# A method for generating double-ring-shaped vector beams*

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We propose a method for generating double-ring-shaped vector beams. A step phase introduced by a spatial light modulator (SLM) first makes the incident laser beam have a nodal cycle. This phase is dynamic in nature because it depends on the optical length. Then a Pancharatnam–Berry phase (PBP) optical element is used to manipulate the local polarization of the optical field by modulating the geometric phase. The experimental results show that this scheme can effectively create double-ring-shaped vector beams. It provides much greater flexibility to manipulate the phase and polarization by simultaneously modulating the dynamic and the geometric phases.

**Keywords:** vector beam, Pancharatnam–Berry phase element, geometric phase

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## 1. Introduction

Spatially inhomogeneous polarization states (radial and azimuthal polarizations are the typical examples), also termed as vector beams, have attracted more and more attention in recent years. In contrast to spatially homogeneous polarizations, such as linear, elliptical, and circular polarizations, vector beams have some unique optical properties. Recently, many types of vector beams have been reported, such as cylindrical vector beam,[1] full Poincaré beam,[2,3] hybridly polarized vector beam,[4,5] etc. The radially polarized (RP) and azimuthally polarized (AP) light beams are two special cases of the cylindrical vector beams and have been investigated by a few researchers. RP and AP beams focused by an objective lens with a high numerical aperture have been reported in the last few years.[6–13] The researchers found that the RP beams have a large longitudinal electric field component, while the AP beams can be focused into a hollow dark spot. Lerman *et al*. carried out a study on hybridly polarized vector beams.[14] Beckley *et al*. explored the change of polarization properties during propagation.[15] Due to their interesting properties, the vector beams have many potential applications in optical communication, optical trapping, particle manipulation, laser processing, super-resolution microscopy, and optical imaging, as well as in optical data storage.[1] Various methods are proposed to generate vector beams. The schemes can be divided into active and passive ones. The active methods force the lasers to oscillate in vector modes with technically designed optical resonators.[16] The passive schemes use either the wavefront reconstruction based on specially designed elements[17–22] or interferometric schemes.[23–25]

More recently, some researchers paid attention to the so-called double-ring-shaped vector beam. Kozawa and Zhang *et al.* theoretically demonstrated that a tightly focused double-ring-shaped RP beam can produce an optical cage in the focal region. It can be used to trap particles with refractive indices smaller than that of the ambient.[26,27] The focusing properties of the double-ring-shaped AP light beam were studied by some researchers.[28,29] It was shown that a subwavelength focal hole and a long depth of focus are acquired near the focus. Due to the potential application in optical trapping and particle manipulation, to efficiently generate double-ring-shaped vector beams is a significative work. Specially designed optical resonators and interferometric schemes are commonly used for generating double-ring-shaped vector beams.[30,31] However, the method with specially designed optical resonators lacks flexibility and the interferometric schemes are not stable enough. In this work, we propose a simple and stable setup to produce double-ring-shaped vector beams by modulating the dynamic phase and the Pancharatnam–Berry phase (PBP). The PBP is one of the two kinds of geometric phases. The other kind of geometric phase is the Rytov–Vladimirskii–Berry phase. The PBP is linked to the local polarization changes of light. The optical elements making use of the geometric PBP to produce a desired wavefront are called PBP optical elements.[32–34] Different from the refractive and diffractive elements, the PBP optical elements introduce the phase by space-variant polarization instead of optical path differences. These particular devices for phase-front shaping have been applied to generate vector or vortex beams, focus, manipulate the photonic spin Hall effect,[35,36] and so on.

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In this paper, we theoretically analyze the manipulation of polarization with PBP elements. On this basis, we propose an experimental setup to realize the polarization mode conversion based on PBP elements at wavelength 632.8 nm. We firstly generate a double-ring-shaped beam with a spatial light modulator by modulating the dynamic phase. Then, a PBP element is employed to manipulate the local polarization of the light field by modulating the geometric phase (PBP). The experimental results show that a linearly polarized (LP) Gauss beam can be converted into a double-ring-shaped vector beam. The local polarization direction of the double-ring-shaped vector beam is determined by the angle between the polarization direction of the incident LP light and the initial optical axis orientation of the PBP element on the $x$ axis. Therefore, we can obtain a double-ring-shaped cylindrical vector beam by rotating the PBP element. This novel method for generating double-ring-shaped vector beams can also be extended to produce other higher-order vector beams.

## 2. Optical PBP element

Optical PBP elements are wavefront shaping devices that introduce the phase by changing the local polarization of light. In this work, the PBP elements are fabricated by writing nanoscale wave plates in fused silica glasses with a femtosecond laser.[37] The intense laser irradiation can decompose the uniform glass ($SiO_2$) into porous glass ($SiO_{2(1-x)}+xO_2$), which results in the change of the refractive index. It means that an artificial birefringence, i.e., form birefringence, is introduced. We can obtain the birefringence by controlling the deposited energy in the silica glasses. The phase retardation can be written as

$$\psi = 2\pi(n_e - n_o)d/\lambda, \tag{1}$$

where $(n_e - n_o)$ is the induced birefringence and $d$ is the writing depth. The effective ordinary and extraordinary refractive indices can be expressed as[38]

$$n_o = \sqrt{fn_1^2 + (1-f)n_2^2}, \quad n_e = \sqrt{\frac{n_1^2 n_2^2}{fn_2^2 + (1-f)n_1^2}}, \tag{2}$$

where $f$ is the filling factor, and $n_1$ and $n_2$ are the refractive indices of the nanoscale wave plates. Here, the filling factor is 0.1–0.2, the writing period is 200–250 nm, and the line width is 30–50 nm.

According to the required phase modulation, these nanoscale wave plates are specially arranged into a prescribed pattern. Due to the manipulation of polarization by these wave plates, a PBP is generated. Here, we consider each wave plate having an identical phase retardation. The orientation of the space-variant slow axis is denoted by $\phi(x,y)$, which is the angle between the slow axis and the fixed reference axis $x$. Now, let us consider a specific pattern geometry described by the following expression:

$$\phi(x,y) = q\varphi, \tag{3}$$

where $\varphi = \arctan(y/x)$, and $q$ is a constant. Figure 1 illustrates schematically the slow axis distributions in the PBP elements with $q = 0.5, 1.0$, and 1.5, respectively. The initial optical axis lies in the $x$ axis.

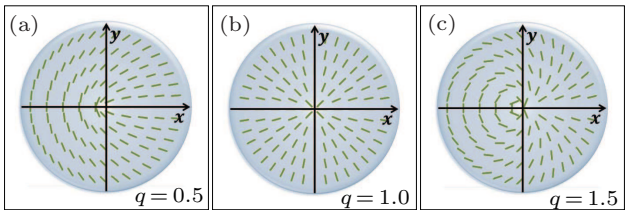



**Fig. 1.** (color online) Slow axis spatial distributions of the PBP elements with (a) $q = 0.5$, (b) $q = 1.0$, and (c) $q = 1.5$.

The Jones matrix is a convenient tool for describing the PBP elements. The Jones matrix $\boldsymbol{M}$ can be easily derived as follows:

$$\begin{aligned}\boldsymbol{M} &= \boldsymbol{R}(-\phi)\boldsymbol{M}_{\rm wp}(\Gamma)\boldsymbol{R}(\phi)\\ &= \begin{pmatrix} \cos\left(\frac{\Gamma}{2}\right) - \mathrm{i}\cos(2\phi)\sin\left(\frac{\Gamma}{2}\right) & -\mathrm{i}\sin(2\phi)\sin\left(\frac{\Gamma}{2}\right) \\ -\mathrm{i}\sin(2\phi)\sin\left(\frac{\Gamma}{2}\right) & \cos\left(\frac{\Gamma}{2}\right) + \mathrm{i}\cos(2\phi)\sin\left(\frac{\Gamma}{2}\right) \end{pmatrix}\\ &= \cos\frac{\Gamma}{2}\begin{pmatrix}1&0\\0&1\end{pmatrix} - \mathrm{i}\sin\frac{\Gamma}{2}\begin{pmatrix}\cos 2\phi & \sin 2\phi\\ \sin 2\phi & -\cos 2\phi\end{pmatrix},\end{aligned} \tag{4}$$

where $\boldsymbol{R}(\phi)$ is a $2\times 2$ rotation matrix, $\boldsymbol{M}_{\rm wp}$ is the Jones matrix of the wave plate, and $\Gamma$ is the phase retardation of the wave plate.

Let us now consider the case that an LP Gauss beam illuminates a PBP element. The Jones vector of the input field is then given by

$$\boldsymbol{E}_{\rm in}(r,\varphi) = E_0(r)\begin{pmatrix}\cos\theta_0\\ \sin\theta_0\end{pmatrix}\exp\left(-\frac{r^2}{w_0^2}\right), \tag{5}$$

where $\theta_0$ is the polarization angle of light, $E_0(r)$ is the radially-dependent amplitude, and $w_0$ is the waist width. At the PBP element output plane, we obtain the following field:

$$\begin{aligned}
&\boldsymbol{E}_{\text{out}}(r,\varphi)\\
&= \frac{1}{2}E_0(r)\exp(-\mathrm{i}\theta_0)\left[\cos\frac{\Gamma}{2}+\mathrm{i}\sin\frac{\Gamma}{2}\exp(-\mathrm{i}\psi)\right]\begin{pmatrix}1\\ \mathrm{i}\end{pmatrix}\\
&\quad\times\exp\left(-\frac{r^2}{w_0^2}\right)+\frac{1}{2}E_0(r)\exp(+\mathrm{i}\theta_0)\\
&\quad\times\left[\cos\frac{\Gamma}{2}+\mathrm{i}\sin\frac{\Gamma}{2}\exp(+\mathrm{i}\psi)\right]\begin{pmatrix}1\\ -\mathrm{i}\end{pmatrix}\exp\left(-\frac{r^2}{w_0^2}\right), \quad (6)
\end{aligned}$$

where $\psi = 2(\phi-\theta_0) = 2(q\varphi-\theta_0)$ is an additional space-variant PBP depending on the local optical axis orientation of the PBP element.

Let us assume that the phase retardation $\Gamma$ is equal to $\pi$ (corresponding to a half-wave plate). According to Eq. (6), the output field should be rewritten as

$$\begin{aligned}
\boldsymbol{E}'_{\text{out}}(r,\varphi) &= \frac{\mathrm{i}}{2}E_0(r)\exp[-\mathrm{i}(\psi+\theta_0)]\begin{pmatrix}1\\ \mathrm{i}\end{pmatrix}\exp\left(-\frac{r^2}{w_0^2}\right)\\
&\quad+\frac{\mathrm{i}}{2}E_0(r)\exp[+\mathrm{i}(\psi+\theta_0)]\begin{pmatrix}1\\ -\mathrm{i}\end{pmatrix}\exp\left(-\frac{r^2}{w_0^2}\right)\\
&= E_0(r)\begin{pmatrix}\cos(m\varphi-\theta_0)\\ \sin(m\varphi-\theta_0)\end{pmatrix}\exp\left(-\frac{r^2}{w_0^2}\right), \quad (7)
\end{aligned}$$

where $m = 2q$ is the polarization order number of the polarized light field. In fact, $m$ is the number of polarization rotations per round trip. It is clear from Eq. (7) that the output wave is a cylindrical vector beam. The local polarization orientation depends on the orientation of the PBP element in the $xy$ plane and the polarization angle of the incident light. When the polarization angle of the incident beam is fixed, the local polarization orientation of the cylindrical vector beam is only determined by the orientation of the PBP element. That is, we can obtain cylindrical vector beams with different local polarizations by rotating the PBP element.

## 3. Experimental setup and results

The experimental setup is demonstrated in Fig. 2(a). The above theoretical analysis shows that the PBP elements can manipulate the local polarization of light beams. To obtain a double-ring-shaped intensity distribution, we also need to modulate the input Gauss beam into the double-ring shape. For this purpose, we employ a reflective phase only spatial light modulator (SLM) (Holoeye Pluto-Vis, Germany). Each pixel of the SLM can produce an individually programmable phase shift in the interval between 0 and $2\pi$. Here, we let the SLM provide 0 phase shift in the central circular zone and $\pi$ phase shift in the surrounding zone. It means that the reflective light wave acquires a step phase from the SLM. The step phase makes the reflective wave produce a nodal cycle in the azimuthal direction where the electric field is zero. Due to the $\pi$ phase difference, the polarization directions of the central electric field and the outer rings are opposite. The SLM can be replaced by a step phase plate in practical applications. It is notable that the phase modulation introduced by the SLM or step phase plate is purely dynamic in nature, because the phase is dependent on the optical length. The grey level image of the step phase loaded in the SLM is shown in Fig. 2(b). Figure 2(c) is the intensity distribution of the beam reflected by the SLM. As an example, the intensity distribution of the beam emerging from the PBP element with $q = 0.5$ is shown in Fig. 2(d). We can see that the intensity at the center of the beam is zero due to the existence of polarization singularity.

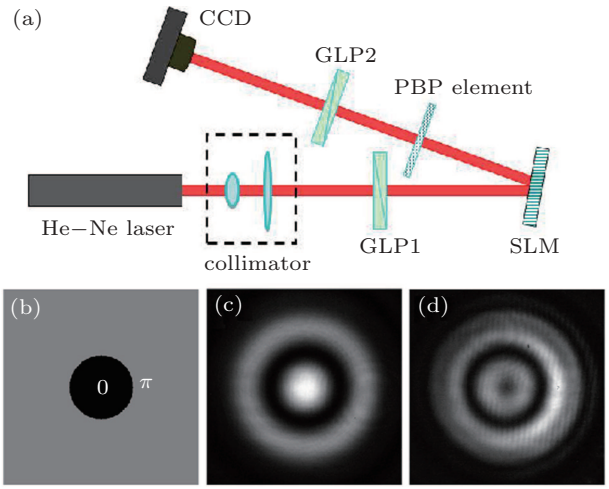



**Fig. 2.** (color online) (a) Schematic diagram of the experimental setup for generating double-ring-shaped vector beams, where GLP is a Glan laser polarizer, and CCD is a charge-coupled device. (b) The grey level image displayed on the SLM. (c) The intensity distribution of the laser beam reflected by the SLM. (d) The intensity distribution of the beam emerging from the PBP element with $q = 0.5$.

Figure 3 shows the created double-ring-shaped vector beams with polarization order $m = 1$. A horizontally polarized light firstly impinges on the SLM and generates a nodal cycle. A PBP element with $q = 1/2$ is then used to manipulate the local polarization of the double-ring beam based on the modulation of geometric phase. To analyze the polarization distribution, a polarizer (GLP2) is inserted behind the PBP element and rotated to different polarization angles. The first row in Fig. 3 is the experimental results when the PBP element is placed at 0°. We can see that the intensity pattern rotates with the polarizer in the same direction and the nodal line is perpendicular to the polarization axis of the polarizer. Therefore, we can reconstruct the local polarization structure. The vector field is plotted in the last column. It is obvious that the output beam is a radially polarized double-ring-shaped vector beam. The second row in Fig. 3 is the experimental results when the PBP element is placed at +90°. The measured intensity pattern rotates with the polarizer in the opposite direction and the

nodal line is parallel to the polarization axis of the polarizer. Obviously, the output field is an azimuthally polarized double-ring-shaped vector beam.

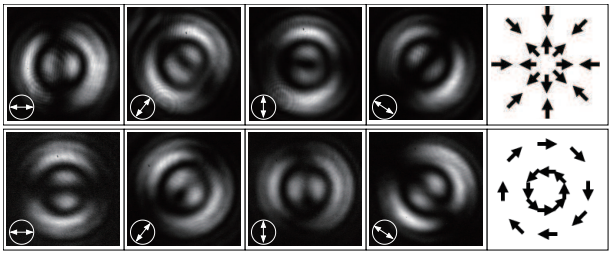


**Fig. 3.** (color online) The experimentally generated double-ring-shaped vector beams with polarization order $m = 1$ by a PBP element with $q = 1/2$. The first row shows the respective intensity distributions behind a polarizer (GLP2) at different polarization angles when the PBP element is placed at 0°. The second row shows the intensity cross-sections when the PBP element rotates +90°.

Next, we respectively adopt two PBP elements with $q = 1.0$ and $q = 1.5$ in our experiments. The experimental results are shown in Figs. 4 and 5, respectively. It is easy to find that the intensity patterns in Fig. 4 have two nodal lines in the azimuthal direction and one nodal cycle in the radial direction. Therefore, the intensity spot is segmented into eight segments. It means that the PBP element with $q = 1.0$ can be used to generate a double-ring-shaped vector beam with polarization order $m = 2$. Similarly, we find that the intensity patterns in Fig. 5 have three nodal lines in the azimuthal direction and the PBP element with $q = 1.5$ can be used to generate a double-ring-shaped vector beam with polarization order $m = 3$.

The above experimental results show that the proposed method for generating double-ring-shaped vector beams is effective. A dynamic step phase introduced by the SLM makes the reflected laser beam have a nodal cycle in the radial direction. A PBP element is used to manipulate the polarization state of the optical field by introducing a geometric phase. This method can also be extended to generate other higher-order vector beams.

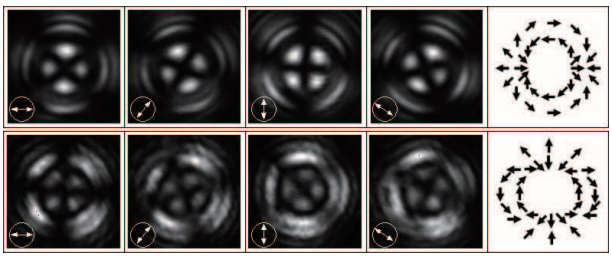


**Fig. 4.** (color online) The experimentally generated double-ring-shaped vector beams with polarization order $m = 2$ by a PBP element with $q = 1.0$. The first row shows the respective intensity distributions behind a polarizer (GLP2) at different polarization angles when the PBP element is placed at 0°. The second row shows the intensity cross-sections when the PBP element rotates +90°.

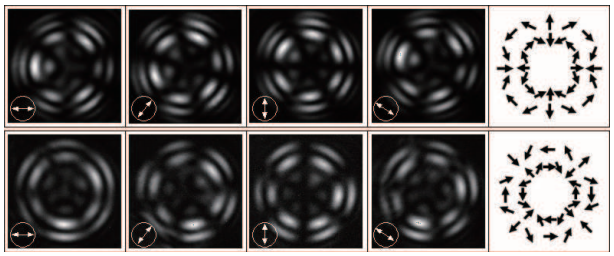


**Fig. 5.** (color online) The experimentally generated double-ring-shaped vector beams with polarization order $m = 3$ by a PBP element with $q = 1.5$. The first row shows the respective intensity distributions behind a polarizer (GLP2) at different polarization angles when the PBP element is placed at 0°. The second row shows the intensity cross-sections when the PBP element rotates +90°.

## 4. Conclusion

We have proposed a method to generate double-ring-shaped vector beams. A step phase modulation introduced by an SLM makes the incident Gauss beam generate a nodal cycle. This phase is dynamic in nature because it depends on the optical path. A PBP element introduces a geometric phase to realize the modulation of the local polarization state. We adopt three PBP elements with $q = 1.0$, 1.0, and 1.5 to generate double-ring-shaped vector beams with polarization order $m = 1$, 2, and 3, respectively. The experimental results show that the proposed method can create high-quality double-ring-shaped vector beams. It also provides a new method to manipulate the phase and polarization of the optical field by simultaneously introducing dynamic phase and geometric phase modulation.

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