Self-organized phenomena of pedestrian counterflow through a wide bottleneck in a channel*

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(Received 6 March 2016; revised manuscript received 10 April 2016; published online 20 July 2016)

The pedestrian counterflow through a bottleneck in a channel shows a variety of flow patterns due to self-organization. In order to reveal the underlying mechanism, a cellular automaton model was proposed by incorporating the floor field and the view field which reflects the global information of the studied area and local interactions with others. The presented model can well reproduce typical collective behaviors, such as lane formation. Numerical simulations were performed in the case of a wide bottleneck and typical flow patterns at different density ranges were identified as rarefied flow, laminar flow, interrupted bidirectional flow, oscillatory flow, intermittent flow, and choked flow. The effects of several parameters, such as the size of view field and the width of opening, on the bottleneck flow are also analyzed in detail. The view field plays a vital role in reproducing self-organized phenomena of pedestrian. Numerical results showed that the presented model can capture key characteristics of bottleneck flows.

Keywords: counterflow, bottleneck, cellular automaton, flow patterns

PACS: 45.70.Mg, 05.65.+b, 07.05.Tp **DOI:** 10.1088/1674-1056/25/9/098901

1. Introduction

Recently, pedestrian flows have attracted more attention due to many interesting collective behaviors in various situations.[1] A typical scenario is pedestrian flow along a channel which is easy to observe experimentally and simulate numerically. If there is not a bottleneck available, the simplest flow is the unidirectional pedestrian flow which is similar to the multilane traffic with lane-changing behaviors. One may obtain basic characteristics of pedestrian flow, such as the fundamental diagram and the transition from free flow to congestion. However, the more interesting case is the pedestrian counterflow which is significantly different from vehicular traffic. Lanes are formed where pedestrians move in one direction, therefore pedestrians can move with higher speeds since strong interactions with oncoming ones are reduced significantly. Kretz *et al.*[2] studied the pedestrian counterflow in a corridor experimentally. They found that the sum of flow and counterflow in any case turns out to be larger than the flow in all situations without counterflow. Lane formation in pedestrian counterflow has been reproduced by many models, such as the social force model[3] and the centrifugal force model.[4] The original lattice gas model seems not to provide an efficient mechanism to make such a phenomenon reoccur.[5] However, Tajima *et al.*[6] extended the lattice gas model by introducing the view field to achieve the end goal. They studied the effect of others in the same and opposite directions in each pedestrian's view field, respectively. They have shown that both mechanisms lead to lane formation but they are quite different. The effect on lane formation of the latter mechanism is weaker than the former. Li *et al.*[7] have considered both the effect of others in the same and opposite directions in each pedestrian's view field. Pedestrian counterflow in a channel has been extensively investigated by researchers with different models.[8–14]

In the presence of bottlenecks (e.g., doors), a variety of phenomena have been revealed by experiments,[15–17] e.g., the formation of lanes at the entrance to the bottleneck, clogging and blockages at narrow bottlenecks, and the oscillatory flows where the passing direction of pedestrian flow changes at bottlenecks occasionally. Helbing *et al.* have investigated both unidirectional and bidirectional pedestrian flow passing the bottlenecks with different lengths.[18] They found an unexpected result that bidirectional flows are more efficient than unidirectional flows. When the opposing flows interrupt each other at a narrow bottleneck, they found irregular oscillations of the passing direction. The switch of the passing direction is explained by the pressure difference between the crowds at the two sides of the bottleneck. Another typical bottleneck flow should be noted, i.e., the evacuation from a room, which has also been investigated by many researchers.[19–28] In most cases, the crowds pass through exits in the same direction during evacuation process. Tajima *et al.*[29] have used the lattice gas model to study unidirectional pedestrians flowing through a bottleneck in a channel. They showed the transition from free flow to choked flow and the existence of scaling behavior. However, the most interesting and challenging issue is the

*Project supported by the National Basic Research Program of China (Grant No. 2012CB725404) and the National Natural Science Foundation of China (Grant Nos. 11172164 and 11572184).

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bidirectional pedestrian flow through bottlenecks. Helbing *et al*. have exhibited the oscillatory flow at a bottleneck by the social force model.[3] Bursteded *et al*.[19] suggested a novel cellular automaton (CA) model by introducing the so-called floor field. They also reported that this model can reproduce lane formation in pedestrian counterflow and oscillatory flows at bottlenecks. However, it seems that force-based models, e.g., the social force model, gave better descriptions about the bidirectional bottleneck flow than rule-based models (e.g., CA models), especially in the case of narrow bottlenecks. Most of the previous investigations on this issue were performed by force-based models.[4,30,31] It is partly due to the reduction of freedom in CA models, in which walkers have to move in discrete grids with discrete velocities. Therefore, it is easy to produce complete congestion of pedestrians passing through the bottleneck from opposite directions. On the other hand, most of the related works mainly showed that their models can make the oscillating flow at a bottleneck reoccur. When the bottleneck is narrow, the incoming flow usually exceeds its capacity. Therefore, the only possible flow may be the oscillating one.

When the bottleneck is wide, the bottleneck flow will exhibit various self-organized phenomena. In a certain density range, the bidirectional pedestrian flow may have the characteristics of both the channel flow and the bottleneck flow. Therefore, these patterns of bottleneck flows at different densities and the effect of the width of bottlenecks on capacity drop are of concern and merit further investigations. However, the bottleneck flow in the case has not been investigated entirely in a quantitative way yet, especially using cellular automaton models.

In this paper, a cellular automaton model is proposed to investigate pedestrian counterflow through a bottleneck in a channel under periodic boundary condition. The presented model is based on the floor field CA model with the introduction of the view field. It can reproduce well not only the lane formation of bidirectional pedestrian flow in a channel, but also the oscillating flow at a narrow bottleneck. More attention was paid to the case of the bottleneck flow with a wider opening which shows various flow patterns in different density regimes.

In Section 2, the floor-field cellular automaton model is improved to include the effect of the view field in order to mimic the oscillating flow. Numerical simulations are carried out. Both flow patterns and quantitative results with different parameters are given in Section 3. Conclusions and suggestions for further research are presented in Section 4.

2. Double-field model

The presented cellular automata model is defined on the square lattice of $L \times W$ sites, where L and W are the length and width of the channel, respectively. Figure 1 shows the schematic illustration of the pedestrian counterflow through a bottleneck in a horizontal channel. The top and bottom sides are solid walls, therefore walkers never go out through them. A vertical wall with an opening, i.e., a bottleneck of width w, is located at the middle of channel. The thickness of the bottleneck is just the length of a cell. There are two types of walkers in the system. Each walker moves to the preferential direction with back step, i.e., from left to right or from right to left. The right (left) walker going to the right (left) is indicated by the open (full) circle in Fig. 1. Each site can be occupied by only a single walker. The boundary condition in the horizontal direction is periodic. Thus, once a right (left) walker leaves the system from the right (left) boundary, he enters the domain from left (right) boundary immediately. Therefore, the total number of walkers is conserved.

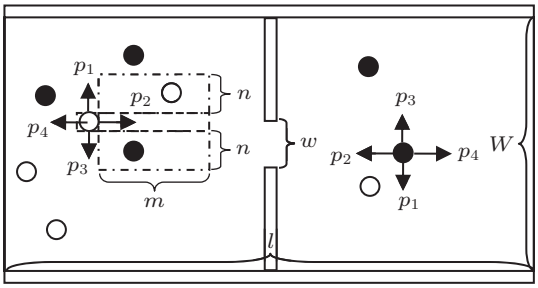


Fig. 1. Schematic illustration of the pedestrian counter flow through a bottleneck in a channel. The view field of a right walker is shown together.

The preferential direction of a walker is determined by the static floor field which was first suggested by Burstedde *et al*.[13] In this case, there exist two static floor fields, namely, S^{R} and S^{L}, generated by the right and left boundary respectively, which drive right and left walkers to move forward. The bottleneck is treated as an obstacle, hence the floor field is calculated by the method suggested by Huang *et al*., which is suitable for complex structures, such as a room with obstacles. The static floor field S^{R} is shown in Fig. 2, which reflects the influence of the right boundary and the topography of the domain. The maximum gradient indicates the most probable direction of a walker for a given cell. If a walker always follows such a direction, it will be the shortest way to his target. The static floor field does not change with time and the presence of walkers.

A walker can shift left or right, move forward or backward to his nearest neighbors in a single time step according to the corresponding probabilities (see Fig. 1). The probability $p_{r,s}$ for a walker moving form cell (i, j) to its neighboring cell $(i+r, j+s)$ is obtained by

$$p_{r,s} = C\exp(k_{\mathrm{S}} S_{i+r,j+s})(1 - n_{i+r,j+s}), \tag{1}$$

where $n_{i+r,j+s}$ is the occupation number of the target cell, i.e., $n_{i+r,j+s}=0$ for an empty cell and $n_{i+r,j+s}=1$ for an occupied cell or an obstacle; C is the normalization factor to ensure $\sum_{(r,s)} p_{r,s} = 1$, and k_{S} is the sensitivity parameter of the static floor field. Figure 3 shows the transition probabilities at a given cell from its original position to his nearest neighbors.

Here the von Neumann neighborhood is adopted. For simplicity, p_1, p_2, p_3, and p_4 are used to represent $p_{0,1}$, $p_{1,0}$, $p_{0,-1}$, and $p_{-1,0}$ ($p_{0,-1}$, $p_{-1,0}$, $p_{0,1}$, and $p_{1,0}$) of a right (left) walker, respectively.

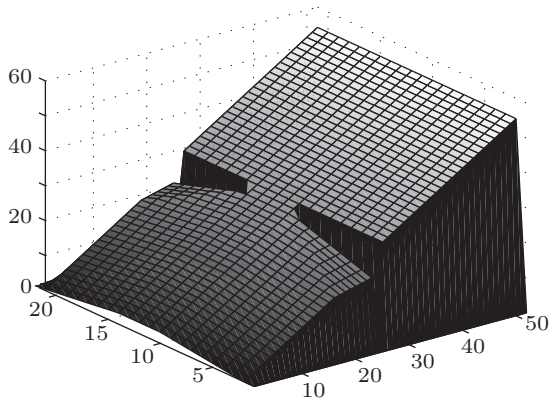


Fig. 2. The floor field S^{R} generated by the right boundary.

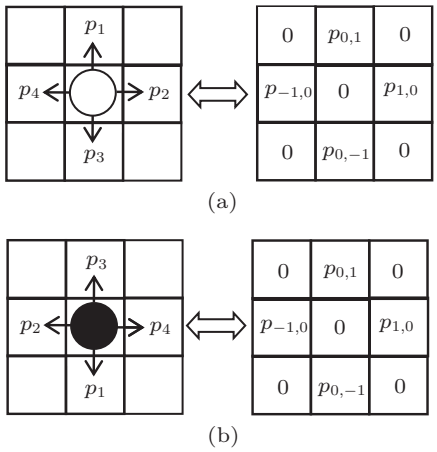


Fig. 3. Transition probabilities for a (a) right and (b) left walker.

In fact, a walker interacts not only with his nearest-neighbor walkers but also with others in his perception range. Therefore, the view field is introduced to extend the range of local interaction among walkers, which has a similar effect to the dynamic floor field to some degree but is more effective. In this study, the view field of a walker is represented by a rectangle area, which is divided into three parts, i.e., front left, front, and front right. Both front left and front right parts have the same size of $m \times n$, where m (n) is the length (width) of the corresponding part. The front part has the size of $m \times 1$. The three parts do not overlap with each other. Thus, the total size of the view field is $m \times (2n+1)$. It is generally known that a walker tends to follow the leaders in the same direction and keep away from those in the opposite direction in reality. Such a behavior is modeled as follows: walkers will adjust their transition probabilities according to the number of others with the same and opposite directions in their view fields. With the help of view field, walkers take proper actions in advance before they are in direct contact with others. Therefore, it is expected to alleviate conflicts among walkers and then enhance the efficiency of walking. Since the presented model includes both global floor field and local view field, it is named double-field (DF) model in the paper.

The movement of each walker is determined by the following rules.

(i) Calculate the transition probabilities p_1, p_2, p_3, and p_4 of each walker according to Eq. (1);

(ii) count the number of walkers of the same (opposite) direction in the front left, front, and front right of view field respectively, which is denoted by T_1 (R_1), T_2 (R_2), and T_3 (R_3);

(iii) calculate the final transition probabilities P_1, P_2, P_3, and P_4 according to the following formula:

$$P_i = \frac{p_i(T_i+1)/(R_i+1)}{p_4 + \sum_{j=1}^{3} p_j(T_j+1)/(R_j+1)}, \quad i = 1,2,3, \tag{2}$$

$$P_4 = \frac{p_4}{p_4 + \sum_{j=1}^{3} p_j(T_j+1)/(R_j+1)}, \tag{3}$$

Equation (2) means that walkers prefer such a direction with more walkers in the same direction and less walkers in the opposite direction. Although the probability of moving backward is not influenced directly by the presence of view field, it will be changed due to the variations of the other three probabilities.

(iv) Each walker chooses a target cell according to the above calculated transition probabilities.

(v) When two or more walkers share the same cell, one of them will be selected to enter it with equal probability and the others keep still. If all nearest neighbors are occupied, he has to stay at his original position. It should be noted that a walker is permitted to move backward in the presented model. This may lead to loosely congested area around the bottleneck at high density, therefore it is possible for a walker to go out of the crowd.

All walkers update their movements at the same time, i.e., the parallel updating is adopted in this study.

3. Simulation and discussion

The simulation of pedestrian counterflow through a bottleneck in a channel is performed under periodic boundary condition. The size of a cell corresponds to approximately 0.4 m×0.4 m, which is the typical space occupied by a person in a dense crowd.[13] Initially, walkers with the given density are distributed randomly in the studied area. The total density ρ is defined as the number of all walkers divided by the capacity of the area. The density of left (right) walkers is determined by calculating the fraction of left walkers f, i.e.,$\rho_l = \rho f$ and $\rho_r = \rho(1-f)$. The average velocity in each time step is defined as the number of walkers moving towards their preferential direction divided by the total number of walkers existing

in the area. The average flux is defined as the number of walkers passing the bottleneck per unit of time. The following parameters were used in the simulation unless otherwise stated: $W = 20$, $L = 50$, $w = 5$, $k_s = 1$, $m = 10$, $n = 2$, and $f = 0.5$. The view field covers a rectangle area of 4 m $\times$ 2 m which is the approximate perception range of a pedestrian in normal conditions. Each run of a simulation takes 2×10^3 time steps. The final results are the average of 100 runs of simulations.

3.1. Flow patterns

First we identify typical flow patterns at different densities (see Fig. 4). In most cases, they have symmetric distributions, that is, the numbers of both sides are the same, i.e., $f = 0.5$ except in Fig. 4(d) where $f = 0.2$. As seen in Fig. 4(a), it is the rarefied flow that there are few walkers available in the area since the density is rather low. Most of the walkers move along the middle line. It is a kind of free flow where nearly everyone has enough space to move forward. Each walker's view field only contains very few walkers which changes in a random way. Therefore, it is not easy to form an ordered structure. If the size of view field is further enlarged, the interaction among walkers will be enhanced (see Fig. 4). In Fig. 4(b), it is another kind of free flow where lane formation can be observed. The right and left walkers are completely separated into two groups and hence they can pass the bottleneck more efficiently. Borrowing the ideas from fluid mechanics, this kind of flow is named the laminar flow. Although the density is low, the number of walkers is large enough to interact with each other efficiently. With the consideration of walkers in the same and opposite directions together in the view field, long-range interaction among walkers leads to lane formation, i.e., lanes of uniform direction are segregated effectively from pedestrian counter flow. In the above two kinds of free flow, the bottleneck flow increases with density (see Figs. 5 or 6). As seen in Fig. 4(c), it is the interrupted bidirectional flow that walkers with opposite directions can pass the bottleneck continuously. However, small congested areas appear on both sides of the bottleneck which may disturb the opposite flow. Therefore, the bottleneck flow begins to reduce with increasing density. Hence, there exists a critical density which indicates a transition beginning from free flow to congested flow. An asymmetric case is shown in Fig. 4(d) that the dominant flow appears due to the difference of the number of walkers from both sides. Here the number of right walkers is significantly larger than that of left walkers. The right walkers pass the bottleneck continuously, but the left walkers are suppressed and gather at the right side of the bottleneck. Figures 4(e) and 4(f) show the oscillatory flow from right (left) to left (right) which were obtained at different instants of a single simulation. This self-organized phenomenon has been reported by many researchers in the case of narrow bottlenecks. Walkers in one direction

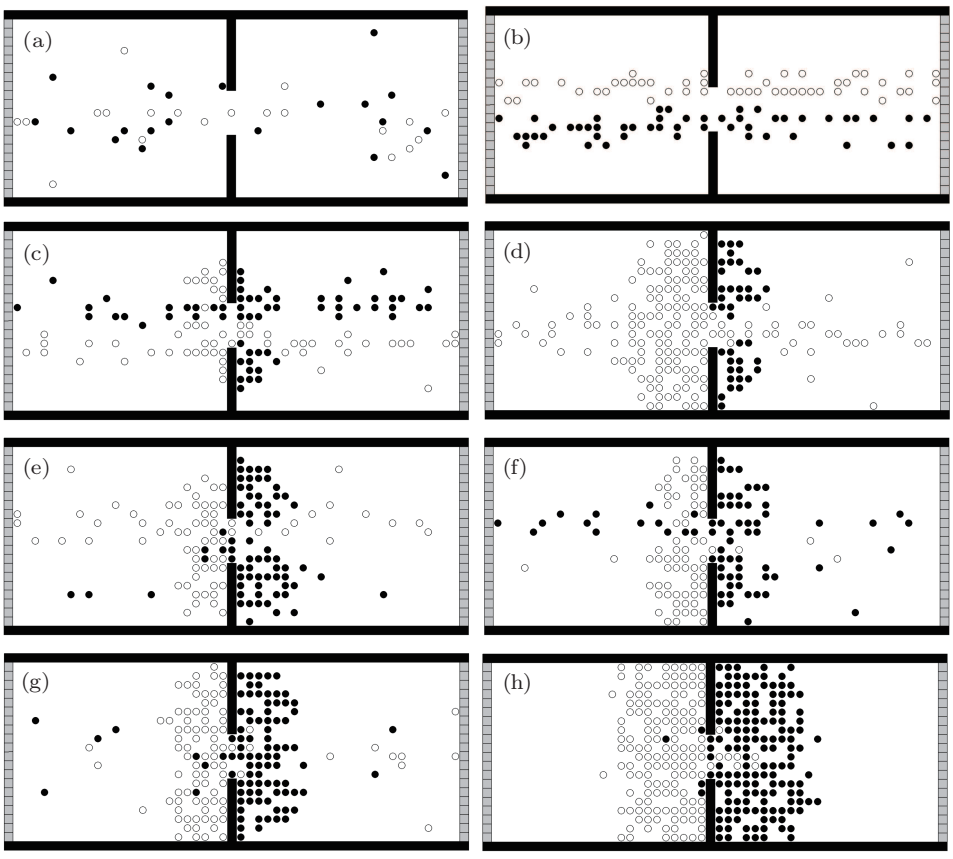


Fig. 4. Flow patterns appear at different densities (a) $\rho = 0.04$, rarefied flow, (b) $\rho = 0.10$, laminar flow, (c) $\rho = 0.12$, interrupted bidirectional flow, (d) $\rho = 0.20$, $f = 0.2$, dominant flow, (e) and (f) $\rho = 0.14$, oscillatory flow, (g) $\rho = 0.18$, intermittent flow and (h) $\rho = 0.30$, choked flow.

pass the bottleneck continuously, meanwhile those in another direction are suppressed to a large extent. That is to say, one of the two opposite flows becomes dominant temporarily. This kind of directional flow continues for some time and changes irregularly. When the width of bottleneck becomes wide, the weak one will hardly be suppressed completely. In Fig. 4(g), it is the intermittent flow that the bottleneck flow occurs in fits and starts. Sometimes fewer walkers can pass the bottleneck and go out of the crowd, sometimes the bottleneck was blocked temporally. In the choked flow, the density is large enough and the bottleneck flow almost stops (see Fig. 4(h)). It is difficult or takes an extreme long time for walkers to pass through the heavy congestions from one side to another side. As a consequence, these flow patterns at different densities provide concise physical pictures to understand the underlying mechanism behind these collective behaviors.

3.2. Effect of the size of view field

As we know the view field enhances the range of interaction among walkers, therefore it is necessary to study how the size of the view field influences the walkers' movement. Figures 5 and 6 show numerical results for different sizes of the view field. The result in the case of no view field is also shown in the same figure for comparison. In most cases, the average velocity decreases monotonously with density (see Fig. 5(a)). More precisely, the average velocity decreases gradually at first, then decreases quickly (beginning from the critical density), and finally decreases gradually again. Here the critical density is defined as the density for which the flux reaches its maxima (see Fig. 5(b)). It is interesting to note the case of larger view fields, e.g., $m = 20$, that the average velocity increases first at the very low density. When the length of the view field is large enough, even walkers are rather few but the number of walkers in the view field is enough to increase the probability to move forward. As a result, the average velocity increases with density correspondingly. It is obvious that a phase transition from free flow to congestion occurs as the density exceeds the critical value. These critical densities in the case with the view field are significantly larger than that in the case without the view field. As can be seen in Fig. 5, the average velocity, the flux, and the critical density increase with the length of the view field. The two quantities are considerably larger than those in the case without a view field. It is shown that the variations in length for the view field have a significant influence on the bottleneck flow.

As shown in Fig. 6, it seems that both the average velocity and the flux are nearly independent of the width of the view field for a given length of the view field ($m = 10$), therefore the critical densities are almost the same. It is significantly different from those in the channel without a bottleneck, see Ref. [7], where both the average velocity and the flux change with the width of the view field. The reason may be the pedestrian flow fully determined by the bottleneck which leads to capacity drop. The width of the view field is rather large com-

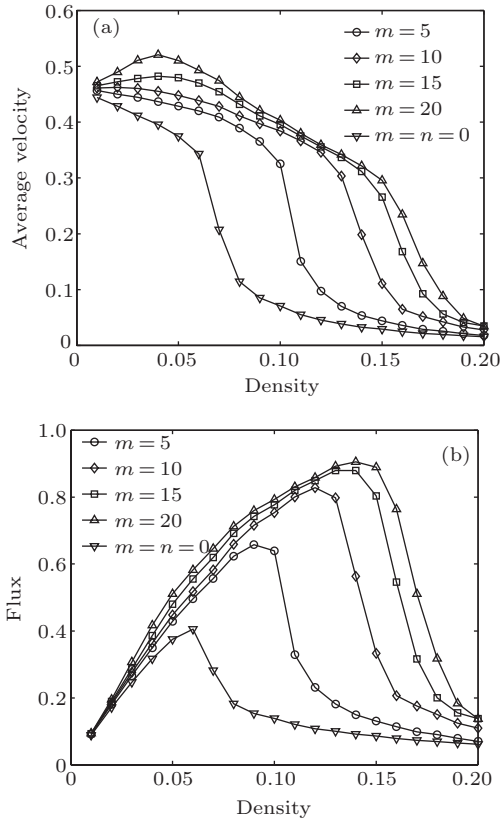


Fig. 5. Plot of the average velocity (a) and the flux (b) against density with various m and fixed $n(= 2)$. The curve in the case of no view field (i.e., $m = n = 0$) is shown together for comparison.

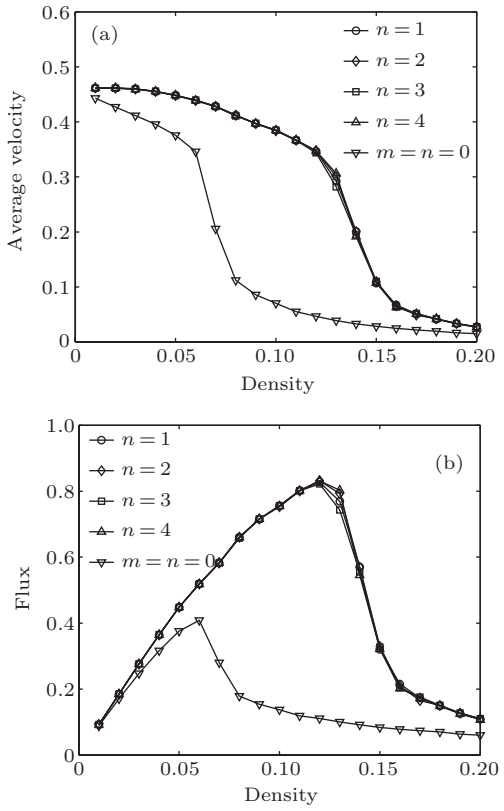


Fig. 6. Plot of the average velocity (a) and the flux (b) against density with fixed $m(= 10)$ and various n. The curve in the case of no view field (i.e., $m = n = 0$) is shown together for comparison.

pared to the width of the bottleneck. Therefore, the variation of the width of the view field only has negligible effect on the bottleneck flow.

3.3. Effect of fraction

We investigate the effect of the fraction of left walkers, i.e., f, on the bottleneck flow. It is found that the average velocity and the flux in the symmetric case of $f = 0.5$ are larger than those in the asymmetric cases before the critical density. The maximum flux appears in the symmetric case of $f = 0.5$ (see Fig. 7(b)), which indicates that the bidirectional flow is more efficient than the unidirectional flow in the free flow state. This is due to the existence of the view field which is essential to self-organize pedestrian counterflow to reduce the oncoming conflicts. On the other hand, walkers are impeded by their predecessors especially when the unidirectional flow exceeds the capacity of the bottleneck. This finding coincides with previous observations in pedestrian counter flow in a channel.[2,18] A clear transition from free flow to congested flow can be observed in the symmetric case. When the density is larger than the critical value, the bottleneck flow decreases quickly to a rather small value. However, in the asymmetric cases, the dominant flow maintains for a long time especially when f is small (see Fig. 4(d)). After the flux reaches its maxima, the dominant flow decreases gradually with density. Complete congestion will not occur even when the density is rather high.

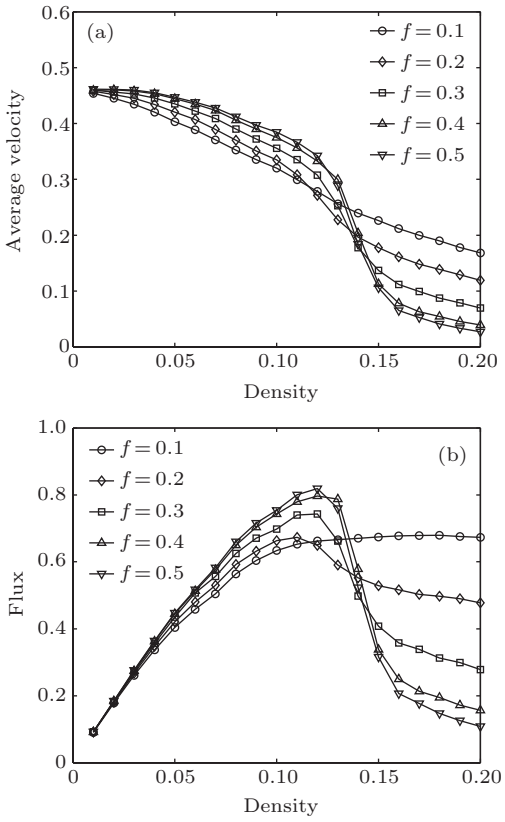


Fig. 7. Plot of the average velocity (a) and the flux (b) against density with various f.

3.4. Effect of sensitivity parameter

Then we investigate the effect of the sensitivity parameter of the static floor field k_S on the bottleneck flow (see Fig. 8). It is found that the average velocity in the free flow state increases with k_S and so does the maximum flux. However, the critical density decreases with k_S. The sensitivity parameter of the static floor field has the following property: the more the value of k_S is, the larger the probability for a walker moving to his preferred direction becomes. Thus, the larger k_S leads to higher velocity and therefore larger incoming flow. It is easy to exceed the capacity of the bottleneck and induce congestion right there. In the case of $k_S = 2.5$, a sharp transition occurs at the critical density where both the average velocity and the flux decrease abruptly. The faster the walkers try to pass the bottleneck, the longer time they need to take, i.e., the faster-is-slower effect.

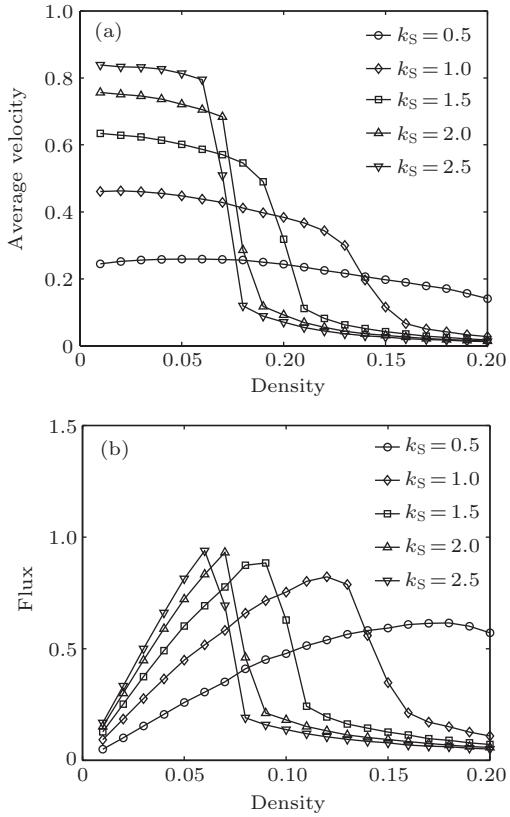


Fig. 8. Plot of the average velocity (a) and the flux (b) against density with various k_S.

3.5. Effect of bottleneck width

Finally, we investigate the effect of the width of the bottleneck shown in Fig. 9. As expected, both the flux and the critical density increase with the width of the bottleneck. It is evident that it is easy for walkers to pass a wide bottleneck and is unlikely to cause congestion as well. It is found that the flux increases continuously with the width of the bottleneck, then the growth slows down gradually. Obviously, the width of the bottleneck has significant influence on flow patterns at the bottleneck. If the width of the bottleneck is too small, the

bidirectional stratified flow becomes impossible. On the contrary, it is not easy to maintain the oscillatory flow for a wide bottleneck and is more likely to form bidirectional flow. It is worth noting that the DF model can make the oscillating flow reoccur even in the case of w= 1. On the other hand, it can reproduce lane formation quickly when there is no bottleneck, i.e., $w = W$.

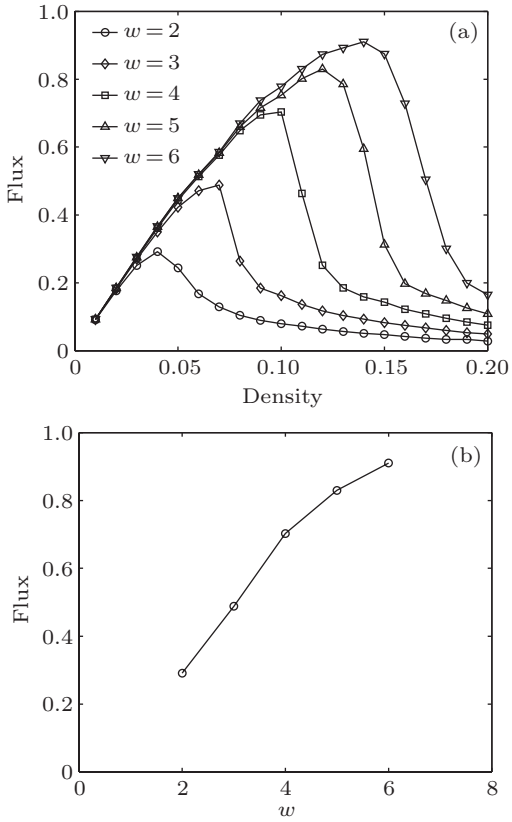


Fig. 9. (a) The flux against density with various w. (b) The relation between the maxima of flux and the width of bottleneck.

4. Conclusions

The purpose of this paper is to get a better understanding of the bidirectional pedestrian flow through a bottleneck at various densities. To achieve the goal, we have proposed a double-field (DF) cellular automaton model which incorporates both the floor field and the view field. This model can reproduce typical self-organized phenomena in pedestrian flow, such as lane formation. Then we adopted this model to investigate dynamical collective behaviors of pedestrian counterflow passing a wide bottleneck in a channel. More attention is paid to the flow patterns at different densities in the symmetric case of $f = 0.5$. Six typical flow patterns are identified as rarefied flow, laminar flow, interrupted flow, oscillatory flow, intermittent flow, and choking flow. In the asymmetric cases, the dominant flow is the most striking phenomena. It is found that the view field plays a vital rule in reducing the conflicts among walkers, and therefore the efficiency of walking is enhanced. It can be viewed as an example of optimal self-organization. It turns out that this model provides a more realistic description of pedestrian flow and can be further applied to other cases.

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