Influence of the colored noise on determining the period of a torsion pendulum*

Jie Luo(罗杰)[1], Wen-Ze Zhan(占文泽)[1], Wei-Huang Wu(巫伟皇)[1], Cheng-Gang Shao(邵成刚)[2,†], and Dian-Hong Wang(王典洪)[1]

[1] School of Mechanical Engineering and Electronic Information, China University of Geosciences, Wuhan 430074, China
[2] MOE Key Laboratory of Fundamental Physical Quantities Measurement, School of Physics, Huazhong University of Science and Technology, Wuhan 430074, China

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Based on statistical properties, two typical models are considered to calculate the uncertainties for some random noise sequences on the period extraction of a torsion pendulum, which is important and instructive in the measurement of gravitational constant G with the time-of-swing method. An expression of the uncertainty for the period measurement is obtained, which is dependent on the ratio $\Delta t/(1/\lambda)$ where Δt is the interval of the sample time and $1/\lambda$ is the length of the correlation time. The result of processing experimental data shows that as the interval of the sample time Δt gradually shortens, the uncertainty of the period becomes smaller, and further when the ratio $\Delta t/(1/\lambda)$ is less than 1, the uncertainty remains substantially unchanged.

Keywords: torsion pendulum, Gaussian colored noise, uncertainty, correlation time

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1. Introduction

The period determination of a torsion pendulum with high precision is of great importance for some gravitational experiments.[1–3] For example, in the measurement of the Newtonian gravitational constant G with the time-of-swing method, one needs to determine the periods of the torsion pendulum at different configurations of source masses with high precision.[4–10] The measurement accuracy of G is directly restricted by the period extraction accuracy.

The torsion pendulum is one of the most sensitive physical measurement devices, and has been recognized and applied widely over the last two centuries since Cavendish's measurement of the gravitational constant G.[11] As a highly sensitive device, the sensitivity and precision of the torsion pendulum are inevitably limited by a variety of noises, and thus the precision of period estimation is also limited by these noises. Based on statistical properties, some random noise sequences obey the Gaussian probability density distribution and have definite correlation time,[12,13] which could be generally divided into two categories. One is named the Gaussian white noise, which is a statistically independent random variable. In fact, the random noise sequence of which the correlation time length is far smaller than the sampling interval could be viewed as the Gaussian white noise. Maybe the Gaussian white noise is not a perfect model for torsion balance experiment, but it is useful and advisable for theoretical analysis to judge whether a method is good or not.[14] The other is defined as the Gaussian colored noise, whose correlation time length is larger than the sampling interval. For example, the random noise resulting from the angle-deflection readout system in the experiment should be viewed as the Gaussian colored noise.[15] Obviously, the effect of the Gaussian colored noise on the period extraction of the pendulum is well worth studying.

The influences of many noises on the period extraction of a torsion pendulum have been studied in detail before.[14–19] These relevant studies focused on the effects caused by specific noises of different origins and corresponding contributions of these noises to the uncertainty of period extraction. Besides, with the correlation time obtained, the use of the noise model and the setting of a suitable interval of the sample time are significant issues which need solving. In this condition, it is necessary to analyze the relation of the uncertainty of the pendulum period with the correlation time and the interval of the sample time.

In this paper, two typical noise models are considered to analyze the influence of the random noise sequence on the period extraction of a torsion pendulum. Though there are a variety of methods to determine the period of the pendulum,[1,14,15,17,20] the correlation method, which is used in our analysis, is one of the most efficient and accurate methods for subtle signal processing since it is insensitive to higher order harmonics, nor the damping effect nor the slow drift of the period.[14] The uncertainty expression of the Gaussian colored noise on the period extraction is derived in detail and compared with that of the Gaussian white noise. According to the formula of the uncertainty, we can conclude that the un-

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†Corresponding author. E-mail: cgshao@mail.hust.edu.cn
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certainty expressions of the Gaussian white and colored noises are dependent on the sampling interval and length of the correlation time respectively. Generally, with the interval of sample time far smaller than the length of the correlation time, the Gaussian colored noise model is used to analyze the random noise sequence, and otherwise the Gaussian white noise model is used. Meanwhile, if the length of the correlation time of a random noise sequence is known, the interval of the sample time can be determined directly according to the requirement for the period accuracy. Consequently, the applicable noise model could be chosen to obtain the uncertainty correspondingly. Finally, an analysis of typical experimental data of the measurement of G with the time-of-swing method further shows that as the interval of the sample time gradually shortens, the uncertainty of the period becomes smaller consequently, and when the ratio between the sampling interval and the correlation time is less than 1, the uncertainties remain substantially unchanged.

2. Influence of the random noise sequence on period

Due to the sensitivity of measurement device and change of external experiment environment, the actual period of torsion pendulum is inevitably influenced by varieties of noises. Based on statistical properties, the Gaussian white noise model is usually used to analyze the influences of some random noise sequences.[14,19,21] In fact, there may be no strict white noise in the nature environment, and many noises have a definite correlation time each. Generally, when the correlation time of noise is far less than the relevant time of the system, the Gaussian white noise model is used to deal with the noise.[22] However, there are many noises with a relatively long correlation time in the experimental environment. On this condition, the Gaussian white noise model cannot be used any more, and hence it is necessary to use the Gaussian colored noise model to analyze the influence of this kind of noise.

2.1. The Gaussian colored noise model

The Gaussian colored noise is one of the most popular and applicable colored noise models. Although the physical origin of this kind of noise is as yet unknown, which usually has a definite correlation time and complies with Gaussian distribution. The expectation value and correlation function of the noise ε_{ic} are expressed, respectively, as[12,13]

$$\begin{cases} \langle \varepsilon(t) \rangle = 0, \\ \langle \varepsilon(t_1)\varepsilon(t_2) \rangle = \sigma^2 \exp(-\lambda\,|t_1 - t_2|), \end{cases} \tag{1}$$

where $\langle\ \rangle$ is the expectation operator of population mean, σ is the root mean square of the Gaussian colored noise, and $1/\lambda$ is the length of correlation time which, in general, satisfies the inequality $1/\lambda \ll \tau_0$ in the experiments of torsion pendulum, where τ_0 is the period of the pendulum. For convenience in calculation, the ratio between the sampling interval and the correlation time is set as $\gamma = \Delta t/(2/\lambda)$. Generally, in the Gaussian colored model $\gamma \ll 1$, while $\gamma \gg 1$ in the Gaussian white model.[15]

In experiment, the experimental signal y_i of torsion pendulum is expressed as

$$\begin{aligned} y_i &= \theta_0 \cos(\omega_0 t_i + \phi_0) + \varepsilon_{ic} \\ &= a\cos(\omega_0 t_i) + b\sin(\omega_0 t_i) + \varepsilon_{ic}, \end{aligned} \tag{2}$$

where θ_0, ω_0, and ϕ_0 denote the amplitude, angular frequency and initial phase of the pendulum, respectively. The coefficients a and b are the decomposed coefficients of the amplitude. The parameter ε_{ic} represents the Gaussian colored noise sequence. According to the correlation method, a standard sinusoidal function $\cos(\omega_{\rm ref}t)$ or $\sin(\omega_{\rm ref}t)$ is chosen as the reference signal with $\omega_{\rm ref} \approx \omega_0$ and the reference period $\tau_{\rm ref} = 2\pi/\omega_{\rm ref}$. Then we should compare the sample data $\{y_i\}$ with the standard sinusoidal function, and calculate the phase angle sequence ϕ_j $(j = 0, 1, 2, \ldots, m-1)$, where m is the number of periods. Assume that k is the number of sampling points in each period, then the total number of sample points will be $N = mk$. For each period, the phase angle ϕ_j is determined by the cross-correlation function of the reference signal and the sample data, and expressed as

$$\phi_j = \tan^{-1}\frac{b_j}{a_j} = \tan^{-1}\frac{\dfrac{2}{\tau_{\rm ref}}\displaystyle\int_{j\tau_{\rm ref}}^{(j+1)\tau_{\rm ref}} y(t)\sin(\omega_{\rm ref}t)\,{\rm d}t}{\dfrac{2}{\tau_{\rm ref}}\displaystyle\int_{j\tau_{\rm ref}}^{(j+1)\tau_{\rm ref}} y(t)\cos(\omega_{\rm ref}t)\,{\rm d}t}. \tag{3}$$

For convenience of numerical computation, the integral should be replaced by the sum. In order to keep the phase unambiguous, the frequency $\omega_{\rm ref}$ should be sufficiently close to the real frequency of the pendulum. Besides, $\omega_{\rm ref}$ can be easily given by the fast Fourier transform method.[14] Generally, the initial phase is chosen as $|\phi_0| \ll 1$, and then the phase difference $\Delta\phi_j$ is approximately expressed as

$$\Delta\phi_j = -\phi_j \approx \phi_0 + 2\pi\frac{\omega_{\rm ref} - \omega_0}{\omega_{\rm ref}}\,j + \delta\phi_j, \tag{4}$$

where $\phi_0 = \tan^{-1}(b/a)$ is the initial phase, and $\delta\phi_j$ is a random variable denoting the Gaussian colored noise. Since the reference period satisfies $\tau_{\rm ref} \approx \tau_0$, $\delta\phi_j$ can be written as

$$\delta\phi_j \approx \frac{2}{\tau_0\theta_0}\int_{j\tau_0}^{(j+1)\tau_0} \varepsilon(t)\sin(\omega_0 t)\,{\rm d}t. \tag{5}$$

Then, the statistical property of $\delta\phi_j$ can be expressed as follows:

$$\begin{cases} \langle \delta\phi_j \rangle = 0, \\ \langle \delta\phi_i \delta\phi_j \rangle = \dfrac{4\sigma^2\delta_{ij}}{\theta_0^2\lambda\,\tau_0}. \end{cases} \tag{6}$$

By performing the linear fitting of the phase difference sequence $\{\Delta\phi_j\}$ and number sequence $\{j\}$, the frequency ω can be estimated from

$$\omega = \omega_{\rm ref} - \frac{K\omega_{\rm ref}}{2\pi}, \tag{7}$$

where K is the estimate for the slope of Eq. (4). Thus the central value of the period is estimated to be

$$\tau_0 = \tau_{\rm ref}\left(1+\frac{K}{2\pi}\right). \tag{8}$$

Based on the least-squares fitting method, the linear regress model can be written as $\Delta\hat{\phi}_j = \hat{\phi}_0 + j\hat{K}$, and then the intercept and slope are obtained by

$$\begin{aligned}\begin{pmatrix}\hat{\phi}_0\\ \hat{K}\end{pmatrix} &= (C^{\rm T}C)^{-1}\begin{pmatrix}\sum\limits_{i=0}^{m-1}\Delta\phi_i\\ \sum\limits_{i=0}^{m-1} i\Delta\phi_i\end{pmatrix}\\ &= \begin{pmatrix}\dfrac{4m-2}{m(m+1)} & \dfrac{-6}{m(m+1)}\\ \dfrac{-6}{m(m+1)} & \dfrac{-12}{m(1-m^2)}\end{pmatrix}\begin{pmatrix}\sum\limits_{i=0}^{m-1}\Delta\phi_i\\ \sum\limits_{i=0}^{m-1} i\Delta\phi_i\end{pmatrix},\end{aligned} \tag{9}$$

with the matrix $C = \begin{bmatrix}1 & 1 & \cdots & 1\\ 0 & 1 & \cdots & m-1\end{bmatrix}^{\rm T}$. Since the period of the torsion pendulum is inevitably disturbed by some noise sequences, the deviation of the slope, $\Delta\hat{K}$, which is caused by the phase noise $\delta\phi_j$, can be approximately expressed as

$$\begin{aligned}\Delta\hat{K} &\approx \frac{6}{m^2}\begin{pmatrix}-1 & 2/m\end{pmatrix}\begin{pmatrix}\sum\limits_{i=0}^{m-1}\delta\phi_i\\ \sum\limits_{i=0}^{m-1} i\delta\phi_i\end{pmatrix}\\ &= \frac{6}{m^2}\sum_{i=0}^{m-1}\left(\frac{2i}{m}-1\right)\delta\phi_i.\end{aligned} \tag{10}$$

The deviation of the slope is given as the standard deviation, and expressed as

$$\sigma(\Delta\hat{K}) = \sqrt{E[(\Delta\hat{K})^2]} = \sqrt{\frac{48\sigma^2}{m^3\theta_0^2\lambda\tau_0}}. \tag{11}$$

Therefore the uncertainty of the angular frequency of the pendulum based on the Gaussian colored noise is

$$\delta\omega_0\Big|_{\gamma\ll1} = \frac{\omega_{\rm ref}}{2\pi}\sigma(\Delta\hat{K}) \approx \frac{1}{\tau_0}\sqrt{\frac{48\sigma^2}{m^3\theta_0^2\lambda\tau_0}}. \tag{12}$$

Finally, the relative uncertainty of the period estimation is

$$\frac{\delta\tau_0}{\tau_0}\Big|_{\gamma\ll1} \sim \frac{\delta\omega_0}{\omega_0} = \frac{2\sqrt{3}}{\pi}\frac{\sigma}{\theta_0 m}\sqrt{\frac{1}{\lambda m\tau_0}}. \tag{13}$$

Equation (13) shows that the uncertainty of the period estimation is a function of the initial oscillation amplitude, the period of torsion pendulum, the number of periods during the whole measurement time, the amplitude of the noise and the length of the correlation time.

2.2. Gaussian white noise model

Unlike the Gaussian colored noise, the Gaussian white noise is a kind of ideal mathematical model. In fact, the random noise sequence, of which the correlation time length is far smaller than the sampling interval, could be modeled as the Gaussian white noise. For example, the environmental noise and the thermal noise caused by the Brown motion can be viewed as the white noise model.[15,19] The expectation value and correlation function can be given, respectively, as[14]

$$\begin{cases}\langle\varepsilon_i\rangle = 0,\\ \langle\varepsilon_i\varepsilon_j\rangle = \sigma^2\delta_{i,j},\end{cases} \tag{14}$$

where σ^2 is the variance of the Gaussian white noise. In this case, the series of the experiment data y_i of the pendulum can be expressed as

$$y_i = \bar{y}_i + \varepsilon_{iw} = \theta_0\cos(\omega_0 t_i + \phi_0) + \varepsilon_{iw}, \tag{15}$$

where $\bar{y}_i$ and ε_{iw} represent the real signal and the sequence of Gaussian white noise, respectively. Like the analysis of the Gaussian colored noise, the statistical property of the phase noise $\delta\phi_j$, which is modeled as the Gaussian white noise, can be expressed as

$$\begin{cases}\langle\delta\phi_j\rangle = 0,\\ \langle\delta\phi_i\delta\phi_j\rangle = \dfrac{2\sigma^2\Delta t\,\delta_{ij}}{\theta_0^2\tau_0},\end{cases} \tag{16}$$

where Δt is the interval of the sample time, $t_m = N\Delta t$ is the total sample time.

Similarly, the standard deviation is used to calculate the error of the slope, and expressed as

$$\sigma(\Delta\hat{K}) = \sqrt{E[(\Delta\hat{K})^2]} = \sqrt{\frac{24\sigma^2\Delta t}{m^3\theta_0^2\tau_0}}, \tag{17}$$

and then the uncertainty of the frequency caused by the Gaussian white noise is

$$\delta\omega_0\Big|_{\gamma\gg1} = \frac{\omega_{\rm ref}}{2\pi}\sigma(\Delta\hat{K}) \approx \frac{1}{\tau_0}\sqrt{\frac{24\sigma^2\Delta t}{m^3\theta_0^2\tau_0}}. \tag{18}$$

Finally, the relative uncertainty of the period estimation is

$$\frac{\delta\tau_0}{\tau_0}\Big|_{\gamma\gg1} \sim \frac{\delta\omega_0}{\omega_0} = \frac{\sqrt{6}}{\pi}\frac{\sigma}{\theta_0 m}\sqrt{\frac{\Delta t}{m\tau_0}}. \tag{19}$$

It is obvious that the uncertainty of the period estimation is dependent on the initial oscillation amplitude, the period of torsion pendulum, the number of the periods during the whole measurement time, the amplitude of the noise and the interval of the sample time, which is similar to the result of Eq. (13). Namely, the uncertainty expressions of the Gaussian white and colored noises are dependent on the sampling interval and length of the correlation time respectively. When the length of the correlation time is small enough, the Gaussian colored noise model in the noise analysis should be replaced by the Gaussian white noise model.

2.3. Combination of the above two typical models

According to Eqs. (13) and (19), the relative uncertainty of the random noise sequence on the period estimation can be combined into

$$\frac{\delta\tau_0}{\tau_0} = f(\gamma)\frac{\delta\tau_0}{\tau_0}\bigg|_{\gamma\ll 1} \quad \text{with} \quad f(\gamma) = \begin{cases} 1, & \gamma \ll 1, \\ \sqrt{\gamma}, & \gamma \gg 1, \end{cases} \tag{20}$$

where $f(\gamma)$ is the normalized relative uncertainty of the period, and $\gamma = \Delta t/(2/\lambda)$ denotes the ratio between the sampling interval and correlation time. Obviously, we can conclude that for a random noise sequence, the used noise model and corresponding uncertainty are dependent on the ratio $\Delta t/(1/\lambda)$. When $\gamma \ll 1$, namely the interval of the sample time is far smaller than the length of the correlation time, the Gaussian colored noise model should be used to analyze the random noise sequence. When $\gamma \gg 1$, the Gaussian white noise model should be used to model the random noise sequence. However, there is no rigorous formula when the value of γ belongs to the other range. On this condition, the uncertainties can be obtained from the simulation results, which are normalized as Eq. (20), as shown in Fig. 1.

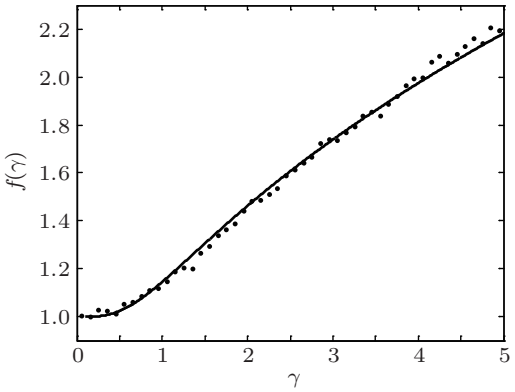


Fig. 1. Relationship between the normalized uncertainty and the interval of the sample time. $f(\gamma)$ is the normalized uncertainty of the period, $\gamma = \Delta t/(2/\lambda)$ represents the different interval of the sample time on the condition that the length of the correlation time is set as a constant. Each point denotes the normalized statistical uncertainty. The solid line is obtained by the polynomial fitting between simulated uncertainties and the different ratio γ. Some points in the plot are covered by the solid line. Referring to the real experimental data in the experiment of measuring G with the time-of-swing method,[23] the simulation parameters are set respectively.

It is obvious that the simulation results are in good agreement with the theoretical uncertainty in Eq. (20). Hence, for a random noise sequence, as long as the length of the correlation time is known, combined with the experimental Δt, the noise model is chosen to directly obtain the uncertainty of the period precisely. The fitting function shown in Fig. 1 can be used to approximately obtain the normalized uncertainty when the value of γ belongs to the other range. The value of the normalized uncertainty can be obtained from the simulation results as shown in Fig. 1, where $\gamma = \Delta t/(2/\lambda)$ represents the different intervals of the sample time, with the correlation time length known. Therefore, for a random noise sequence, with the correlation time length known, according to the requirement for the period extraction accuracy, combined with the predicted uncertainty $f(\gamma)$, the interval of the sample time can be determined directly from the value of γ. Namely, for an experiment signal, when the total sample time is known, as the interval of the sample time gradually shortens, the uncertainty of the period becomes smaller consequently, and when the ratio between the sampling interval and the correlation time is less than 1, the uncertainties remain substantially unchanged.

3. Experiment data analysis for a torsion pendulum

In the experiment of measuring G with the time-of-swing method,[23] each data set is taken in a regular total sample time of about 3 days in intervals of 0.5 s (i.e., the sample time $\Delta t = 0.5$ s). The total sample time cannot be too long because of the damping effect of torsion pendulum and the influence of external experimental environment. Figure 2 shows a section of a raw experiment data of the pendulum. The period of the pendulum at the far position is about 392.3 s, with a total period number being about 650 in the correlation method. The slope of the monotonic drift is $d \approx -5.2 \times 10^{-10}$ rad/s while the amplitude of the pendulum is $\theta_0 \approx 5 \times 10^{-3}$ rad, which can be ignored. Figure 3 shows a spectrum of the amplitude of the data shown in Fig. 2. We can know that the major noise is from the higher harmonics, which may be caused by the nonlinearity of the torsion fiber. Since the correlation method is insensitive to higher harmonics, the influences of these harmonics can be removed, and hence the model we considered can be used to calculate the uncertainty of the period. Figure 4 shows that the uncertainty of the period varies with the interval of sample time. While the correlation time length in the real experiment data is unknown, the normalized processing cannot be given. It is clear that as the interval of the sample time gradually shortens, the uncertainty of the period consequently becomes smaller, whose trend is consistent with the numerical simulation results in Fig. 1. Obviously, in some gravitational experiments, when $\Delta t \gg (1/\lambda)$, the uncertainty of the random noise becomes larger rapidly as the Δt shortens gradually, and with the ratio between the sampling interval and the correlation time being less than 1, the uncertainty of the random noise remains nearly constant. Thus, a suitable interval of the sample time is highly important for improving the period extraction accuracy and optimizing the data acquisition system. At the same time, via comparing Fig. 1 with Fig. 4, the correlation time length in the experiment can be approximately obtained from the ratio between the sampling interval and the correlation time, which is about 1.5 s.

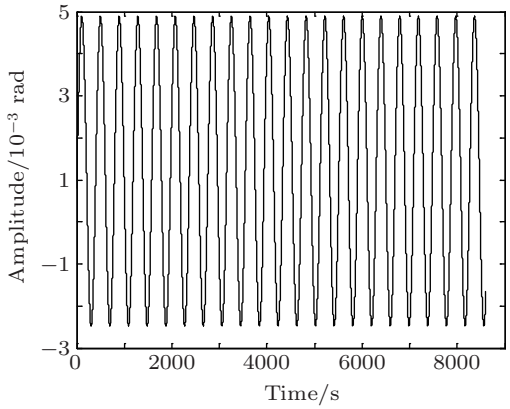


Fig. 2. Section of raw experimental data for a torsion pendulum (about 17000 samples). The total sample time is about 3 days, the slope of the monotonic drift is $d = -5.2 \times 10^{-10}$ rad/s, and the sample time is 0.5 s.

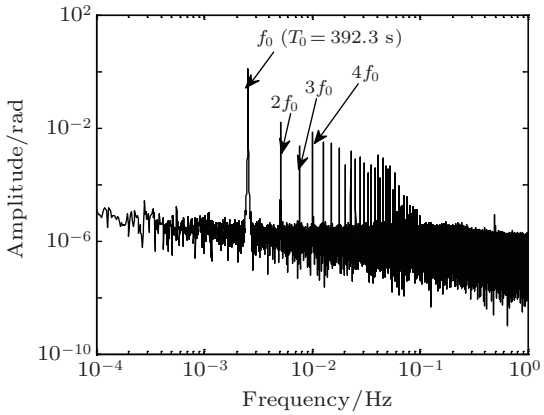


Fig. 3. Spectrum of the experimental signal shown in Fig. 2, displaying that higher harmonics are the primary noise, but the correlation method is insensitive to these harmonics. Note that there is a peak at frequency $f = 2.55 \times 10^{-3}$ Hz, which is the frequency of the pendulum.

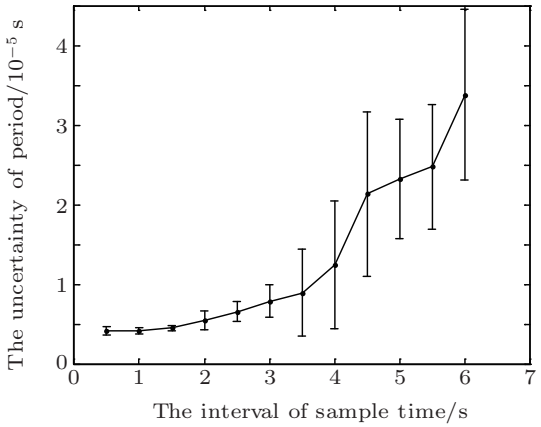


Fig. 4. Uncertainty of the period varying with sample time. Each point represents the uncertainty acquired by the correlation method with six pairs of experimental data sets. Though the correlation time length in the real experimental data is unknown, by comparing Fig. 4 with Fig. 1, the correlation time length can be obtained approximately, which is about 1.5 s.

4. Conclusions

In this work, the influences of the Gaussian white and colored noises on the period determination of a torsion pendulum are discussed respectively. According to the correlation method, the uncertainties of random noise sequences on the period extraction are formalized. Corresponding numerical simulations indicate that for a random noise sequence, with the correlation time length known, according to the requirement for the period extraction accuracy, the suitable interval of the sample time can be determined directly, and thus the applicable noise model is chosen to obtain the uncertainty correspondingly. If the interval of the sample time $\Delta t \ll (1/\lambda)$, the random noise sequences should be modeled as a Gaussian colored noise, and if $\Delta t \gg (1/\lambda)$, the random noise sequences should be modeled as a Gaussian white noise. An experimental data analysis for the measurement of G with the time-of-swing method further shows that as the sample time gradually shortens, the uncertainty of the period becomes smaller consequently, and further when the ratio is less than 1, the uncertainty remains substantially unchanged. Though some of the above conclusions are based on numerical simulation and mathematical analysis, it is instructive for the improvement of the period extraction accuracy and the optimization of data acquisition system. Since many kinds of noises have not been fully understood so far, there is still much work to do in this field.

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