{3}+1)/2\sqrt{3})$, x should be larger than $(R-R^2)/(6R-6R^2-1)$ in order to ensure that V_{sl} exceeds 0.5.

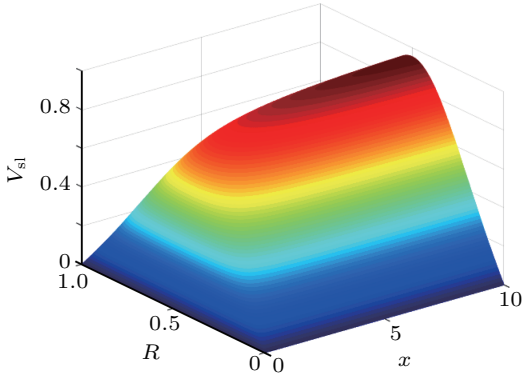


Fig. 8. (color online) Visibility of the second-order interference pattern of photons emitted by a single-photon source and a single-mode laser.

2.6. The second-order interference of photons emitted by a single-photon source and a thermal source

If the photons emitted by the single-photon source and the thermal source are indistinguishable, the second-order coherence function of photons emitted by these two sources in Fig. 1 is

$$G^{(2)}_{st}(\boldsymbol{r}_1,t_1;\boldsymbol{r}_2,t_2) = \langle | \sqrt{P_{1b}P_{2b}} \left(\frac{1}{\sqrt{2}}\, e^{i(\varphi_b+\frac{\pi}{2})} K_{b1}\, e^{i\varphi'_b} K_{b2} + \frac{1}{\sqrt{2}}\, e^{i\varphi_b} K_{b2}\, e^{i(\varphi'_b+\frac{\pi}{2})} K_{b1} \right) + \sqrt{P_{1a}P_{2b}}\, e^{i\varphi''_a} K_{a1}\, e^{i\varphi''_b} K_{b2} + \sqrt{P_{1b}P_{2a}}\, e^{i(\varphi''_b+\frac{\pi}{2})} K_{b1}\, e^{i(\varphi''_a+\frac{\pi}{2})} K_{a2} |^2 \rangle, \quad (20)$$

where S_a is assumed to be the single-photon source and S_b is assumed to be the thermal source. The visibility for the second-order interference pattern is

$$V_{st} = \frac{2xR(1-R)}{x(1-2R+2R^2)+2R(1-R)}. \quad (21)$$

Figure 9 shows the visibility of the second-order interference pattern, which is similar to the one in Fig. 8 except that the visibility in Fig. 9 is less than the one in Fig. 8 for any fixed x and R. The visibility in Fig. 9 can also exceed 0.5. When R is not in the interval of $((\sqrt{3}-1)/2\sqrt{3},(\sqrt{3}+1)/2\sqrt{3})$, it is impossible for V_{st} to exceed 0.5 no matter what the value of the ratio might be. When R is in the interval of $((\sqrt{3}-1)/2\sqrt{3},(\sqrt{3}+1)/2\sqrt{3})$, x should be larger than $(2R-2R^2)/(6R-6R^2-1)$ in order to ensure that V_{st} exceeds 0.5.

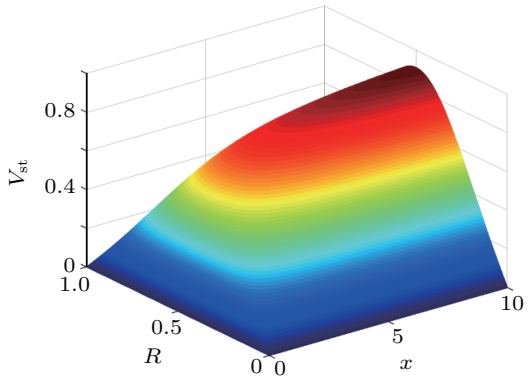


Fig. 9. (color online) Visibility of the second-order interference pattern of photons emitted by a single-photon source and a thermal source.

3. Discussion

In the last section, we calculated the visibility for the second-order interference pattern of two independent light beams at an asymmetrical BS. Table 1 summarizes the maximal visibility of the second-order interference pattern. Here, l, t, and s are short for laser, thermal, and single-photon light, respectively, V_{max} is the maximal visibility, R_{max} and x_{max} are the reflectivity and ratio when the visibility reaches V_{max}, respectively.

Table 1. The maximal visibility of the second-order interference pattern of two independent light beams at an asymmetrical BS. In this table, l: laser light; t: thermal light; s: single-photon light; V_{max}: maximal visibility; R_{max}: the reflectivity when the visibility reaches V_{max}; x_{max}: the ratio when the visibility reaches V_{max}.

Light	V_{max}	R_{max}	x_{max}
lt	$1/(\sqrt{2}+1)$	0.5	$\sqrt{2}/2$
ll	0.5	0.5	1
tt	1/3	0.5	1
ss	1	0.5	$(0,\infty)$
sl	$\to 1$	0.5	$\to\infty$
st	$\to 1$	0.5	$\to\infty$

It was predicted that the maximal visibility for the second-order interference pattern of two independent classical light beams is 0.5.[32] When the two independent single-mode laser light beams are superposed at a symmetrical BS, the visibility of the second-order interference pattern can reach 0.5. When the two single-mode laser light beams are superposed at an asymmetrical BS ($R \neq 0.5$), the visibility is always less than 0.5. When the thermal and laser light beams are superposed at BS, no matter symmetrical or asymmetrical, the visibility is less than 0.5. The maximal visibility of the second-order interference pattern of these two independent thermal light beams is 1/3. When the nonclassical light is employed, the visibility of the second-order interference pattern can be larger than 0.5. For instance, the visibility can reach 1 when the photons emitted by two independent single-photon sources are superposed at a symmetrical BS. The $(0,\infty)$ in row 5 of Table 1 means that the visibility in this case gets its maximal for any value of x when R equals to 0.5. The visibility of the second-order interference pattern in this case can be larger than 0.5 when the reflectivity of the asymmetrical BS is in the range of $((\sqrt{3}-1)/2\sqrt{3},(\sqrt{3}+1)/2\sqrt{3})$. The visibility can be larger than 0.5 when the single-photon light is superposed with the laser or thermal light. In row 6 of Table 1, the maximal visibility approaches 1 when the ratio between the intensities of light beams emitted by the single-photon source and laser approaches infinity. The visibility of the second-order interference of photons emitted by the single-photon source and the laser (or thermal source) is dependent on the ratio. When the visibility of the second-order interference of photons emitted by the single-photon source and the laser is required to be larger than 0.5, the reflectivity and ratio should satisfy some requirements as pointed out in Section 3. Only satisfying one condition cannot guarantee the visibility exceeding 0.5.

4. Conclusions

In conclusion, we have employed two-photon interference in Feynman's path integral theory to calculate the second-order temporal interference of two independent light beams at an asymmetrical BS. It is confirmed that the visibility of the second-order interference pattern with laser and thermal light beams cannot exceed 0.5. On the other hand, when the nonclassical light is employed, the visibility can be larger than 0.5. The maximal visibility of the second-order interference pattern is always obtained when the reflectivity of the asymmetrical BS equals to 0.5, which explains why the symmetrical BS instead of the asymmetrical BS is employed in most of the interference experiments.

Although the conclusions in this paper are based on the second-order temporal interference, it is straightforward to generalize the method to the spatial part and prove that similar conclusions are true for the second-order spatial interference. The studies in this paper are helpful to understand the second-order interference of classical and nonclassical light at

an asymmetrical BS, which is important for the applications in quantum optics and quantum information when the asymmetrical BS is employed.

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