Entanglement of movable mirror and cavity field enhanced by an optical parametric amplifier*

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A scheme to generate entanglement in a cavity optomechanical system filled with an optical parametric amplifier is proposed. With the help of the optical parametric amplifier, the stationary macroscopic entanglement between the movable mirror and the cavity field can be notably enhanced, and the entanglement increases when the parametric gain increases. Moreover, for a given parametric gain, the degree of entanglement of the cavity optomechanical system increases with increasing input laser power.

Keywords: optomechanical system, optical parametric amplifier, stationary macroscopic entanglement

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1. Introduction

Entanglement is one of the most attractive topics of quantum mechanics, which has been widely applied in diverse embranchments of physics. It is of basic and practical significance that we can generate entanglement among mesoscopic and even macroscopic systems. With the advance of technology, the Schrödinger cat state which describes the superposition states of macroscopic systems has been realized in experiments.[1] A macroscopic entanglement state can be prepared by optomechanical coupling via radiation pressure. Since the larger number of photons can lead to stronger radiation pressure, enhancing the photon number to increase the radiation pressure can acquire robust entanglement of optomechanical systems. A cavity optomechanical system is an available candidate system to study the entanglement of macroscopic states. Recently, there has been a growing interest in investigating entanglement in macroscopic systems.[2–13] In optomechanical systems, the quantum measurement theory and its applications have been explored,[3] a large variety of nonclassical states of both the cavity field and the movable mirror can be generated,[4] and the robust stationary entanglement can be produced between the intracavity mode and the mechanical mirror.[6] Reference [8] obtained a steady state entanglement of the motion of two dielectric membranes, which are suspended inside a Fabry–Perot cavity. The properties of optomechanical entanglement in a coupled cavity array with a movable mirror have been studied in Ref. [10]. Barzanjeh *et al.*[11] proposed a scheme for the realization of stationary continuous-variable entanglement in a hybrid tripartite system formed by an optical cavity and a microwave cavity, both interacting with a mechanical resonator. Chiara *et al.*[12] investigated the mirror–light entanglement in a hybrid optomechanical device formed by a Bose–Einstein condensate. In recent years, the experiments have realized the macroscopic entanglement.[14,15]

With the improvement of the qualities of the nonlinear crystals, optical parametric amplifiers (OPAs) including degenerate OPA and nondegenerate OPA, have shown tremendous applications in generating squeezed and entangled states.[16–24] Agarwal[17] investigated interferences in the quantum fluctuations of the output of an OPA, and demonstrated that the interferences can be manipulated by choosing the squeezing of the input field. Reference [23] obtained broadband entangled light through cascading nondegenerate OPAs, and the application of the entangled light from the cascading nondegenerate OPAs to broadband teleportation has been discussed.

Recently, the enhancement of the entanglement has been investigated by introducing nondegenerate OPAs.[25–27] Chen *et al.*[25] theoretically showed that the degree of the correlation of the input entangled beams can be improved by the nondegenerate OPA inside an optical cavity. In Refs. [26] and [27], it was experimentally demonstrated that the entanglement degree of the entangled state can be improved and manipulated by using nondegenerate OPAs. In this paper, we propose a scheme to produce the stationary macroscopic entanglement between a cavity field and a movable mirror in an optomechanical system filled with OPA. With the existence of the OPA, the entanglement can be pronouncedly increased compared to that without OPA, and the entanglement increases with the increasing parametric gain of the OPA. Furthermore, for a fixed parametric gain, the larger entanglement of the optomechanical system corresponds to the larger input laser power.

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2. Model and steady-state entanglement for the system

As shown in Fig. 1, we consider an OPA confined inside an optical cavity with a fixed mirror and a perfectly reflective movable mirror driven by a laser. The Hamiltonian of the system can be given as

$$\begin{aligned} H &= \hbar\omega_{\rm c}a^{\dagger}a+\frac{1}{2}\hbar\omega_{\rm m}(P^2+Q^2)+\hbar\chi Qa^{\dagger}a \\ &\quad +{\rm i}\hbar\varepsilon(a^{\dagger}{\rm e}^{-{\rm i}\omega_{\rm L}t}-a{\rm e}^{{\rm i}\omega_{\rm L}t}) \\ &\quad +{\rm i}\hbar G({\rm e}^{{\rm i}\theta}a^{\dagger 2}-{\rm e}^{-{\rm i}\theta}a^2). \end{aligned} \tag{1}$$

The first term describes the energy of the cavity, where a ($a^{\dagger}$) is the annihilation (creation) operator of the cavity fields with frequency $\omega_{\rm c}=2\pi c/L$, and L is the cavity length. The second term is the energy of the mechanical oscillator, where Q (P) is the dimensionless position (momentum) operator of the mechanical oscillators with frequency $\omega_{\rm m}$. The third term represents the radiation pressure with the coupling rate

$$\chi=\frac{\omega_0}{L}\sqrt{\frac{\hbar}{m\omega_{\rm m}}}.$$

The fourth term gives the coupling between the input laser field and the cavity mode, where ε is related to the input power $\wp$ by $\varepsilon=\sqrt{2\wp\kappa/m\omega_{\rm L}}$ and $\omega_{\rm L}$ is the frequency of the driving laser. The last term is the coupling between the OPA and the cavity mode, where G is the nonlinear gain of the OPA and θ is the phase of the field driving the OPA. In the interaction picture, the Hamiltonian is given as

$$\begin{aligned} H_{\rm I} &= \hbar(\omega_{\rm c}-\omega_{\rm L})a^{\dagger}a+\frac{1}{2}\hbar\omega_{\rm m}(P^2+Q^2)+\hbar\chi Qa^{\dagger}a \\ &\quad +{\rm i}\hbar\varepsilon(a^{\dagger}-a)+{\rm i}\hbar G({\rm e}^{{\rm i}\theta}a^{\dagger 2}-{\rm e}^{-{\rm i}\theta}a^2). \end{aligned} \tag{2}$$

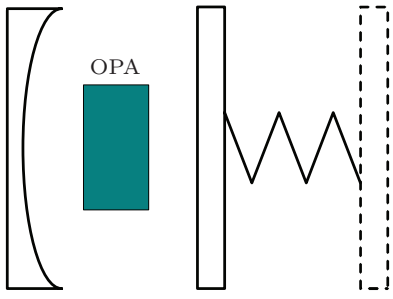


Fig. 1. Sketch of the system. An optical parametric amplifier is inserted in a Fabry–Perot cavity with a fixed mirror and a movable mirror.

Considering all of the noise and damping terms, we can investigate the dynamics of the system by using the quantum Langevin equation. By introducing phenomenologically the damping and noise terms into the Heisenberg operator equation, the quantum Heisenberg–Langevin equation can be obtained as

$$\dot{A}=\frac{1}{{\rm i}\hbar}[A,H_{\rm I}]-\kappa_A A+A_{\rm in}, \tag{3}$$

where κ_A is the decay of the cavity field and $A_{\rm in}$ describes the input noise. From Eq. (2), we have

$$\begin{aligned} \dot{Q} &= \omega_{\rm m}P, \\ \dot{P} &= -\omega_{\rm m}Q-\chi a^{\dagger}a-\gamma_{\rm m}P+\xi, \\ \dot{a} &= -[\kappa-{\rm i}\Delta]a+2G{\rm e}^{{\rm i}\theta}a^{\dagger}+\varepsilon+\sqrt{2\kappa}a_{\rm in}, \end{aligned} \tag{4}$$

where $\Delta=\omega_{\rm c}-\omega_{\rm L}+\chi Q$. The quantum Brownian noise operator ξ with zero mean value comes from the coupling of the movable mirror to the environment, and has the following correlation function

$$\begin{aligned} &\langle\xi(t)\xi(t')\rangle \\ &=\frac{\gamma}{2\pi\omega_{\rm m}}\int\omega{\rm e}^{-{\rm i}\omega(t-t')}\left[1+\coth\left(\frac{\hbar\omega}{2k_{\rm B}T}\right)\right]{\rm d}\omega, \end{aligned}$$

where $k_{\rm B}$ is the Boltzmann constant, and T is the temperature of the mechanical mirror. When $\omega_{\rm m}/\gamma_{\rm m}\gg 1$, the correlation function of the noise $\xi(t)$ can be written as $\langle\xi(t)\xi(t')+\xi(t')\xi(t)\rangle/2\cong\gamma_{\rm m}(2N+1)\delta(t-t')$, where $N=[\exp(\hbar\omega_{\rm m}/k_{\rm B}T)-1]^{-1}$ is the mean thermal excitation number of the resonator. The noise operator $a_{\rm in}$ satisfies $\langle a_{\rm in}(t)\rangle=0$ and the nonzero time-domain correlation functions $\langle a_{\rm in}^{\dagger}(t)a_{\rm in}(t')\rangle=n\delta(t-t')$, $\langle a_{\rm in}(t)a_{\rm in}^{\dagger}(t')\rangle=(n+1)\delta(t-t')$, where $n=[\exp(\hbar\omega_0/k_{\rm B}T)-1]^{-1}$ is the equilibrium mean thermal photon number.

In order to study the steady state entanglement of the system, we adopt the standard methods of quantum optics[28] to solve Eq. (4), namely, the steady state solutions of the system can be obtained by setting all the time derivatives of Eq. (4) to zero. Consequently, we obtain the steady state values of the system

$$\begin{aligned} P_{\rm s} &= 0, \\ Q_{\rm s} &= -\frac{\chi|a^{\rm s}|^2}{\omega_{\rm m}}, \\ a_{\rm s} &= \frac{\kappa-{\rm i}\Delta+2G{\rm e}^{{\rm i}\theta}}{\kappa^2+\Delta^2-4G^2}\varepsilon. \end{aligned} \tag{5}$$

When the cavity is very intensely driven, i.e., $a^{\rm s}\gg 1$, each operator of the system can be expanded as the sum of its steady-state mean value and a small fluctuation with zero mean value, namely, the operator can be written as $\hat{o}=o^{\rm s}+\delta\hat{o}$, and the dynamics of the small fluctuations is around the steady state of the system. We have linearized Langevin equations as follows:

$$\begin{aligned} \delta\dot{Q} &= \omega_{\rm m}\delta P, \\ \delta\dot{P} &= -\omega_{\rm m}\delta Q-\chi X^{\rm s}\delta X-\chi Y^{\rm s}\delta Y-\gamma_{\rm m}\delta P+\xi, \\ \delta\dot{X} &= \chi Y^{\rm s}\delta Q+(-\kappa+2G\cos\theta)\delta X \\ &\quad +(\Delta+2G\sin\theta)\delta Y+\sqrt{2\kappa}\delta X_{\rm in}, \\ \delta\dot{Y} &= -\chi X^{\rm s}\delta Q+(-\Delta+2G\sin\theta)\delta X \\ &\quad +(-\kappa-2G\cos\theta)\delta Y+\sqrt{2\kappa}\delta Y_{\rm in}, \end{aligned} \tag{6}$$

where we have defined quadrature operators for the cavity field as

$$X=\frac{1}{\sqrt{2}}(a+a^{\dagger}),\ \ Y=\frac{1}{\sqrt{2}\mathrm{i}}(a-a^{\dagger}). \tag{7}$$

Equation (6) can be written in a compact form

$$\dot{\boldsymbol{f}}=\boldsymbol{A}\boldsymbol{f}+\boldsymbol{B}, \tag{8}$$

where

$$\boldsymbol{f}^{\mathrm{T}}=(\delta Q,\delta P,\delta X,\delta Y)$$

and

$$\boldsymbol{B}^{\mathrm{T}}=(0,\xi,\sqrt{2\kappa}\delta X_{\mathrm{in}},\sqrt{2\kappa}\delta Y_{\mathrm{in}})$$

are the column vectors of the fluctuations and the noise sources, respectively. The drift matrix $\boldsymbol{A}$ is written as

$$\boldsymbol{A}=\begin{bmatrix}0 & \omega_{\mathrm{m}} & 0 & 0\\ -\omega_{\mathrm{m}} & -\gamma_{\mathrm{m}} & -\chi X^{\mathrm{s}} & -\chi Y^{\mathrm{s}}\\ \chi Y^{\mathrm{s}} & 0 & -\kappa+2G\cos\theta & \Delta+2G\sin\theta\\ -\chi X^{\mathrm{s}} & 0 & -\Delta+2G\sin\theta & -\kappa-2G\cos\theta\end{bmatrix}. \tag{9}$$

When the real parts of all the eigenvalues of matrix $\boldsymbol{A}$ are negative,[6,29] the system is stable and reaches its steady state as $t\to\infty$. It is not easy to obtain analytically the eigenvalues of the drift matrix, so we will guarantee steady state conditions via a numerical method. We define the element of the covariance matrix of the quantum fluctuations $V_{ij}(\infty)=[\langle f_i(\infty)f_j(\infty)+f_j(\infty)f_i(\infty)\rangle]/2$, and investigate the nature of the steady state of the cavity optomechanical system. When the system is stable, the steady-state correlation matrix satisfies the Lyapunov equation[30]

$$\boldsymbol{A}\boldsymbol{V}+\boldsymbol{V}\boldsymbol{A}^{\mathrm{T}}=-\boldsymbol{D}, \tag{10}$$

where $\boldsymbol{D}=\mathrm{diag}[0,\gamma_{\mathrm{m}}(2n+1),\kappa(2N+1),\kappa(2N+1),0,0]$ is the noise correlation matrix. We can directly solve the covariance matrix.

We aim to investigate the properties of entanglement of the cavity optomechanical system containing OPA. We employ the logarithmic negativity to quantify the entanglement. The logarithmic negativity can be defined as[31]

$$E_{\mathrm{N}}=\max\{0,-\ln 2\eta^{-}\}, \tag{11}$$

where $\eta^{-}=\sqrt{\Sigma(\boldsymbol{V})/2-\sqrt{(\Sigma(\boldsymbol{V})^2-4\det\boldsymbol{V})/2}}$ with $\Sigma(\boldsymbol{V})=\det\boldsymbol{A}+\det\boldsymbol{B}-2\det\boldsymbol{C}$, and it is the smallest symplectic eigenvalue of the partially transposed covariance matrix given by

$$\boldsymbol{V}=\begin{bmatrix}\boldsymbol{A} & \boldsymbol{C}\\ \boldsymbol{C}^{\mathrm{T}} & \boldsymbol{B}\end{bmatrix}.$$

According to the definition of Eq. (11), the system is entangled if $E_{\mathrm{N}}>0$, or equivalently, $\eta^{-}<1/2$.

We now discuss the properties of the entanglement of the cavity optomechanical system. The entanglement of the system is displayed in Figs. 2 and 3, and the logarithmic negativity E_{N} is plotted as a function of the normalized detuning $\Delta/\omega_{\mathrm{m}}$. In Fig. 2, we show the logarithmic negativity of the system for three values of G, i.e., $G=0$, 0.8κ, and 0.9κ. From Fig. 2, we notice that the larger entanglement between the movable mirror and the cavity mode can be obtained when the OPA exists, and the entanglement increases with the increase of G. This is because increasing the parametric gain makes the coupling between the cavity field and the movable mirror stronger due to the increase of the photon number inside the cavity. The photon number inside the cavity is 6.29×10^8, 8.02×10^9, and 10.34×10^9 for $G=0$, $G=0.8\kappa$, and $G=0.9\kappa$, respectively. Moreover, in Fig. 2, it can be observed that the entanglement with the OPA (dashed and solid lines) can be notably enhanced compared to that without OPA (dashed-dotted line) between the movable mirror and the cavity field.

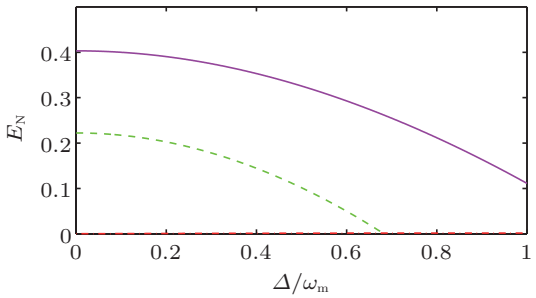


Fig. 2. (color online) Entanglement between the cavity field and the mechanical mirror, and the dashed-dotted, dashed, and solid lines correspond to $G=0$, 0.8κ, and 0.9κ, respectively. For all plots, the parameters are $L=5$ mm, $m=20$ ng, $\omega_{\mathrm{m}}=2\pi\times10^7$ Hz. The laser has wavelength $\lambda=810$ nm and power $\wp=10$ mW, the mechanical quality factor $Q'=\omega_{\mathrm{m}}/\gamma_{\mathrm{m}}=6700$.

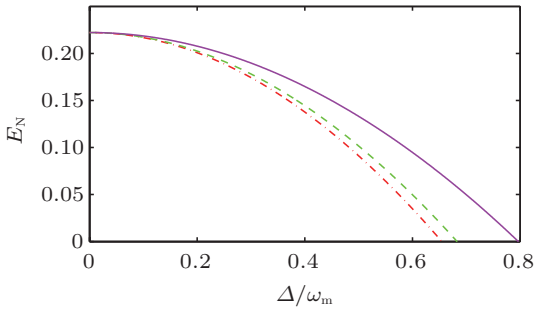


Fig. 3. (color online) Entanglement between the cavity field and the mechanical mirror when $G=0.8\kappa$, the dashed-dotted, dashed, and solid lines correspond to $\wp=7$ mW, 10 mW, and 15 mW, respectively. The other parameters are the same as those in Fig. 2.

Figure 3 describes E_{N} of the cavity optomechanical system for different input laser power $\wp$ at $G=0.8\kappa$. From Fig. 3, it can be observed that the entanglement of the optomechanical system increases with the increase of the laser power when parametric gain G is fixed. The reason is that the coupling

between the cavity field and the mirror is increased with increasing input laser power due to the increase of the photon number.

3. Discussion and conclusion

We give a brief discussion on the experimental feasibility of our program. To numerically calculate the logarithmic negativity of the optomechanical system, we have chosen feasible parameters based on the present experiment.[32] In fact, it is an experimental challenge to detect the entanglement of macroscopic mechanical systems. However, some promising schemes have been proposed[6,33] to relatively easily detect the quantum correlation by homodyne measurement techniques, so quantum entanglement can be indirectly detected by quantum correlation. We believe that our scheme is realizable in experiments with the development of science and technology.

In conclusion, we have proposed a scheme to generate stationary macroscopic entanglement of an optomechanical device filled with OPA. Our study shows that the entanglement of the cavity optomechanical system can be pronouncedly enhanced due to the introduction of the OPA, and the larger entanglement corresponds to the larger parametric gain G. Moreover, for a given G, the degree of entanglement increases when the input laser power increases.

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