Optimal quantum parameter estimation of two-qutrit Heisenberg XY chain under decoherence*

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Adopting the Milburn decoherence model, we investigate the performance of quantum Fisher information of the two-qutrit isotropic Heisenberg XY chain under decoherence. We find that the quantum Fisher information with respect to the decoherence rate and the magnetic field decreases exponentially in the long-time limit, which significantly reduces the precision of optimal quantum estimation. We also show that with the increase of the decoherence rate or the magnetic field, the QFIs go down considerably. Furthermore, we find that the precision of optimal quantum estimation can be enhanced by the entanglement in the input state.

Keywords: intrinsic decoherence, quantum Fisher information, spin chain

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1. Introduction

Quantum metrology,[1,2] investigating the limit of the precision of parameter estimation bounded by quantum mechanics, has excited wide interest recently.[3–9] Quantum Fisher information (QFI) plays an important role in quantum metrology. According to the quantum Cramér–Rao inequality,[10] QFI determines the lower bound of the variance of the unbiased estimator for a parameter.[10,11] Moreover, QFI has also been applied in quantum frequency standards,[12] detection of entanglement,[13] probing the quantum phase transition,[14] and so on. Recently, many schemes have been proposed to improve the QFI in a noisy environment. Reference [15] showed that the QFI can be strengthened by the strategy of dynamical decoupling. The enhancement of the QFI employing weak measurement and measurement reversal has been reported. Other methods, such as quantum feedback,[16] quantum screening,[17] and non-Markovian environment,[18] are also useful to enhance the parameter estimation precision.

The spin chain[19] is a natural candidate for quantum estimation,[20–22] because as a solid-state system it has good scalability, and its exchange coupling can produce a many-body entanglement state. In this paper, we adopt the quantum parameter estimation, specifically the QFI to investigate the model of the two-qutrit isotropic Heisenberg XY chain in a uniform magnetic field under the intrinsic decoherence. We focus on two estimating parameters, the decoherence rate and the magnetic field strength. We find that the QFIs with respect to the decoherence rate and the magnetic field strength increase with the time in the initial evolution, and decrease exponentially in the long-time limit. We also show that with the increase of the decoherence rate or the magnetic field, the QFIs go down considerably. In addition, we display that the estimation precision can be enhanced by the degree of the entanglement in the input state, which is consistent with the results in Refs. [23] and [24].

The paper is organized as follows. In the next section, we review the main aspects of the QFI. In Section 3, we address our model, the two-qutrit isotropic Heisenberg XY chain in a uniform magnetic field under the intrinsic decoherence. In Section 4, we study the QFI of our model with respect to the decoherence rate and the magnetic field strength. The conclusions are given in Section 5.

2. Quantum Fisher information

We briefly review the main aspects of the QFI. In classical statistical inference theory, Fisher information is used to measure the amount of information about an unknown parameter θ from the measurement results of an observable variable X. The classical Fisher information with respect to the parameter θ is defined as[25]

$$G(\theta)=\sum_i p_i(x|\theta)\left[\frac{\partial}{\partial\theta}\ln p_i(x|\theta)\right]^2, \tag{1}$$

where $p_i(x|\theta)$ denotes the probability density of the measurement outcome $\{x_i\}$ conditioned on the fixed parameter θ. By

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extending the Fisher information to the quantum regime, the QFI is defined as[26]

$$F(\theta)=\mathrm{Tr}\left(\rho_\theta L_\theta^2\right)=\mathrm{Tr}\left[\left(\frac{\partial}{\partial\theta}\rho_\theta\right)L_\theta\right], \tag{2}$$

with L_θ being the symmetric logarithmic derivative operator, which is determined by

$$\frac{\partial\rho_\theta}{\partial\theta}=\frac{1}{2}\left(\rho_\theta L_\theta+L_\theta\rho_\theta\right). \tag{3}$$

By diagonalizing the density matrix $\rho_\theta=\sum_i\omega_i|\varphi_i\rangle\langle\varphi_i|$, the QFI is transformed into[27]

$$F(\theta)=\sum_{i'}\frac{\left(\partial_\theta\omega_{i'}\right)^2}{\omega_{i'}}+\sum_{i\neq j,\omega_i+\omega_j\neq0}\frac{2\left(\omega_i-\omega_j\right)^2}{\omega_i+\omega_j}\left|\langle\varphi_i|\partial_\theta\varphi_j\rangle\right|^2. \tag{4}$$

Here the first term on the right-hand side is just the classical Fisher information, and the second term can be considered as the contribution from the quantum coherence. According to the quantum Cramér–Rao inequality[10]

$$\mathrm{Var}(\hat{\theta})\geq\frac{1}{MF(\theta)}, \tag{5}$$

the achievable lower bound of the mean-square error for an unbiased estimator $\hat{\theta}$ is determined by the QFI. Here $\mathrm{Var}(\cdot)$ is the variance, and M is the number of the repeated experiment. With a larger quantum Fisher information, the parameter would be estimated with a higher precision.

3. Model of system

We consider a two-qutrit isotropic Heisenberg XY chain in a uniform magnetic field.[28] The Hamiltonian of the system is

$$H=J\left(S_{\mathrm{A}}^x S_{\mathrm{B}}^x+S_{\mathrm{A}}^y S_{\mathrm{B}}^y\right)+B\left(S_{\mathrm{A}}^z+S_{\mathrm{B}}^z\right). \tag{6}$$

Here S_k^α $(\alpha=x,y,z;\ k=\mathrm{A},\mathrm{B})$ are the spin-1 operators, given as

$$S_x=\frac{1}{\sqrt{2}}\begin{bmatrix}0&1&0\\1&0&1\\0&1&0\end{bmatrix},\quad S_y=\frac{1}{\sqrt{2}}\begin{bmatrix}0&-\mathrm{i}&0\\\mathrm{i}&0&-\mathrm{i}\\0&\mathrm{i}&0\end{bmatrix},$$
$$S_z=\begin{bmatrix}1&0&0\\0&0&0\\0&0&-1\end{bmatrix}.$$

The J and B are the two-body interaction and the magnetic field strength, respectively. The eigenvalues of the Hamiltonian are

$$\begin{aligned}&E_1=0,\quad E_{2,3}=\mp2B,\quad E_{4,5}=\mp\sqrt{4B^2+2J^2},\\&E_{6,7}=\mp B-\sqrt{B^2+J^2},\quad E_{8,9}=\mp B+\sqrt{B^2+J^2},\end{aligned} \tag{7}$$

with the corresponding eigenvectors

$$\begin{aligned}&|\phi_1\rangle=N_1\left(-J|1,-1\rangle+2B|0,0\rangle+J|-1,1\rangle\right),\\&|\phi_2\rangle=|-1,-1\rangle,\quad|\phi_3\rangle=|1,1\rangle,\\&|\phi_4\rangle=N_2\left(a_-|1,-1\rangle+b_-|0,0\rangle+|-1,1\rangle\right),\\&|\phi_5\rangle=N_3\left(a_+|1,-1\rangle+b_+|0,0\rangle+|-1,1\rangle\right),\\&|\phi_6\rangle=N_4\left(c_-|0,-1\rangle+|-1,0\rangle\right),\\&|\phi_7\rangle=N_4\left(c_-|1,0\rangle+|0,1\rangle\right),\\&|\phi_8\rangle=N_5\left(c_+|0,-1\rangle+|-1,0\rangle\right),\\&|\phi_9\rangle=N_5\left(c_+|1,0\rangle+|0,1\rangle\right).\end{aligned} \tag{8}$$

Here $N_1,N_2,\ldots$ are the normalization constants and

$$\begin{aligned}a_\pm&=\frac{4B^2+J^2\pm2B\sqrt{4B^2+2J^2}}{J^2},\\b_\pm&=\frac{2B\pm\sqrt{4B^2+2J^2}}{J},\\c_\pm&=\frac{B\pm\sqrt{B^2+J^2}}{J}.\end{aligned} \tag{9}$$

For the decoherence of our system, we consider the Milburn model, which gives a simple modification of the Schrodinger equation. Under the Markovian approximation, the master equation of the density matrix $\rho(t)$ reduces to[29]

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho(t)=-\mathrm{i}\left[H,\rho(t)\right]-\frac{\gamma}{2}\left[H,\left[H,\rho(t)\right]\right], \tag{10}$$

where γ is the decoherence rate. The solution of the Milburn equation can be written as[30]

$$\rho(t)=\sum_{k=0}^{\infty}\frac{\gamma t}{k!}M(t)^k\rho(0)M^\dagger(t)^k. \tag{11}$$

Here $M(t)^k=H^k\,\mathrm{e}^{-\mathrm{i}Ht}\,\mathrm{e}^{-\gamma tH^2/2}$, and $\rho(0)$ is the initial state. With the eigenvalues E_n and eigenvectors $|\varphi_n\rangle$, the solution of Eq. (10) can be explicitly written as[31]

$$\begin{aligned}\rho(t)\ =&\sum_{m,n}\mathrm{e}^{(-\gamma t/2)(E_m-E_n)^2-\mathrm{i}(E_m-E_n)t}\\&\times\langle\varphi_m|\rho(0)|\varphi_n\rangle|\varphi_m\rangle\langle\varphi_n|.\end{aligned} \tag{12}$$

For the initial state $|\varphi_0\rangle=\sin(\alpha)|1,1\rangle+\cos(\alpha)|-1,-1\rangle$, the non-zero elements of $\rho(t)$ are obtained as

$$\begin{aligned}&\rho_{11}=\sin(\alpha)^2,\quad\rho_{99}=\cos(\alpha)^2,\\&\rho_{19}=\rho_{91}^*=\sin(\alpha)\cos(\alpha)\,\mathrm{e}^{-4\mathrm{i}Bt-8B^2\gamma t}.\end{aligned} \tag{13}$$

The eigenvalues of $\rho(t)$ are given as

$$f_{1,2,3,4,5,6,7}=0,\quad f_{8,9}=\frac{1}{4}(2\mp\mathrm{e}^{-8B^2\gamma t}\Lambda), \tag{14}$$

with the corresponding eigenvectors

$$\begin{aligned}&|\varphi_1\rangle=|-1,0\rangle,\quad|\varphi_2\rangle=|-1,1\rangle,\\&|\varphi_3\rangle=|0,-1\rangle,\quad|\varphi_4\rangle=|0,0\rangle,\\&|\varphi_5\rangle=|0,1\rangle,\quad|\varphi_6\rangle=|1,-1\rangle,\quad|\varphi_7\rangle=|1,0\rangle,\end{aligned}$$

$$|\varphi_8\rangle = \frac{d_+}{\sqrt{d_+^2+1}}|1,1\rangle + \frac{1}{\sqrt{d_+^2+1}}|-1,-1\rangle,$$
$$|\varphi_9\rangle = \frac{d_-}{\sqrt{d_-^2+1}}|1,1\rangle + \frac{1}{\sqrt{d_-^2+1}}|-1,-1\rangle. \quad (15)$$

Here

$$d_\pm = \pm 2\mathrm{e}^{4\mathrm{i}Bt+2B^2\gamma t}\cos(2\alpha) + \mathrm{e}^{4\mathrm{i}Bt}\Lambda,$$
$$\Lambda = \sqrt{2\left(1-\cos(4\alpha)+\mathrm{e}^{16B^2\gamma t}\left(1+\cos(4\alpha)\right)\right)}. \quad (16)$$

4. Numerical results and discussion

4.1. QFI with respect to the decoherence rate

Making use of Eq. (4), we can obtain the QFI of the density matrix $\rho(t)$ given in Eq. (13). As its expression is very complicated, we will display the results mainly by numerical simulations. Here, we focus on the dynamics of QFI with respect to the decoherence rate $F(\gamma)$.

In Fig. 1, we exhibit the dynamics of $F(\gamma)$ with respect to time t and parameter α. It shows that $F(\gamma)$ has a maximum at a certain time for a given α. Moreover, the maximum of the QFI is obtained when α is close to $\alpha = \pi/4$. This implies that the input state is vital to enhance the precision of quantum estimation.

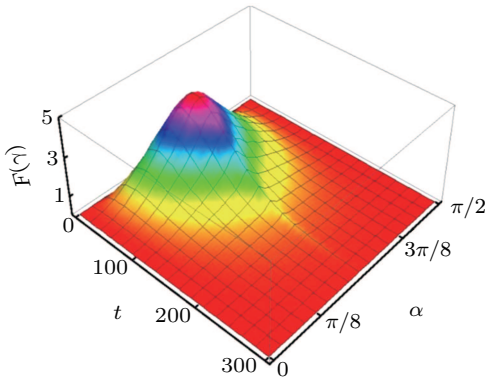


Fig. 1. (color online) The evolution of QFI $F(\gamma)$ as a function of time t and parameter α. The other parameters are $B = 0.1$ and $\gamma = 0.2$.

To obtain a better understanding of the effect of the input state on the QFI, we study the negativity entanglement,[32] which measures the degree of the entanglement of two spin-1 particles. The negativity of a given state ρ is defined as[33]

$$N = \frac{\left\|\rho^{\mathrm{T_A}}\right\| - 1}{2} = -\sum_i g_i, \quad (17)$$

where $\left\|\rho^{\mathrm{T_A}}\right\|$ is the trace norm of the partial transpose density matrix $\rho^{\mathrm{T_A}}$, and g_i is the negative eigenvalues of $\rho^{\mathrm{T_A}}$. The negativity of the input state $\rho(0)$ is obtained as

$$N = \frac{\sqrt{1-\cos(4\alpha)}}{2\sqrt{2}}. \quad (18)$$

We plot the negativity entanglement N of the input state versus the parameter α in Fig. 2. This figure shows that with the increase or decrease of the entanglement in the input state, $F(\gamma)$ correspondingly goes up or down. For $\alpha = \pi/4$, corresponding to the maximum entanglement, the QFI also has the largest value. Generally, it is well-known that the precision of parameter estimation will be enhanced with a large entanglement in the input state.[19,20] Our result is consistent with this conclusion.

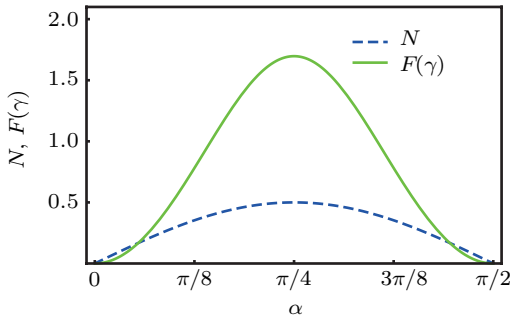


Fig. 2. (color online) The variations of the QFI $F(\gamma)$ at time $t = 10$ and the negativity entanglement N of the input state with respect to parameter α. The other parameters are $B = 0.1$ and $\gamma = 0.2$.

For the input state with $\alpha = \pi/4$, the QFI is obtained as

$$F(\gamma) = \frac{64B^4\mathrm{e}^{-16B^2\gamma t}t^2}{1-\mathrm{e}^{-8B^2\gamma t}} + \frac{64B^4\mathrm{e}^{-16B^2\gamma t}t^2}{1+\mathrm{e}^{-8B^2\gamma t}}. \quad (19)$$

We can consider two asymptotic limits of Eq. (19). In the limit $t \to 0$, the QFI becomes

$$F(\gamma) \propto \frac{8B^2t}{\gamma}. \quad (20)$$

This shows that the QFI is an increasing function of time. In the limit $t \to \infty$,

$$F(\gamma) \propto 128B^2\mathrm{e}^{-16B^2\gamma t}t^2. \quad (21)$$

In this case, the term $\mathrm{e}^{-16B^2\gamma t}$ makes the QFI exponentially decay into zero. Thus, the QFI must have a maximum value at a certain time.

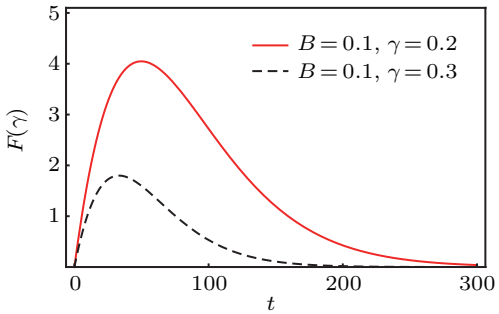


Fig. 3. (color online) The time evolution of QFI $F(\gamma)$ with different decoherence rates.

Figure 3 shows the effect of the decoherence rate on the QFI. It is easy to see that with a larger decoherence rate, the QFI becomes smaller. This phenomenon can be understood

as follows. On one side, $F(\gamma)$ is proportional to $8B^2t/\gamma$ in the initial evolution, which is a decreasing function of the decoherence rate. On the other side, $F(\gamma)$ is proportional to $e^{-16B^2\gamma t}t^2$ in the long-time limit. Thus $F(\gamma)$ will exponentially decrease to zero with a rate $B^2\gamma$.

As the maximum QFI $F_{\gamma,\mathrm{m}}$ implies the largest estimating precision to the parameter γ, we plot $F_{\gamma,\mathrm{m}}$ as a function of γ in Fig. 4. Here the maximum QFI $F_{\gamma,\mathrm{m}}$ is defined as

$$F_{\gamma,\mathrm{m}} = \max_t \left[F_\gamma(t)\right]. \tag{22}$$

Figure 4 shows that $F_{\gamma,\mathrm{m}}$ decreases considerably with the increase of parameter γ. We also display the relationship between $F_{\gamma,\mathrm{m}}$ and the parameter α in Fig. 5. This figure shows that a large entanglement in the input state can enhance the maximum QFI $F_{\gamma,\mathrm{m}}$.

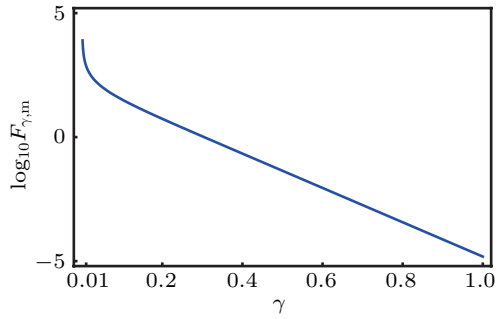


Fig. 4. (color online) The variation of $F_{\gamma,\mathrm{m}}$ with respect to γ.

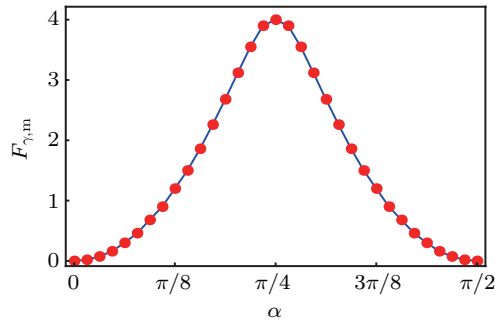


Fig. 5. (color online) The variation of $F_{\gamma,\mathrm{m}}$ with respect to parameter α. The other parameters are $B = 0.3$ and $\gamma = 0.2$.

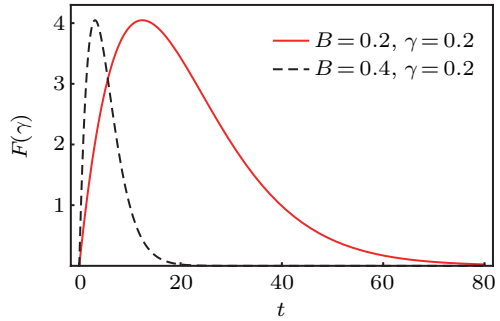


Fig. 6. (color online) The time evolution of QFI $F(\gamma)$ with different magnetic fields.

We also address the role of the magnetic field on the dynamics of the QFI in Fig. 6. It is apparent that a larger magnetic field accelerates the decay of the QFI. More interestingly, we also find that the maximum QFI is independent of the magnetic field.

4.2. Estimation of the magnetic field

For the estimated parameter being the magnetic field, its QFI $F(B)$ is expressed as

$$F(B) = -\frac{16(1 - e^{-16B^2\gamma t} + 32B^2\gamma^2)t^2}{1 - e^{16B^2\gamma t}}. \tag{23}$$

Similarly, in the limit $t \to 0$, the QFI $F(B)$ is proportional to $16\gamma t$, which is an increasing function of time. While in the limit $t \to \infty, F(B)$ is proportional to $e^{-16B^2\gamma t}t^2$. Here the term $e^{-16B^2\gamma t}$ plays a dominant role, which makes the QFI $F(B)$ decrease very quickly.

Figure 7 displays the effect of the decoherence rate γ on the QFI. It shows that $F(B)$ decreases significantly with a small increase of the decoherence rate. The reason is that with a smaller decoherence rate, the coherence of the system can be preserved in a longer time, and the precision of estimation can be enhanced by decreasing the decoherence rate.

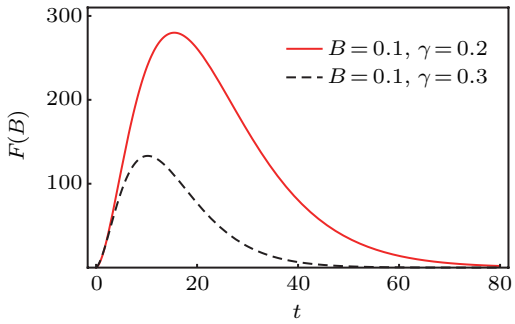


Fig. 7. (color online) The time evolution of $F(B)$ with different decoherence rates.

Figure 8 shows the effect of the magnetic field on the QFI. We can see from this figure that with the increase of the magnetic field, $F(B)$ decreases. From Eq. (13), the effect of the magnetic field on the QFI is obvious: with a larger B, the off-diagonal elements or the coherence of the system should decrease exponentially. Therefore, it is reasonable that the QFI goes down with the increase of the magnetic field.

The maximum quantum Fisher information $F_{B,\mathrm{m}}$, similarly defined as Eq. (22), can be approximated as

$$F_{B,\mathrm{m}} \approx \frac{192B^2\gamma^2 + 2}{300B^4\gamma^2}. \tag{24}$$

As shown in Fig. 9, we find that $F_{B,\mathrm{m}}$ decreases very quickly with the increase of the magnetic field. This result implies that a larger B decreases the estimation precision significantly, which is consistent with our above argument. At the same time, the maximum value $F_{B,\mathrm{m}}$ also goes down with the increase of the decoherent rate. Moreover, it is easy to see that

$F_{B,\mathrm{m}}$ is also symmetric with respect to $\alpha = \pi/4$ in Fig. 10. This result implies that one can enhance the QFI by making use of the entanglement in the input state even with the decoherent environment.

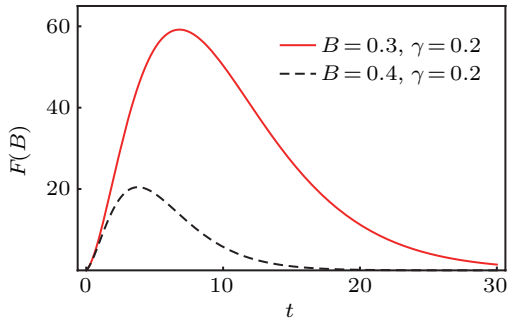


Fig. 8. (color online) The time evolution of $F(B)$ with different magnetic fields.

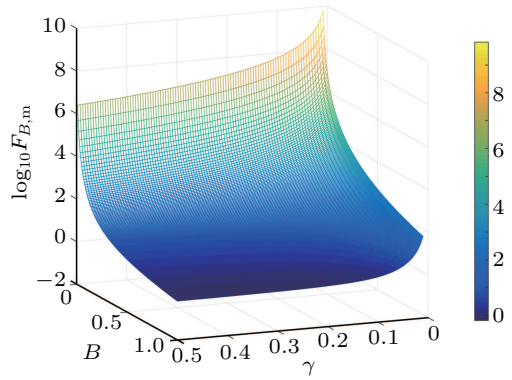


Fig. 9. (color online) The variation of $F_{B,\mathrm{m}}$ with respect to the decoherent rate γ and the magnetic field B.

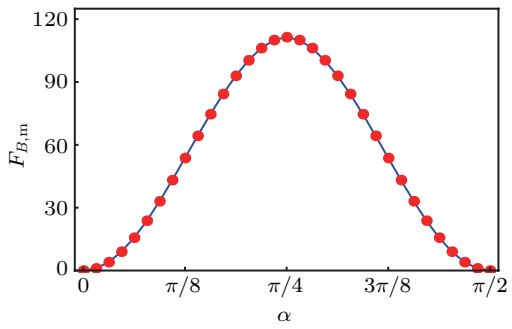


Fig. 10. (color online) The variation of $F_{B,\mathrm{m}}$ with respect to parameter α. The other parameters are $\gamma = 0.1$ and $B = 0.3$.

We also study the effect of the entanglement in the input state on the QFI in Fig. 11. This figure shows that both the QFI and the negativity are symmetrical with respect to $\alpha = \pi/4$. Moreover, the larger entanglement in the initial state, the higher precision of parameter estimation, which is consistent with the results reported in Refs. [23] and [24].

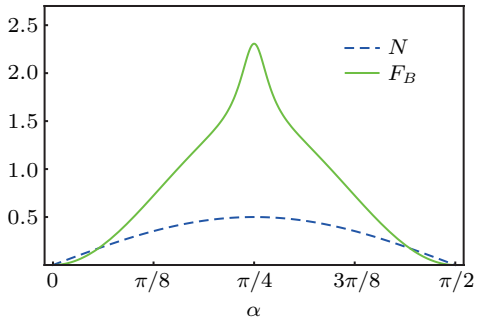


Fig. 11. (color online) The variation of the QFI F_B at time t=2 and the negativity N of the input state with respect to parameter α. The other parameters are $\gamma = 0.8$ and $B = 0.4$.

5. Conclusion

According to the quantum Cramér–Rao inequality, the QFI plays an important role in quantum estimation theory. In this paper, we discuss the QFI of the two-qutrit isotropic Heisenberg XY chain in a uniform magnetic field. Making use of the Milburn intrinsic decoherence model, we study the dynamics of the QFIs with respect to the decoherent rate and the magnetic field, respectively. We find that the QFIs increase in the initial time evolution, and go down exponentially in the long-time limit. With the increase of the decoherent rate and the magnetic field, the QFIs decrease considerably. We also show that the optimal estimation precision can be enhanced by improving the entanglement in the input state. In the future, it will be interesting to discuss the multiple-parameter estimation using a spin chain.

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