Transferring information through a mixed-five-spin chain channel

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We initially introduce one-dimensional mixed-five-spin chain with Ising-XY model which includes mixture of spins-1/2 and spins-1. Here, it is considered that nearest spins $(1,1/2)$ have Ising-type interaction and nearest spins $(1/2,1/2)$ have both XY-type and Dzyaloshinskii–Moriya (DM) interactions together. Nearest spins $(1,1)$ have XX Heisenberg interaction. This system is in the vicinity of an external homogeneous magnetic field B in thermal equilibrium state. We promote the quantum information transmitting protocol verified for a normal spin chain with simple model (refer to Rossini D, Giovannetti V and Fazio R 2007 *Int. J. Quantum Infor.* **5** 439) (widely in reference: Giovannetti V and Fazio R 2005 *Phys. Rev. A* **71** 032314) by means of considering the suggested mixed-five-spin chain as a quantum communication channel for transmitting both qubits and qutrits ideally. Hence, we investigate some useful quantities such as quantum capacity and quantum information transmission rate for the system. Finally, we conclude that, when the DM interaction between spins $(1/2,1/2)$ increases the system is a more ideal channel for transmitting information.

Keywords: Dzyaloshinskii–Moriya interaction, XX Heisenberg interaction, communication channel, transmission rate

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1. Introduction

Transferring information between distinct systems is an important problem in quantum information theory. Many researchers involved themselves to investigate this problem carefully and tried to introduce some useful methods for better understanding transferring quantum states protocol theoretically (and practically in the futurity[1]) through ideal channels.[2–7] Recently, many profound studies have been devoted to quantum communication by using spin chains as suitable channels, and it was shown that transferring quantum information can be reliably achieved using chains of permanently coupled spins.[8–14]

Most works on quantum communication assume that channels have noise in transmission protocols, also somewhere, quantum channels with memory[7] have received considerable attention (The problem of quantum channels with memory was initially introduced in Ref. [2]). In the last two decades, quantum communication has not only been successful in characterizing these subjects but has also opened new problems. Another essential problem concerns the additivity of channel capacity when entangled states are used as signals to transfer quantum information.[15] It was proved that, quantum systems can be used for quantum communication channels[15,16] for carrying both classical[17,18] and quantum information.[5,19] Namely, different capacities for a quantum channel exist that must be separably defined. The classical capacity $\mathcal{C}$[6,20,21] (also it has been mentioned of quasi-classical capacity in Ref. [21]) and the quantum capacity $\mathcal{Q}$[3,6,7,16,22] of a quantum channel are defined as the maximum number of bits and qubits respectively that can be reliably transmitted per channel use.

The transmission protocol can be conveniently described in terms of quantum operation formalism.[6,10,23,24] Hence, any input quantum state ρ is mapped onto the output state $\rho' = \mathcal{M}[\rho]$ via a linear completely positive trace preserving (CPTP) map $\mathcal{M}$. The simplest searchable models of quantum channels are the class of memoryless channels as $\mathcal{M}_n = \mathcal{M}^{\otimes n}$ which describes n channel uses. In this paper, we confine ourselves to verify the quantum spin chains as memoryless channels with capacity. Furthermore, in the transferring quantum information process, efficiency of the communication can be characterized with the transmission rate $\mathcal{R}$. This is a quantity which can be introduced as the maximum number of quantum states that one can transfer per unit of time with almost maximum fidelity ($\mathcal{F} \approx 1$) in the limit of large transmission time,[7,15,25] or in other words, the transmission rate can be (loosely speaking) defined as the fraction of states that can be recovered at the output with high fidelity over the total number of transmitted states.

Here, it should be noted that transmitting information by use of the spin systems is not restricted to the system evolution, in other words, one can also use the teleportation protocol (see review article[26]) and cloning for transferring quantum information. Cloning can be done commonly in classical systems, while for quantum systems it is not possible (due to the existence of the pure states). But on the other hand, we can try to clone a quantum state with almost maximum fidelity, or it can be ideally done with maximum probability. Cloning and teleportation protocols have been precisely investigated by Heng Fan *et al.* in Ref. [27].

From quantum information theory and quantum correlations view points, many investigations have been done on

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the mixture of coupled different spins with various models and many interesting results have been reported.[28–34] In the present work, we intend to introduce a new class of the quantum communication channels through which qutrits can be reliably transmitted as well as qubits. Hence, we use a mixed-five-spin chain with special Ising-XY model for which the nearest spins $(1,1/2)$ have Ising-type interaction and nearest spins $(1/2,1/2)$ have both XY-type and Dzyaloshinskii–Moriya (DM) interactions together. Nearest spins $(1,1)$ have XX Heisenberg interaction, as a quantum communication channel then, investigate its quantum capacity and transmission rate $\mathcal{R}$.

The outline of the paper is as follows. In Section 2 we introduce our favorite spin model with an analytical Hamiltonian. The transmission protocol is explained in Section 3. In Section 4, as a preliminary for review of the aforementioned quantities, we investigate quantum capacity and transmission rate for such a spin model as a communication channel according to the statements in Ref. [11]. Some numerical calculations and simulations are done in this section. Section 5 is devoted to summary and discussions.

2. Spin model and theoretical background

We introduce a coupled mixed-M-spin chain with Ising-XY model (spin chain presented in Fig. 1) which is in an external homogeneous magnetic field B with the following Hamiltonian

$$H=\sum_{i=1}^{N}\left((1+\gamma)S_{i,1}^{x}S_{i,3}^{x}+(1-\gamma)S_{i,1}^{y}S_{i,3}^{y}\right)+\sum_{i=1}^{N}\boldsymbol{B}\cdot(\boldsymbol{S}_{i,1}+\boldsymbol{S}_{i,3})+\sum_{i=1}^{N}\boldsymbol{D}\cdot(\boldsymbol{S}_{i,1}\times\boldsymbol{S}_{i,3})$$
$$+\sum_{i=1}^{N}J(S_{i,1}^{z}J_{i,2}^{z}+J_{i,2}^{z}S_{i,3}^{z})+\sum_{i=1}^{N>1}\Lambda(J_{i,2}^{x}J_{i+1,2}^{x}+J_{i,2}^{y}J_{i+1,2}^{y})+\sum_{i=1}^{N}\boldsymbol{B}\cdot\boldsymbol{J}_{i,2}, \tag{1}$$

where i is the number of triangular cells in the chain, γ is anisotropy parameter, J is the Ising coupling constant between nearest spins $(1,1/2)$, D is the DM interaction between spin-half of any cell and Λ is the XX Heisenberg interaction parameter between spin-integer of any two adjacent cells. $\boldsymbol{S}=\{S^x,S^y,S^z\}$ and $\boldsymbol{J}=\{J^x,J^y,J^z\}$ are spin operators (with $\hbar=1$) which are introduced as the following matrices

$$S^x=\frac{1}{2}\begin{pmatrix}0&1\\1&0\end{pmatrix},\quad S^y=\frac{1}{2}\begin{pmatrix}0&-\mathrm{i}\\\mathrm{i}&0\end{pmatrix},$$
$$S^z=\frac{1}{2}\begin{pmatrix}1&0\\0&-1\end{pmatrix}, \tag{2}$$
$$J^x=\sqrt{2}\begin{pmatrix}0&1&0\\1&0&1\\0&1&0\end{pmatrix},\quad J^y=\sqrt{2}\begin{pmatrix}0&-\mathrm{i}&0\\\mathrm{i}&0&-\mathrm{i}\\0&\mathrm{i}&0\end{pmatrix},$$
$$J^z=\begin{pmatrix}1&0&0\\0&0&0\\0&0&-1\end{pmatrix}. \tag{3}$$

Here we consider $N=2$ which represents the first two adjacent triangular cells of the chain containing five mixed spins. $\boldsymbol{B}=B_z$ and $\boldsymbol{D}=D_z$ are, respectively, homogeneous magnetic field and DM interaction in the z direction. Note that here all of the introduced parameters are considered dimensionless.

3. Transmission protocol

Assume that a quantum state ρ is propagated through a quantum channel, hence, we can characterize this propagation by a mapping protocol as

$$\mathcal{M}:\rho_i\longrightarrow\rho_f=\mathcal{M}[\rho_i], \tag{4}$$

where ρ_i is the initial state and ρ_f is the external mapped state through the channel. Such mapping is performed by unitary transformation operator U related to the feature of the channel as

$$\mathcal{M}[\rho]=U\rho U^{\dagger}. \tag{5}$$

A diagram of this protocol is shown in Fig. 1.

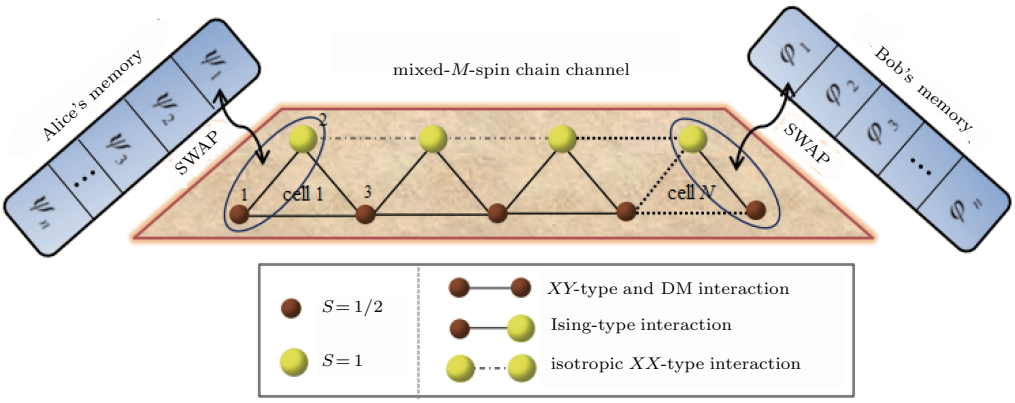


Fig. 1. (color online) Schematic diagram of a mixed M-spin chain with Ising-XY model as quantum communication channel.

We explain the transmission protocol step by step in the following.

(i) Sender (Alice) applies an SWAP operation $S_{\rm A}$ for the set of unknown states $|\psi_n\rangle_{\rm A} =| \psi_n,\ldots,\psi_3,\psi_2,\psi_1\rangle$, and the first two parts of the mixed-five-spin chain. Here, it is assumed that the mixed-five-spin chain is initially in a separable state $|\psi\rangle_C =|\downarrow\circlearrowleft\downarrow\circlearrowleft\downarrow\rangle$ in which states of the spin-half and the spin-integer are set up in the S_z and J_z down states $|0\rangle =|-1/2\rangle =|\downarrow\rangle$ and $|-1\rangle =|\circlearrowleft\rangle$ respectively;

(ii) Quantum states are propagated through the channel in a particular time interval $t=\tau$;

(iii) After time interval $t=\tau$, the receiver (Bob) extracts the information sent by applying an SWAP operator $S_{\rm B}$. This operator couples Bob's memory with the latest two parts of the chain. In fact, after time $t=\tau$, this protocol can be described as a completely positive trace preserving (CPTP) mapping from input state density matrices to output state density matrices, in the form

$$\rho_{\rm A} \longrightarrow \mathcal{M}[\rho_{\rm A}] = {\rm Tr}^{\rm T_B}[U(\rho_{\rm A}\otimes\rho_{\rm C})U^{\dagger}], \tag{6}$$

where $\mathcal{M}$ represents the mapping scenario of a real memoryless channel and U is the unitary transformation which describes joint evolution of the composite Hamiltonian $H_{\rm AC} = H^{\rm A}\otimes H^{\rm C}$, ${\rm Tr}^{T_{\rm B}}$ denotes the trace over all spins (sender A and the chain C) except receiver B. According to the definition of the mapping $\mathcal{M}$, it should be mentioned that the capacities of this map may not provide a good quantification of the communication scenarios of the spin chain, they can only be used for those scenarios where many parallel copies of the same state are simultaneously operated by the sender and receiver of the communication protocols, while they may fail to describe those scenarios where multiple uses of the same spin chain is applied if the spin chain system is not reset after each use.[14]

In the present work, a scenario is considered where the sender and the receiver use their spins belonging respectively to the first and last pairs' spins $(1,1/2)$ of the mixed-five-spin chain system for encoding and decoding information (classical or quantum). From the communication theory viewpoint, this consideration is not persuasive enough, but on the other hand, the consequences can be treated analytically.

For doing the transmission protocol, suppose that Alice has a memory with state $|\Psi\rangle_A = \cdots|\psi_3\rangle\otimes|\psi_2\rangle\otimes|\psi_1\rangle$, where $|\psi_j\rangle = \left(\alpha_j'|\downarrow\rangle+\beta_j'|\uparrow\rangle\right)\otimes|\circlearrowleft\rangle$ $(j=\{1,2,3,\ldots\}$ and $|\alpha_j'|^2+|\beta_j'|^2=1)$. By starting the transmission protocol $(t=0)$ by Alice via coupling first memory element $|\psi_1\rangle$ with the state of the first pair spins $(1,1/2)$ of the mixed-five-spin chain through SWAP gate $S_{\rm A}(1)$, the memory element $|\psi_1\rangle$ will replace with the state of the first two parts of the mixed-five-spin chain.

This procedure can be characterized as

$$\begin{aligned}&\left(\cdots|\psi_3\rangle\otimes|\psi_2\rangle\otimes|\psi_1\rangle\right)_{\rm A}\otimes|\downarrow\circlearrowleft\downarrow\circlearrowleft\downarrow\rangle_{\rm C}\otimes\left(|\downarrow\rangle\otimes|\downarrow\rangle\otimes|\downarrow\rangle\cdots\right)_{\rm B}\xrightarrow{S_{\rm A}(1)}\\ &\left(\cdots|\psi_3\psi_2\downarrow\circlearrowleft\rangle\right)_{\rm A}\left(\alpha_1'|\downarrow\circlearrowleft\downarrow\circlearrowleft\downarrow\rangle+\beta_1'|\uparrow\circlearrowleft\downarrow\circlearrowleft\downarrow\rangle\right)_{\rm C}\left(|\downarrow\rangle\otimes|\downarrow\rangle\otimes|\downarrow\rangle\cdots\right)_{\rm B}.\end{aligned} \tag{7}$$

After time evolution τ, the first memory element $|\psi_1\rangle$ is embedded in the first two parts of the chain after first SWAP $S_{\rm A}(1)$, then, spreads along the mixed-five-spin chain, thereby, the total state (7) becomes

$$\left(\cdots|\psi_3\psi_2\downarrow\circlearrowleft\rangle\right)_{\rm A}\otimes\left(\left(\alpha_1'|\downarrow\circlearrowleft\downarrow\circlearrowleft\downarrow\rangle+\beta_1'\Upsilon_{13}(\tau)|\downarrow\circlearrowleft\uparrow\circlearrowleft\downarrow\rangle+\beta_1'\Upsilon_{35}(\tau)|\downarrow\circlearrowleft\downarrow\circlearrowleft\uparrow\rangle\right)_{\rm C}\otimes\left(|\downarrow\downarrow\downarrow\cdots\rangle\right)_{\rm B}, \tag{8}$$

where

$$\begin{aligned}\Upsilon_{13}(\tau) &= {}_{\rm C}\langle\downarrow\circlearrowleft\downarrow\circlearrowleft\downarrow|\exp\left(\frac{-{\rm i}H\tau}{\hbar}\right)|\downarrow\circlearrowleft\uparrow\circlearrowleft\downarrow\rangle_{\rm C},\\ \Upsilon_{35}(\tau) &= {}_{\rm C}\langle\downarrow\circlearrowleft\uparrow\circlearrowleft\downarrow|\exp\left(\frac{-{\rm i}H\tau}{\hbar}\right)|\downarrow\circlearrowleft\downarrow\circlearrowleft\uparrow\rangle_{\rm C},\end{aligned} \tag{9}$$

in which, $\Upsilon_{13}(\tau)$ and $\Upsilon_{35}(\tau)$ are the probability amplitude of finding the spin up $(|\uparrow\rangle)$, respectively, in the 3rd and 5th parts of the mixed-five-spin chain, where ${\rm i}=\sqrt{-1}$. In the next section, we will use these statements to verify the transmission rate of the channel numerically.

4. Channel capacity and transmission rate

Quantum transmission rate for the introduced protocol in Fig. 1 can be obtained with regard to the quantum channel capacity of the memoryless channel map $\mathcal{M}[\rho]$. This capacity that was defined as maximum amount of the quantum information reliably transmitted per use of the channel,[22] was used for a spin chain channel in Ref. [11] and is given by

$$\mathcal{Q}(\eta) = \max_{p\in[0,1]}\{H_2(\eta p)-H_2((1-\eta)p)\}, \tag{10}$$

where $H_2(X) = -X\log_2(X)-(1-X)\log_2(1-X)$ is the dyadic Shannon entropy.

The rate of this channel for ε used in the time interval $T=\varepsilon\tau$ can be defined as

$$\mathcal{R} \equiv \lim_{\varepsilon\to\infty}\frac{\varepsilon\mathcal{Q}(\eta)}{\varepsilon\tau} = \frac{\mathcal{Q}(\eta)}{\tau}. \tag{11}$$

For our favorite mixed-five-spin system of the introduced model with Hamiltonian (1) as a communication channel, the transfer amplitude is a sinusoidal function of τ just the same as the one in Ref. [11], but with changeable period $\pi/2(1-{\rm i}D)$,

namely

$$\eta = |\Upsilon_{13}(\tau)\cdot\Upsilon_{35}(\tau)|^2 = |\sin(2(1-\mathrm{i}D)\tau)\cdot\sin(2(1-\mathrm{i}D)\tau)|^2. \quad (12)$$

By setting Eq. (12) in Eq. (10), we can get the quantum transmission rate (11).

Figure 2 shows the transmission rate of the mixed-five-spin chain where sender (Alice) has prepared her first memory element as state $|\psi_1\rangle = (\alpha_1'|\downarrow\rangle + \beta_1'|\uparrow\rangle)\otimes|\circlearrowleft\rangle$ and replaced it with the state of the first two parts of the mixed-five-spin chain (pair spins with labels 1 and 2 in Fig. 1) by means of the SWAP operation $S_{\mathrm{A}}(1)$. It should be noted that, equation (10) introduced dyadic Shannon entropy which is used for two different parts, for satisfying that here we consider first pair spins $(1,1/2)$ of the mixed-five-spin chain as one part and latest pair spins $(1,1/2)$ of it as another part.

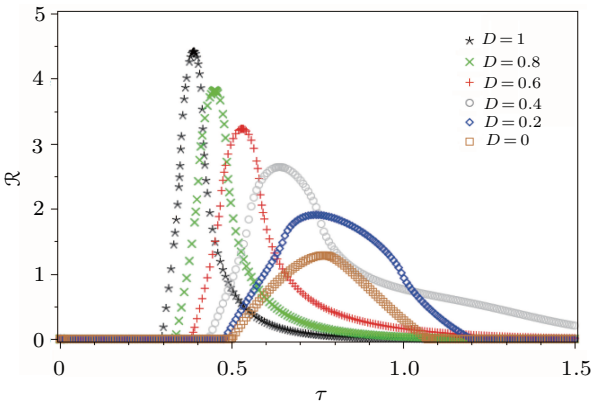


Fig. 2. (color online) Transmission rate for the mixed-five-spin chain where the first element of Alice's memory is state $|\psi_1\rangle = (\alpha_1'|\downarrow\rangle + \beta_1'|\uparrow\rangle)\otimes|\circlearrowleft\rangle$ which is propagated throughout the chain wholly.

With regard to this figure, for $D=0$ (boxed diagram) transmission rate starts from $\tau = 0.5$ and reaches its maximum amount at $\tau \approx 0.75$, then vanishes in $\tau \approx 1.05$. Here, we see a good compatibility between transmission rate of our favorite system at $D=0$ and that represented in Ref. [11]. With increase of the DM interaction $|D|$ (note that the transmission rate for $D<0$ is the same for $D>0$) from zero, transmission rate arises at the less time and reaches its maximum amount in the less time interval, but on the other hand, it vanishes at the longer time. It is clear that range of the maximum is proportional to $|D|$, namely, by increasing the DM interaction, the maximum amount of the transmission rate in addition to occur in the less time interval, increases.

Now, we investigate the transmission rate of the system when Alice prepares the state $|\psi_1\rangle = (\alpha_1'|\downarrow\rangle + \beta_1'|\uparrow\rangle)\otimes|\bigcirc\rangle$ as the first element of her memory. After applying the SWAP operator $S_{\mathrm{A}}(1)$ transfer amplitude can be characterized as

$$\eta' = |\Upsilon_{13}(\tau)\cdot\Upsilon_{35}(\tau)\cdot\Upsilon_{24}'(\tau)|^2 = |\sin(2(1-\mathrm{i}D)\tau)\cdot\sin(2(1-\mathrm{i}D)\tau)\cdot\sin(2\Lambda\tau)|^2. \quad (13)$$

By substituting Eq. (13) into Eq. (10), one can get the quantum transmission rate (11) for the mixed-five-spin chain. This protocol is precisely illustrated in Fig. 3. With regard to this figure, in the absence of the DM interaction (Fig. 3(a)), for various fixed values of Λ, transmission rate has different behavior in the time interval $0.5 < \tau < 1.1$. For better understanding the subject, we divide the transmission protocol into two stages. Stage one corresponds to the interval $0 \le \Lambda \le 1$. Here, for $\Lambda \approx 0$, any qutrit will not be transferred through the mixed-five-spin chain channel, therefore, transmission rate is zero. In $0 < \Lambda \le 1$ by increasing Λ, transmission rate starts at a particular transmission starting time and increases by passing of the time. With further passing of the time, the transmission rate decreases and ultimately finishes at a transmission stopping time. Moreover, in this region, the maximum amount of the transmission rate increases and shifts to the less time. Also, the time interval between starting and stopping of the transmission increases (from the asterisk diagram to the diagonal-cross one). The second stage corresponds to the interval $1 \le \Lambda \le 2$. Here, the maximum amount of the transmission rate and also time interval between starting and stopping of the transmission decrease simultaneously (from the diagonal-cross diagram to the next), until, at $\Lambda \approx 2$ any qutrit cannot be transferred through the channel again. The maximum amount of the transmission rate is shifted to the lesser times continuously.

Transmission rate in the vicinity of a weak DM interaction $D = 0.2$ is shown in Fig. 3(b). By inspecting this figure, one can observe that the behavior of the transmission rate is similar to the former case but here, in the second stage, by increasing Λ, the maximum amount of the transmission rate is shifted to the longer time, also, it is higher than the case of Fig. 3(a). Moreover, the transmission rate gradually vanishes and is stopped in longer time for all of Λ. Indeed, the existence of the DM interaction between spin-half of any cell causes the transmission to occur for the higher XX Heisenberg interaction Λ as well (see the red circular line related to $\Lambda = 2$). In the stronger DM interaction, equation (11) has amazing behavior, namely, at $D = 0.4$ (Fig. (3(c)) for fixed values of $\Lambda = 1.5$ and $\Lambda = 2$, the transmission rate has more than one maxima and with further increase of the DM interaction, these maxima vanish gradually (Figs. 3(c)–3(f)), on the other hand, the number of the maxima initially increases for the case where $\Lambda = 1$ (Fig. 3(d)) then decreases for the DM interaction stronger than $D = 0.6$. Despite all this, the range of the transmission rate increases with the increase of the DM interaction. In conclusion, by inspecting Fig. 3, one can precisely realize that the DM interaction between spin-half of any cell can have profound influence in the transmission rate of the mixed-five-spin chain.

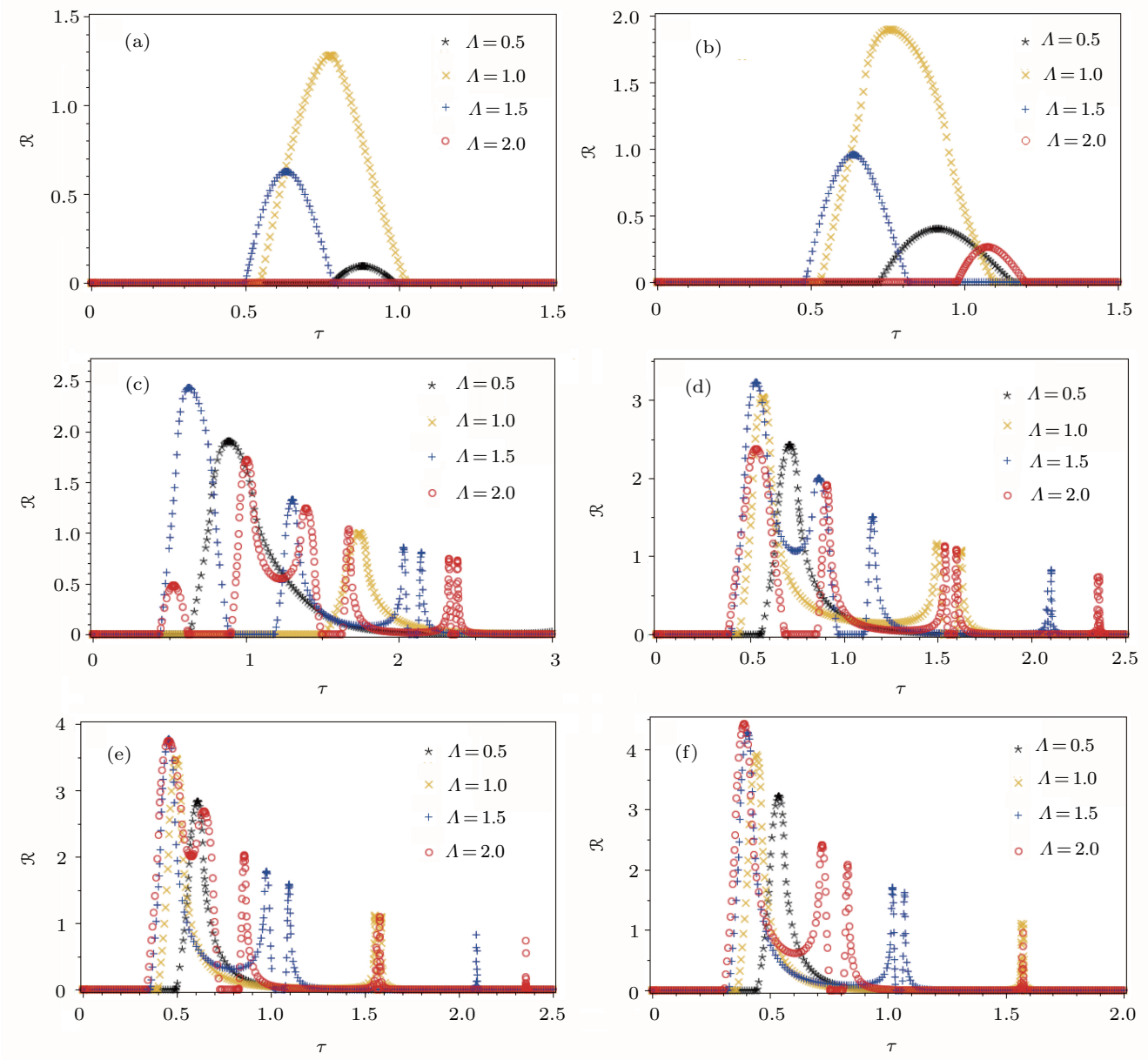


Fig. 3. (color online) Transmission rate for the mixed-five-spin chain where the first element of Alice's memory is state $|\psi_1\rangle = (\alpha_1'|\downarrow\rangle + \beta_1'|\uparrow\rangle) \otimes |\bigcirc\rangle$, which is propagated throughout the chain wholly for different D values: (a) $D = 0$; (b) $D = 0.2$; (c) $D = 0.4$; (d) $D = 0.6$; (e) $D = 0.8$; and (f) $D = 1$.

In the introduction of this favorite model, we consider some irrelevant coefficients such as a homogeneous magnetic field B in the z direction, XY-type interaction for spin-half of any triangular cell and Ising-type interaction between spins $(1,1/2)$ of any cell, which actually did not play any role to calculate and simulate the transmission rate, our motivation for citing them is introducing theoretically an upstanding spin system with a specific structure, by which its components have some special interactions together. Of course, by means of this citation, we interestingly realized that the transmission rate for this specific spin system is independent of the magnetic field B, anisotropy parameter γ, and Ising interaction coefficient J. It must be noted that these coefficients are ineffective until the initial state of the system is pure product state $|\psi\rangle_C = |\downarrow\bigcirc\downarrow\bigcirc\downarrow\rangle$, with change of this state some other ineffective coefficients may be involved in the computation of the transmission rate.

5. Summary and conclusion

In this paper, we tried to introduce theoretically a new class of spin chain channel for transferring qutrits. In this way, we have introduced a one-dimensional mixed-five-spin chain with Ising-XY model which contains both permanently coupled spins-1/2 and spins-1 states. It has been assumed that nearest spins-1/2 have both XY-type and Dzyaloshinskii–Moriya (DM) interactions, nearest spins $(1,1/2)$ have Ising-type interaction and nearest spins-1 (spins-1 of two adjacent triangular cells) have XX Heisenberg interaction. The system is in the vicinity of a homogeneous magnetic field and has an analytical Hamiltonian. Then, we have considered this system as a memoryless quantum communication channel with

an initial pure product state and explained quantum information transmission protocol, in what follows we have calculated the quantum capacity of the channel and, finally, obtained the quantum information transmission rate.

According to our investigations, we concluded that for such a mixed-five-spin chain, the transmission rate is solely dependent on the DM interaction D between nearest spins-1/2 when the sender Alice prepared her memory with state $|\psi_1\rangle = (\alpha_1'|\downarrow\rangle + \beta_1'|\uparrow\rangle) \otimes |\circlearrowleft\rangle$, and it is dependent on both DM interaction D and XX Heisenberg interaction parameter Λ related to the nearest spins-1 when Alice prepared her memory with state $|\psi_1\rangle = (\alpha_1'|\downarrow\rangle + \beta_1'|\uparrow\rangle) \otimes |\bigcirc\rangle$. Also, we realized that the behavior of the transmission rate is different for various fixed values of the DM and XX Heisenberg interactions. Indeed, by increasing the DM interaction $|D|$, the maximum amount of the transmission rate increases and shifts to the less time for both Alice's memory-prepared states. On the other hand, for all the fixed values of the DM interaction $|D|$, with increase of the XX Heisenberg interaction Λ the transmission rate behavior in interval $0 \leq \Lambda \leq 1$ is different from $1 \leq \Lambda \leq 2$ when sender Alice prepared her memory with state $|\psi_1\rangle = (\alpha_1'|\downarrow\rangle + \beta_1'|\uparrow\rangle) \otimes |\bigcirc\rangle$. Moreover, we found that with increase of the DM interaction from zero, the transmission rate can also occur for the higher XX Heisenberg interaction $\Lambda > 1.5$. The multiplicity of the transmission rate maxima at different times increases with the increase of the DM interaction $|D| > 0.2$ as well. With further increase of the DM interaction, the range of these maxima increases, while their multiplicity decreases and tends to the less time. This rate has no dependence on the magnetic field, anisotropy parameter γ between nearest spins-half and Ising interaction between spins $(1, 1/2)$ of any cell.

Therefore, if one chooses such a mixed-five-spin chain system with initial pure product state as a communication channel for transferring quantum qutrits states ideally, he (or she) should investigate the transmission protocol in the strong DM interaction regardless of existence of any magnetic field in the z direction, anisotropy related to the nearest spins-half and Ising-type interaction between nearest spins $(1, 1/2)$. Moreover, since the transmission rate of the suggested model is sensitive to the XX interaction between spins-1, use of this model as quantum communication channel might be more attractive.

It is more interesting to investigate this model with more spins ($N > 2$) and different interactions between its members. Also, one can verify the transmission rate for such a model but with mixed-six-spin which contains 3 spins-integer and 3 spins-half and gets some novel and attractive outcomes.

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