Scheme for preparation of multi-partite entanglement of atomic ensembles*

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We show a scheme of preparing multipartite W type of maximally entangled states among many atomic ensembles with the generation time increasing with the party number only polynomially. The scheme is based on laser manipulation of atomic ensembles and single-photon detection, and fits well the status of the current experimental technology. We also show one of the applications of this kind of W state, demonstrating Bell theorem without inequalities.

Keywords: multi-partite entanglement, atomic ensembles

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1. Introduction

Entanglement has been studied very extensively as it impacts both foundational as well as applied aspects of quantum theory.[1] Moreover, multi-partite entanglement plays a central role in a host of quantum technologies, including metrology, imaging, communication, and quantum information processing.[2–8] Many schemes for generation of the multi-particle entangled state have been proposed.[9–19] Among all the physical systems which have been considered for generating multi-particle entangled states, atomic ensembles are well developed and regarded as ideal candidates for quantum information processing, especially for quantum state engineering. Recently, breakthrough experimental achievements on atomic ensembles[20–27] have been reported, which attracts more attention to this physical system.

We present a scheme of preparing multipartite W type of maximally entangled states among many atomic ensembles with the generation time increasing with the party number only polynomially. In addition, a proof of Bell theorem[28] without inequalities specifically for the W states can be realized experimentally. The scheme involves the following features: firstly, it is robust to realistic noise and imperfections. As a result, the physical requirements of this scheme are moderate and well fit the experimental technique. Secondly, in contrast to the belief that an entanglement preparation scheme based on post-selection will suffer from the fast exponential degradation of the efficiency,[9,10] we design this scheme with an efficient scaling and make it possible to maximally entangle many ensembles with the current technology. Finally, by making use of quantum memory available in atomic internal levels, the W states can be put into many applications such as demonstrating quantum non-locality, and a quantum channel for perfect teleportation.[29]

2. The protocol for preparation of the W-type multi-particle entangled states

Let us have a look at the generalized form $|W_M\rangle$ of the W state in multi-qubit systems. In Ref. [30], the state is defined as

$$|W_M\rangle = \frac{1}{\sqrt{M}}|M-1,1\rangle, \tag{1}$$

where $|M-1,1\rangle$ denotes the totally symmetric state including $M-1$ zeros and 1 one. For example, we obtain $M=3$,

$$|W_3\rangle = \frac{1}{\sqrt{3}}(|001\rangle + |010\rangle + |100\rangle). \tag{2}$$

The basic element of this scheme is an ensemble of many identical alkali atoms with a Raman type Λ-level configuration coupled by a pair of optical fields, shown as Fig. 1.

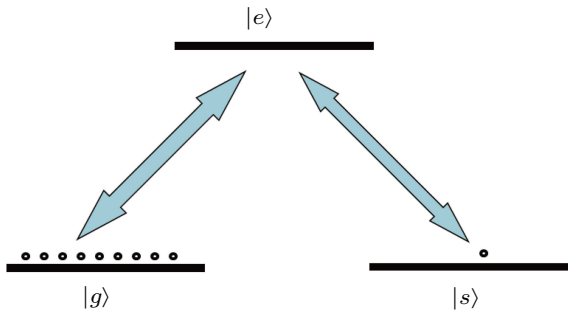


Fig. 1. (color online) The relevant type Λ-level structure of the alkali atoms in the ensembles. A pair of meta-stable lower states $|g\rangle$ and $|s\rangle$ can be achieved, for example, in hyperfine or Zeeman sublevels of electronic ground states, and $|e\rangle$ is the excited state.

The experimental realization can be either a room-temperature dilute atomic gas[31–33] and a sample of cold trapped atoms.[34,35] We continue to use the symbols and corresponding definitions shown in Refs. [36] and [37]. There are

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two kinds of laser pulses (pumping laser and repumping laser) applied to the atomic ensembles, which correspond to Raman transition $|g\rangle \rightarrow |s\rangle$ and anti-Raman transition $|s\rangle \rightarrow |g\rangle$, respectively. A weak pumping laser is shone on all atoms so that each atom has an equal small probability to be excited into the state $|s\rangle$ through the Raman transition. After the atomic gas interacts with the pumping laser, there will be a special atomic mode $\hat{S}$ called the symmetric collective atomic mode

$$\hat{S} = \frac{1}{\sqrt{N_a}} \sum_{i=1}^{N_a} |g\rangle_i \langle s|, \tag{3}$$

where $N_a \gg 1$ is the total atom number. In particular, an emission of the single Stokes photon in a forward direction results in the state of atomic ensembles given by $\hat{S}^\dagger|\text{vac}\rangle$, where the ensemble ground state $|\text{vac}\rangle = \otimes_i |g\rangle_i$. The scheme for preparation of W type of maximally entangled states between atomic ensembles works in the following way shown in Fig. 2. We choose the case in which three atomic ensembles are entangled in the W state $|W\rangle_{123}$ shown as Eq. (2) by way of example.

Firstly, the three atomic ensembles are prepared to the ground state $|g\rangle$, i.e., $|\text{vac}\rangle_{123} = |\text{vac}\rangle_1 \otimes |\text{vac}\rangle_2 \otimes |\text{vac}\rangle_3$ and illuminated by the classical pumping laser pulses instantaneously. Each ensemble will be excited to be in the state $\hat{S}^\dagger|\text{vac}\rangle$ with probability p_c. The forward scattering Stokes photons from optical excitations of the ensembles are combined at the three BSs with transmittances and reflectances T_i and R_i (here $i = 1,2,3$, and $T_i^2 + R_i^2 = 1$) after some filters which filter out the pumping laser pulses, and the outputs are detected by three single-photon detectors. We find that through selecting the certain BSs with the proper transmittances and reflectances, the three ensembles are entangled in the W state if there is a click in either one of the detectors D_2 and D_3.

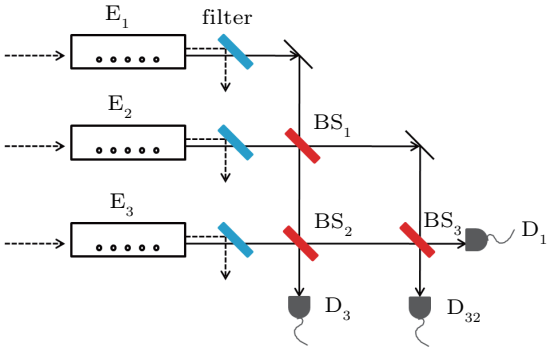


Fig. 2. (color online) Setup of the modified preparation scheme. The classical pumping laser pulses are applied to the three atomic ensembles instantaneously. If either one of the detectors D_2 and D_3 clicks, the ensembles are entangled on the W state.

If the detector D_1 clicks, the state of the atomic ensembles is given in

$$\left[(R_3R_1 + T_3R_2T_1)\hat{S}_1^\dagger + e^{i\phi_{12}}(R_3T_1 + T_3R_2R_1)\hat{S}_2^\dagger + e^{i\phi_{13}}T_3T_2\hat{S}_3^\dagger\right]|\text{vac}\rangle_{123}. \tag{4}$$

If the detector D_2 clicks, the state is given in

$$\left[(T_3R_1 + R_3R_2T_1)\hat{S}_1^\dagger + e^{i\phi_{12}}(T_3T_1 + R_3R_2R_1)\hat{S}_2^\dagger + e^{i\phi_{13}}R_3T_2\hat{S}_3^\dagger\right]|\text{vac}\rangle_{123}. \tag{5}$$

Otherwise, if D_3 clicks, the ensembles are put in the state shown as

$$\left[T_2T_1\hat{S}_1^\dagger + e^{i\phi_{12}}T_2R_1\hat{S}_2^\dagger + e^{i\phi_{13}}R_2\hat{S}_3^\dagger\right]|\text{vac}\rangle_{123}. \tag{6}$$

Once the transmittances and reflectances of the BSs are chosen as $T_1 = R_1 = \sqrt{2}/2$, $T_2 = \sqrt{6}/3$, $R_2 = \sqrt{3}/3$, and $T_3 = 1/2$, $R_3 = \sqrt{3}/2$, there are only two cases in which either D_2 or D_3 clicks and D_1 does not click and we can obtain the W state $|W\rangle_{123}$ among the atomic ensembles.

To realize an arbitrary ratio of transmittance and reflectance of the BSs experimentally, one only needs to add a tunable quarter-wave plate (QWP) and half-wave plate (HWP) in front of the 50/50 BS, changing the circular polarization of the Stokes photons from optical excitation into linear polarization. That is, the Stokes photons are in $T|H\rangle + e^{i\varphi}R|V\rangle$, where H and V, respectively, represent the horizontal and vertical polarizations, and T, R, and φ are fixed by the relative inclination between the optical axes of the QWP and HWP. The outputs go through a polarizing beam splitter (PBS) before the single-photon detectors. By adjusting the optical axes of the QWP and HWP, one can obtain an arbitrary ratio of the Stokes photons from the optical excitation with the horizontal and vertical polarizations, respectively.

The scheme of generation of three-particle W-type entangled states can be extended to generate n-particle W states via BSs with proper transmittances and reflectances. If either of the detectors D_i $(i = 2,\ldots,n)$ clicks, the rest and D_1 do not click, we obtain an n-particle W state $|W_n\rangle$.

Now we calculate the efficiency of this scheme for preparing three-party W state. In the generation process, the dominant noise is the photon loss, which includes the contributions from the channel attenuation, the spontaneous emissions in the atomic ensembles, the coupling inefficiency of Stokes light into and out of the channel, and the inefficiency of the single-photon detectors which cannot perfectly distinguish between one and two photons. All the above noise is described by an overall loss probability η. Since the dominant realis-

tic noise — loss of excitations — only has influence on the success probability to register an excitation from each ensemble, once the excitation is registered, we obtain a perfect W state (2). Hence, the entanglement preparation time should be $t_1 \sim 3t_\Delta/2(1-\eta)p_c$ as high as that of the bipartite EPR state.[36] To entangle n ensembles in the W state, we only need more BSs with proper transmittances and reflectances and n single-photon detectors. However, the total generation time is approximated by $T \sim t_\Delta/[f(n)(1-\eta)p_c]$, where $f(n) \geq 1/n$ $(n \geq 3)$, which increases with n polynomially. Compared to the previous scheme of generation of W-type multi-particle entangled states,[9] in which the total generation time is $\sim t_0/(1-\eta)^{2n-1}$ and increases with the number of ensembles exponentially by the factor $1/[(1-\eta)^2 p_c]$, the efficiency of the present scheme is much improved.

Also with the noise, the state of the ensembles is actually described by

$$\rho_n = \frac{1}{c_n+1}(c_n\rho_{\rm vac} + |W\rangle_{1\cdots n}\langle W|), \tag{7}$$

where the vacuum coefficient c_n is basically given by the conditional probability for the inherent mode-mismatching noise contribution and $\rho_{\rm vac}$ stands for the vacuum component with no excitation in the ensembles. As we mentioned before, the dominant realistic noise is loss of excitations which only has influence on the success probability to register an excitation from each ensemble, the coefficient c_n can be estimated from the decoherence rate which is caused by the imperfection of the interferometer built by BSs and the imperfections of the optical elements. We use three-qubit W state as an example. The accuracy of each BS with an arbitrary ratio of transmittance and reflectance which is implemented by wave-plates, 50/50 BS and PBSs is about 0.997. The visibility of the interferometer built by three BSs with certain ratios of transmittance and reflectance can reach 0.993.[38] Thus the fidelity of the three-qubit W state can be estimated numerically as about 0.989.

We briefly discuss the practical implication of this proposal. With the improved scheme, for example, we can generate a high fidelity W entanglement over $n = 20$ ensembles in a time $T \sim 300$ μs with a notable loss $\eta \sim 1/3$ and a typical choice of $p_c \sim 0.01$, $t_\Delta \sim 1/f_p$, where $f_p = 10$ MHz is the repetition frequency of the Raman pulses. With such a short preparation time, the noise that we have not included, such as the non-stationary phase, drift induced by the pumping laser or by the optical channel, the single bit rotation error ($< 10^{-4}$) and the dark count probability (about 10^{-5} in a typical detection time window 0.1 μs) of the single-photon detectors can be negligible.

3. Demonstration of Bell theorem

Since the W state shown in Eq. (2) is entangled in the Fock basis, it is experimentally hard to do certain single-bit rotation. In the following we will show how the W state can be used to realize the communication protocols, such as demonstration of quantum non-locality, with simple experimental configurations shown as Fig. 3.

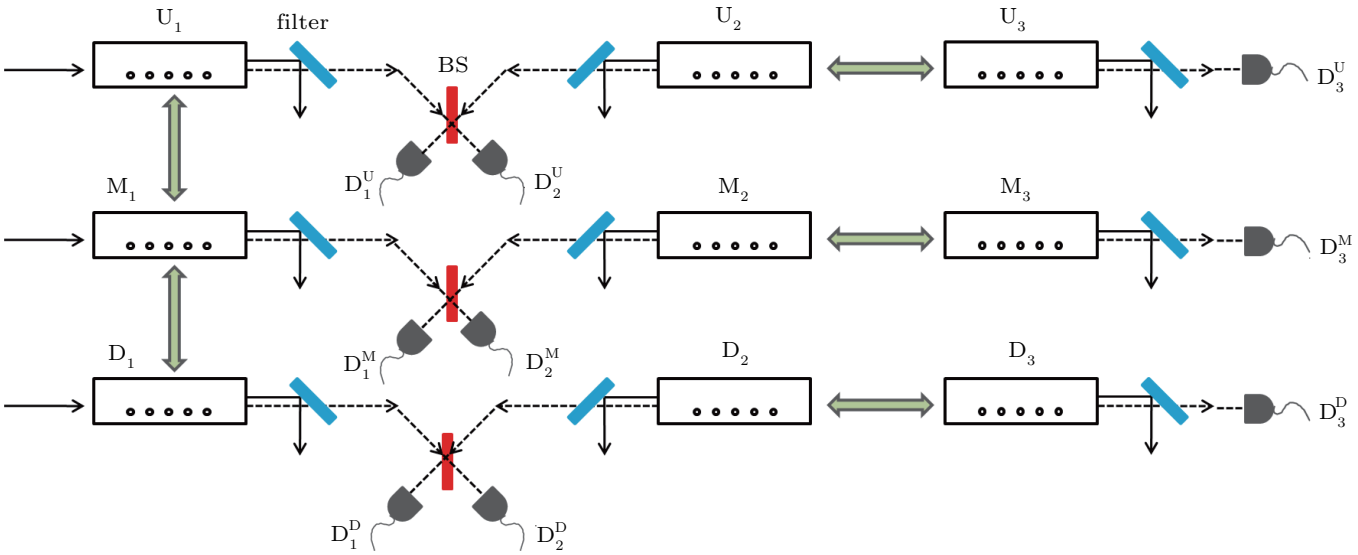


Fig. 3. (color online) Schematic setup for the realization of demonstration of Bell theorem with the W state.

The first step is to share an EPR type of entangled state

$$|\psi\rangle_{\Lambda_2\Lambda_3} = (\hat{S}^\dagger_{\Lambda_2} + e^{i\phi_{\Lambda_2\Lambda_3}}\hat{S}^\dagger_{\Lambda_3})/\sqrt{2}|{\rm vac}\rangle_{\Lambda_2\Lambda_3}$$

(here Λ = U, M, D) between three pairs of distant ensembles U_2 and U_3, M_2 and M_3, D_2 and D_3. Then the other ensembles U_1, M_1, and D_1 are prepared in the W state

$$|W\rangle_{U_1M_1D_1} = \frac{\hat{S}^\dagger_{U_2} + e^{i\phi_{U_1M_1}}\hat{S}^\dagger_{M_1} + e^{i\phi_{U_1D_1}}\hat{S}^\dagger_{D_1}}{\sqrt{3}|{\rm vac}\rangle_{U_1M_1D_1}}$$

shown in Eq. (2). Through the post-selection technique, the

product state of the whole system $\prod_{\Lambda}|\psi\rangle_{\Lambda_2\Lambda_3}\otimes|W\rangle_{\mathrm{U_1M_1D_1}}$ can be easily prepared, and whenever it is put into application, the effective "polarized" W state is yielded. In analogy to the language for photons, we can define a basis of the "polarization" qubit from the states

$$|H\rangle_{\Lambda}=\hat{H}^{\dagger}_{\Lambda}|\mathrm{vac}\rangle_{\Lambda}=\hat{S}^{\dagger}_{\Lambda_2}|\mathrm{vac}\rangle_{\Lambda_1\Lambda_2\Lambda_3},$$

$$|V\rangle_{\Lambda}=\hat{V}^{\dagger}_{\Lambda}|\mathrm{vac}\rangle_{\Lambda}=\hat{S}^{\dagger}_{\Lambda_1}\hat{S}^{\dagger}_{\Lambda_3}|\mathrm{vac}\rangle_{\Lambda_1\Lambda_2\Lambda_3}, \tag{8}$$

where $|\mathrm{vac}\rangle_{\Lambda}=|\mathrm{vac}\rangle_{\Lambda_1\Lambda_2\Lambda_3}$. In the expansion of the product state, there are only three components which have one and only one excitation on each pair of ensembles Λ_1 and Λ_2. This component state is given by

$$|W\rangle_{\mathrm{eff}}=\left[\mathrm{e}^{\mathrm{i}(\phi_{\mathrm{U_1D_1}}+\phi_{\mathrm{D_2D_3}})}\hat{H}^{\dagger}_{\mathrm{U}}\hat{H}^{\dagger}_{\mathrm{M}}\hat{V}^{\dagger}_{\mathrm{D}}+\mathrm{e}^{\mathrm{i}(\phi_{\mathrm{U_1M_1}}+\phi_{\mathrm{M_2M_3}})}\hat{H}^{\dagger}_{\mathrm{U}}\hat{V}^{\dagger}_{\mathrm{M}}\hat{H}^{\dagger}_{\mathrm{D}}+\mathrm{e}^{\mathrm{i}\phi_{\mathrm{U_2U_3}}}\hat{V}^{\dagger}_{\mathrm{U}}\hat{H}^{\dagger}_{\mathrm{M}}\hat{H}^{\dagger}_{\mathrm{D}}\right]|\mathrm{vac}\rangle_{\mathrm{UWD}}, \tag{9}$$

which is exactly the three-particle W type of maximally entangled state in the "polarization" basis. Note that for any practical application of the W type of entanglement, the state preparation should be succeeded by a measurement of the "polarization" of the excitation on each pair of ensembles Λ_1 and Λ_2, which can be done for our system by combining single-bit rotation (such as "σ_x") and the number detection of the single photon detectors.

The preparation of the EPR state is probabilistic, however, due to the available quantum memory provided by the meta-stable atomic mode $\hat{S}$, the preparation time of the state is at most $3t_0$ if the EPR states are prepared one by one, and can be reduced to t_0 if the EPR states are prepared independently at the same time. Also, the W state is prepared synchronously with the generation time $t_1>t_0$. After the preparation of the product state, the projection efficiency (success probability) from the product state to the effective "polarized" W state is given by $(1-\eta)^3/8$. So the total time from registering the three-party "polarized" W entanglement is $T\sim 8t_1/(1-\eta)^3$.

To demonstrate quantum non-locality, firstly, we perform an operation "σ_x" on each of the three qubits resulting in the value $x_{\Lambda}\in\{1,-1\}$ with equal probability. The repumping lasers with the frequencies ω_{repump} are applied on the ensembles Λ_1 and Λ_2, respectively, and the outputs are combined at the 50/50 BSs. We only consider the cases in which there is one excitation on each pair of ensembles Λ_1 and Λ_2. If a forward scattering Stokes photon is detected by the detector D^{Λ}_1, we get the value $x_{\Lambda}=1$; otherwise, if the detector D^{Λ}_2 clicks, the result of the operation is -1. At the same time, we prove that the state preparation succeeds. Then we go on with the next step. Otherwise, we repeat all the above process until we succeed. For the operation "σ_z", we perform a repumping laser on the ensembles Λ_3. If the detector D^{Λ}_3 clicks, we assume $z_{\Lambda}=1$; otherwise, if there is no photon detected, the value is -1. According to Ref. [28], the proof can be presented in the relevant properties of the W state

$$P_{\mathrm{W}}(z_i=-1,z_j=-1)=1,$$
$$P_{\mathrm{W}}(x_j=x_k|z_i=-1)=1,$$
$$P_{W}(x_i=x_k|z_j=-1)=1,$$

$$P_{\mathrm{W}}(x_i=x_j=x_k)=\frac{3}{4}, \tag{10}$$

where $i,j,k\in\{\mathrm{U},\mathrm{M},\mathrm{D}\}$, and $i\neq j\neq k$. P_{W} $(z_i=-1,z_j=-1)$ means the probability of two qubits giving the result -1 when measuring σ_z on all three qubits, and P_W $(x_j=x_k|z_i=-1)$ means the conditional probability of σ_{xj} and σ_{xk} having the same result given that the result of σ_{zi} is -1. In EPR's criterion of elements of reality, the equation $x_i=x_j=x_k$ is always satisfied. However, according to quantum mechanics, when measuring σ_x on all qubits, we get the probability $3/4$. Then we can easily realize a proof of Bell theorem for the W state in experiment.

4. Conclusion

Finally, we have a brief conclusion. In this paper, we describe a scheme of preparation of entangling many atomic ensembles in the W type of maximally entangled states through laser manipulation. Remarkably, the generation time increases with the party number only polynomially. Due to the efficient scaling of this scheme, one can use it to steadily entangle many atomic ensembles in the W state with the current experimental technology. Hence, such an extraordinary possibility opens up prospects for many exciting experiments and applications.

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