Discord and entanglement in non-Markovian environments at finite temperatures*

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The dynamic evolutions of the discord and entanglement of two atoms immersed in two independent Lorentzian reservoirs at zero and finite temperatures have been investigated by using the time-convolutionless master-equation method. Our results show that, nonzero temperature can induce the entanglement sudden death and accelerate the decays of discord and entanglement. The discord and the entanglement have different robustness for different initial states and their robustness may change under certain conditions. When both the non-Markovian effect and detuning are present simultaneously, due to the memory and feedback effect of non-Markovian reservoirs, the discord and entanglement can be effectively protected even at nonzero temperature by increasing the non-Markovian effect and the detuning.

Keywords: discord, entanglement, non-Markovian environment, temperature

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1. Introduction

Entanglement, as a kind of quantum correlation,[1,2] is considered to be not only a vital concept in physics but also a prime resource for quantum information processing.[3,4] Therefore, a great deal of attention has been devoted to the experimental generation and manipulation of entangled systems and the theoretical study of the entanglement evolution.[5,6] In particular, since Yu and Eberly[7] discovered that the Markovian entanglement dynamics of two qubits exposed to local noisy environments may markedly differ from the single-qubit decoherence evolution, the analysis of entanglement decay and its relation with decoherence induced by an unavoidable interaction between a system and its environment has become an important topic. Many important progresses have been acquired in experimental and theoretical researches on entanglement dynamics in open quantum systems. The investigations show that environmental noises can not only be helpful to protect entanglement, but also can make entanglement sudden death (ESD), entanglement sudden birth (ESB), and entanglement swapped.[8–10]

However, entanglement is not the only type of quantum correlation useful for quantum information processing. Ollivier and Zurek introduced another concept of quantum correlation, termed discord,[11,12] which captures nonclassical correlations, including but not limited to entanglement, that is, separability of a density matrix describing a two-qubit system does not guarantee vanishing of discord. Discord has significant applications in deterministic quantum computation with one pure qubit (DQC1)[13] and estimation of quantum correlations in the Grover search algorithm.[14] On the other hand, discord has also been extensively used in studies of quantum phase transition[15,16] and to measure the quantum correlation between relatively accelerated observers.[17]

A realistic physical system always suffers from some unwanted interactions with its outside environments,[18] causing decoherence and destroying entanglement and discord. This interaction of a quantum system with its environment may be described using either a classical or a quantum mechanical picture of the environment. Understanding whether and in which conditions the two descriptions are equivalent is still a debated topic.[19–22] When the environment has many degrees of freedom and/or a structured noise spectrum, a classical description may be convenient and also more accurate. Several systems of interest belong to these categories and many efforts have been devoted to study situations where quantum systems are affected by the classical noise. Examples include the dynamics of quantum correlations[23–26] and the decoherence in solid state qubits.[27,28] But a quantum mechanical description of an open system is usually thought of as being a more general approach. In recent years, many methods and important progresses have been acquired in studying open systems using the quantum mechanical picture of the environment. For instance, Bellomo *et al.* studied the entanglement dynamics of two independent qubits in non-Markovian environments.[29,30] Xiao *et al.*[31] discussed the discord dynamics of two qubits in this model. In Refs. [30]–[33], the same model is adopted, in which the whole system can be divided into two parts and a formal solution for the evolution may be obtained by exactly

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solving the time-dependent Schrödinger equation in the subspace with one excitation.

In this paper, we apply the time-convolutionless (TCL) master-equation method to investigate discord and entanglement of two atoms immersed in two independent Lorentzian reservoirs, in which the two atoms and the two reservoirs are included as a whole system. We focus on the influences of temperatures, atomic initial states, and non-Markovian effect as well as detuning on discord and entanglement. Firstly, we find that the nonzero temperature can induce the entanglement sudden death and accelerate the decays of discord and entanglement. Secondly, the discord and entanglement have different robustness for the different initial states and their robustness may change under certain conditions. Thirdly, when both the non-Markovian effect and detuning are present simultaneously, due to the memory and feedback effect of the non-Markovian reservoirs, the discord and entanglement can be effectively protected even at the nonzero temperature by increasing the non-Markovian effect and the detuning.

The report is structured as follows. We present a physical model in Section 2. In Section 3, we introduce discord and entanglement of a two-qubit system. Results and a discussion are given in Section 4. We conclude the report briefly in Section 5.

2. Physical model

We consider two two-level atoms, where each atom couples with a structured reservoir.[34] The Hamiltonian is ($\hbar = 1$) $H = H_0 + \alpha H_{\rm I}$, where $H_0 = \omega_0 \sum_{j=A}^{B} S_j^z + \sum_n \omega_{n,A} b_{n,A}^\dagger b_{n,A} + \sum_m \omega_{m,B} b_{m,B}^\dagger b_{m,B}$ is the free Hamiltonian of the combined system, ω_0 is the transition frequency of the atom, S_j^z is the inversion operators describing the atom j ($j = A$ or B), $b_{n,A}^\dagger (b_{m,B}^\dagger)$ and $b_{n,A}(b_{m,B})$ are the creation and annihilation operators of the reservoir with the frequency $\omega_{n,A}(\omega_{m,B})$. The parameter α is a dimensionless expansion parameter. In the interaction picture, the Hamiltonian $\alpha H_{\rm I}$ reads

$$\alpha H_{\rm I}(t) = \sum_{j=A}^{B} \left(S_j^+ \sum_n g_{n,j} b_{n,j} \, {\rm e}^{{\rm i}(\omega_0 - \omega_{n,j})t} + {\rm h.c.} \right), \tag{1}$$

where $g_{n,j}$ is the coupling constant between the atom and its reservoir, S_j^+ and S_j^- are the upward and downward operators of the atom, respectively.

In the second-order approximation, the TCL master equation[18] of $\rho_{AB}(t)$ has the form ${\rm d}\rho_{AB}(t)/{\rm d}t = -\alpha^2 \int_0^t {\rm d}\tau {\rm Tr}_E([H_{\rm I}(t), [H_{\rm I}(\tau), \rho_{AB}(t) \otimes \rho_E]])$ with the environment state ρ_E. Here, we have supposed that $\rho(t) = \rho_{AB}(t) \otimes \rho_E$ and ${\rm Tr}_E([H_{\rm I}(t), \rho_{AB}(0) \otimes \rho_E]) = 0$, thus the TCL master equation may be written as

$$\frac{\rm d}{{\rm d}t}\rho_{AB}(t) = \mathcal{L}^{(A)}\rho_{AB}(t) + \mathcal{L}^{(B)}\rho_{AB}(t), \tag{2}$$

where $\mathcal{L}^{(j)}$ is the Liouville super-operator[35] defined by

$$\begin{aligned} \mathcal{L}^{(j)}\rho_{AB}(t) = {} & f_j(t)[S_j^- \rho_{AB}(t), S_j^+] + f_j^*(t)[S_j^-, \rho_{AB}(t) S_j^+] \\ & + k_j^*(t)[S_j^+ \rho_{AB}(t), S_j^-] + k_j(t)[S_j^+, \rho_{AB}(t) S_j^-]. \end{aligned} \tag{3}$$

The correlation functions $k_j(t)$ and $f_j(t)$ are given by

$$k_j(t) = i \sum_n |g_{n,j}|^2 \langle b_{n,j}^\dagger b_{n,j} \rangle_{E_j} \frac{1 - {\rm e}^{{\rm i}(\omega_0 - \omega_{n,j})t}}{\omega_0 - \omega_{n,j}}, \tag{4}$$

and

$$f_j(t) = i \sum_n |g_{n,j}|^2 \langle b_{n,j} b_{n,j}^\dagger \rangle_{E_j} \frac{1 - {\rm e}^{{\rm i}(\omega_0 - \omega_{n,j})t}}{\omega_0 - \omega_{n,j}}, \tag{5}$$

where $\langle O \rangle_{E_j} \equiv {\rm Tr}_{E_j}(O \rho_{E_j})$.

Assuming that the two reservoirs are identical and initially prepared in a thermal state with temperature T, the correlation functions reduce to

$$\begin{aligned} k(t) &= {\rm i} \sum_n |g_n|^2 \frac{1}{{\rm e}^{\hbar\omega_n / k_{\rm B} T} - 1} \frac{1 - {\rm e}^{{\rm i}(\omega_0 - \omega_n)t}}{\omega_0 - \omega_n}, \\ f(t) &= {\rm i} \sum_n |g_n|^2 \frac{{\rm e}^{\hbar\omega_n / k_{\rm B} T}}{{\rm e}^{\hbar\omega_n / k_{\rm B} T} - 1} \frac{1 - {\rm e}^{{\rm i}(\omega_0 - \omega_n)t}}{\omega_0 - \omega_n}. \end{aligned} \tag{6}$$

Here, $k_{\rm B}$ is the Boltzmann constant. For a sufficiently large environment, we can replace the sum over the discrete coupling constants with an integral over a continuous distribution of frequencies of the environmental modes, i.e., $\sum_n |g_n|^2 \to \int_0^\infty {\rm d}\omega J(\omega)$.

Let the spectral density have a Lorentzian form

$$J(\omega) = \frac{1}{2\pi} \frac{\gamma_0 \lambda^2}{(\omega_0 - \omega - \delta)^2 + \lambda^2}, \tag{7}$$

where δ is the detuning between ω_0 and the reservoir center frequency ω. The parameter λ defines the spectral width of the coupling, which is connected to the reservoir correlation time $\tau_{\rm R}$ by $\tau_{\rm R}$=λ^{-1} and the parameter γ_0 is related to the relaxation time scale $\tau_{\rm S}$ by $\tau_{\rm S}$=γ_0^{-1}. In the subsequent analysis, typically weak and strong coupling regimes can be distinguished. For a weak regime, $\lambda > 2\gamma_0$ (i.e., $\tau_{\rm S} > 2\tau_{\rm R}$), the dynamical behavior of the system is essentially a Markovian exponential decay controlled by γ_0. In the strong coupling regime, $\lambda < 2\gamma_0$ (i.e., $\tau_{\rm S} < 2\tau_{\rm R}$), the non-Markovian effects become relevant.[36]

Inserting Eq. (7) into Eq. (6), we obtain the correlation functions

$$\begin{aligned} k(t) &= \frac{\gamma_0 \lambda (1 - {\rm e}^{({\rm i}\delta - \lambda)t})}{2(\lambda - {\rm i}\delta)} \frac{1}{{\rm e}^{\hbar(\omega_0 - \delta - {\rm i}\lambda)/k_{\rm B} T} - 1}, \\ f(t) &= \frac{\gamma_0 \lambda (1 - {\rm e}^{({\rm i}\delta - \lambda)t})}{2(\lambda - {\rm i}\delta)} \frac{{\rm e}^{\hbar(\omega_0 - \delta - {\rm i}\lambda)/k_{\rm B} T}}{{\rm e}^{\hbar(\omega_0 - \delta - {\rm i}\lambda)/k_{\rm B} T} - 1}. \end{aligned} \tag{8}$$

3. Discord and entanglement

For any two-qubit system ρ_{AB}, its total correlation can be written as $\mathcal{I}(\rho_{AB}) = S(\rho_A) + S(\rho_B) - S(\rho_{AB})$, where $S(\rho) = -\mathrm{Tr}(\rho \log_2 \rho)$ is the von Neumann entropy[2] of ρ. Its classical correlation is $\mathcal{C}(\rho_{AB}) = \max_{\{\Pi_k\}}[S(\rho_A) - S(\rho_{A|B})]$, here the maximum represents the most information gained about subsystem A as a result of the perfect measurement $\{\Pi_k\}$ on subsystem B. The $S(\rho_{A|B}) = \sum_{k=1}^{2} p^k S(\rho_A^k)$ is the quantum conditional entropy of subsystem A: $p_A^k = \mathrm{Tr}_{AB}(\Pi_k \rho_{AB} \Pi_k)$ and $\rho_A^k = \mathrm{Tr}_B(\Pi_k \rho_{AB} \Pi_k)/p_A^k$ are the probability and the state of subsystem A for measurement outcome k. Discord is defined as the difference between the total correlation ($\mathcal{I}(\rho_{AB})$) and the classical correlation ($\mathcal{C}(\rho_{AB})$), i.e.,

$$\mathcal{D}(\rho_{AB}) = \mathcal{I}(\rho_{AB}) - \mathcal{C}(\rho_{AB}), \tag{9}$$

and is interpreted as a measurement of quantum correlations.[11] For the X state, described by the following density matrix:

$$\rho_{AB} = \begin{pmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & \rho_{22} & \rho_{23} & 0 \\ 0 & \rho_{32} & \rho_{33} & 0 \\ \rho_{41} & 0 & 0 & \rho_{44} \end{pmatrix}. \tag{10}$$

Discord can be calculated,[37] analytically, as

$$D(\rho_{AB}) = \min\{Q_1, Q_2\}, \tag{11}$$

where $Q_j = h(\rho_{11} + \rho_{33}) + \sum_{k=1}^{4} \lambda_k \log_2 \lambda_k + D_j$, with λ_k being the eigenvalues of ρ_{AB} and $h(x) = -x \log_2 x - (1-x)\log_2(1-x)$ is the binary entropy. Here, $D_1 = h(\tau)$, where $\tau = \{1 + \sqrt{[1 - 2(\rho_{33} + \rho_{44})]^2 + 4(|\rho_{14}| + |\rho_{23}|)^2}\}/2$ and $D_2 = -\sum_{k=1}^{4} \rho_{kk} \log_2 \rho_{kk} - h(\rho_{11} + \rho_{33})$.

In addition to discord, another quantum correlation is entanglement, which can be calculated through entanglement of formation (EoF), as[38]

$$E(\rho_{AB}) = H\left(\frac{1 + \sqrt{1 - C^2}}{2}\right), \tag{12}$$

where $H(x) = -x \log_2 x - (1-x)\log_2(1-x)$, $C = \max(0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4})$ is Wootter's concurrence and λ_i are the eigenvalues, organized in descending order, of the matrix $\tilde{\rho} = \rho(\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$. For the X state such as Eq. (10), the concurrence of ρ_{AB} is $C = 2\max(0, |\rho_{14}| - \sqrt{\rho_{22}\rho_{33}}, |\rho_{23}| - \sqrt{\rho_{11}\rho_{44}})$.[9]

Discord is equal to entanglement for pure states, but their relation is complicated for the mixed states, that is, there are separable mixed states with nonzero discord.[39] In the following, our main concern will be the dynamic behaviors of discord and entanglement of two atoms in Lorentzian reservoirs at zero and finite temperatures.

4. Results and discussion

In the section, we discuss in detail the dynamic behaviors of discord (D) and entanglement (E) of two atoms in Lorentzian reservoirs. If the initial atomic states are

$$|\psi(0)\rangle = \frac{1}{2}(|00\rangle + |11\rangle), \quad |\phi(0)\rangle = \frac{1}{2}(|01\rangle + |10\rangle), \tag{13}$$

the atomic density matrix $\rho_{AB}(t)$ can remain an X structure under the master equation Eq. (2). Utilizing Eqs. (11) and (12), we can calculate the discord and entanglement of $\rho_{AB}(t)$. In the following discussion, we first give the dynamic evolutions of discord and entanglement in the resonant case, then we analyze those in the non-resonant case.

4.1. Resonant case

We assume that the two atoms couple resonantly to the Lorentzian reservoirs (i.e., $\delta = 0$) and study the influence of the temperature, the non-Markovian effect, and the initial states on the discord and entanglement.

4.1.1. Case I: The Markovian regime

Figure 1 presents the dynamics of discord and entanglement for different initial states and different temperatures under $\lambda = 5\gamma_0$ and $\delta = 0$.

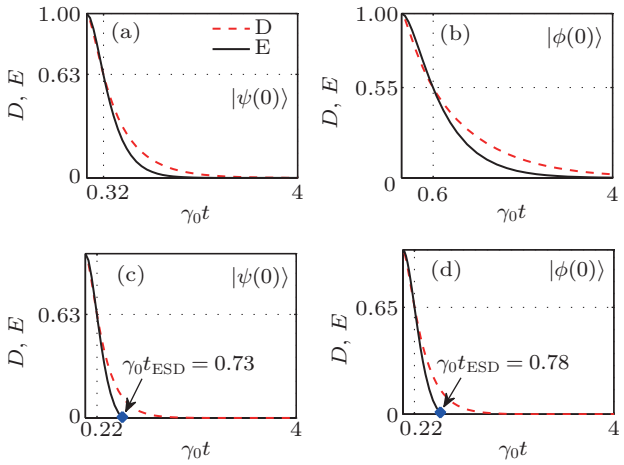


Fig. 1. (color online) For different initial states and different temperatures, D (dashed line) and E (solid line) versus $\gamma_0 t$ under $\delta = 0$ and $\lambda = 5\gamma_0$. (a) and (b) $T = 0$. (c) and (d) $k_B T/\hbar\omega_0 = 1$. Other $\omega_0 = 10\gamma_0$.

Figure 1(a) displays the dynamics of discord and entanglement for $|\psi(0)\rangle$ at $T = 0$. The results show that, both the discord and entanglement decay exponentially and vanish asymptotically, and their robustness can change at the critical time, which is expressed by t_{cri} and $\gamma_0 t_{\mathrm{cri}} = 0.32$, namely, the discord reduces slightly faster than the entanglement when $\gamma_0 t < 0.32$ while the discord is more robust than the entanglement when $\gamma_0 t > 0.32$. Figure 1(b) gives the dynamics of discord and entanglement for $|\phi(0)\rangle$ and $T = 0$. Comparing Fig. 1(a) with Fig. 1(b), we find that the discord for $|\phi(0)\rangle$ is a little more robust than that for $|\psi(0)\rangle$, and likewise for the entanglement. Also $\gamma_0 t_{\mathrm{cri}} = 0.6$, namely, the t_{cri} for $|\phi(0)\rangle$ is bigger than that for $|\psi(0)\rangle$.

When $k_BT/\hbar\omega_0 = 1$, the dynamic behaviors of discord and entanglement for $|\psi(0)\rangle$ and $|\phi(0)\rangle$ are respectively plotted in Figs. 1(c) and 1(d). In Fig. 1(c), there is $\gamma_0 t_{cri} = 0.22$, and it is worth noting that the discord still reduces quickly and asymptotically to zero but the entanglement can vanish completely in a very short time, i.e., ESD, whose time is expressed by t_{ESD} and $\gamma_0 t_{ESD} = 0.73$. In Fig. 1(d), there are $\gamma_0 t_{cri} = 0.22$ and $\gamma_0 t_{ESD} = 0.78$, so the entanglement behavior in Fig. 1(d) is very similar to that in Fig. 1(c) except that the entanglement decay in Fig. 1(d) is a little slower than that in Fig. 1(c), and likewise for the discord. Namely, the initial states hardly affect the discord and entanglement at nonzero temperature, which is markedly different from that in the zero temperature case.

Comparing Figs. 1(c) and 1(d) with Figs. 1(a) and 1(b) respectively, we can see that, nonzero temperature can accelerate the decays of the discord and entanglement and shorten t_{cri}. More importantly, nonzero temperature can yet induce the ESD. The reason is that the thermal effect of the environment can enhance the atomic dissipation so that the discord and entanglement reduce more quickly and the discord is more robust against temperature than the entanglement.

Therefore, under the resonance and the Markovian regime, the discord for $|\phi(0)\rangle$ is more robust than that for $|\psi(0)\rangle$, and likewise for the entanglement. Nonzero temperature can accelerate the decays of the discord and entanglement and induce the ESD.

4.1.2. Case II: The non-Markovian regime

Figure 2 depicts the evolutions of discord and entanglement versus $\gamma_0 t$ for different initial states and different temperatures under $\lambda = 0.1\gamma_0$ and $\delta = 0$.

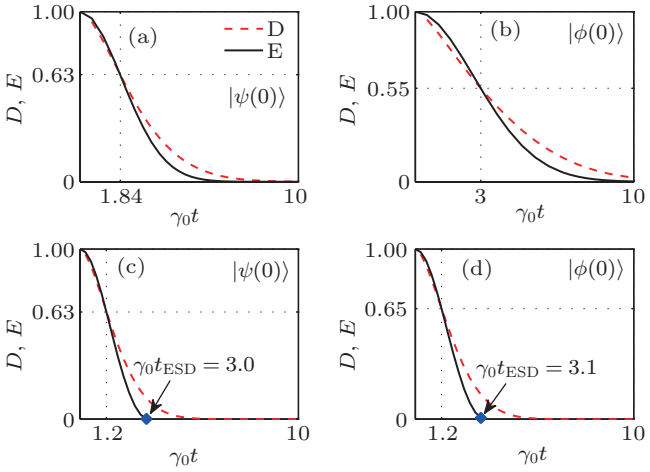


Fig. 2. (color online) For different initial states and different temperatures, D (dashed line) and E (solid line) versus $\gamma_0 t$ under $\delta = 0$ and $\lambda = 0.1\gamma_0$. (a) and (b) $T = 0$. (c) and (d) $k_BT/\hbar\omega_0 = 1$. Other $\omega_0 = 10\gamma_0$.

Figure 2(a) shows that, for $T = 0$ and $|\psi(0)\rangle$, both the discord and entanglement decay exponentially and vanish asymptotically after a short time, and $\gamma_0 t_{cri} = 1.84$. Figure 2(b) describes that, for $T = 0$ and $|\phi(0)\rangle$, the discord and entanglement will also reduce asymptotically to zero and $\gamma_0 t_{cri} = 3.0$. Comparing Fig. 2(b) with Fig. 2(a), we find that, the discord for $|\phi(0)\rangle$ is also more robust than that for $|\psi(0)\rangle$, and likewise for the entanglement. Figures 2(c) and 2(d) respectively give the discord and entanglement for $|\psi(0)\rangle$ and $|\phi(0)\rangle$ at $k_BT/\hbar\omega_0 = 1$. Figure 2(c) tells us that, $\gamma_0 t_{cri} = 1.2$ and $\gamma_0 t_{ESD} = 3.0$. In Fig. 2(d), there are $\gamma_0 t_{cri} = 1.2$ and $\gamma_0 t_{ESD} = 3.1$, so the entanglement behavior in Fig. 2(d) is very similar to that in Fig. 2(c) except that the decay of the entanglement in Fig. 2(d) is a little slower than that in Fig. 2(c), and likewise for the discord. Comparing Figs. 2(c) and 2(d) with Figs. 2(a) and 2(b) respectively, it can be observed that, nonzero temperature can also accelerate the decays of the discord and entanglement and induce the ESD.

Comparing Figs. 2(a)–2(d) with Figs. 1(a)–1(d) respectively, we know that, in the resonance, the non-Markovian effect can reduce the decay rates of the discord and entanglement and prolong t_{cri} and t_{ESD}, but these influences are very small.

4.2. Non-resonant case

Above, we have restricted our discussion to the resonant case, but the detuning case (i.e., $\delta \neq \gamma_0$) would be more reasonable.

4.2.1. Case I: The Markovian regime

Figure 3 displays the dynamics of discord and entanglement for different initial states and different temperatures under $\lambda = 5\gamma_0$ and $\delta = \gamma_0$. Comparing Fig. 3 with Fig. 1, we see that, the decay rates of discord in Figs. 3(a)–3(d) are approximately equal to those in Figs. 1(a)–1(d) respectively, and likewise for the entanglement. However, the t_{ESD} in Figs. 3(c) and 3(d) is a little bigger than that in Figs. 1(c) and 1(d), respectively. As a result, in the Markovian regime, the detuning hardly impacts on the discord and entanglement.

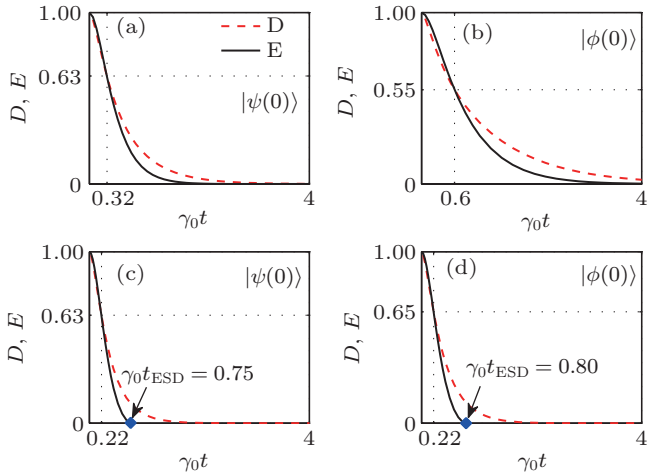


Fig. 3. (color online) For different initial states and different temperatures, D (dashed line) and E (solid line) versus $\gamma_0 t$ under $\delta = \gamma_0$ and $\lambda = 5\gamma_0$. (a) and (b) $T = 0$. (c) and (d) $k_BT/\hbar\omega_0 = 1$. Other $\omega_0 = 10\gamma_0$.

4.2.2. Case II: The non-Markovian regime

Figure 4 exhibits the influence of initial states and temperatures on the discord and entanglement when both the non-Markovian effect and the detuning are present simultaneously (i.e., $\lambda = 0.1\gamma_0$ and $\delta = \gamma_0$).

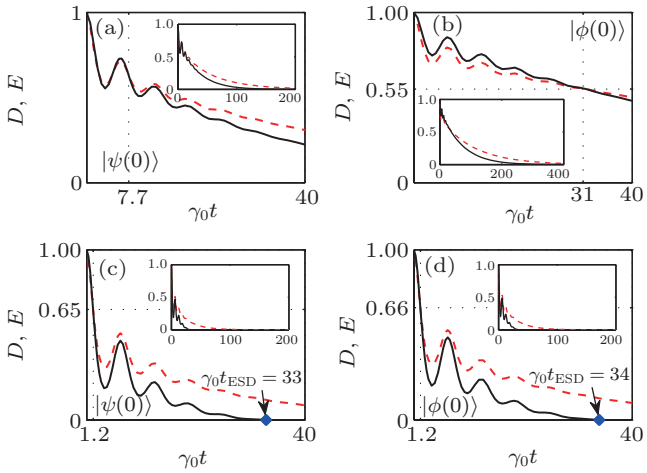


Fig. 4. (color online) The influence of initial states and temperatures on D (dashed line) and E (solid line) under $\delta = \gamma_0$ and $\lambda = 0.1\gamma_0$. (a) and (b) $T = 0$. (c) and (d) $k_B T/\hbar\omega_0 = 1$. The insets show the long-time behaviors. Other $\omega_0 = 10\gamma_0$.

Figure 4(a) gives the dynamic behaviors of discord and entanglement for $|\psi(0)\rangle$ at $T = 0$. In this case, due to the memory and feedback effect of reservoirs, both the discord and entanglement can oscillate damply then reduce asymptotically to zero over a long time (see the inset in Fig. 4(a)) and $\gamma_0 t_{cri} = 7.7$. In a short time ($\gamma_0 t \leq 40$), both the discord and entanglement are obviously protected. The relations of the discord and entanglement versus $\gamma_0 t$ for $|\phi(0)\rangle$ at $T = 0$ are plotted in Fig. 4(b). It is seen that, $\gamma_0 t_{cri} = 31$, and both the discord and entanglement are also obviously protected. Comparing Figs. 4(b) and 4(a), it is very evident that the discord for $|\phi(0)\rangle$ reduces much slower than that for $|\psi(0)\rangle$, and likewise for the entanglement.

In Figs. 4(c) and 4(d), we draw the dynamics of discord and entanglement for $|\psi(0)\rangle$ and $|\phi(0)\rangle$ at $k_B T/\hbar\omega_0 = 1$. Figure 4(c) shows that, for $|\psi(0)\rangle$, there is $\gamma_0 t_{cri} = 1.2$ and the discord and entanglement all oscillate damply then the discord decreases asymptotically to zero but the entanglement will be sudden death at $\gamma_0 t_{ESD} = 33$. In Fig. 4(d) (for $|\phi(0)\rangle$), there are $\gamma_0 t_{cri} = 1.2$ and $\gamma_0 t_{ESD} = 34$, that is, the discord behavior in Fig. 4(d) is very similar to that in Fig. 4(c), and likewise for the entanglement. Comparing Figs. 4(c) and 4(d) with Figs. 4(a) and 4(b) respectively, it can show that, nonzero temperature can accelerate the decays of discord and entanglement, enlarge the oscillating amplitudes of discord and entanglement and induce the ESD even if both the non-Markovian effect and detuning are present simultaneously.

Comparing Fig. 4 with Figs. 2 and 3, we find that they are essentially different. If there is only the non-Markovian effect or only the detuning, the discord and entanglement will monotonously and rapidly decline to zero. However, if the non-Markovian effect and the detuning are present simultaneously, the discord and entanglement will oscillate damply and their decay rates also evidently become smaller. The physical interpretation is that, when only $\lambda < 2\gamma_0$ (the non-Markovian effect) or only $\delta \neq 0$ (the detuning), the quantum information can flow out from the atomic system but is hardly returned from the reservoirs so that the discord and entanglement monotonously and rapidly reduce, as shown in Figs. 2 and 3. However, when $\lambda < 2\gamma_0$ and $\delta \neq 0$, due to the memory and feedback effect of the non-Markovian reservoir, the quantum information flowing to the reservoir will be partly returned to the atom so that the discord and entanglement oscillate damply and their decay rates become smaller, as shown in Fig. 4.

Figure 5 reveals the dynamics of discord and entanglement under $\lambda = 0.1\gamma_0$ and $\delta = 4\gamma_0$. From Fig. 5(a), we see that, for $|\psi(0)\rangle$ and $T = 0$, the discord and entanglement can be effectively protected and the entanglement is more robust than the discord when $\gamma_0 t \leq 40$ although they can reduce asymptotically to zero over a very long time (see the inset in Fig. 5(a)). Figure 5(b) shows that the discord for $|\phi(0)\rangle$ is more robust than that for $|\psi(0)\rangle$, and likewise for the entanglement. Figure 5(c) states clearly that, the discord and entanglement can still be effectively protected at $k_B T/\hbar\omega_0 = 1$, but their decay rates and oscillating amplitudes are a little bigger than those at $T = 0$. The entanglement in Fig. 5(d) is similar to that in Fig. 5(c) expect that the decay rate in Fig. 5(d) is a little smaller than that in Fig. 5(c), and likewise for the discord.

Comparing Figs. 5(a)–5(d) with Fig. 4(a)–4(d) respectively, we know that, in the non-Markovian regime, increasing the detuning can prolong t_{cri} and t_{ESD} (see the insets in Fig. 5), enlarge the oscillating frequency and reduce the decays of discord and entanglement. More importantly, even at nonzero temperature, the discord and entanglement can also be effectively protected by means of the big detuning. The physical explanation is that, in the non-Markovian regime, with δ increasing, the atom can more quickly exchange the quantum information with its reservoir so that the atomic decay becomes slower and the quantum information returned from the reservoir is more, hence the discord and entanglement can be more effectively protected.

Figure 6 exhibits the dynamics of discord and entanglement under $\lambda = 0.01\gamma_0$ and $\delta = \gamma_0$. From Fig. 6(a), we see that, for $|\psi(0)\rangle$ and $T = 0$, the discord and entanglement can be effectively protected when $\gamma_0 t \leq 40$ although the discord and entanglement yet reduce asymptotically to zero in a very long time (see the inset in Fig. 6(a)). Figure 6(b) reveals that, for $|\phi(0)\rangle$ and $T = 0$, the discord for $|\phi(0)\rangle$ is more robust than that for $|\psi(0)\rangle$, and likewise for the entanglement. Figure 6(c) tells us that, the discord and entanglement can also be effectively protected at $k_B T/\hbar\omega_0 = 1$, but their decay rates and oscillating amplitudes are a little larger than those at $T = 0$. The entanglement in Fig. 6(d) is very similar to that in Fig. 6(c) expect that the decay rate in Fig. 6(d) is a little smaller than that in Fig. 6(c), and likewise for the discord.

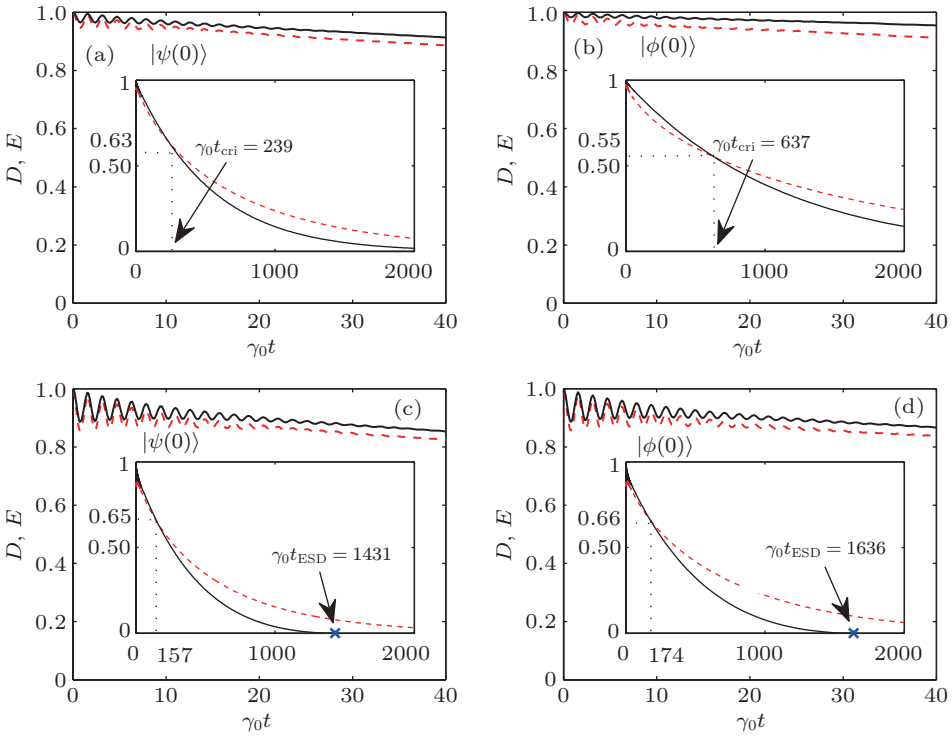


Fig. 5. (color online) The influence of the big detuning on D (dashed line) and E (solid line) in the non-Markovian regime (i.e., $\delta = 4\gamma_0$ and $\lambda = 0.1\gamma_0$). (a) and (b) $T = 0$. (c) and (d) $k_{\rm B}T/\hbar\omega_0 = 1$. The insets show the long-time dynamics. Other $\omega_0 = 10\gamma_0$.

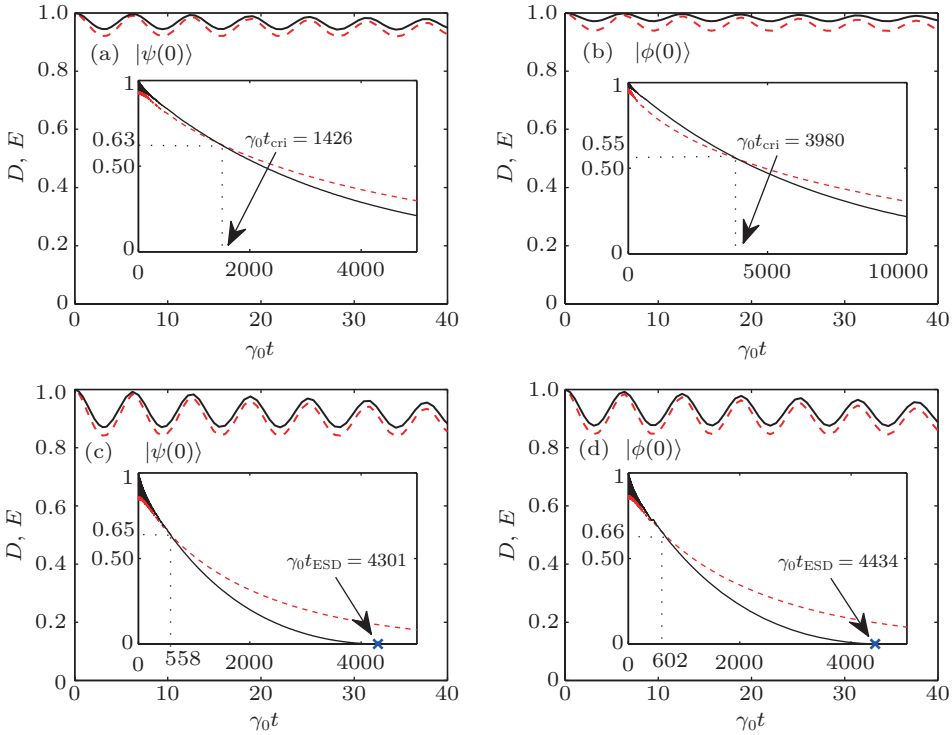


Fig. 6. (color online) The influence of the strong non-Markovian effect on D (dashed line) and E (solid line) with the detuning (i.e., $\delta = \gamma_0$ and $\lambda = 0.01\gamma_0$). (a) and (b) $T = 0$. (c) and (d) $k_{\rm B}T/\hbar\omega_0 = 1$. The insets show the long-time dynamics. Other $\omega_0 = 10\gamma_0$.

Comparing Figs. 6(a)–6(d) with Figs. 4(a)–4(d) respectively, we know that, under the detuning, increasing the non-Markovian effect can prolong $t_{\rm cri}$ and $t_{\rm ESD}$ (see the insets in Fig. 6) and reduce the decays of discord and entanglement. More importantly, even at nonzero temperature, the discord and entanglement can also be effectively protected by means of the strong non-Markovian effect. The physical explanation is that, under the detuning, with λ decreasing, the memory and feecback effect of the non-Markovian reservoir can become strong so that the atomic decay becomes slower and the quantum information returned from the reservoir is more, hence the discord and entanglement can be more effectively protected.

5. Conclusion

In the present work, we have investigated the dynamics of discord and entanglement of two atoms in Lorentzian reservoirs at zero and nonzero temperatures by using the time-convolutionless master-equation method. We find that, the discord and entanglement markedly rely on the non-Markovian effect, the detuning, and the temperature as well as the atomic initial states. The entanglement is more robust than the discord when $t < t_{\rm cri}$ while the discord is more robust than the entanglement when $t > t_{\rm cri}$, and the value of $t_{\rm cri}$ relies on the initial states, the temperature, the detuning, and the non-Markovian effect. The discord for $|\phi(0)\rangle$ is always more robust than that for $|\psi(0)\rangle$, specially, this difference is very evident at the zero temperature, and likewise for the entanglement. The nonzero temperature can accelerate the decays of discord and entanglement and induce the entanglement sudden death. More importantly, if there is only the detuning or only the non-Markovian effect, both the discord and entanglement will monotonously and rapidly decline to zero. However, if the non-Markovian effect and the detuning are present simultaneously, due to the memory and feedback of the non-Markovian reservoir, the discord and entanglement will oscillate damply and both of them can be effectively protected even at the nonzero temperature by increasing the detuning and the non-Markovian effect. These results will be useful in quantum computation and quantum information processing.

The current experimental technologies[40] show that our proposals have a certain feasibility. For example, a circular Rydberg atom with the two circular levels with principal quantum numbers 51 and 50 which are called $|e\rangle$ and $|g\rangle$ respectively, the $|e\rangle \Leftrightarrow |g\rangle$ transition is at 51.1 GHz corresponding to the atomic decay rate $\gamma_0 = 33.3$ Hz. Indeed, in the above discussion, the detuning $\delta = 2\gamma_0 = 66.6$ Hz is very small so that it could be realized by Stark-shifting the frequency with a static electric field. The typical Stark shift is about 200 kHz,[41] which is far more than 66.6 Hz. This shift is therefore large enough to effectively preserve the discord and entanglement. Moreover, in cavity QED experiments, ultrahigh finesse Fabry–Perot super-conducting resonant cavities with quality factors $Q = 4.2 \times 10^{10}$, corresponding to the spectral width $\lambda = 7$ Hz, have been realized.[42] These values correspond to $\lambda/\gamma_0 \approx 0.2$ which represents a good non-Markovian regime. Moreover, from Figs. 5 and 6, we may know that, the discord and entanglement can be effectively protected when $\gamma_0 t \leq 40$ (meaning a corresponding time on the scale of seconds) and $k_{\rm B}T/\hbar\omega_0 = 1$ (meaning the order of $T \sim 10^{-8}$ K), these conditions may be realized in the current experimental technologies.

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