Entanglement in a two-spin system with long-range interactions

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The quantum entanglement between two spins in the Ising model with an added Dzyaloshinsky–Moriya (DM) interaction and in the presence of the transverse magnetic field is studied. The exchange interaction is considered as a function of the distance between spins. The negativity as a function of magnetic field, exchange and DM interaction is calculated. The effect of the distance between spins is studied based on the negativity. In addition, the effect of the thermal fluctuation on the negativity is also investigated.

Keywords: entanglement, Ising model, DM interaction, negativity, long range interaction

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1. Introduction

Quantum entanglement plays an important role in quantum information processing,[1,2] teleportation,[3] communication systems,[4–6] quantum computers,[7–9] quantum spin networks,[10] and security cryptography.[11] It has achieved many successes both in theory and experiment. Specially, it is known that the particles can be entangled in a system with long range interactions.[12,13]

Significant research has been performed to understand the quantum entanglements in different spin systems, such as XY,[14–16] XX,[17] XXZ,[18] XYZ models.[19,20] The effects of Dzyaloshinsky–Moriya (DM) interaction[21–25] and magnetic field[14,26–29] on the quantum entanglement are also studied. Negativity and its cocurrence are the most widely used measures in entanglement study. The negativity was first used by Zyczkowski *et al.*[30] and subsequently introduced as an entanglement measure by Vidal and Werner.[31]

In recent years, long range interactions have attracted a great deal of attention since they can produce interesting new phenomena.[32–36] A new type of long-range interaction has been introduced in Ref. [37], where the exchange interaction is inversely proportional to the power of the distance between the two spins. In 2014, it was observed that interactions in natural systems may fall into a class of long range interaction.[38] In Ref. [39], a region of logarithmic growth of entropy in a system with long-range interaction was reported. In Ref. [40], a local quench long-range Ising model in the transverse field with the resulting spread of quasi-particles was analyzed.

In the present study, we employ the negativity to analyze the entanglement in a spin system with long-range interactions. The considered model is the one-dimensional (1D) Ising model in the presence of a transverse magnetic field (ITF model) and the DM interaction. We consider the exchange interaction as a function of the distance between spins. Recently, the entanglement was studied by using this model in the case of constant exchange interaction.[41] Here, we consider exchange interaction proportional to the inverse square of distance between spins, R. We calculate the negativity and analyze the behavior of the entanglement as a function of R. Results show that the quantum correlations are very sensitive to the mutual effects of the distance and the DM interaction.

2. Model

Here, we consider a set of two localized spin-1/2 particles coupled through the exchange $J(R)$ and the DM interactions, subjected to an external magnetic field h

$$H = J(R)S_1^xS_2^x + D(S_1^xS_2^y - S_1^yS_2^x) + h(S_1^z + S_2^z), \tag{1}$$

where S denotes the spin-1/2 operator. The matrix form of the Hamiltonian is obtained as

$$H = \begin{bmatrix} h & 0 & 0 & J/4 \\ 0 & 0 & J/4 + \mathrm{i}D/2 & 0 \\ 0 & J/4 - \mathrm{i}D/2 & 0 & 0 \\ J/4 & 0 & 0 & -h \end{bmatrix}. \tag{2}$$

By diagonalizing the Hamiltonian, eigenvalues and eigenstates are obtained as

$$E_1 = \frac{1}{4}\sqrt{J^2 + 4D^2}, \tag{3}$$

$$E_2 = -\frac{1}{4}\sqrt{J^2 + 4D^2}, \tag{4}$$

$$E_3 = \frac{1}{4}\sqrt{16h^2 + J^2}, \tag{5}$$

$$E_4 = -\frac{1}{4}\sqrt{16h^2 + J^2}, \tag{6}$$

$$|1\rangle = \frac{1}{\sqrt{2}}\left[|\uparrow\downarrow\rangle + \alpha|\downarrow\uparrow\rangle\right], \tag{7}$$

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$$|2\rangle = \frac{1}{\sqrt{2}}\left[|\uparrow\downarrow\rangle - \alpha|\downarrow\uparrow\rangle\right], \tag{8}$$

$$|3\rangle = \frac{1}{\sqrt{1+\beta^2}}\left[\beta|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle\right], \tag{9}$$

$$|4\rangle = \frac{1}{\sqrt{1+\beta^2}}\left[\beta'|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle\right], \tag{10}$$

where

$$\alpha = \frac{J-2\mathrm{i}D}{\sqrt{J^2+4D^2}}, \tag{11}$$

$$\beta = \frac{4}{J}\left(-\frac{\sqrt{16h^2+J^2}}{4}+h\right), \tag{12}$$

$$\beta' = \frac{4}{J}\left(\frac{\sqrt{16h^2+J^2}}{4}+h\right). \tag{13}$$

Using energy eigenvalues, the partition function and the density matrix are obtained as

$$Z = \mathrm{tr}\left[\exp(-\beta H)\right] = 2\cosh(\beta E_1) + 2\cosh(\beta E_3). \tag{14}$$

On the other hand, to study the entanglement, one should obtain the density matrix of the system

$$\rho = \frac{1}{Z}\sum \mathrm{e}^{-\beta\varepsilon_i}|i\rangle\langle i| = \begin{bmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & \rho_{22} & \rho_{23} & 0 \\ 0 & \rho_{32} & \rho_{33} & 0 \\ \rho_{41} & 0 & 0 & \rho_{44} \end{bmatrix}. \tag{15}$$

Using the eigenvalues of the partial transpose of the density matrix, one can obtain the negativity of this model. The negativity is given by the following equation

$$N = \frac{||\rho^{\mathrm{T}}|| - 1}{2}. \tag{16}$$

where $||\rho^{\mathrm{T}}||$ is the trace norm or the sum of the absolute values of the operator ρ^{T}.

3. Results and discussion

From the previous section, we can easily find the negativity. Here, we consider the exchange interaction as an inverse square function of the distance between spins, $= J_0/R^2$. For simplicity, $J = 1$ is considered. Using Eqs. (15) and (16), the negativity is calculated as a function of magnetic field, DM interaction, and distance between spins.

First, in the absence of the magnetic field nor the DM interaction, the effect of distance between two spins on their quantum entanglement is investigated. In this situation, the ground state energy is degenerated twice and corresponding eigenstates are

$$|GS_1\rangle = \frac{1}{\sqrt{2}}\left[-|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right],$$

$$|GS_2\rangle = \frac{1}{\sqrt{2}}\left[-|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle\right].$$

Therefore, the ground state of the system is a linear combination of these two degenerated states. Apparently, this linear combination leads to an un-entangled ground state. Moreover, the increasing of the temperature cannot create the entanglement in the system.

Then, we apply the DM interaction (the magnetic field yet is zero). Figure 1 shows the plots of the negativity versus R at two different temperatures. At temperature near zero ($T = 0.01$) and small values of R, the entanglement is zero because the exchange interaction (J) is very large compared with the DM interaction. In this case, the system state is similar to in the previous situation and then the ground state is degenerated again. Therefore, there is not any entanglement at very small value of R. When R increases, the effect of J is reduced and, instead of it, the role of DM interaction is significant and then the entanglement is increased. By raising the temperature, the destructive effect of temperature on DM interaction is dominated and then the entanglement becomes zero.

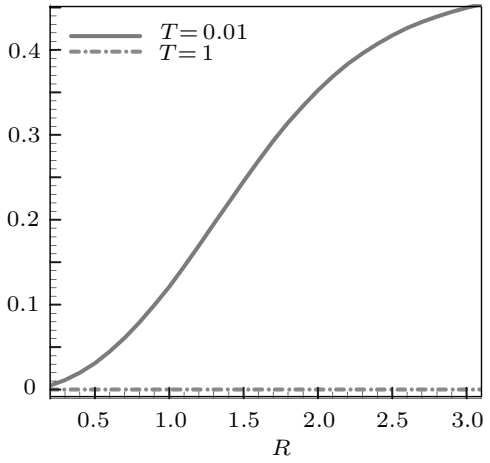


Fig. 1. Plots of negativity versus R for the two-spin system with exchange interaction $1/R^2$ at two different temperatures.

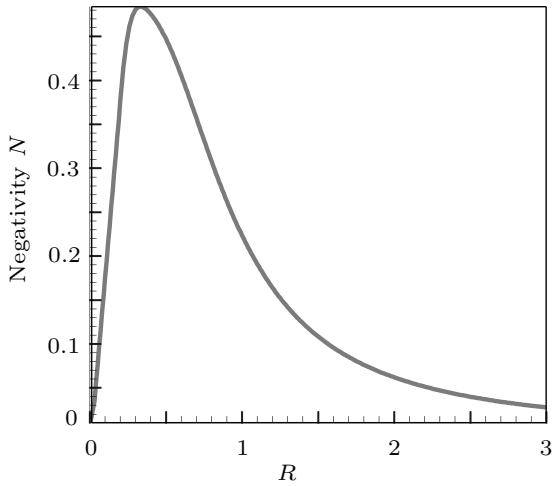


Fig. 2. Negativity as a function of R for the two-spin system with exchange interaction $1/R^2$. Here, $D = 0.1$, $h = 0.5$, and $T \sim 0$.

In Fig. 2, the R dependence of negativity is plotted for the system with DM interaction and in the presence of the magnetic field. At the small value of R, as explained in Fig. 1, the entanglement is zero. At large value of R, although the destroying effect of J is reduced, the magnetic field, which is larger than D, causes a high drop. The peak of entanglement might be due to the expansion coefficient of the ground state.

If a magnetic field is applied, the dependence of the negativity on the distance for values of magnetic field less than the DM interaction will be different from that for values of magnetic field larger values of DM.

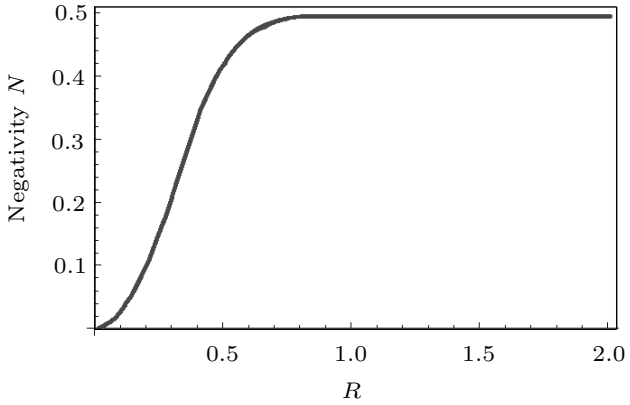


Fig. 3. Negativity as a function of R for the two-spin system with exchange interaction $1/R^2$. Here, $D = 0.6$, $h = 0.2$, and $T \sim 0$.

Figure 3 shows this scenario for $D = 0.6$ and $h = 0.2$ in the vicinity of the zero temperature. For small value of R, the created Ising model makes zero entanglement. But, by increasing the value of the R, the effect of exchange interaction is reduced and the effect of DM interaction is highlighted instead. Because h is lower than D, the entanglement cannot be destroyed.

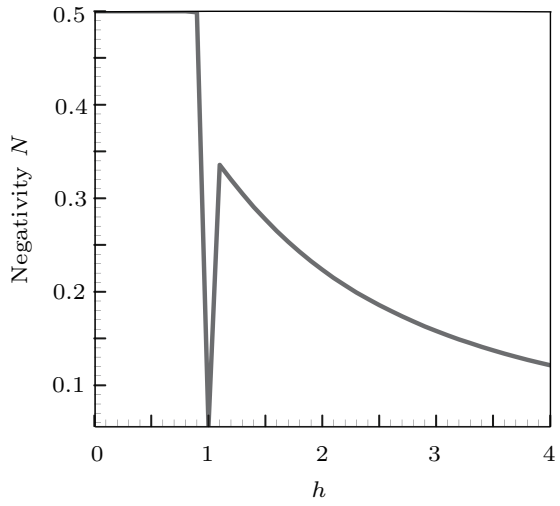


Fig. 4. Magnetic field dependence of negativity N for the two-spin system with exchange interaction $1/R^2$. Here, $D = 2$, $R = 0.5$, and $T \sim 0$.

The study of the effect of the magnetic field on the entanglement is significant because it represents a critical magnetic field $h_c = D/2$. In fact, at this point the ground state of the system is degenerated. For $h < h_c$, the energy of the ground state of the system is

$$E_2 = -(1/4)\sqrt{J^2 + 4D^2},$$

therefore the change of h does not have any effect on the system, but for $h > h_c$, the energy of the ground state is

$$E_4 = -(1/4)\sqrt{16h^2 + J^2},$$

then, the destroying effect of h reduces the entanglement of the system sharply (see Fig. 4).

4. Conclusions

In this work, we studied the quantum entanglement between two spins in the Ising model with the addition of the Dzyaloshinsky–Moriya (DM) interaction and a transverse magnetic field. We considered the exchange interaction as an inverse square function of the distance between spins (R). Also we focused on the negativity to investigate the entanglement between spins. Our results show that spins are unentangled in the absence of the DM interaction nor the magnetic field and by adding the DM interaction, the system remains unentangled for a short distance. When we increase the distance between spins, the entanglement is induced due to the quantum effect of the DM interaction. In the presence of the magnetic field, the entanglement will be changed and in a critical field the entanglement will be zero.

References

[1] Schumacher B 1996 *Phys. Rev. A* **54** 2614
[2] Imamoghlu A, Awschalom D D, Burkard G, DiVincenzo D P, Loss D, Sherwin M and Small A 1999 *Phys. Rev. Lett.* **83** 4204
[3] Bennett C H, Brassard G, Crépeau C, Jozsa R, Peres A and Wootters W K 1993 *Phys. Rev. Lett.* **70** 1895
[4] Bennett C H and Wiesner S J 1988 *Phys. Rev. Lett.* **69** 2881
[5] Han L 2015 *Acta Phys. Sin.* **64** 160307 (in Chinese)
[6] Qing Q, Kui F and Cheng C 2015 *Acta Phys. Sin.* **64** 160306 (in Chinese)
[7] Loss D and Divincenzo D P 1998 *Phys. Rev. A* **57** 120
[8] Santos L F 2003 *Phys. Rev. A* **67** 062306
[9] Bukard G, Loss D and Divincenzo D P 1999 *Phys. Rev. B* **59** 2070
[10] Dvincenzo D P 1995 *Science* **270** 255
[11] Deutsch D, Ekert A, Jozsa R, Macchiavello C, Popescu S and Sanpera A 1996 *Phys. Rev. Lett.* **77** 2818
[12] Bose S and Contemp J 2007 *Physica* **48** 13
[13] Key A 2010 *Int. J. Quantum Inform.* **8** 641
[14] Kamta G L and Starace A F 2002 *Phys. Rev. Lett.* **88** 107901
[15] Sun Y, Chen Y and Chen H 2003 *Phys. Rev. A* **68** 044301
[16] Wang X 2001 *Phys. Rev. A* **64** 012313
[17] Yeo Y 2002 *Phys. Rev. A* **66** 062312
[18] Santos L F 2003 *Phys. Rev. A* **67** 062306
[19] Zhou L Song H S, Guo Y Q and Li C 2003 *Phys. Rev. A* **68** 024301
[20] Abliz A et al. 2006 *Phys. Rev. A* **74** 052105
[21] Zhang G F 2007 *Phys. Rev. A* **75** 034304
[22] Soltani M R, Vahedi J, Sadremomtaz A R and Aboulhasni M R 2012 *Indian J. Phys.* **86** 1073
[23] Han S D and Aydiner E 2014 *Chin. Phys. B* **23** 050305

[24] Vahedi J, Soltani M R and Mahdavifar S 2012 *Journal of Superconductivity and Novel Magnetism* **15** 1159
[25] Li D C, Li X M, Li H, Tao R, Yang M and Cao Z L 2015 *Chin. Phys. Lett.* **32** 050302
[26] Arnesen M C, Bose S and Vedral V 2001 *Phys. Rev. Lett.* **87** 017901
[27] Soheilian F and Soltani M R 2010 *Indian J. Phys.* **84** 257
[28] Wang L C, Yan J Y and Yi X X 2011 *Chin. Phys. B* **20** 040305
[29] Asoudeh M and Karimipour V 2005 *Phys. Rev. A* **71** 022308
[30] Zyczkowski K, Horodecki P, Sanpera A and Lewenstein M 1998 *Phys. Rev. A* **58** 883
[31] Vidal G and Werne R F 2002 *Phys. Rev. A* **65** 032314
[32] Sutherland B 1971 *Phys. Rev. A* **4** 2019
[33] Haldane F D M 1988 *Phys. Rev. Lett.* **60** 635
[34] Haldane F D M 1991 *Phys. Rev. Lett.* **66** 1529
[35] Shastry B S 1992 *Phys. Rev. Lett.* **69** 164
[36] Kawakami N 1992 *Phys. Rev. B* **46** 1005
[37] Han S D, Tufekci T, Spiller T P and Aydiner E 2012 arXiv: 1111.2694 quanta-ph
[38] Jurcevic P and Lanyon B P 2014 *Nature* **511** 202
[39] Schachenmayer J, Lanyon B P, Roos C F and Daley A 2013 *Phys. Rev. X* **3** 031015
[40] Hauke P and Tagliacozzo L 2013 *Phys. Rev. Lett.* **111** 207202
[41] Divyamani1 B G 2013 *Chin. Phys. Lett.* **30** 120301
[42] Shastry B S 1988 *Phys. Rev. Lett.* **60** 639