Controllable soliton propagation based on phase-front curvature in asymmetrical nonlocal media*

Huafeng Zhang(张华峰)[1,†], Hua Lü(吕华)[2], Jianghua Luo(罗江华)[1], and Lihui Sun(孙利辉)[1]

[1] *Institute of Quantum Optics and Information Photonics, Yangtze University, Jingzhou* 434023, *China*
[2] *Experimental Teaching Center, Guangdong University of Technology, Guangzhou* 510006, *China*

(Received 24 January 2016; revised manuscript received 20 March 2016; published online 25 June 2016)

The influence of phase-front curvature on the dynamical behavior of the fundamental mode soliton during its transmission in asymmetrical nonlocal media is studied in detail and the phase-front curvature can be imposed on the fundamental mode soliton by reshaping or phase imprinting technologies. By changing the phase-front curvature or its imposed position, controllable soliton propagation in asymmetrical nonlocal media can be achieved.

Keywords: phase-front curvature, asymmetrical nonlocality, soliton controlling

PACS: 42.65.Jx, 42.65.Tg **DOI:** 10.1088/1674-1056/25/8/084210

1. Introduction

The dynamics of localized nonlinear waves is an attractive subject in a wide range of real physical systems.[1–3] In nonlinear optics, the theory and experiments on all-optical control have been developing at a fast pace in recent decades,[4–10] and the dynamical properties of localized nonlinear optical waves (optical solitons) in nonlinear media open a new door to all-optical control.[11–14] In general, the medium approximately presents local nonlinearity because the beamwidth is often considerably larger than the characteristic width, which is also width of nonlinear response function of medium.[15–18] However, the nonlinearity of the media may also be remarkable nonlocal under the right conditions.[19–21] In recent years, the nonlocality has become one of the hot topics in nonlinear optics due to its unique properties different from the locality and potential applications in all-optical controlling field.[22–30]

In the general case, nonlocal nonlinear medium is homogeneous, and its response function is a symmetric function such as Gauss function, exponential function, and so on.[25,31] Nevertheless, in some cases, nonlocal nonlinear medium can also be inhomogeneous, correspondingly, its response function may be asymmetric.[32] Several studies have shown that asymmetric nonlocality significantly affects the soliton properties and their dynamical behaviors.[33–36] Particularly, in Kerr media, the asymmetric nonlocal response can result in the tunable self-bending of soliton light beams,[33] and can alter the existence domains and the stabilities of asymmetric nonlocal solitons.[34]

In this paper, dynamical properties of a spatial soliton with phase-front curvature, which propagates in asymmetric nonlocal medium, are investigated in details. The results suggest that the phase-front curvature plays an important role in propagation and evolution of the asymmetric nonlocal soliton. More specifically, controllable soliton propagation can be achieved in asymmetrical nonlocal media by controlling the phase-front curvature of the asymmetric nonlocal soliton. These properties are of value to practical applications in all-optical controlling and steering fields.

2. Theoretical model

We consider the propagation of a laser beam along the z axis of a slab waveguide with symmetric local and asymmetric nonlocal components of the focusing nonlinear response in the presence of an imprinted optical lattice. The dynamics of propagation is described by the modified nonlinear Schrödinger equation for the dimensionless complex light field amplitude $q(z,x)$[32,33]

$$\mathrm{i}\frac{\partial q}{\partial z}+\frac{1}{2}\frac{\partial^2 q}{\partial x^2}+|q|^2 q-\mu q\frac{\partial}{\partial x}|q|^2+pR(x)q=0. \qquad (1)$$

Here x and z stand for the transverse and longitudinal coordinates scaled to the beam width and diffraction length, respectively; the parameter p characterizes the lattice depth; the function $R(x)=\cos^2(\Omega x)$ describes lattice profile, where Ω is the modulation frequency; the nonlocality parameter μ represents the magnitude of the nonlocal component of nonlinear response, and in most realistic situations $\mu<1$. It should be pointed out that the total energy flow $U=\int_{-\infty}^{\infty}|q|^2\,\mathrm{d}x$ is conserved. The change of the refractive index, induced by the laser beam with intensity $I(x,z)=|q(x,z)|^2$, can be expressed

*Project supported by the National Natural Science Foundation of China (Grants Nos. 11547007 and 11304024), the Innovation Personnel Training Plan for Excellent Youth of Guangdong University Project (Grant No. 2013LYM_0023), and the Yangtze Fund for Youth Teams of Science and Technology Innovation (Grant No. 2015cqt03).

†Corresponding author. E-mail: zhhf72@126.com

© 2016 Chinese Physical Society and IOP Publishing Ltd

as

$$n(x,z) = |q|^2 - \mu \frac{\partial}{\partial x} |q|^2 .$$

Equation (1) has stationary soliton solutions in the form of $q_0(x,z) = w(x)\exp(\mathrm{i}bz)$, where $w(x)$ is a real function and b is the propagation constant. Then, plugging it into Eq. (1), one can obtain the following equation

$$bw = \frac{1}{2}\frac{\mathrm{d}^2 w}{\mathrm{d}x^2} + w^3 - 2\mu w^2 \frac{\mathrm{d}w}{\mathrm{d}x} + pR(x)w. \tag{2}$$

3. Effect of initial phase-front curvature on soliton propagation in local nonlinear system

When other parameters keep unchanged, there is a critical value μ_{cr} of the nonlocality parameter. Stationary solitons can exist in the nonlinear system described by Eq. (2) only for $\mu < \mu_{\mathrm{cr}}$.[32] We can search for these stationary soliton solutions of Eq. (1) by solving Eq. (2) numerically with standard relaxation method,[37] imaginary time method[38,39] or shooting method.[40] The properties of these stationary soliton solutions have been studied in Refs. [32] and [33] in great detail. Therefore, in the following section, we consider the dynamical properties of spatial soliton with initial phase-front curvature during its propagation in an asymmetric nonlocal nonlinear medium. To do so, the spatial soliton with initial phase-front curvature must first be obtained. By reshaping and phase imprinting technologies, the soliton, which can stably exist in the system described by Eq. (1), can have an initial phase-front curvature imposed on it.[41,42]

Thus, in order to study the dynamical properties of the asymmetric nonlocal spatial soliton with an initial phase-front curvature during its propagation in the system (1), one should first obtain the stationary soliton $q_0(x,z) = w(x)\exp(\mathrm{i}bz)$ which exists in the system (1); and then impose an initial phase-front curvature C on it at $z = 0$ by reshaping and phase imprinting technologies; finally, investigate its dynamical properties during the propagation by taking $q(x,0) = q_0(x,0)\exp\left(-\mathrm{i}Cx^2/2\right)$ as an initial input.

In particular, in the case when $\mu = 0$, the system (1) is in essence a local one. The research results show that the propagation properties of localized optical field, which is stationary in local system, were remarkably influenced by the initial phase-front curvature C. Figure 1 presents typical evolution characteristics of fundamental mode solitons with different initial phase-front curvatures in local media. Obviously, one can see that initial phase-front curvature C does not change the trajectory of the light beam center by comparing Figs. 1(a), 1(b), and 1(c). However, it may make the beam width oscillatorily vary during propagation of the beam, as shown in Figs. 1(a) and 1(c). Furthermore, the effect of positive initial phase-front curvature on soliton propagation in local nonlinear system is slightly different from that of the negative one (see Figs. 1(a) and 1(c)). More specifically, one can see from Fig. 1(c) that the positive initial phase-front curvature makes the beam initially shrink; whereas the reverse occurs for the negative phase-front curvature, as shown in Fig. 1(a). Finally, it is worth pointing out that the beam may keep unchanged in waveform when C is equal to zero.

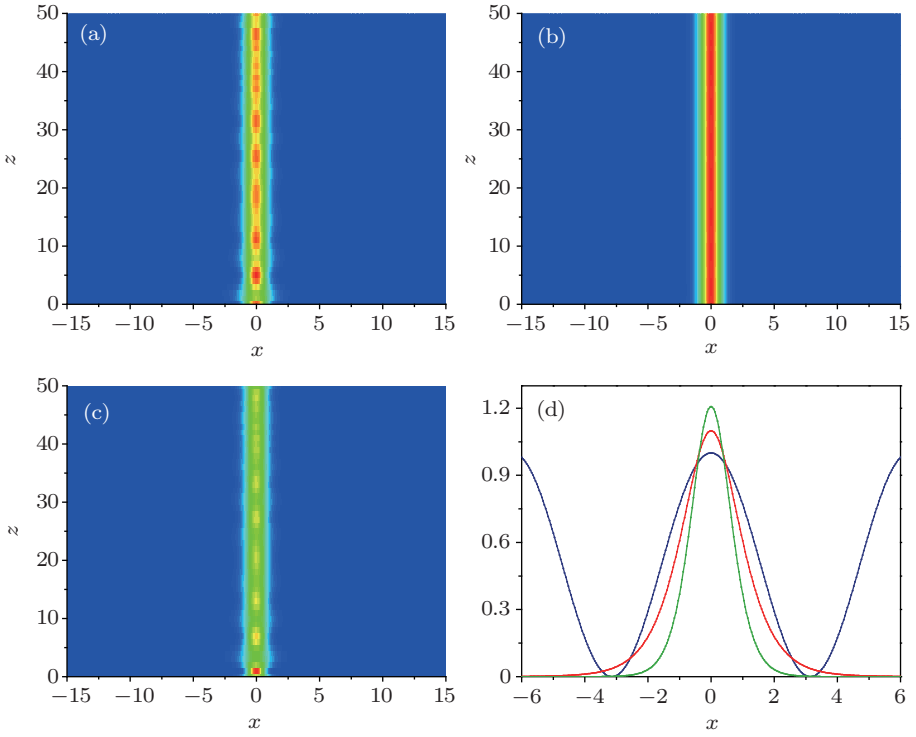


Fig. 1. (color online) Typical evolution plots of fundamental mode solitons with different initial phase-front curvatures (a) $C = -0.5$, (b) $C = 0$, (c) $C = 0.5$; (d) profile of fundamental mode soliton (red line), lattice profile (blue line), and nonlinear refractive index distribution (green line) in the system (1). Other parameters are $p = 0.5$, $\Omega = 0.5$, $\mu = 0$, and $U = 2$.

4. Effect of phase-front curvature on soliton propagation in asymmetrical nonlocal nonlinear system

In general, the nonlocality parameter μ is usually nonzero, the corresponding system described by Eq. (1) is an asymmetrical nonlocal medium. In such a system, the effect of the initial phase-front curvature C on propagation of the stationary optical field is markedly different from that in local system. As illustrated in Fig. 2, the initial phase-front curvature C not only induces the oscillation of beam width, but it also can synchronously change the trajectory of the light beam during its propagation in an asymmetrical nonlocal medium. By comparing Fig. 2(b) with Fig. 2(c), one can easily discover that the oscillating amplitude of the light beam center increases in a transverse direction with an increase of the initial phase-front curvature C [see Fig. 3(a)]. One can define propagation distance of the light beam center along the z direction within a periodic transverse oscillation as its oscillating period, and results showed that the oscillating period increases when its initial phase-front curvature C increases, as shown in Fig. 3(b). Noticeably, the sign of initial phase-front curvature plays the same roles in width variation of light beam as the local case. However, two initial phase-front curvatures, which have the same absolute value and the opposite signs, make the light beam propagate along two distinct paths, as shown in Fig. 4.

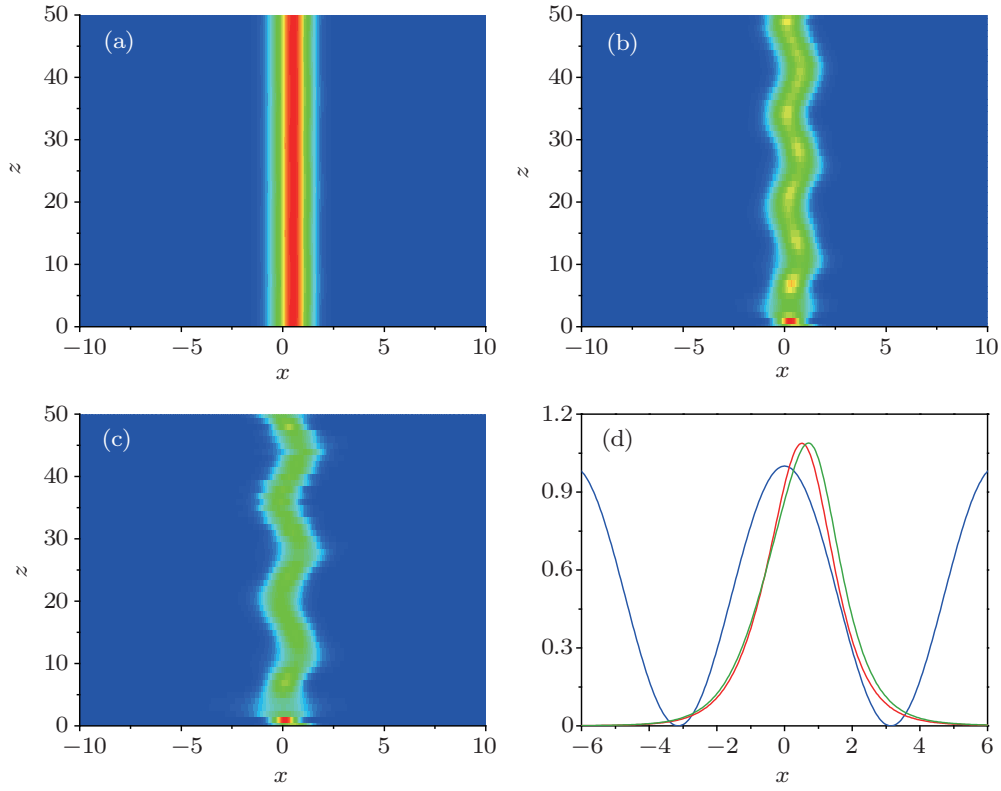


Fig. 2. (color online) Typical evolution plots of fundamental mode solitons with different initial phase-front curvatures (a) $C = 0$, (b) $C = 0.5$, (c) $C = 1$; (d) profile of fundamental mode soliton (red line), lattice profile (blue line), and nonlinear refractive index distribution (green line) in the system (1). Other parameters are $p = 0.5$, $\Omega = 0.5$, $\mu = 0.1$, and $U = 2$.

As can be seen from the above, in an asymmetrical nonlocal medium, the initial phase-front curvature can significantly affect the trajectory of the light beam during its propagation. The main reason for this phenomenon originates from asymmetrical nonlocal nonlinear response of material to optical field. In more specific terms, the soliton light beam can induce an asymmetrical distribution of nonlinear refractive index due to asymmetrical nonlocal nonlinearity, and the position of the maximum value of nonlinear refractive index does not coincide with that of the maximum value of the amplitudes of localized light field, as shown in Fig. 2(d). So, the localized light field can be dragged towards the transverse direction by the asymmetrical distribution of nonlinear refractive index. On the other hand, the optical lattices have bound effect on localized optical field.[32] When $C = 0$, the initial phase-front curvature C cannot cause the oscillation of beam width, the distribution of nonlinear refractive index induced by the beam is invariable during its propagation. Then, the drag effect can always be balanced by the bound effect originated from optical lattices (the distribution of linear refractive index), which makes the light beam propagate in a straight line with constant beam width [see Fig. 2(a)]. However, for $C \neq 0$, the beam width os-

cillates in a period when the light beam transmits ahead, thus it causes a periodically changing distribution of nonlinear refractive index, the drag force also varies periodically. But the optical lattices keeps constant, so that the bound force on localized optical field is invariable. Finally, the light beam oscillations in transverse direction during its forward propagation due to the combined effect of invariable bound force and periodically changing drag force, as shown in Figs. 2(b), 2(c), and Fig. 3. This property can be used to achieve effective control of the light beam, and to provide a new way to realize all-optical switches, optical routers, and optical networks.

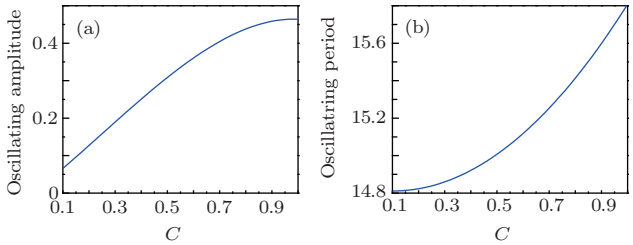


Fig. 3. (color online) Dependence of (a) the oscillating amplitude and (b) the oscillating period for fundamental mode soliton on the initial phase-front curvature C during its propagation in the system (1). Other parameters are $p = 0.5$, $\Omega = 0.5$, $\mu = 0.1$ and $U = 2$.

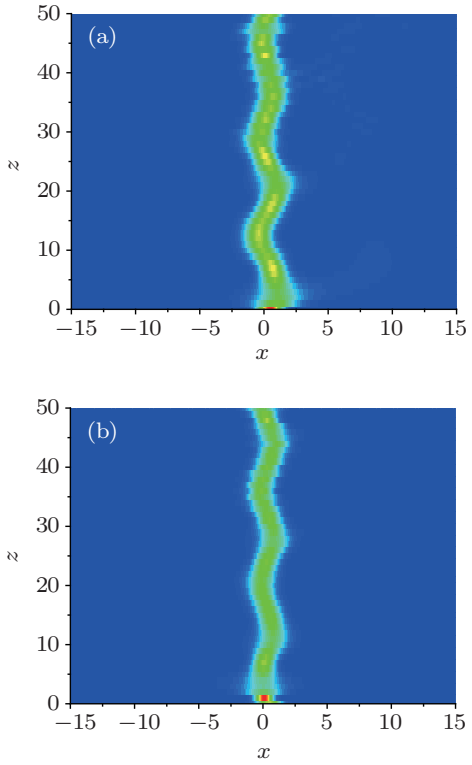


Fig. 4. (color online) Typical evolution plots of fundamental mode solitons with different initial phase-front curvatures (a) $C = -1$ and (b) $C = 1$ in the system (1). Other parameters are $p = 0.5$, $\Omega = 0.5$, $\mu = 0.1$, and $U = 2$.

5. Controllable soliton propagation based on phase-front curvature in asymmetrical nonlocal media

In order to achieve controllable soliton propagation based on phase-front curvature in asymmetrical nonlocal media, one can truncate the optical lattice to a finite length along the longitudinal direction. In the following discussion, the function $R(x,z)$ described lattice profile is expressed as

$$\begin{aligned} R(x,z) &= \cos^2(\Omega x), \quad \text{for} \quad z < z_0, \\ R(x,z) &= 0, \qquad\qquad \text{for} \quad z > z_0. \end{aligned} \tag{3}$$

Furthermore, the phase-front curvature can be imposed on a light beam at any location z_i of the optical lattices (clearly, $z_i \in [0, z_0]$) by employing phase imprinting technologies. We still take the stationary soliton $q_0(x,0) = w(x)$ as an initial input; and then impose a phase-front curvature C on it when it propagates to $z = z_i$; finally, the light beam is outputted at $z = z_{\text{end}}$. For the sake of analysis, z_0 and z_{end} are taken as 20 and 60, respectively, in the numerical simulations. To explore soliton steering, one can define Δ as a deviation distance of emission position relative to incident position of the soliton beam along x axis, and define α_{end} as an escape angle between incidence direction and final direction of motion.

The effect of the initial phase-front curvature C on the propagation of the fundamental mode soliton is important for soliton steering and controlling. In this case, a phase-front curvature C is imposed on the fundamental mode soliton at $z = 0$ (i.e., $z_i = 0$). Numerical results show that the initial phase-front curvature C has a significant impact on dynamical behaviors of the fundamental mode soliton in the system described by Eqs. (1) and (3), as shown in Figs. 5(a) and 5(b). Specifically, figures 5(c) and 5(d) present the dependence of deviation distance Δ and escape angle α_{end} on the initial phase-front curvature C. As one can see from Figs. 5(a) and 5(b), with varying the initial phase-front curvature C, the fundamental mode soliton can observably change its trajectory, this causes deviation distance Δ and escape angle α_{end} to be simultaneously changed. It means that one can effectively control soliton propagation by adjusting the parameter C.

Similarly, in the system described by Eqs. (1) and (3), dynamical behavior of the fundamental mode soliton can also be effectively controlled by changing the position z_i at which the phase-front curvature C is imposed on it. Figures 6(a) and 6(b) present typical evolution plots of fundamental mode soliton for different positions z_i. Obviously, by changing the parameter z_i, one can make a fundamental mode soliton move along different trajectories during its transmission. Deviation distance Δ and escape angle α_{end} also change with the variation of the parameter z_i, as illustrated in Figs. 6(c) and 6(d). As is shown in Figs. 6(c) and 6(d), with an increase of the parameter z_i, the variation of the deviation distance Δ and the escape angle α_{end} is non-monotonous. Notably, the tendency of the variation of escape angle is similar to that of the deviation distance except several oscillations.

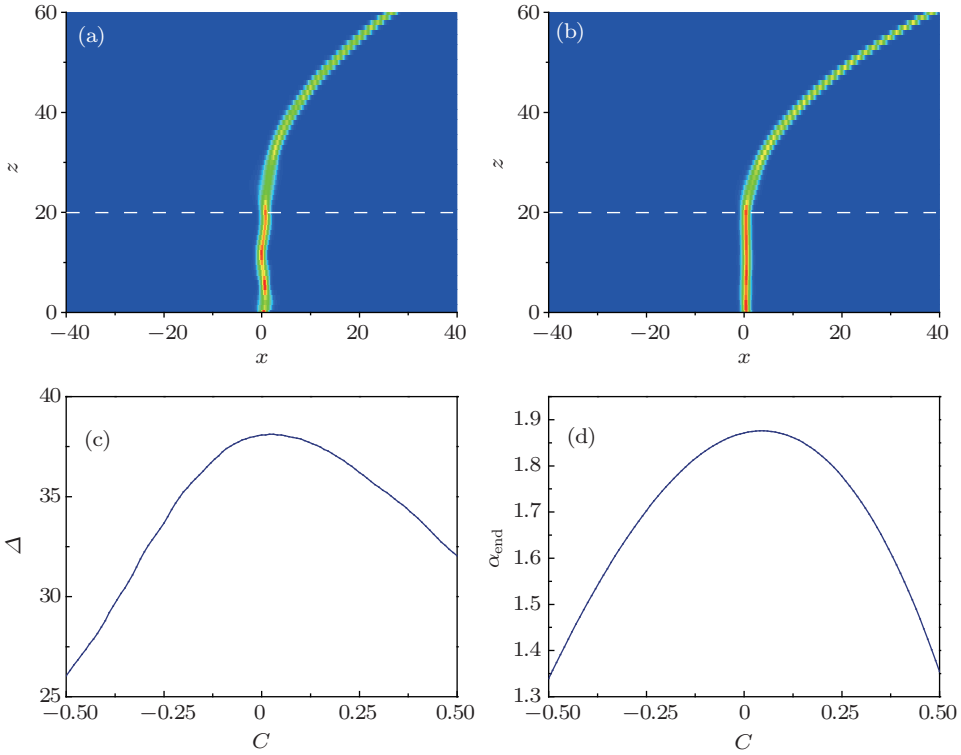


Fig. 5. (color online) Typical evolution plots of fundamental mode solitons with different initial phase-front curvatures (a) $C = -0.5$ and (b) $C = 0.1$ in the system described by Eqs. (1) and (3); dependence of (c) deviation distance Δ and (d) escape angle α_{end} on initial phase-front curvature C for fundamental mode solitons. Other parameters are $p = 0.5$, $\Omega = 0.5$, $\mu = 0.1$, $z_0 = 20$, and $U = 2$.

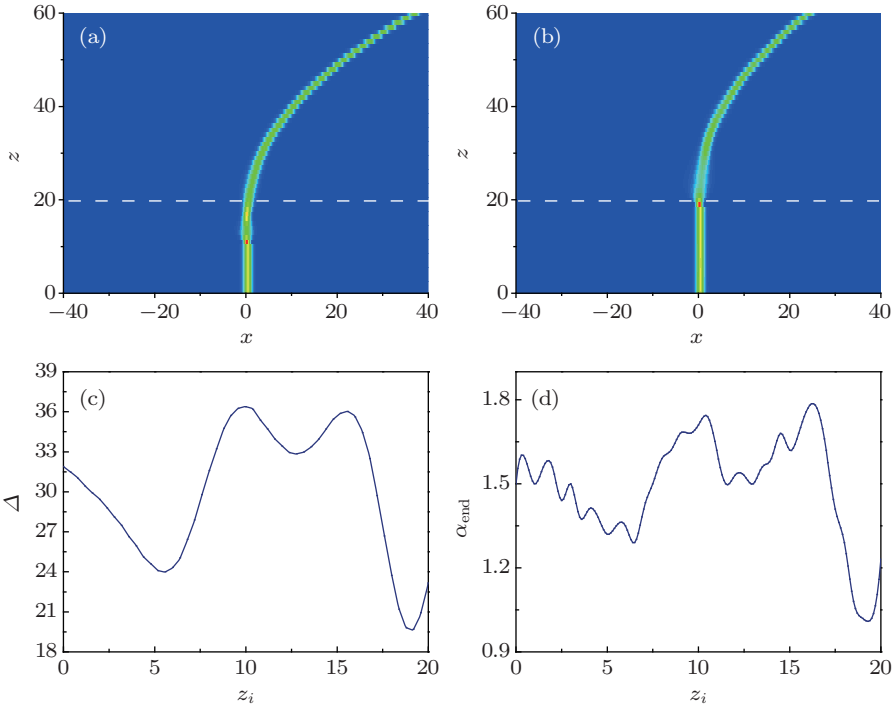


Fig. 6. (color online) Typical evolution plots of fundamental mode soliton when phase-front curvatures are imposed on it at different positions (a) $z_i = 10$ and (b) $z_i = 18$ in the system described by Eqs. (1) and (3); dependence of (c) deviation distance Δ and (d) escape angle α_{end} on the position z_i for fundamental mode solitons. Other parameters are $p = 0.5$, $\Omega = 0.5$, $\mu = 0.1$, $z_0 = 20$, $C = 0.5$, and $U = 2$.

6. Conclusion

In summary, the phase-front curvature C has an important influence on the dynamical behavior of the fundamental mode soliton during its transmission in asymmetrical nonlocal media. The phase-front curvature C can be imposed on the fundamental mode soliton by reshaping or phase imprinting technologies. By changing the phase-front curvature C or its imposed position z_i, controllable soliton propagation in asym-

metrical nonlocal media can be achieved. It is very useful for further understanding the properties of optical beams in asymmetrical nonlocal media and may be applied to all-optical controlling field.

References

[1] Ciracì C, Scalora M and Smith D R 2015 *Phys. Rev. B* **91** 205403
[2] Aubourg Q and Mordant N 2015 *Phys. Rev. E* **91** 041201(R)
[3] Strinati M C and Conti C 2014 *Phys. Rev. A* **90** 043853
[4] Kartashov Y V, Zelenina A S, Torner L and Vysloukh V A 2004 *Opt. Lett.* **29** 766
[5] Kartashov Y V, Vysloukh V A and Torner L 2014 *Opt. Lett.* **39** 933
[6] Zhang H F, Zhu D S, Xu D H, Cai C M, Zeng H and Tian Y H 2013 *J. Opt. Soc. Am. B* **30** 319
[7] Wang H C, Ling D X and He Y J 2015 *Chin. Phys. Lett.* **32** 074203
[8] Szameit A, Kartashov Y V, Dreisow F, Heinrich M, Pertsch T, Nolte S, Tünnermann A, Vysloukh V A, Lederer F and Torner L 2009 *Phys. Rev. Lett.* **102** 153901
[9] Yang J K 2014 *Opt. Lett.* **39** 1133
[10] Verma O N and Dey T N 2015 *Phys. Rev. A* **91** 013820
[11] Dai Z, Wang Y and Guo Q 2008 *Phys. Rev. A* **77** 063834
[12] Hu W, Ouyang S, Yang P, Guo Q and Lan S 2008 *Phys. Rev. A* **77** 033842
[13] Peccianti M, Dyadyusha A, Kaczmarek M and Assanto G 2006 *Nat. Phys.* **2** 737
[14] Peccianti M, Conti C, Assanto G, Luca A D and Umeton C 2004 *Nature* **432** 733
[15] Snyder A W and Mitchell D J 1997 *Science* **276** 1538
[16] Kivshar Y S and Agrawal G P 2003 *Optical Solitons: From Fibers to Photonic Crystals* (San Diego: Academic)
[17] Wu L, Zhang J F, Li L, Tian Q and Porsezian K 2008 *Opt. Express* **16** 6352
[18] Wu L, Li L and Zhang J F 2008 *Phys. Rev. A* **78** 013838
[19] Króikowski W and Bang O 2000 *Phys. Rev. E* **63** 016610
[20] Zhang H F, Li L and Jia S T 2007 *Phys. Rev. A* **76** 043833
[21] Wu X F, Deng D M and Guo Q 2011 *Chin. Phys. B* **20** 084201
[22] Wang J, Li Y, Guo Q and Hu W 2014 *Opt. Lett.* **39** 405
[23] Assanto G, Minzoni A A and Smyth N F 2014 *Opt. Lett.* **39** 509
[24] Gomes R M, Salles A, Toscano F, Souto R P H and Walborn S P 2009 *Phys. Rev. Lett.* **103** 033602
[25] Krolikowski W, Bang O, Rasmussen J J and Wyller J 2001 *Phys. Rev. E* **64** 016612
[26] Bang O, Krolikowski W, Wyller J and Rasmussen J J 2002 *Phys. Rev. E* **66** 046619
[27] Briedis D, Edmundson D, Bang O and Krolikowski W 2005 *Opt. Express* **13** 435
[28] Hutsebaut X, Cambournac C, Haelterman M, Adamski A and Neyts K 2004 *Opt. Commun.* **233** 211
[29] Nikolov N I, Neshev D, Królikowski W, Bang O, Rasmussen J J and Christiansen P L 2004 *Opt. Lett.* **29** 286
[30] Alberucci A, Jisha C P, Smyth N F and Assanto G 2015 *Phys. Rev. A* **91** 013841
[31] Yang Z J, Ma X K, Zheng Y Z, Gao X H, Lu D Q and Hu W 2011 *Chin. Phys. Lett.* **28** 074213
[32] Zhang H F, Xu F, Zhu D S, Zhang L, Xu D H and Tian Y H 2014 *Opt. Express* **22** 995
[33] Kartashov Y V, Vusloukh V A and Torner L 2004 *Phys. Rev. Lett.* **93** 153903
[34] Xu Z Y, Kartashov Y V and Torner L 2006 *Opt. Lett.* **31** 2027-2029
[35] Shi X, Malomed B A, Ye F W and Chen X 2012 *Phys. Rev. A* **85** 053839
[36] Lobanov V E, Kartashov Y V, Vysloukh V A and Torner L 2012 *Opt. Lett.* **37** 4540
[37] Press W H, Teukolsky S A, Vetterling W T and Flannery B P 2001 *Numerical Recipes in Fortran* 77*: The Art of Scientific Computing* (Cambridge: Cambridge University) pp. 753–63
[38] Chiofalo M L, Succi S and Tosi M P 2000 *Phys. Rev. E* **62** 7438
[39] Bao W Z, Chern I L and Lim F Y 2006 *J. Comput. Phys.* **219** 836
[40] Chapra S C 2012 *Applied Numerical Methods with MATLAB for Engineers and Scientists*, 3rd edn. (McGraw-Hill) pp. 621–28
[41] Li L, Zhao X S and Xu Z Y 2008 *Phys. Rev. A* **78** 063833
[42] Zhao X S, Li L and Xu Z Y 2009 *Phys. Rev. A* **79** 043827