Speeding up transmissions of unknown quantum information along Ising-type quantum channels*

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Quantum teleportation with entanglement channels and a series of two-qubit SWAP gates between the nearest-neighbor qubits are usually utilized to achieve the transfers of unknown quantum state from the sender to the distant receiver. In this paper, by simplifying the usual SWAP gates we propose an approach to speed up the transmissions of unknown quantum information, specifically including the single-qubit unknown state and two-qubit unknown entangled ones, by a series of entangling and disentangling operations between the remote qubits with distant interactions. The generic proposal is demonstrated specifically with experimentally-existing Ising-type quantum channels without transverse interaction; liquid NMR-molecules driven by global radio frequency electromagnetic pulses and capacitively-coupled Josephson circuits driven by local microwave pulses. The proposal should be particularly useful to set up the connections between the distant qubits in a chip of quantum computing.

Keywords: unknown quantum information transfers, Ising-type quantum chains, NMR molecules, superconducting circuits

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1. Introduction

Reliably transporting quantum information (encoded by various quantum states) from one place to another is one of the crucial tasks in quantum information processing.[1] This is particularly important for both secure communications over long distances, and quantum logic operations between the distant qubits within a quantum computing register. It is well-known that unknown quantum information can be transmitted from a sender to its entangled receiver by quantum teleportation technique along quantum entangled channels. Analogous to that the conductive wires (e.g., microwave transmission lines and optical fibers) are utilized to transport the usual electromagnetic data, certain quantum channels (e.g., optical fibers, lattices, and quantum wires, etc.) can be used to propagate quantum information stored in various flying individual carriers (e.g., photons,[3,4] phonons,[5–7] ballistic electrons,[8,9] etc.). Indeed, the quantum information could be automatically dispersing[10–13] along certain quantum channels with transverse interactions, although the robustness of their propagations are obviously limited by lossy environments and the difficulties of interfacing with the relevant stationary quantum objects.

In principle, an arbitrary quantum state could be transferred along the quantum channels by successively applying two-qubit SWAP gates.[18,19] Strictly speaking, an ideal two-qubit SWAP gate should not depend on the quantum states of the other qubits. Therefore, many additional pulses are required to refocus the unwanted (but practically-existing) untunable interactions between the operated qubits and the other ones.[20–22] As a consequence, the number of the required operations significantly increases.

To overcome this difficulty, in this paper, we propose a simplified method to transmit unknown quantum states along an Ising-type quantum chain,[24] with untunable interbit interactions, by applying a series of entangling and disentangling operations to the remote qubits. Since the long-range interactions between the distant qubits are utilized, the number of the operations decreases significantly, compared with the usual method by successively applying two-qubit SWAP gates. Specifically, the proposal is demonstrated with the experimentally-existing Ising-type chains without transverse interactions, such as liquid NMR-molecules driven by global radio frequency (rf) magnetic pulses[20] and capacitively-coupled Josephson circuits driven by local microwave pulses.[25,26]

2. Driving Ising-type quantum chains as quantum channels to highly efficiently transfer unknown quantum states

A generic driven Ising-type quantum chain can be described by the Hamiltonian

$$H = H_0 + H_{\rm d}(t), \quad H_0 = \sum_{i=1}^{N} \omega_i \sigma_i^z + \frac{1}{2}\sum_{i\neq j} J_{ij}\sigma_i^z\sigma_j^z, \qquad (1)$$

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with ω_i and σ_i^z being the eigenfrequency and Pauli operator of the i-th qubit, respectively, J_{ij} is the coupling constant between the i-th qubit and the distant j-th one. Note that, here, the bare Hamiltonian H_0 without any transverse interbit coupling and thus could not automatically transport quantum state along the quantum chain. Therefore, to implement the quantum state transport, a pulse-deriving term $H_{\rm d}(t)$ should be introduced to successively flip the selected qubits along the chain. Indeed, quantum information can be transferred from the sender to the receiver by entangling and then disentangling them. For example, by applying a pair of entangling–disentangling operations, one can transfer an unknown quantum state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ (with $|\alpha|^2 + |\beta|^2 = 1$) from the i-th qubit to the j-th one

$$|\psi_i\rangle \otimes \prod_{k\neq i} |e_k\rangle \xrightarrow{\hat{U}_{i,j}^{(N)}} |\psi_{ij}\rangle \otimes \prod_{k\neq i,j} |e_k\rangle \xrightarrow{\tilde{U}_{i,j}^{(N)}} |\psi_j\rangle \otimes \prod_{k\neq j} |e_k\rangle. \quad (2)$$

Here, the entangling operation $\hat{U}_{i,j}^{(N)}$, which conditionally flips the known state $|0_j\rangle$ ($|1_j\rangle$) of the j-th qubit if the i-th qubit is in the state $|1_j\rangle$ ($|0_j\rangle$). This operation entangles the j-th qubit (whose quantum state $|e_j\rangle$, $e_j = 0, 1$ is known) to the i-th one (whose state is unknown, i.e., $|\psi_i\rangle = \alpha|0_i\rangle + \beta|1_i\rangle$). Correspondingly, the operation $\tilde{U}_{i,j}^{(N)}$ conditionally flips the state $|1_i\rangle$ in the case of the j-th qubit being in the state $|1_j\rangle$. This operation disentangles the i-th qubit from the unknown two-qubit entangled state $|\psi_{ij}\rangle = \alpha|0_i 0_j\rangle + \beta|1_i 1_j\rangle$. Note that, during the above entangling and disentangling operations (they flipped the j-th and i-th qubits, respectively), all the remaining qubits are unchanged and kept in their initial known states. Obviously, the operations $\hat{U}_{i,j}^{(N)}$ and $\tilde{U}_{k,l}^{(N)}$ are just the well-known two-qubit CNOT gates (the corresponding target qubits are the j-th and the k-th qubits, respectively) for switchable-coupling systems. However, these evolutions in the N-qubit quantum chain with fixed interbit couplings are practically N-qubit operations (without using flying qubit[27]), which depend on the unchanged states of all the remaining qubits.

Certainly, if the direct interaction between the sender and the receiver is not sufficiently strong, limiting the speed of communications due to long durations of the above entangling and disentangling operations, then certain qubits in the chain could act as the intermediators. For example, by using the nearest-neighbor interbit couplings, unknown quantum state $|\psi_1\rangle$ could be transferred to the N-th qubit along the quantum chain by a series of entangling and disentangling operations: $|\psi_1\rangle \otimes \prod_{i\neq 1} |e_i\rangle \xrightarrow{\tilde{U}_{1,2}^{(N)}\hat{U}_{1,2}^{(N)}} |\psi_2\rangle \prod_{i\neq 2} |e_i\rangle \ldots \xrightarrow{\tilde{U}_{N-1,N}^{(N)}\hat{U}_{N-1,N}^{(N)}} \prod_{i\neq N} |e_i\rangle \otimes |\psi_N\rangle$; or $|\psi_1\rangle \otimes \prod_{i\neq 1} |e_i\rangle \xrightarrow{\hat{U}_{12\ldots N}^{(N)}} |\psi_{12,\ldots,N}\rangle \xrightarrow{\tilde{U}_{12\ldots N}^{(N)}} \prod_{i\neq N} |e_i\rangle \otimes |\psi_N\rangle$, with $\hat{U}_{12\ldots N}^{(N)} = \hat{U}_{12\ldots N-1,N}^{(N)} \ldots \hat{U}_{12,3}^{(N)} \hat{U}_{1,2}^{(N)}$ and $\tilde{U}_{12\ldots N}^{(N)} = \tilde{U}_{1,2\ldots N}^{(N)} \tilde{U}_{2,3\ldots,N}^{(N)} \ldots \tilde{U}_{N-1,N}^{(N)}$. Again, we emphasize that only for the switchable-coupling qubits, the above a series of entangling–disentangling operations are simply equivalent to a series of two-qubit SWAP gates. Next, the above transfer procedures might also be simplified by applying the operations of simultaneously flipping multiple qubits, e.g., $\hat{U}_{ij,kl}^{(N)}$ entangles the k-th and l-th qubits (their states are known) simultaneously to the i-th and j-th ones; and $\tilde{U}_{ij,kl}^{(N)}$ disentangles the i-th and j-th qubits simultaneously from a four-qubit unknown entangled state $|\psi_{ijkl}\rangle$, etc.

The above scheme for transferring single-qubit states could be easily generalized to transfer unknown multi-qubit entangled states. For example, an unknown two-qubit entangled state, e.g., $|\psi_{12}\rangle = \alpha|0_1 0_2\rangle + \beta|1_1 1_2\rangle$ of the first and second qubits, can be transferred to two other qubits, e.g., the $(N-1)$-th and N-th qubits, by the following two-pair operations:

$$|\psi_{12}\rangle \otimes \prod_{i\neq 1,2} |e_i\rangle \xrightarrow{\tilde{U}_{1,2(N-1)}^{(N)}\hat{U}_{12,(N-1)}^{(N)}} |\psi_{2(N-1)}\rangle \otimes \prod_{i\neq 2,N-1} |e_i\rangle \xrightarrow{\tilde{U}_{2,(N-1)N}^{(N)}\hat{U}_{2(N-1),N}^{(N)}} \prod_{i\neq N-1,N} |e_i\rangle \otimes |\psi_{(N-1)N}\rangle. \quad (3)$$

Therefore, by sequentially applying a series of entangling and disentangling operations defined above, arbitrarily unknown entanglement could be non-dispersively transported along the above driven Ising-type quantum chain. For example, an unknown two-qubit entanglement $|\psi_{12}\rangle$, shared by the first qubit and the second one, could controllably "flow" along the Ising-type quantum chain without dispersion: the first qubit could only entangle to the third, fourth, ..., and the N-th qubit, sequentially. This process is represented by the following successive evolutions of the N-qubit states:

$$|\psi_{12}\rangle \otimes \prod_{i\neq 1,2} |e_i\rangle \xrightarrow{\tilde{U}_{2,13}^{(N)}\hat{U}_{12,3}^{(N)}} |\psi_{13}\rangle \otimes \prod_{i\neq 1,3} |e_i\rangle \ldots \xrightarrow{\tilde{U}_{N-1,1N}^{(N)}\hat{U}_{1(N-1),N}^{(N)}} |\psi_{1N}\rangle \otimes \prod_{i\neq 1,N} |e_i\rangle. \quad (4)$$

It is emphasized that, here, the quantum state transfers are controllable, instead of passively waiting[10,13] their dispersions from the sending site to the receiving end. Furthermore, for the system with untunable interbit couplings all the applied operations are non-local; although they are designed to only flip the selected qubit(s) and keep the remaining qubits unchanged. For example, the operation $\hat{U}_{i,j}^{(N)}$, for entangling the j-th qubit to the i-th qubit, certainly depends on the states of the other qubits (which should be kept unchanged during this operation). Finally, the basic protocols introduced above could be easily generalized to transfer unknown multi-qubit quantum states.

Many physical systems with switchable interbit couplings (e.g., atoms in cavities connected by flying photons and trapped ions communicating via phonons) provide trivial realizations of the quantum chain proposed above, as the required entangling/disentangling operations in these systems

could be simply generated by a series of two-qubit CNOT or SWAP gates. Below, without these two-qubit gates, we specifically show how to drive the experimentally existing quantum Ising chains with untunable long-range interactions, typically NMR-molecules driven by global rf-magnetic pulses and capacitively coupled Josephson circuits driven by local microwave pulses, to physically realize the basic protocols described above.

3. Transferring unknown quantum states along NMR-molecules by applying global rf pulses

Since the wavelengths of the applied rf magnetic pulses (e.g., $\lambda > 0.6$ m for $\nu < 500$ MHz) are much longer than the separations of the qubits (typically in nanometers), the driving of the the NMR-molecules with N qubits are global. Thus, the driven term generated the applied radio frequency magnetic pulse reads[20]

$$H_{\rm d}^{\rm NMR}(t) = \frac{1}{2}\sum_{i=1}^{N}[\Omega_i \exp({\rm i}\nu t)\sigma_i^{\dagger} + {\rm H.c.}], \tag{5}$$

with $\sigma_i^{\dagger}$ and Ω_i being Pauli ladder operator and the Rabi frequency of the i-th qubit interacting with the applied rf field of frequency ν, respectively. Under such a driving, the system is manipulated within the space $\Re$ expanded by the eigenstates of H_0: $\Re = \{|n\rangle = \prod_{i=1}^{N}|e_i\rangle\}, \sum_n |n\rangle\langle n| = 1$; with $n = \sum_{i=1}^{N} e_i 2^{i-1}$, and $H_0|n\rangle = \lambda_n|n\rangle$. Here, the information of fixed interbit couplings are included into the eigenvalues λ_n, and instead are passively refocused. As a specific example, we consider the experimentally demonstrated crotonic acid[28–30] (Fig. 1(a)). Here, four interacting ^{13}C nuclei serve as the four qubits with the parameters: $\omega_1 = 6879.9$ Hz, $\omega_2 = 1882.6$ Hz, $\omega_3 = 4410.1$ Hz, $\omega_4 = -8604.9$ Hz; $J_{12} = 72.4$ Hz, $J_{13} = 1.4$ Hz, $J_{14} = 7.2$ Hz, $J_{23} = 69.6$ Hz, $J_{24} = 1.6$ Hz, and $J_{34} = 41.5$ Hz. Obviously, the present Ising-type quantum chain with four qubits could be globally treated as an "atom" with sixteen levels of energies $\lambda_{e_ie_je_ke_l}$, $e_i, e_j, e_k, e_l = 0, 1$. Flipping a selected qubit, e.g., the j-th qubit, practically equals to driving a transition from level $\lambda_{e_ie_je_ke_l}$ to level $\lambda_{e_ie'_je_ke_l}$ with $e_j \neq e'_j = 0, 1$. Due to the limit of electric-dipole selection rule, a single monochromatic rf-pulse flips only one selected qubit and keeps the remaining ones unchanged. If the system is initialized to a full four-spin pseudopure state $|0000\rangle$, an unknown single-qubit state $|\psi_1\rangle$ could be generated by applying an rf-pulse (P_0) with frequency $\nu_{1000,0000} = \lambda_{1000} - \lambda_{0000} = 2\omega_1 - J_{12} - J_{13} - J_{14} = 13676.8$ Hz. Our protocol for transferring this unknown single-qubit quantum state $|\psi_1\rangle$ from the first qubit to the fourth one only consists of two operations: (i) applying a rf π-pulse (P_1), with frequency $\nu_{1001,1000} = |\lambda_{1001} - \lambda_{1000}| = |2\omega_4 + J_{14} - J_{24} - J_{34}| = 17245.7$ Hz to perform the desirable operation $\hat{U}_{1,4}^{(4)}$; and then (ii) applying an rf 3π-pulse (P_2) (of frequency $\nu_{1001,0001} = \lambda_{1001} - \lambda_{0001} = 2\omega_1 - J_{12} - J_{13} + J_{14} = 13691.2$ Hz $\neq 2\omega_1$) to implement the operation $\tilde{U}_{1,4}^{(4)}$.

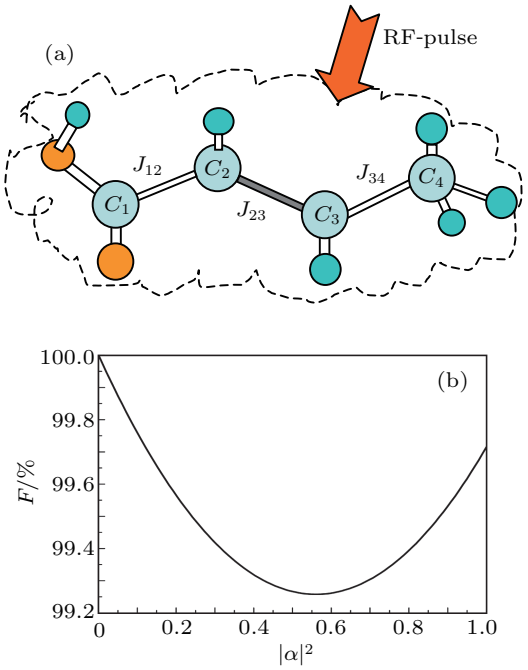


Fig. 1. (color online) (a) Molecule of crotonic acid[28–30] with four fixed-coupling qubits driven by a global rf-magnetic pulse. (b) The fidelity of transferring an unknown quantum state $|\psi_1\rangle = \alpha|0_1\rangle + \beta|1_1\rangle$ (by a pair of entangling-disentangling operations) from the first qubit C_1 to the fourth one C_4 versus $|\alpha|^2$. Here, only the influence from the nearest-resonant transitions with the smallest detunings $\Delta_1 = \Delta_4 = 14.4$ Hz are considered for the typical Rabi frequencies: $\Omega_1 = \Omega_4 = 1$ Hz.

Imperfections of performing the protocols in NMR models (usually with significantly long decoherence time) dominantly arises from unwanted non-resonant transitions.[28–30] During a resonant transition with duration τ_i, the probability of such a non-resonant one (with detuning Δ_j) is

$$P_j^{\rm non} = \frac{\Omega_j^2}{C_j^2}\sin^2\left(\frac{\tau_i C_j}{2}\right), \quad C_j = \sqrt{\Delta_j^2 + \Omega_j^2}, \tag{6}$$

which is negligible for large detuning: $\Delta_j \gg \Omega_j$. Tables 1 and 2 list all possible non-resonant transitions during the pulses P_1 and P_2, respectively.

Table 1. Possible non-resonant transitions and their frequencies during the applied rf-pulse P_1.

Transition	Frequency/Hz
$\|0000\rangle \rightarrow \|1000\rangle$	13678.80
$\|0000\rangle \rightarrow \|0100\rangle$	3621.60
$\|0000\rangle \rightarrow \|0010\rangle$	8707.70
$\|0000\rangle \rightarrow \|0001\rangle$	**17260.10**
$\|1000\rangle \rightarrow \|0000\rangle$	13678.80
$\|1000\rangle \rightarrow \|1100\rangle$	3766.40
$\|1000\rangle \rightarrow \|1010\rangle$	8710.50

Table 2. Possible non-resonant transitions and their frequencies during the applied rf-pulse P_2.

Transition	Frequency/Hz
$\|0000\rangle \to \|1000\rangle$	**13678.80**
$\|0000\rangle \to \|0100\rangle$	3621.60
$\|0000\rangle \to \|0010\rangle$	8707.70
$\|0000\rangle \to \|0001\rangle$	17260.10
$\|1001\rangle \to \|1000\rangle$	17245.70
$\|1001\rangle \to \|1101\rangle$	3769.60
$\|1001\rangle \to \|1011\rangle$	8793.50

Considering the effects of the nearest-resonant transitions (all the other transitions are far from resonances): $|0000\rangle \to |0001\rangle$ (with detuning $\Delta_4 = 14.4$ Hz) during P_1-pulse, and $|0000\rangle \longrightarrow |1000\rangle$ (with detuning $\Delta_1 = 14.4$ Hz) during P_2-pulse, the fidelity of the above transfer is

$$F = \left|(|\alpha|^2 a_1 a_2 + (1-|\alpha|^2)\right|, \tag{7}$$

with

$$a_1 = \left[\cos\left(\frac{\pi A_4}{2}\right) - \mathrm{i}\frac{\Delta_4}{C_4}\sin\left(\frac{\pi A_4}{2}\right)\right]\exp(\mathrm{i}\pi B_4),$$
$$a_2 = \left[\cos\left(\frac{3\pi A_1}{2}\right) - \mathrm{i}\frac{\Delta_1}{C_1}\sin\left(\frac{3\pi A_1}{2}\right)\right]\exp(3\mathrm{i}\pi B_1),$$
$$A_j = \sqrt{1+(\Delta_j/\Omega_j)^2}, \quad B_j = \Delta_j/(2\Omega_j), \quad j = 1,4.$$

Figure 1(b) shows that the above physical implementation of the desired quantum state transfer is almost perfect, the minimal fidelity is 99.25%, typically for $\Omega_1 = \Omega_4 = 1$ Hz. Shorter durations are possible by using stronger rf-drivings, but this will decrease the fidelity of the transfer (as the unwanted effects of non-resonant transitions will enhance for the higher ratio of Δ_j/Ω_j).

It is emphasized that only two global pulses are required in the above implementation to transfer one-qubit unknown quantum state from the first qubit to the fourth one. Compared with the usual approach by successively applying a series of two-qubit SAWP gates implemented by dozens of rf pulses, some of them are applied to realize the SAWP gates between the nearest-neighbor qubits, and the others are necessarily used to refocus the unwanted evolutions related to the untunable interbit couplings, the proposal demonstrated above significantly speeds up the efficiency of the unknown quantum state transmission.

4. Transferring unknown quantum states along capacitively-coupled COOPER-pair boxes by local microwave pulses

Another specific Ising-type quantum chain is the four-qubit capacitively-coupled Josephson circuit shown in Fig. 2, generated by two experimentally existing superconducting circuits,[25,26] could also be used to implement the basics protocols introduced in Section 2.

The dynamics of each SQUID-biased Cooper-pair box can be effectively restricted to the subspace spanned by the computational basis, if each applied gate-voltage is set near its "degenerate" point $C_i^g V_i = (2k+1)e$, $i = 1,2,3,4$; $k = 0,1,2,\ldots$. Thus, the generic Hamiltonian (1) reduces to

$$H_0 = \frac{1}{2}\sum_{i=1}^{4} E_i^{\mathrm{C}}\sigma_i^z + \frac{1}{2}\sum_{i<j} K_{ij}\sigma_i^z\sigma_j^z, \quad \hat{H}_{\mathrm{d}}^i(t) = -\frac{E_i^{\mathrm{J}}(t)}{2}\sigma_i^x, \tag{8}$$

with $E_i^{\mathrm{C}} = 2e^2(2n_i^{\mathrm{g}}-1)/\tilde{C}_i + \sum_{j\neq i} 2e^2(2n_j^{\mathrm{g}}-1)/\tilde{C}_{ij}$ being the effective charging energy of the i-th qubit, whose effective Josephson energy reads $E_i^{\mathrm{J}}(t) = 2\varepsilon_i^{\mathrm{J}}\cos[\pi\Phi_i(t)/\Phi_0]$ with $\varepsilon_i^{\mathrm{J}}$ being the Josephson energy in j-th SQUID-biased qubit. The interbit coupling between the i-th qubit and the j-th one, via the effective coupling capacitance $\tilde{C}_{ij}$, is described by $K_{ij} = 2e^2/\tilde{C}_{ij}$. Above, the effective capacitances $\tilde{C}_i$ connected to various Cooper-pair boxes are defined by $\tilde{C}_i^{-1} = C_1^{-1} + C_{12}^2\bar{C}_3/(C_1^2\bar{C}^2)$, $\tilde{C}_2^{-1} = \bar{C}_3/\bar{C}^2$, $\tilde{C}_3^{-1} = \bar{C}_2/\bar{C}^2$, $\tilde{C}_4^{-1} = C_4^{-1} + C_{34}^2\bar{C}_2/(C_4^2\bar{C}^2)$, with C_i being the sum of all geometrical capacitances connected to the i-th box, and $\bar{C}_2 = C_2 - C_{12}^2/C_1$, $\bar{C}_3 = C_3 - C_{34}^2/C_4$, $\bar{C}^2 = \bar{C}_2\bar{C}_3 - C_{23}^2$. The effective coupling-capacitances $\tilde{C}_{ij}$ among two boxes, including the long-range couplings between the remote qubits, are determined by

$$\tilde{C}_{12}^{-1} = C_{12}\bar{C}_3/(C_1\bar{C}^2), \quad \tilde{C}_{23}^{-1} = C_{23}/\bar{C}^2,$$
$$\tilde{C}_{34}^{-1} = C_{34}\bar{C}_2/(C_4\bar{C}^2), \tag{9}$$
$$\tilde{C}_{13}^{-1} = C_{12}C_{23}/(C_1\bar{C}^2), \quad \tilde{C}_{24}^{-1} = C_{23}C_{34}/(C_4\bar{C}^2), \tag{10}$$
$$\tilde{C}_{14}^{-1} = C_{12}C_{23}C_{34}/(C_1C_4\bar{C}^2), \tag{11}$$

respectively.

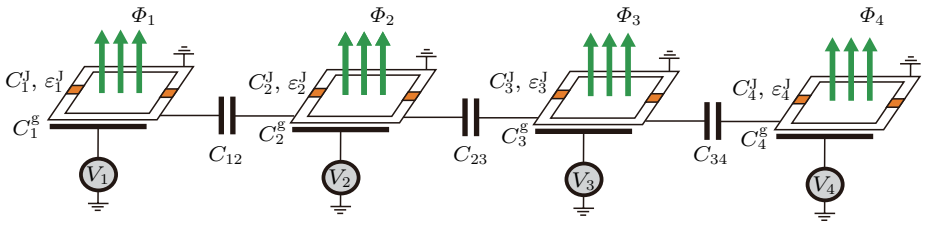


Fig. 2. (color online) Coupled SQUID-biased Cooper-pair boxes as the Ising-type quantum chain driven by local microwave magnetic pulses. Here, C_{ij} is the coupled capacitance between the i-th box and the j-th box, C_i^{J}, $\varepsilon_i^{\mathrm{J}}$, V_i, and Φ_i are the Josephson junction capacitance, junction energy, biased voltage, and microwave magnetic pulse applied to the i-th SQUID-biased Cooper-pair box, respectively.

Differing from the NMR-molecules discussed above, the present microwave pulse applying to switch on/off each effective Josephson energy is local (as the wavelength of the applied driving pulse is shorter than the distance of two nearest-neighbor qubits). Note that, however, the eigenfrequency $\omega_i = E_i^{\rm C}/2$ of the i-th qubit is non-locally controllable, as it depends on both V_i applied to the i-th qubit and $V_j (j \neq i)$ applied to the other qubits. Experimentally, the nearest-neighbor interactions are significantly stronger than those between the remote boxes, i.e.,

$$K_{12},\ K_{23},\ K_{34} > K_{23},\ K_{13},\ K_{24} > K_{14}. \tag{12}$$

However, if the coupling capacitances are fabricated to be sufficiently large, such as $C_m \equiv C_{i(i+1)} \sim 100$ aF (three times of that in the experimental circuit,[25,26]) the non-nearest interactions are still sufficently large yielding non-negligible remote interactions (such as $K_{13} = K_{24} \sim 3$ GHz), which thus could be utilized to directly connect the remote qubits for speeding up the quantum state transfers between them.

At low temperatures, e.g., a few mK, the circuit without any Josephson energy (by properly setting the flux, e.g., $\Phi_i = \pi/2$, to keep $E_i^{\rm J} \equiv 0$) is initialized to its pure ground state $|0000\rangle$. The simplest entangling/disentangling operation, i.e., conditionally flipping only one selected qubit and keeping the remaining ones unchanged, could be implemented by properly setting the gate-voltage and then applying a flux-pulse to switch on the Josephson energy of the selected qubit. For example, if the condition $E_4^{\rm C} = K_{14} - K_{24} - K_{34}$ is set beforehand and $E_4^{\rm J} = 2K_{14}/\sqrt{15}$ is switched on, then the fourth qubit is entangled to the first one prepared at an unknown state with the duration being set as

$$\sin\left(\frac{E_4^{\rm J}\tau_4}{2\hbar}\right) = \cos\left(\frac{\gamma_4\tau_4}{\hbar}\right) = 1, \quad \gamma_4 = \sqrt{K_{14}^2 + \left(\frac{E_4^{\rm J}}{2}\right)^2}. \tag{13}$$

Similarly, to implement a disentangling operation such as $\tilde{U}_{1,234}^{(4)} : \alpha|0000\rangle + {\rm i}\beta|1111\rangle \to |0\rangle \bigotimes (\alpha|000\rangle + \beta|111\rangle)$, which disentangles the first qubit from the remaining three entangled ones, we properly set the gate-voltages to satisfy the condition: $E_1^{\rm C} = \sum_{j=2}^4 K_{1j}$, and then switch on $E_1^{\rm J} = 2\sum_{j=2}^4 K_{1j}/\sqrt{15}$ to evolve the circuit for the time τ_1' determined by

$$\sin\left(\frac{E_1^{\rm J}\tau_1'}{2\hbar}\right) = -\cos\left(\frac{\gamma_1'\tau_1'}{\hbar}\right) = -1, \tag{14}$$

with $\gamma_1' = \sqrt{(\sum_{j=2}^4 K_{1j})^2 + (E_1^{\rm J}/2)^2}$.

One of the experimental challenges for the above demonstrations is to quickly implement the expected operations within the decoherence times, which are typically short, such as at the order of nanoseconds. Indeed, for the experimental coupled capacitance:[25,26] $C_m = 34$ aF yielding $K_{14} \sim 0.4$ μeV, the duration of the above operation should be $\tau_4 = \hbar\pi\sqrt{15}/(2K_{14}) \sim 10$ ns. In principle, this difficulty could be overcome by using the second/third qubits as an intermedia, instead changing the circuit structure. In fact, a π-pulse with duration $\tau_2 \sim 100$ picoseconds is enough to implement the entangling operation $\tilde{U}_{1,2}^{(4)} : |0000\rangle \to |0000\rangle$, $|1000\rangle \to {\rm i}|1100\rangle$, if the gate-voltage is set beforehand to satisfy the condition $E_2^{\rm C} = K_{12} - K_{23} - K_{24}$, and the Josephson energy $E_2^{\rm J} = 2K_{12}/\sqrt{15}$ is switched on. Thus, six such pulses with total duration about 600 picoseconds could finish the transfer of an unknown quantum state from the first qubit to the fourth one, at least theoretically. Similarly, one can estimate that, by using the nearest-neighbor interactions, four microwave pulses (sequentially applied to the third, first, fourth, and second qubits, respectively) with total duration of 400 ps could transfer an unknown entanglement shared by the first and second qubits to that shared by the third and fourth ones. This indicates that the remote interactions could be really utilized to speed up the transfers of unknown quantum states from a qubit to the remote one along the Ising-type quantum chain.

5. Discussion and conclusion

In summary, by conditionally flipping the selected qubits to implement the expected entangling/disentangling operations, we have shown that unknown quantum states (including entangled ones) can be controllably transferred along the driven Ising-type quantum chains with fixed interbit couplings. With this approach by using the distant interactions between the remote qubits, the transmissions of unknown quantum information along the Ising-type quantum chains could be significantly sped up, as the number of the applied operations manifestly decreases.

Specifically, the experimentally-existing NMR-molecules driven by global rf electromagnetic pulses and capacitively-coupled SQUID-biased Cooper-pair boxes driven by local microwave pulses have been demonstrated to act as such driven Ising-type quantum chains to quickly transfer unknown quantum information. In our proposal strict two-qubit CNOT (or SWAP) gates are unnecessary, and the fixed-couplings between qubits are not required to be refocused. Also, our approach is controllable, instead of passively waiting[10–13] quantum states to automatically disperse from the sending site to the receiving end. Hopefully, the proposal is particularly useful to set up the connections between the distant qubits on a chip for implementing the desired quantum computation.

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