Effect of disorders on topological phases in one-dimensional optical superlattices*

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In a recent paper, Lang *et al.* proposed that edge states and topological phases can be observed in one-dimensional optical superlattices. They showed that the topological phases can be revealed by observing the density profile of a trapped fermion system, which displays plateaus with their positions. However, disorders are not considered in their model. To study the effect of disorders on the topological phases, we introduce random potentials to the model for optical superlattcies. Our calculations show that edge states are robust against the disorders. We find the edge states are very sensitive to the number of the sites in the optical superlattice and we propose a simple rule to describe the relationship between the edge states and the number of sites. The density plateaus are also robust against weak disorders provided that the average density is calculated over a long interval. The widths of the plateaus are proportional to the widths of the bulk energy gaps when there are disorders. The disorders can diminish the bulk energy gaps. So the widths of the plateaus decrease with the increase of disorders and the density plateaus disappear when disorders are too strong. The results in our paper can be used to guide the experimental detection of topological phases in one-dimensional systems.

Keywords: edge state, topological phase, optical superlattices

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1. Introduction

Topological properties have long been used to characterize the two-dimensional (2D) electron systems.[1–11] For example, the integer quantum Hall (IQH) effect and the quantum spin Hall (QSH) effect in the 2D system with periodic potentials can both be related to the topological properties of occupied energy bands.[2–7] A topological invariant called the Chern number can be used to characterize the integer quantum Hall effect and the Chern number is directly related to the quantum Hall conductance. The topology of the occupied bands of the 2D QSH system is characterized by a Z_2 topological invariant.[6] The systems exhibit QSH states are topological insulators. Both of the IQH and the QSH systems have bulk energy gaps and topologically protected gapless edge states.[12]

Recently, the ultra cold atomic systems have been used to simulate the topological insulators. Optical superlattices populated with cold atoms are proven to be a powerful tool to study the topological properties of the 2D systems.[13–15] Lang *et al.* proposed that the one-dimensional (1D) quasiperiodic optical lattice populated with trapped fermions can display nontrivial topological properties.[16] The localized edge states appear in the energy gaps of 1D optical lattices with the open boundary condition, which indicates that the bulk 1D system is topologically nontrivial. They also showed that the topological property of the 1D system can be detected by observing the density profile of the trapped fermions, which displays plateaus with positions. Hence, their work opens the possibility to observe the topological phase in the 1D system.

It is generally believed that the topological properties are robust against weak disorders.[17,18] For example, the numerical results show that the edge states remain gapless with weak time reversal symmetric disorders in the QSH state.[18] The edge states become gapped only when the disorders are strong enough to make the bulk gap collapse. In this paper, we check whether the 1D topological phases are robust against disorders. For this purpose, we introduce random potentials to the 1D system. We study the effect of the disorders on both of the bulk and edge properties of the 1D system. The edge states and the density plateaus are simulated with different strengths of disorders. We hope our work may provide some insight into the 1D topological phases.

2. Effect of disorders on the bulk properties of the optical superlattice

We consider the tight-binding model for the 1D polarized Fermi gas loaded in a bichromatic optical lattice as in Refs. [16] and [19]–[21]

$$\hat{H} = -t\sum_i (\hat{c}_i^{\dagger}\hat{c}_{i+1} + \text{H.c.}) + \sum_{i=1}^{L} V_i \hat{n}_i, \tag{1}$$

with

$$V_i = V\cos(2\pi\alpha i + \delta) + V_{\text{r}} r_i, \tag{2}$$

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where L is the number of the lattice sites, $\hat{c}_i^\dagger$ ($\hat{c}_i$) is the creation (annihilation) operator of the fermions, $\hat{n}_i = \hat{c}_i^\dagger \hat{c}_i$ is the fermion number operator for a given site, t is the hopping amplitude which is set to be the unit of the energy ($t = 1$), V is the strength of periodic potential, $\alpha = p/q$ is a rational number (the irrational case is not considered in this paper), p and q are relatively prime integers, q is the minimum period of the potential, which means there are q sites in one primitive cell, and δ is the phase factor. When δ changes by δ_0 the potential will be translated by $\delta_0/(2\pi\alpha)$. The V_r is the strength of the disordered random potentials. r_i is the random numbers. We assume that r_i uniformly distribute between -0.5 to 0.5 in the simulations.

This 1D model can be easily mapped to a 2D model when we identify δ with one component of the wave vector k_y of the 2D system,[16] so the topology of the 1D model can be studied by using the topological concepts for the 2D system. For example the Chern number can still be defined for the 1D system if we consider δ as a variable parameter. In this sense we still use the 2D topological properties to study the topology of our 1D system.

Without disorders, that is $V_r = 0$, and if the periodic boundary condition is used the eigenstates are Bloch waves. In this case the energies and the eigenstates are functions of the crystal momentum k and they form energy bands.[16] When the random potentials are turned on, the translational symmetry will be broken and the crystal momentum will no longer be a good quantum number to describe the eigenstate.

With the periodic boundary condition the numerical results show that the random potentials affect the system in two ways. First, the random potentials broaden the energy bands and thus shrink the energy gaps. The energy spectra for $p = 1$, $q = 3$, and $L = 1000q$ with different random potentials are shown in Fig. 1. The band broadening and the gap shrinking can be seen both from the energy spectrum and the density of states (DOS).

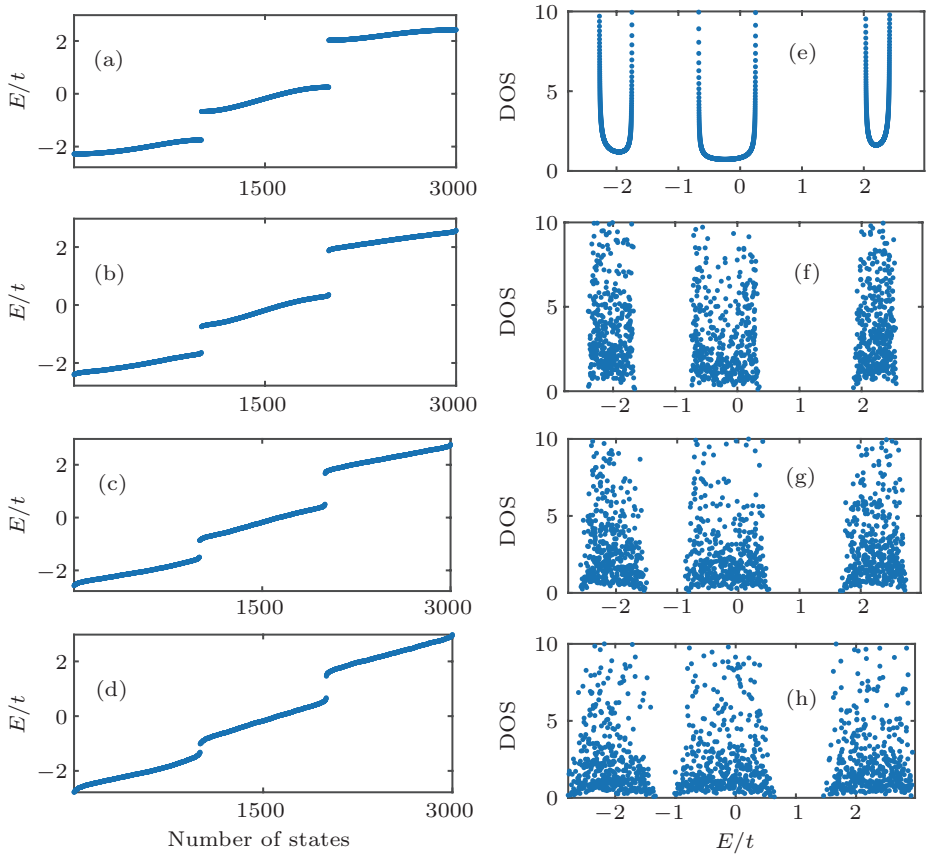


Fig. 1. (color online) (a)–(d) Spectra of the Hamiltonian (1) with $p = 1$, $q = 3$, $V = 1.5$, and $\delta = 2\pi/3$. Periodic boundary conditions are used with 3000 cites. Random strength $V_r = 0$, 0.5, 1, and 1.5 respectively. (e)–(h) The DOS of the corresponding energy spectra.

As long as the random potentials do not become too large there are energy gaps between the energy bands. These results can be understood if we treat the weak random potentials as perturbations. In Fig. 1, only the results for given δ are depicted. However, numerical results show that the conclusion works for all δ. So when we map the 1D model to the 2D model the effect of the disorders on the 2D system can also be studied.

The second effect of random potentials is the eigenstates become localized. Anderson localization is a ubiquitous

phenomenon in the one-dimensional system when disorders present.[19,22] The numerical results show that the bulk states become localized and the localization length decreases when the strength of the disorder increases as is shown in Fig. 2. We find that the localization actually has little effect on the edge states and the density plateaus.

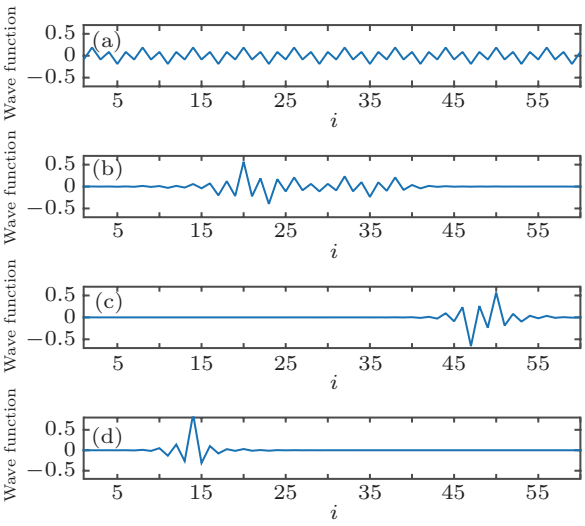


Fig. 2. (color online) (a)–(d) Wave functions at the band edges with $V_r = 0$, 0.5, 1, and 1.5, respectively. Periodic boundary conditions are used with 60 cites. Other parameters are the same as those in Fig. 1. The energy of each state is the highest energy of the lowest band in each case.

3. Effect of disorders on the edge states

If the open boundary condition is used there will be edge states at the ends of the optical superlattices. When there is no disorder the spectrum of the edge states as functions of δ are gapless, that is they continuously connect the upper and lower energy bands. The gapless edge states are related to the topology of the system which can be characterized by the Chern numbers of the occupied energy bands.[2,3,12,23,24]

We find that the edge states remain gapless even when disorders are relatively strong as is shown in Fig. 3. This result can be easily understood: the random potentials tend to make the eigenstates localized, however, the edge states are already localized, so they are less affected by the disorders.

From another point of view, when the energy of the edge state is located at the gap of the bulk energy band, it is energy costly to scatter fermions from the edge states to the bulk. However, we must be cautious when the disorders are strong enough and some edge states are merged to the bulk bands. In this case it is difficult to discriminate the edge states and the bulk states as in Figs. 3(c) and 3(d).

In Ref. [16], it is shown that the 1D system can be mapped to a 2D Hofstadter system. So the robustness of gapless edge states can be explained by the theory for 2D quantum Hall systems. In Ref. [25] it is shown that the Chern number can still be defined for the many-particle ground states when disorders are present as long as there is finite energy gap. So the gaplessness of the edge states can be related to this nontrivial topological invariant. We conclude that the edge states will remain gapless as long as the bulk energy gap is not closed by the disorders.

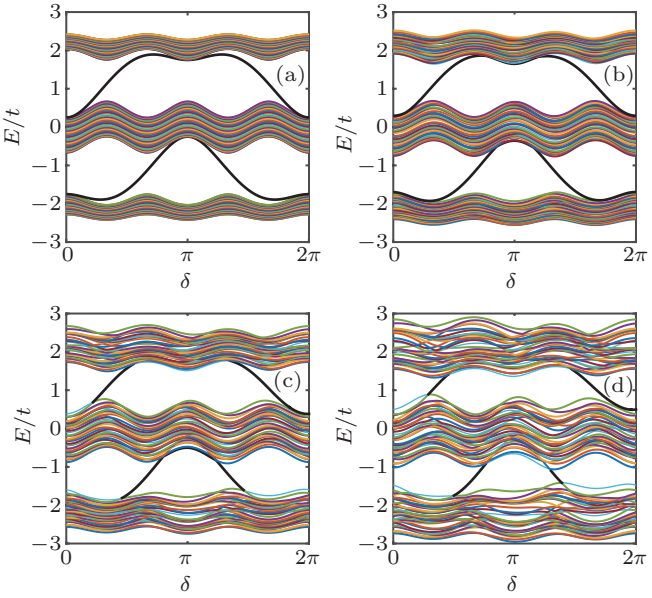


Fig. 3. (color online) (a)–(d) Open boundary energy spectra as functions of δ with $V_r = 0$, 0.5, 1, and 1.5, respectively. Black curves represent the edge state spectra. Chern numbers of the three bands from low to high are 1, −2, and 1 respectively.

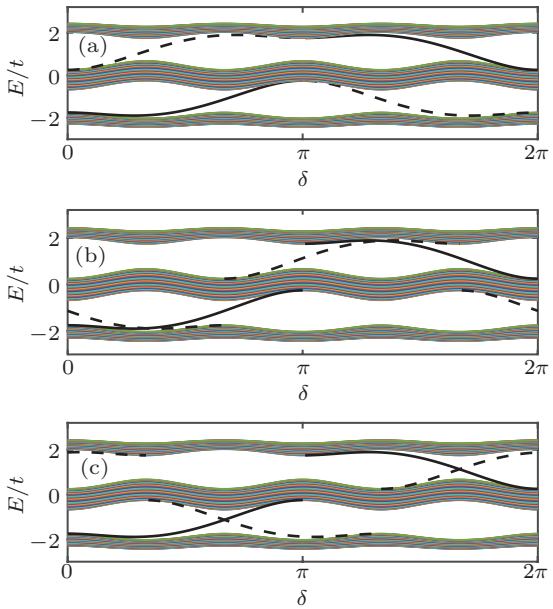


Fig. 4. (color online) (a)–(c) Open boundary energy spectra as functions of δ with 102, 101, and 100 sites respectively. Solid and dashed black curves denote edge states at left and right ends, respectively.

Though the topology of the edge states is not affected by weak disorders, we find that the edge states are very sensitive to the number of sites in the optical superlattice. When the number of sites is not a multiple of q, the missing sites can be considered as point disorders. As is shown in Fig. 4, the energies of the edge states at the right end as a function of δ

are translated to right by $2\pi p/q$ modulo 2π when one site is deleted from the lattices. The numerical results show this simple rule applies when p and q take arbitrary values. This result is not affected by the disorders when they are not too strong.

4. Effect of disorders on the density plateaus

As in Ref. [16], when there is confining potential, the V_i in Eq. (1) is given by

$$V_i = V\cos(\alpha 2\pi i + \delta) + V_{\rm H}(i - i_0)^2 + V_{\rm r} r_i. \tag{3}$$

The density distribution is given by $n_i = \langle \Psi_{\rm F}^{\rm G} | \hat{n}_i | \Psi_{\rm F}^{\rm G} \rangle$ with $\Psi_{\rm F}^{\rm G}$ the ground state wavefunction. The local density is defined by $\bar{n}_i = \sum_{j=0}^{M-1} n_{i+j}/M$.[16] In this paper, M is always chosen to be a multiple of q, that is, the density is averaged within an integer number of prime cells. This choice is justified by the observation that the density distribution is an approximate periodic function with a prime cell as the minimum period in the plateau range.

When there is no disorder, we choose $M = q$. The local density is depicted as a function of sites and the number of Fermions N in Fig. 5(a). We see that the local density form 2D plateaus and as long as N is not too small there is always 1D plateaus for given N. When N increases the number of the plateaus increases discontinuously. However, when N is large enough, the central plateau will dominate and other plateaus become very small. When N equals to the number of the sites, there is only one plateau with height 1. This result can be understood by using the local-density approximation (LDA) as in Ref. [16].

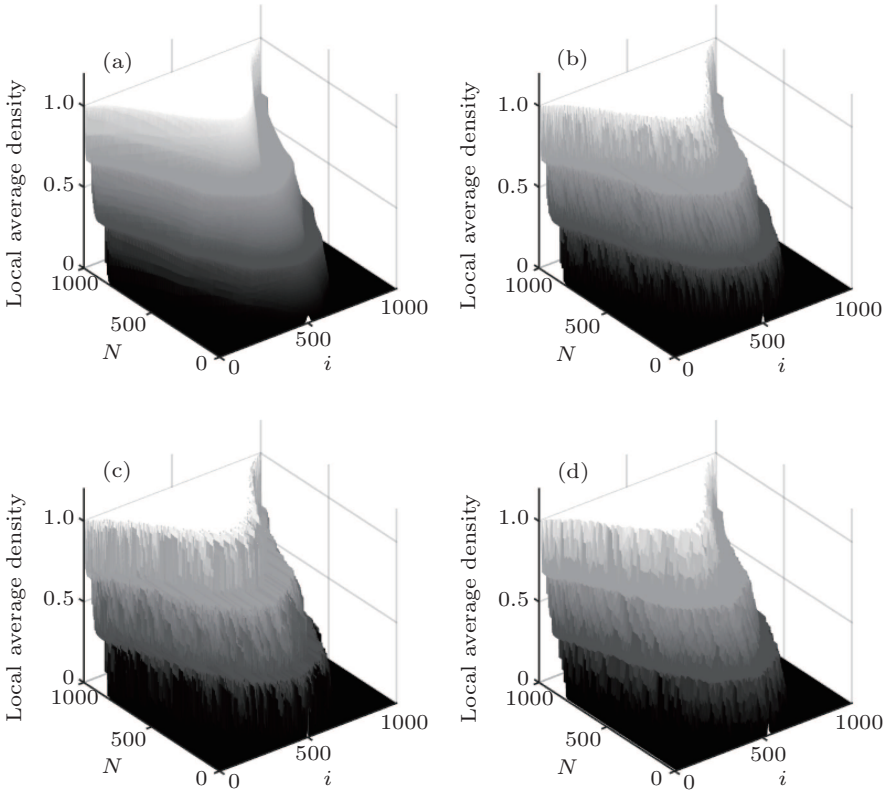


Fig. 5. (a)–(d) Local average densities as functions of both i and Fermion numbers N. Here, $V = 1.5$, $\delta = 0$, and $V_{\rm H} = 3 \times 10^{-5}$. (a)–(c) $M = q = 3$ and $V_{\rm r} = 0$, 0.5, and 1, respectively. (d) $M = 3q = 9$ and $V_{\rm r} = 1$.

To study the effect of the disorders on the density plateaus we plot the local density with different strengths of disorders. As is shown in Figs. 5(b) and 5(c), the 2D plateaus become rougher and rougher when the strength of disorders increases. To get more flat plateaus we use larger M ($M \ll L$),[16,26] that is the density is averaged within a larger number of prime cells. In this way, we can still get relatively flat plateaus (Fig. 5(d)) even when rather strong disorders are present. Thus we confirm that the density plateaus are robust against the weak disorders. However, the widths of the density plateaus are smaller than the clean case (Fig. 5(a)) and when the disorders are large enough the plateaus will disappear.

As in the edge state case, the Anderson localization seems to not have much effect on the density plateaus. Figure 6(a) shows that the eigenstates already become localized when the harmonic trap presents even when there is no disorder, so the decrease of widths of the density plateaus is due to the shrinking of the band gaps by the disorders.

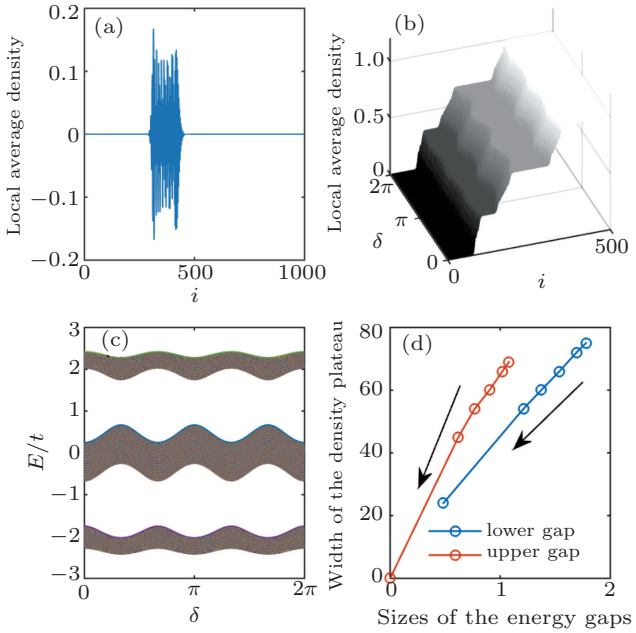


Fig. 6. (color online) (a) Typical localized wave function when the harmonic trap presents. (b) Local average density as a function of δ without disorders ($V_r = 0$). Fermion number $N = 600$. Other parameters are the same as that in Fig. 2. (c) Corresponding energy spectrum as a function of δ with periodic boundary condition. (d) Relationships between the sizes of energy gaps and the widths of the density plateaus with different strengths of disorders. Here, $\delta = \pi$ and $V_r = 0$, 0.25, 0.5, 0.75, 1, and 2. The V_r increases along the directions of the arrows.

To study the relationship between the width of the plateaus and the bulk energy gap we first consider the clean case. For a given fermion number $N = 600$ we plot the local average density as a function of sites and δ with $V_r = 0$. It can be seen from Fig. 6(b) that the width of the plateaus varies with δ. Then we compare the widths of the plateaus and the energy gaps (Fig. 6(c)) both as functions of δ. The result confirms the assertion in Ref. [16]: the widths of the plateaus are approximately in proportion to the size of the energy gaps. However, the proportionality coefficients are different for different energy gaps.

Now we check whether the conclusion on the relationship between the widths of the density plateaus and the bulk energy gap still works when the disorders are present. As is discussed above the bulk energy gaps decrease with disorders increasing. So if the widths of density plateaus still are approximately in proportion to the sizes of the bulk energy gap, the decrease of widths of the plateaus can be explained by the shrinking of energy gaps with disorders. The numerical results confirm our speculation: the widths of the density plateaus approximately are in proportion to the sizes of the bulk energy gaps as is shown in Fig. 6(d). To get more accurate results the sizes of energy gaps and widths of the density plateaus used in Fig. 6(d) are average values of 100 simulations. When the disorders are large enough, e.g., $V_r = 2$, the upper bulk energy gap is closed. The density plateau disappears with the bulk energy gap. So like the 2D systems the topological phases in 1D systems are robust only when the bulk energy gap is not destroyed by the disorders, so an open bulk gap is a necessary condition for measuring the topological phase.

5. Conclusion

In conclusion, by studying the edge states and the density plateaus of the fermions in 1D optical lattices with random potentials, we find that the edge states and density plateaus are robust with weak disorders, which confirms that the edge states and the density plateaus are topologically protected. We qualitatively explain why the Anderson localization has little effect on the topological properties. The edge states are found to be sensitive to the number of sites and a simple rule that describes the relationship between the edge state and the number of the sites is proposed. We find that the relationship between the sizes of energy gaps and the widths of the density plateaus is the same for both the clean and the disordered cases: the widths of the density plateaus are approximately in proportion to the sizes of the bulk energy gaps and they both decrease with the increase of disorders. The density plateaus and the bulk energy gap both disappear when the disorders become too strong.

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