Generating periodic interference in Bose–Einstein condensates*

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The interference between two condensates with repulsive interaction is investigated numerically by solving the one-dimensional time-dependent Gross–Pitaevskii equation. The periodic interference pattern forms in two condensates, which are prepared in a double-well potential consisting of two truncated harmonic wells centered at different positions. Dark solitons are observed when two condensates overlap. Due to the existence of atom–atom interactions, atoms are transferred among the ground state and the excited states, which coincides with the condensate energy change.

Keywords: interference pattern, dark solitons, Bose–Einstein condensates

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1. Introduction

Since the realization of Bose–Einstein condensates (BECs) in dilute and ultracold gases, lots of theoretical and experimental studies on BECs have been reported, such as the creation of solitons,[1–5] vortices,[6,7] and symmetry breaking.[8,9] Since BECs have long coherence time and high controllability, as ideal coherent sources they are widely used to study matter–wave interference.[10–12] In the mean-field approximation, the condensates can be described by the macroscopic wavefunction. This theory paves a way toward studying the interference phenomena of condensates theoretically.

Interference between two condensates was first observed experimentally in 1997.[13] Wallis *et al.*[14] calculated theoretically the macroscopic interference of two independent condensates released from a double-well potential trap. The double-well potential used to form two condensates was created by adding a Gaussian barrier into the center of a harmonic potential.[15] The interference pattern was aperiodic, while dark solitons were generated when two condensates overlapped.[16]

A recent experiment[17] reports a novel pattern of interference between two condensates, which is different from the previous works.[13–15] Motivated by this experiment, the periodic interference pattern of two condensates is demonstrated by numerical simulation in this paper. We also give a theoretical explanation on the generation mechanism of the periodic interference pattern, which has been mentioned by little research up to now.

2. The model

In the zero-temperature limit, single-component BECs trapped in a harmonic potential are described by the nonlinear Schrödinger equation, known as Gross–Pitaevskii (GP) equation[18]

$$\mathrm{i}\hbar\frac{\partial\psi}{\partial t}=\left[-\frac{\hbar^2}{2m}\Delta+V_{\mathrm{h}}(x,y,z)+g\,|\psi|^2\right]\psi, \tag{1}$$

where

$$\Delta=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2},$$
$$V_{\mathrm{h}}(x,y,z)=\frac{1}{2}m(\omega_x^2x^2+\omega_y^2y^2+\omega_z^2z^2),$$

$g=4\pi\hbar^2a_{\mathrm{s}}/m$, and ψ satisfies $\iiint|\psi|^2\,\mathrm{d}x\mathrm{d}y\mathrm{d}z=N$. Here the interaction coupling constant g is characterized by the scattering lengths a_{s} between atoms.

We consider the condensates in a quasi-one-dimensional system, where the harmonic potential $V_{\mathrm{h}}(x,y,z)$ is a cigar-shaped potential with $\omega_z=\omega_y\gg\omega_x$. The three-dimensional (3D) GP equation (1) can be reduced to a 1D GP equation.[19] After a dimensionless process (we denote the order parameter with the same ψ), we have

$$\mathrm{i}\frac{\partial\psi}{\partial t}=\left[-\frac{1}{2}\frac{\partial^2}{\partial x^2}+V(x)+g_{\mathrm{s}}\,|\psi|^2\right]\psi, \tag{2}$$

where $g_{\mathrm{s}}=2N\omega_za_{\mathrm{s}}/(\omega_xL)$, $V(x)=\frac{1}{2}x^2$, $L=\sqrt{\hbar/m\omega_x}$, and ψ satisfies $\int|\psi|^2\,\mathrm{d}x=1$. The time, length, and energy are measured in units of ω_x^{-1}, L, and $\hbar\omega_x$, respectively. The energy of the system is

$$E=\int\left[\frac{1}{2}\left|\frac{\partial\psi}{\partial x}\right|^2+V(x)\,|\psi|^2+\frac{g_{\mathrm{s}}}{2}\,|\psi|^4\right]\mathrm{d}x. \tag{3}$$

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3. Numerical results

As is known to all, two condensates without interaction can form periodic interference.[16] However, there is little research on the periodic interference between two condensates with interaction. Here we first demonstrate the periodic interference between two condensates with repulsive interaction and give a theoretical explanation of this periodic interference pattern. The condensates are initially trapped in a double-well potential, which consists of truncated harmonic wells centered at positions x_i with $i = 1,2$ and $V_i(x) = \frac{1}{2}\min_i (x-x_i)^2$.[20] Here we choose $x_1 = -x_2 = x_a$, where x_a related to the position of the two minima, and then the potential in Eq. (3) is $V(x) = \frac{1}{2}(|x| - x_a)^2$.[21] In the initial system, the condensates are divided into two parts without being overlapped spatially. When $g_s = 0$, the ground state wavefunction can be described by

$$\phi_g(x) = (\pi^{-1/4} e^{-(x-x_a)^2/2} + \pi^{-1/4} e^{-(x+x_a)^2/2})/\sqrt{2},$$

with the corresponding energy

$$\begin{aligned} E_g &= \int \left[\frac{1}{2} \left| \frac{\partial \phi_g(x)}{\partial x} \right|^2 + V(x) \left| \phi_g(x) \right|^2 \right] dx \\ &= \frac{1}{2}(1 + {x_a}^2 - e^{-{x_a}^2}(-1 + {x_a}^2)) \end{aligned}$$

and the normalized coefficient $N_g = \int |\phi_g(x)|^2 dx$. Table 1 gives the relations between these physical quantities and x_a.

Table 1. The relations between the physical quantities $|\phi_g(x=0)|^2$, E_g, N_g and x_a.

x_a	0	1.0	2.0	3.0	4.0	5.0	6.0	7.0
$\lvert\phi_g(x=0)\rvert^2$	1.13	0.42	0.02	1.39×10^{-4}	1.27×10^{-7}	1.57×10^{-11}	2.62×10^{-16}	5.92×10^{-22}
E_g	1.0	1.0	2.4725	4.9995	8.50	13.0	18.5	25.0
N_g	2.0	1.3679	1.0183	1.0001	1.0	1.0	1.0	1.0

From Table 1, we can see that when $x_a = 5$, ϕ_g is normalized to unit 1 with corresponding energy $E_g = \frac{1}{2} + \frac{{x_a}^2}{2} = 13.0$. Besides, $|\phi_g(x=0)|^2 = 1.57\times10^{-11}$ shows that the condensates are divided into two parts without overlap.

When $g_s \neq 0$, the ground state wavefunction of condensates $\psi(x;t=0)$, chosen as the initial state wavefunction for the following interference, is obtained by propagating in imaginary time starting with the test wavefunction $\phi_0(x) = \pi^{-1/4} e^{-x^2/2}$. With $x_a = 5$, the ground state density $|\psi(x;t=0)|^2$ varying with g_s is shown in Fig. 1. Figure 1 shows that when the atom interaction is repulsive, the condensates are divided into two parts without overlap.

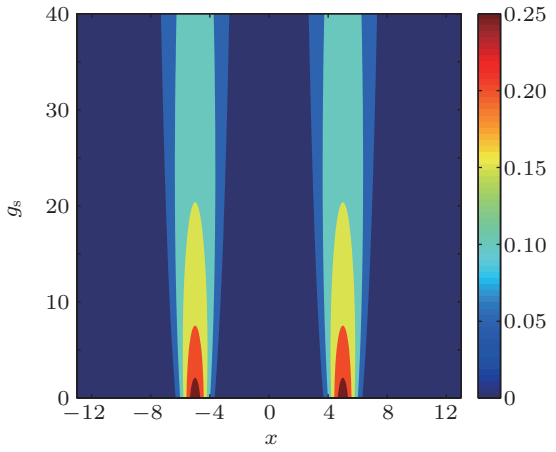


Fig. 1. (color online) The ground density of the system with $x_a = 5$ and varying g_s.

As is known to all, when atom interaction is too strong, the condensates are trapped in one well (i.e., self-trapping), so we choose $g_s = 20$ and $x_a = 5$ to demonstrate the interference between two condensates with repulsive interaction. We aim to analyze the interference between these two condensates. After preparing the initial state, the trap $V(x)$ is suddenly changed into $V(x) = \frac{1}{2}x^2$ at $t = 0$. The time evolution of the GP equation (2) is studied numerically by using the time-splitting spectral method,[22] which was used to solve the GP equation.[3,23] We discretize Eq. (2) with time step $\Delta t = 0.001$ and space step $\Delta x = 0.025$. The evolution of condensates' density distribution is shown in Fig. 2.

From Fig. 2(a), it is surprising that the time evolution of density distribution with $g_s = 20$ is periodic, and the dark and bright stripes appear in the overlapped regions. This interference between two condensates is similar to a Young's double slit experiment.[17] Each condensate executes its own harmonic motion. They have the same kinetic energy and make relative motions. This harmonic motion is similar to that in Refs. [24] and [25], but there are some differences between interference patterns in Fig. 2(a) and that in Refs. [16] and [24]. The most important difference is that the interference pattern here is periodic. Maybe this motion can be described by one-dimensional harmonic oscillator wavefunctions, which will be discussed in the following paper. In the overlapped regions, the density minima shown in Fig. 2(a) along with phase jumps are identified as dark solitons, which is confirmed by the density and phase distributions of two condensates at $t = 1.59$ as shown in Fig. 2(b). From Fig. 2, we can also see that the number and position of dark solitons are not changed at the points of maximum overlap of two condensates. It is noticeable that the 2π phase jumps in Fig. 2(b) are artifacts of the modular arithmetic.[3,26]

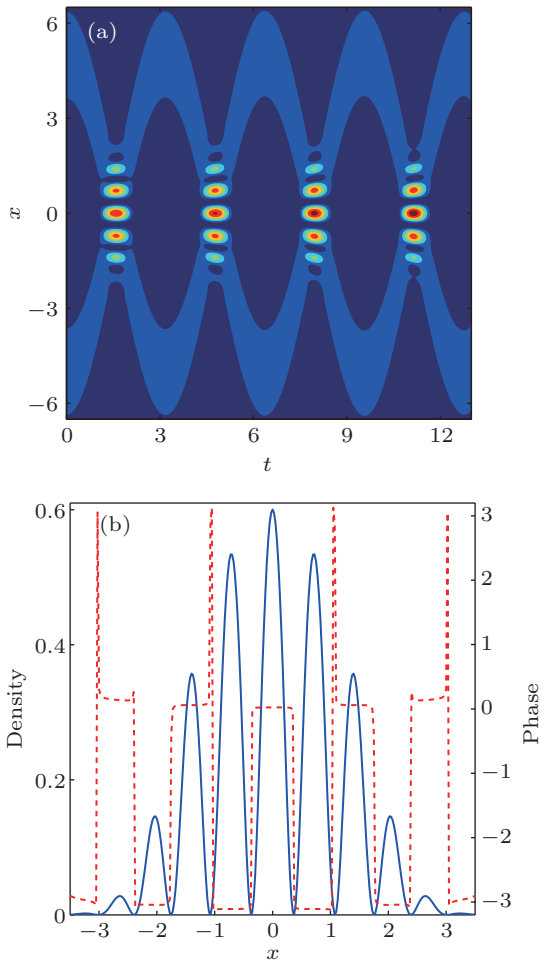


Fig. 2. (color online) (a) Time evolution of density distributions of two condensates with $g_s = 20$. (b) The density (solid line) and phase (dashed line) distributions of the condensates at $t = 1.59$.

Comparing Fig. 2 with the interference between two condensates without interaction in Refs. [16] and [27], we find that (i) the time evolution of density and phase distributions of these two scenarios are both periodic, but with different periods. The period of interference between two condensates with repulsive interaction is greater than 2π as shown in Fig. 3; (ii) dark solitons appear in the overlapped regions.

In order to explain this periodic behavior, we assume that the two condensates execute harmonic motions with their periods T_g, and their momentums satisfy

$$|P(t)| = x_a \left| \sin\left(\frac{2\pi t}{T_g}\right) \right|.$$

We define a new function $\bar{\phi}_n(x;t)$ which contains two wave packets

$$\bar{\phi}_n(x;t) = \frac{1}{\sqrt{2}}\left(e^{ix_a \sin(\frac{2\pi t}{T_g})x} \phi_n\left(x + x_a \cos\left(\frac{2\pi t}{T_g}\right)\right) + e^{-ix_a \sin(\frac{2\pi t}{T_g})x} \phi_n\left(x - x_a \cos\left(\frac{2\pi t}{T_g}\right)\right)\right), \quad (4)$$

where $\phi_n(x) = \sqrt{\frac{1}{\sqrt{\pi}2^n n!}} e^{-x^2/2} H_n(x)$ and $H_n(x)$ is Hermite polynomial. Numerical integration shows that $\int |\bar{\phi}_n(x;t)|^2 dx \approx 1$ and $\int \bar{\phi}_m(x;t)^* \bar{\phi}_n(x;t) dx < 10^{-6}$ $(m \neq n)$, so these wavefunctions $\bar{\phi}_n$ are approximately orthonormal. The total condensates wavefunction $\psi(x;t)$ could be expanded as

$$\psi(x;t) = \sum_n c_n(t) \bar{\phi}_n(x;t). \quad (5)$$

The initial state wavefunction $\psi(x;t=0)$, which consists of two Guassian wave packets, can be written as

$$\psi(x;t=0) = \sum_n c_n(0) \bar{\phi}_n(x;t=0), \quad (6)$$

where $c_0(0) = 0.9299$, $c_2(0) = 0.3564$, $c_4(0) = 0.0906$, $c_6(0) = 9.58 \times 10^{-4}$, $c_1(0) = c_3(0) = c_5(0) = 0$. It shows that atoms are almost distributed at the ϕ_n $(n = 0,2,4)$ states. In the time evolution of two condensates, we assume $c_n(t) \neq 0$ $(n = 0,2,4)$ and the other terms are always equal to zero. In other words, in Eq. (5) we only keep $\bar{\phi}_n$ $(n = 0,2,4)$ terms. To describe whether the wavefunctions $\bar{\phi}_n$ $(x;t)$ $(n = 0,2,4)$ can describe the total condensates wavefunction $\psi(x;t)$, a new function $\Phi(x;t)$ is defined by

$$\Phi(x;t) = \sum_{n=0,2,4} c_n(t) \bar{\phi}_n(x;t), \quad (7)$$

where $c_0(t) = (\bar{\phi}_0(x;t), \psi(x;t))$, $c_2(t) = (\bar{\phi}_2(x;t), \psi(x;t))$, $c_4(t) = (\bar{\phi}_4(x;t), \psi(x;t))$, which can be obtained from Eq. (5).

The period T_g in Eq. (4) can be obtained from the time evolution of total condensates density $|\psi(x;t)|^2$ in $x = 0$. The time evolution of $|\psi(x=0;t)|^2$ and $|\Phi(x=0;t)|^2$ is shown in Fig. 3. It is noted that the total condensate density $|\psi(x=0;t)|^2$ has a maximum value at the point of maximum overlap of two condensates (i.e., at $t = kT_g$, $k = 1/4, 3/4, 5/4, \ldots$). Figure 3 shows that the period of interference pattern shown in Fig. 2 is $T_g = 6.36$, which is greater than 2π, the period of interference between two condensates without interaction. This is due to the existence of the repulsive interaction. At the first quarter of period (i.e., $0 \leq t \leq T_g/4$), the evolution processes arising from interactions produce a resistance force on the condensate tending to reduce its velocity, so the period of interference becomes longer. Figure 3 also demonstrates that during the first quarter of the period (i.e., $0 \leq t \leq T_g/4$), $|\psi(x=0;t)|^2$ and $|\Phi(x=0;t)|^2$ are almost overlapped at $t \leq 1.33$, however, there is obvious difference at $1.33 < t \leq T_g/4$ for the impact of interaction on condensate becomes more obvious.

Then we focus on analyzing the impact of interaction on the density distribution in $\psi(x;t)$ for $0 \leq t \leq T_g/4$. The density distributions in $\psi(x;t)$ and $\Phi(x;t)$ at $t = 0, 1.0, 1.2$, and 1.4 are shown in Fig. 4. From Figs. 4(a) and 4(b), the density distribution in $\psi(x;t)$ and $\Phi(x;t)$ at $t = 0$ and 1.0 are almost overlapped, because for $0 \leq t \leq 1.0$, the centers of two condensates execute simple harmonic motions and the repulsive interaction plays little effect on the momentum of two condensates. From Figs. 4(c) and 4(d), we can see that at $t = 1.2$

and 1.4, the numbers of peak density in $\psi(x;t)$ and $\Phi(x;t)$ are the same, however, there are differences in the strength and position of peak density. In this time interval, the effect of repulsive interaction on the momentum of two condensates becomes more obvious, especially at $t = 1.59$. We will modify the condensates' momentum $P(t)$ and $c_n(t) \neq 0$ $(n = 0,2,4)$ to reduce the differences between $\Phi(x;t)$ and $\psi(x;t)$ at $t = 1.59$.

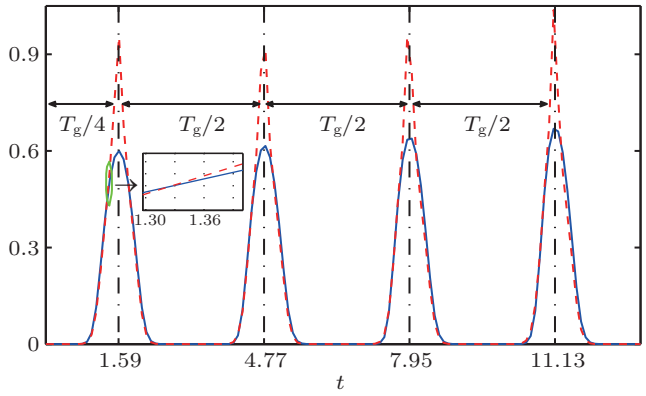


Fig. 3. (color online) The time evolution of $|\psi(x=0;t)|^2$ (solid line) and $|\Phi(x=0;t)|^2$ (dashed line) described by Eqs. (5) and (7), respectively.

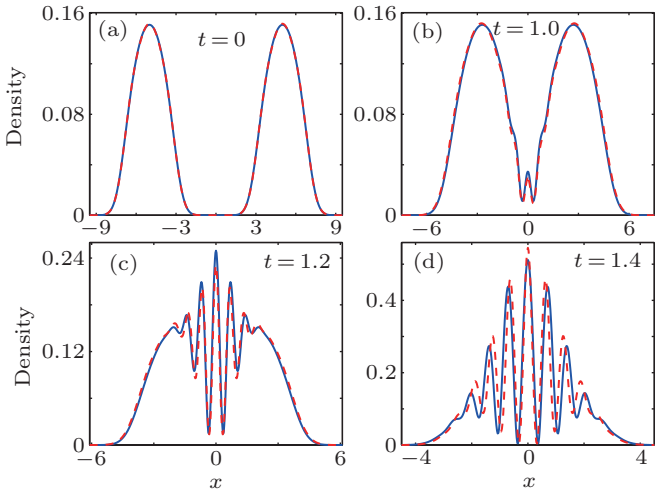


Fig. 4. (color online) The density distribution in $\psi(x;t)$ (solid line) and $\Phi(x;t)$ (dashed line) described by Eqs. (5) and (7) at (a) $t = 0$, (b) $t = 1.0$, (c) $t = 1.2$, and (d) $t = 1.4$.

The density distribution in $\psi(x;t)$ and $\Phi(x;t)$ at $t = 1.59$ is shown in Fig. 5. The coefficient $c_n(t)$ $(n = 0,2,4)$ at $t = 0, 1.0, 1.2, 1.4$, and 1.59 can be given as follows:

$$\begin{aligned}
&c_0(1.0) = 0.9302, \quad c_2(1.0) = 0.3555, \quad c_4(1.0) = 0.0904;\\
&c_0(1.2) = 0.9305, \quad c_2(1.2) = 0.3548, \quad c_4(1.2) = 0.0892;\\
&c_0(1.4) = 0.9475, \quad c_2(1.4) = 0.3035, \quad c_4(1.4) = 0.0411;\\
&c_0(1.59) = 0.9744, \quad c_2(1.59) = 0.1953, \quad c_4(1.59) = 0.0342;
\end{aligned}$$

and the corresponding energies $E(t)$ are $E(1.0) = 14.411$, $E(1.2) = 14.396$, $E(1.4) = 14.394$, and $E(1.59) = 14.398$, which are different from $c_n(0)$ $(n = 0,2,4)$ in Eq. (6). These illustrate that part of the condensates is transferred among three states ϕ_n $(n = 0,2,4)$ due to the existence of interaction, which leads to the internal excitations of atoms. It is noticeable that for $0 \leq t \leq T_g/4$, a small amount of atoms are transferred from excited state ϕ_n $(n = 2,4)$ into ground state ϕ_0, which coincides with the energy change at $t = 1.0$ and $t = 1.2$. Besides, during this time the repulsive interaction tends to hold off atoms, so we can revise the momentum of $\bar{\phi}_n$ $(n = 0,2,4)$ in Eq. (4) to fit $\psi(x;t)$. Here we define a new function Φ_{t_0} to fit $\psi(x;t)$ at $t = 1.59$

$$\Phi_{t_0} = \sum_{n=0,2,4} d_n \frac{1}{\sqrt{2}}(e^{iP_0x} + e^{-iP_0x})\phi_n(x). \tag{8}$$

When $P_0 = 4.41$, $d_0 = 0.9223$, $d_2 = 0.3681$, and $d_4 = 0.1178$, the density distribution $|\Phi_{t_0}|^2$ is shown in Fig. 5 (dashed-dotted line).

At $t = 1.59$, the momentum of $\Phi(x;t = 1.59)$ is $|P(1.59)| = 5.00$. There is obvious difference between the density in $\Phi(x;t = 1.59)$ and $\psi(x;t = 1.59)$. However, when the momentum is changed into $P_0 = 4.41$, and $d_0 = 0.9223$, $d_2 = 0.3681$, $d_4 = 0.1178$, the $|\Phi_{t_0}|^2$ can coincide well with $|\psi(x;t = 1.59)|^2$. This implies that even when two condensates are maximally overlapped, the condensates wavefunction $\psi(x;t)$ can also be expanded with $\bar{\phi}_n$ $(n = 0,2,4)$. It is also noted that there is a small difference between $|\Phi_{t_0}|^2$ and $|\psi(x;t = 1.59)|^2$, which is due to a numerical error in our simulation and the expansion of $\psi(x;t)$ only with $\bar{\phi}_n(x;t)$ $(n = 0,2,4)$. In summary, the condensates wavefunction $\psi(x;t)$ can be well described by $\bar{\phi}_n(x;t)$ $(n = 0,2,4)$ with optimal parameters, seeing Figs. 4(a), 4(b), and 5.

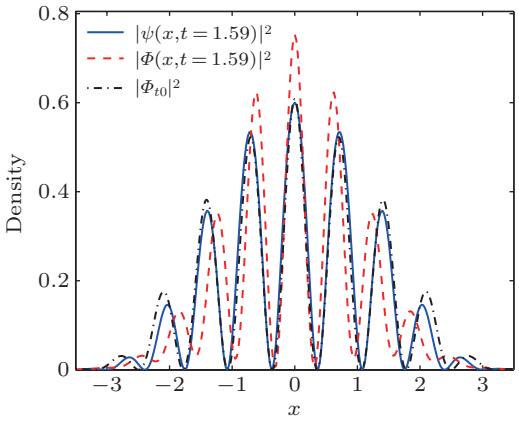


Fig. 5. (color online) The density distribution in ψ $(x;t = 1.59)$ (solid line), Φ $(x;t = 1.59)$ (dashed line), and Φ_{t_0} (dashed-dotted line) described by Eqs. (5), (7), and (8), respectively.

4. Conclusion

In this paper, we demonstrate that two condensates with repulsive interaction execute approximately harmonic motions. They form a periodic interference pattern, which can be expressed in terms of one-dimensional harmonic oscillator wavefunctions with optimal parameters. It is also noticeable that during time evolution of condensates, a small amount of atoms are transferred among three states ϕ_n $(n = 0,2,4)$, which

is due to the existence of interaction leading to the internal excitations of atoms. This atom transfer coincides with the energy change, for which the repulsive interaction tends to hold off atoms. Besides, dark solitons are generated when two condensates overlap. Our method provides a way to study the interaction-leading internal excitations of atoms.

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