Collapse–revival of squeezing of two atoms in dissipative cavities*

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Based on the time-convolutionless master-equation approach, we investigate the squeezing dynamics of two atoms in dissipative cavities. We find that the atomic squeezing is related to initial atomic states, atom–cavity couplings, non-Markovian effects and resonant frequencies of an atom and its cavity. The results show that a collapse–revival phenomenon will occur in the atomic squeezing and this process is accompanied by the buildup and decay of entanglement between two atoms. Enhancing the atom–cavity coupling can increase the frequency of the collapse–revival of the atomic squeezing. The stronger the non-Markovian effect is, the more obvious the collapse–revival phenomenon is. In particular, if the atom–cavity coupling or the non-Markovian effect is very strong, the atomic squeezing will tend to a stably periodic oscillation in a long time. The oscillatory frequency of the atomic squeezing is dependent on the resonant frequency of the atom and its cavity.

Keywords: collapse–revival, atomic squeezing, atom–cavity coupling, non-Markovian effect

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1. Introduction

Based on the Heisenberg uncertainty relation[1,2] which quantified uncertainties by standard deviations of two observables X and Y, i.e., $\Delta X \Delta Y \geq |\langle [X,Y] \rangle|/2$, the atomic squeezing theory has been brought forward. That is to say, its spin or angular momentum has often been regarded as squeezed if the uncertainty of one spin component of an atom, say $(\Delta S_1)^2$ or $(\Delta S_2)^2$, is smaller than $|\langle S_3 \rangle|/2$.[3,4] The past decades progress in researches on atomic squeezing showed that the atomic squeezing effect is one of the important properties of quantum dynamics and has been applied widely in quantum science. For example, the atomic squeezing effects cannot only enhance the sensitivities of gravity-wave detectors,[5] high-resolution spectroscopy,[6] high-precision measurements[7] and atomic clocks,[8] but also can be useful in optical communications[9] and the tiny mechanical detection of a superconducting nano-mechanical resonator both theoretically[10,11] and experimentally.[12] Most of these achievements obtained are related to spin squeezed atomic ensembles[13–15] while the study of two-atom squeezing dynamics is relatively less.

On the other hand, because any real quantum systems unavoidably interact with their environments,[16,17] more and more attention has been paid to the influences of non-Markovian environments on atomic squeezing dynamics. In recent years, researchers have done many works in atomic squeezing dynamics of noisy environments and have had a lot of progress. For example, Xue[18] investigated the spin squeezing dynamics of N independent spin-1/2 particles with exchange symmetry by the Ising-type Hamiltonian. The authors in Ref. [19] studied spin squeezing of an ensemble of N independent spin-1/2 particles under non-Markovian channels by the Hierarchy equation method. Xiao *et al.*[20] considered the squeezing of a two-level atom coupled to a non-Markovian reservoir by the exact solution method. The authors in Ref. [21] studied the squeezing dynamics of two independent two-level atoms off-resonantly coupled to two non-Markovian reservoirs by the time-convolutionless master-equation approach, and so on.

Although many important progresses have been acquired in experimental and theoretical research on the squeezing dynamics of quantum systems, these investigations mentioned above are mainly focused on the two classes of model. One is the model of open quantum systems of qubits directly interacting with non-Markovian environments. The other is the model of closed quantum systems of qubits interacting with cavities, in which the influences of environments on the cavities are neglected. In this paper, we will focus our attention on the squeezing dynamics of two atoms in dissipative cavities by the time-convolutionless master-equation approach. The results show that a collapse–revival phenomenon will occur in the atomic squeezing and this process is accompanied by the buildup and decay of entanglement between two atoms. Enhancing the atom–cavity coupling can increase the frequency of the collapse–revival of the atomic squeezing. The stronger the non-Markovian effect is, the more obvious the collapse–revival phenomenon is. The oscillatory frequency of the atomic squeezing is dependent on the resonant frequency of the atom and its cavity. These results provide a potential method to extend the available time of the atomic squeezing,

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which would be useful in the high precision applications.

The outline of the paper is as follows. In Section 2, we introduce a dissipative two-atom system. In Section 3, we review the squeezing dynamics of two atoms. Then we discuss in detail the two-atom squeezing dynamics in Section 4. Finally, a brief summary is presented in Section 5.

2. Dissipative two-atom system

We consider a composite system of two two-level atoms (A,B) interacting respectively with two cavities (a,b), where each cavity is coupled to a bosonic environment $(1,2)$, and there is no interaction between the partition "$Aa1$" and "$Bb2$".[22] The Hamiltonian is written as

$$H = H_{Aa1} + H_{Bb2}, \tag{1}$$

and

$$\begin{aligned} H_{Aa1} &= \frac{1}{2}\omega_0\sigma_z + \omega_0 a^\dagger a + \sum_k \omega_k c_k^\dagger c_k \\ &\quad + \Omega(a\sigma_+ + a^\dagger\sigma_-) + (a^\dagger + a)\sum_k g_k(c_k^\dagger + c_k), \end{aligned} \tag{2}$$

where $\sigma_+ = |e\rangle\langle g|$ and $\sigma_- = |g\rangle\langle e|$ are the raising and lowering operators of the qubit and $\sigma_z = |e\rangle\langle e| - |g\rangle\langle g|$ is a Pauli operator for the qubit with transition frequency ω_0,[26] $a^\dagger$ and a are the creation and annihilation operators of the cavity, $c_k^\dagger$ and c_k are the creation and annihilation operators of the environment, Ω is the coupling between the atom–cavity, and g_k is the coupling between the cavity–environment.

First, we solve the density matrix of the atom–cavity system satisfying Eq. (2). Let us suppose that there is one initial excitation in the atom–cavity system and the environment is at zero temperature. Neglecting the atomic spontaneous emission and the Lamb shifts, in the dressed-state basis $\{|\varphi_2\rangle, |\varphi_1\rangle, |\varphi_0\rangle\}$, using the second order of the time convolutionless (TCL) expansion,[16] the non-Markovian master equation for the density operator $\rho(t)$ is

$$\begin{aligned} \dot\rho(t) &= -\mathrm{i}[H_0, \rho(t)] \\ &\quad + \gamma_2(t)\left(\frac{1}{2}|\varphi_0\rangle\langle\varphi_2|\rho(t)|\varphi_2\rangle\langle\varphi_0| - \frac{1}{4}\{|\varphi_2\rangle\langle\varphi_2|, \rho(t)\}\right) \\ &\quad + \gamma_1(t)\left(\frac{1}{2}|\varphi_0\rangle\langle\varphi_1|\rho(t)|\varphi_1\rangle\langle\varphi_0| \right. \\ &\quad \left. - \frac{1}{4}\{|\varphi_1\rangle\langle\varphi_1|, \rho(t)\}\right), \end{aligned} \tag{3}$$

where $H_0 = \omega_0\sigma_z/2 + \omega_0 a^\dagger a + \Omega(a\sigma_+ + a^\dagger\sigma_-)$, $|\varphi_2\rangle = (|1g\rangle + |0e\rangle)/\sqrt{2}$ and $|\varphi_1\rangle = (|1g\rangle - |0e\rangle)/\sqrt{2}$ are the eigenstates of H_0 with energy $\omega_0/2 + \Omega$ and $\omega_0/2 - \Omega$ as well as $|\varphi_0\rangle = |0g\rangle$ is the ground state with energy $-\omega_0/2$. The timedependent decay rates for $|\varphi_1\rangle$ and $|\varphi_2\rangle$ are $\gamma_1(t)$ and $\gamma_2(t)$ respectively.

We take a Lorentzian spectral density of the environment, i.e.,

$$J(\omega) = \frac{1}{2\pi}\frac{\gamma_0\lambda^2}{(\omega_1 - \omega)^2 + \lambda^2}, \tag{4}$$

where λ defines the spectral width of the coupling which is connected to the reservoir correlation time τ_R by $\tau_\mathrm{R} = \lambda^{-1}$ and γ_0 is related to the relaxation time scale τ_S by $\tau_\mathrm{S} = \gamma_0^{-1}$. If $\lambda > 2\gamma_0$; the relaxation time is greater than the reservoir correlation time and the dynamical evolution of the system is essentially Markovian. For $\lambda < 2\gamma_0$, the reservoir correlation time is greater than or of the same order as the relaxation time and non-Markovian effects become relevant.[27–29] When the spectrum is peaked on the frequency of the state $|\varphi_1\rangle$, i.e., $\omega_1 = \omega_0 - \Omega$, the decay rates for the two dressed states $|\varphi_1\rangle$ and $|\varphi_2\rangle$ are respectively expressed as[24]

$$\begin{aligned} \gamma_1(t) &= \gamma_0(1 - \mathrm{e}^{-\lambda t}), \\ \gamma_2(t) &= \frac{\gamma_0\lambda^2}{4\Omega^2 + \lambda^2}\left\{1 + \left[\frac{2\Omega}{\lambda}\sin(2\Omega t) - \cos(2\Omega t)\right]\mathrm{e}^{-\lambda t}\right\}. \end{aligned}$$

If the initial state of the atom–cavity is

$$\rho(0) = \begin{pmatrix} \rho_{11}(0) & \rho_{12}(0) & \rho_{13}(0) \\ \rho_{21}(0) & \rho_{22}(0) & \rho_{23}(0) \\ \rho_{31}(0) & \rho_{32}(0) & \rho_{33}(0) \end{pmatrix}, \tag{5}$$

we can acquire the matrix elements at all time from Eq. (3) as

$$\begin{aligned} \rho_{11}(t) &= d_{11}^{11}\rho_{11}(0), \quad \rho_{12}(t) = d_{12}^{12}\rho_{12}(0), \\ \rho_{13}(t) &= d_{13}^{13}\rho_{13}(0), \\ \rho_{22}(t) &= d_{22}^{22}\rho_{22}(0), \quad \rho_{23}(t) = d_{23}^{23}\rho_{23}(0), \\ \rho_{33}(t) &= d_{33}^{11}\rho_{11}(0) + d_{33}^{22}\rho_{22}(0) + d_{33}^{33}\rho_{33}(0). \end{aligned} \tag{6}$$

Here,

$$\begin{aligned} d_{11}^{11} &= \mathrm{e}^{-\frac{1}{2}f_2}, \quad d_{12}^{12} = \mathrm{e}^{-2\mathrm{i}\Omega t}\,\mathrm{e}^{-\frac{1}{4}(f_2+f_1)}, \\ d_{13}^{13} &= \mathrm{e}^{-\mathrm{i}(\omega_0+\Omega)t}\,\mathrm{e}^{-\frac{1}{4}f_2}, \\ d_{22}^{22} &= \mathrm{e}^{-\frac{1}{2}f_1}, \quad d_{23}^{23} = \mathrm{e}^{-\mathrm{i}(\omega_0-\Omega)t}\,\mathrm{e}^{-\frac{1}{4}f_1}, \\ d_{33}^{11} &= 1 - d_{11}^{11}, \quad d_{33}^{22} = 1 - d_{22}^{22}, \quad d_{33}^{33} = 1, \end{aligned} \tag{7}$$

where

$$\begin{aligned} f_1 &= \gamma_0 t + \frac{\gamma_0}{\lambda}(\mathrm{e}^{-\lambda t} - 1), \\ f_2 &= \frac{\gamma_0\lambda^2}{4\Omega^2 + \lambda^2}\left[t - \frac{4\Omega\,\mathrm{e}^{-\lambda t}\sin(2\Omega t)}{4\Omega^2 + \lambda^2}\right. \\ &\quad \left. + \frac{(\lambda^2 - 4\Omega^2)(\mathrm{e}^{-\lambda t}\cos(2\Omega t) - 1)}{\lambda(4\Omega^2 + \lambda^2)}\right]. \end{aligned}$$

From the results above, we can obtain the density matrix of the composite system satisfying Eq. (1). Following the procedure described in Ref. [27], we construct the density matrix $\rho^\mathrm{T}(t)$ for the two atom–cavity system. In the dressed-state basis $\mathcal{D}^\mathrm{T} = \{|1\rangle \equiv |\varphi_2\varphi_2\rangle, |2\rangle \equiv |\varphi_2\varphi_1\rangle, |3\rangle \equiv |\varphi_2\varphi_0\rangle, |4\rangle \equiv |\varphi_1\varphi_2\rangle, |5\rangle \equiv |\varphi_1\varphi_1\rangle, |6\rangle \equiv |\varphi_1\varphi_0\rangle, |7\rangle \equiv |\varphi_0\varphi_2\rangle, |8\rangle \equiv |\varphi_0\varphi_1\rangle, |9\rangle \equiv |\varphi_0\varphi_0\rangle\}$, using Eq. (6), we can obtain the reduced density matrix $\rho^\mathrm{T}(t)$ at all time in the identical case, in which the two atoms are identical, and likewise for the two cavities and for the two Bose reservoirs,[22] i.e.,

$$\rho^{\mathrm{T}}(t) = \begin{pmatrix} \rho_{11}^{\mathrm{T}}(t) & \rho_{12}^{\mathrm{T}}(t) & \rho_{13}^{\mathrm{T}}(t) & \rho_{14}^{\mathrm{T}}(t) & \rho_{15}^{\mathrm{T}}(t) & \rho_{16}^{\mathrm{T}}(t) & \rho_{17}^{\mathrm{T}}(t) & \rho_{18}^{\mathrm{T}}(t) & \rho_{19}^{\mathrm{T}}(t) \\ \rho_{21}^{\mathrm{T}}(t) & \rho_{22}^{\mathrm{T}}(t) & \rho_{23}^{\mathrm{T}}(t) & \rho_{24}^{\mathrm{T}}(t) & \rho_{25}^{\mathrm{T}}(t) & \rho_{26}^{\mathrm{T}}(t) & \rho_{27}^{\mathrm{T}}(t) & \rho_{28}^{\mathrm{T}}(t) & \rho_{29}^{\mathrm{T}}(t) \\ \rho_{31}^{\mathrm{T}}(t) & \rho_{32}^{\mathrm{T}}(t) & \rho_{33}^{\mathrm{T}}(t) & \rho_{34}^{\mathrm{T}}(t) & \rho_{35}^{\mathrm{T}}(t) & \rho_{36}^{\mathrm{T}}(t) & \rho_{37}^{\mathrm{T}}(t) & \rho_{38}^{\mathrm{T}}(t) & \rho_{39}^{\mathrm{T}}(t) \\ \rho_{41}^{\mathrm{T}}(t) & \rho_{42}^{\mathrm{T}}(t) & \rho_{43}^{\mathrm{T}}(t) & \rho_{44}^{\mathrm{T}}(t) & \rho_{45}^{\mathrm{T}}(t) & \rho_{46}^{\mathrm{T}}(t) & \rho_{47}^{\mathrm{T}}(t) & \rho_{48}^{\mathrm{T}}(t) & \rho_{49}^{\mathrm{T}}(t) \\ \rho_{51}^{\mathrm{T}}(t) & \rho_{52}^{\mathrm{T}}(t) & \rho_{53}^{\mathrm{T}}(t) & \rho_{54}^{\mathrm{T}}(t) & \rho_{55}^{\mathrm{T}}(t) & \rho_{56}^{\mathrm{T}}(t) & \rho_{57}^{\mathrm{T}}(t) & \rho_{58}^{\mathrm{T}}(t) & \rho_{59}^{\mathrm{T}}(t) \\ \rho_{61}^{\mathrm{T}}(t) & \rho_{62}^{\mathrm{T}}(t) & \rho_{63}^{\mathrm{T}}(t) & \rho_{64}^{\mathrm{T}}(t) & \rho_{65}^{\mathrm{T}}(t) & \rho_{66}^{\mathrm{T}}(t) & \rho_{67}^{\mathrm{T}}(t) & \rho_{68}^{\mathrm{T}}(t) & \rho_{69}^{\mathrm{T}}(t) \\ \rho_{71}^{\mathrm{T}}(t) & \rho_{72}^{\mathrm{T}}(t) & \rho_{73}^{\mathrm{T}}(t) & \rho_{74}^{\mathrm{T}}(t) & \rho_{75}^{\mathrm{T}}(t) & \rho_{76}^{\mathrm{T}}(t) & \rho_{77}^{\mathrm{T}}(t) & \rho_{78}^{\mathrm{T}}(t) & \rho_{79}^{\mathrm{T}}(t) \\ \rho_{81}^{\mathrm{T}}(t) & \rho_{82}^{\mathrm{T}}(t) & \rho_{83}^{\mathrm{T}}(t) & \rho_{84}^{\mathrm{T}}(t) & \rho_{85}^{\mathrm{T}}(t) & \rho_{86}^{\mathrm{T}}(t) & \rho_{87}^{\mathrm{T}}(t) & \rho_{88}^{\mathrm{T}}(t) & \rho_{89}^{\mathrm{T}}(t) \\ \rho_{91}^{\mathrm{T}}(t) & \rho_{92}^{\mathrm{T}}(t) & \rho_{93}^{\mathrm{T}}(t) & \rho_{94}^{\mathrm{T}}(t) & \rho_{95}^{\mathrm{T}}(t) & \rho_{96}^{\mathrm{T}}(t) & \rho_{97}^{\mathrm{T}}(t) & \rho_{98}^{\mathrm{T}}(t) & \rho_{99}^{\mathrm{T}}(t) \end{pmatrix}. \quad (8)$$

3. Squeezing dynamics of two-atom system

To analyze the squeezing properties of the system with two two-level atoms, we introduce the collective operators S_1, S_2, and S_3 as[30]

$$\begin{aligned} S_1 &= \frac{1}{2}(S_+ + S_-), \\ S_2 &= \frac{1}{2\mathrm{i}}(S_+ - S_-), \\ S_3 &= S_1^z + S_2^z, \end{aligned} \quad (9)$$

where $S_+ = S_1^+ + S_2^+$, $S_- = S_1^- + S_2^-$, and $[S_1, S_2] = \mathrm{i}S_3$. The Heisenberg uncertainty relation is given by

$$(\Delta S_1)^2 (\Delta S_2)^2 \geq \frac{1}{4}\langle S_3\rangle^2, \quad (10)$$

where $(\Delta S_j)^2 = \langle S_j^2\rangle - \langle S_j\rangle^2$ $(j = 1, 2)$. Consequently, the fluctuation in the component S_j of the collective operators is said to be squeezed if S_j satisfies the condition

$$(\Delta S_j)^2 < \frac{1}{2}|\langle S_3\rangle|, \quad (11)$$

or

$$F_j = (\Delta S_j)^2 - \frac{1}{2}|\langle S_3\rangle| < 0. \quad (12)$$

Assume that $\rho_{AB}(t)$ expresses the density matrix of the two-atom system, we can obtain $\langle S_j\rangle = \mathrm{Tr}(S_j\rho_{AB}(t))$ and $\langle S_j^2\rangle = \mathrm{Tr}(S_j^2\rho_{AB}(t))$. Then, we insert $\langle S_j\rangle$ and $\langle S_j^2\rangle$ into Eq. (11) or Eq. (12) and can analyze numerically the squeezing properties of the two-atom system.

4. Results and discussion

In the standard basis $\mathcal{B}^{\mathrm{T}} = \{|11\rangle, |10\rangle, |01\rangle, |00\rangle, |ee\rangle, |eg\rangle, |ge\rangle, |gg\rangle\}$, we set that the initial state of the two atom–cavity is

$$|\psi(0)\rangle = \left(\cos\left(\frac{\theta}{2}\right)|gg\rangle + \mathrm{e}^{\mathrm{i}\varphi}\sin\left(\frac{\theta}{2}\right)|ee\rangle\right)_{AB}|00\rangle_{ab}, \quad (13)$$

where A and B indicate two atoms, a and b express two cavities, θ is an amplitude parameter, and φ is a phase parameter. We can obtain the reduced density matrix $\rho_{ABab}(t)$ using Eq. (8), then $\rho_{AB}(t)$ can also be obtained by taking a partial trace of $\rho_{ABab}(t)$ over the other two degrees of freedom. In order to quantify the entanglements of $\rho_{AB}(t)$, we use Wootter's concurrence,[27,31] which is defined as

$$C = \max(0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}), \quad (14)$$

where λ_i are the eigenvalues (in a descending order) of the matrix $\tilde{\rho} = \rho(\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$. In the following, we analyze the influences of the initially atomic state, the atom–cavity coupling, the non-Markovian effect, and the resonant frequency of the atom–cavity on the atomic squeezing dynamics.

In Fig. 1, we describe the influence of initial atomic states on the atomic squeezing in the Markovian regime ($\lambda = 5\gamma_0$). Figure 1(a) shows the dynamics revolution of the atomic squeezing factor F_2 as functions of φ and $\gamma_0 t$ when $\theta = \pi/4$. From Fig. 1(a), we see that the F_2 is obvious t dependent but is φ independent. For a certain value of φ, the F_2 oscillates damply when time t increases, namely, the squeezing effect of the initial atomic state disappears quickly but it is again revived in a very short time. For different φ, the F_2 always oscillates damply in the same rule as time t increases. Figure 1(b) depicts the dynamics revolution of the squeezing factor F_2 as functions of θ and $\gamma_0 t$ when $\varphi = 0$. From Fig. 1(b), we find that, the F_2 is not only dependent on t but also dependent on θ. For a certain time t, the periodic oscillation will occur in the atomic squeezing as θ increases. For a certain value of θ, the F_2 oscillates damply when time t increases. For different θ, there are the diverse evolution curves of F_2, for example, there is not any squeezing phenomena when $\theta = \pi/2$ while the maximum atomic squeezing is $F_2 = -0.2$ when $\theta = \pi/4$.

In Fig. 2, we plot the influence of the atom–cavity coupling Ω on the atomic squeezing in the Markovian regime ($\lambda = 5\gamma_0$). From Fig. 2, we can see that the F_2 is obviously dependent on the atom–cavity coupling. Figure 2(a) exhibits a quick oscillation of F_2 in a short time scale as time t increases due to the interaction between the atom with the dissipative cavity. The maximum atomic squeezing is $F_2 = -0.2$. In a longer time scale, the oscillation of F_2 is modulated by the coupling parameter Ω (see the envelope line expressed by the red-dotted line in Fig. 2(a)), i.e., the F_2-oscillation collapses to zero first and then is revived in a very short

time. The collapse–revival of F_2-oscillation is observable over $\gamma_0 t \in [0,8]$ and the periods are π/Ω, and this collapse–revival phenomenon is always accompanied by the buildup and decay of entanglement between two atoms. The collapse of F_2-oscillation appears when the entanglement C reduces to zero from 0.707. However, the revival of F_2-oscillation again arises when the C again rises from zero. Finally, the collapse–revival of F_2-oscillations will disappear at $\gamma_0 t = 7.4$ due to the dissipation of the cavities coupling with the Markovian environments. Comparing Figs. 2(a) and 2(b), we can know that their squeezing dynamics is similar for different Ω. The difference is in the collapse–revival frequency of F_2-oscillation and in their decay rates. The collapse–revival frequency of F_2-oscillation in the latter case is twice the former. The decay rate of the atomic squeezing is obviously smaller than that in the first one. In particular, if $\Omega \geq 50\gamma_0$, the collapse–revival frequency of F_2-oscillation will become very large and the F_2 will tend to a stably periodic oscillation when $\gamma_0 t > 16$, that is, this collapse–revival phenomena will disappear in a long time so that the F_2 exhibits a periodical oscillatory behavior between -0.052 to 0.124, as shown in Fig. 2(c). Hence, strengthening Ω can increase the collapse–revival frequency of F_2-oscillation and reduce the decay of F_2.

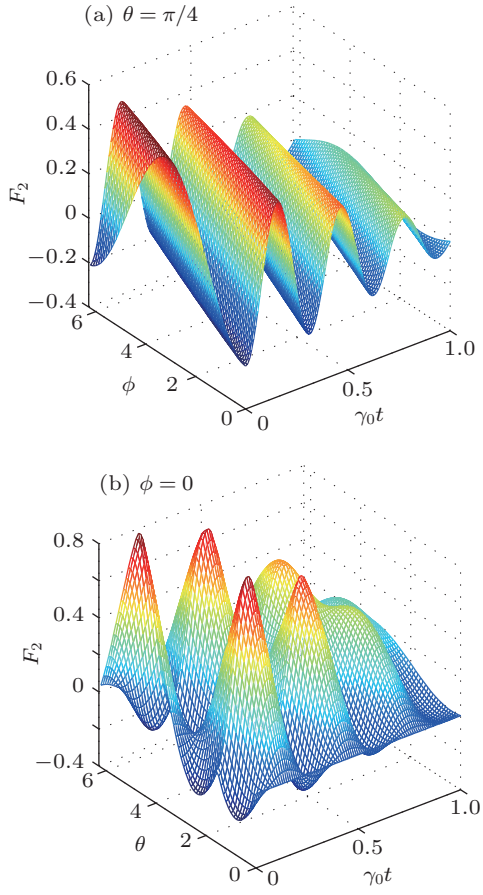


Fig. 1. (color online) The influence of the initially atomic state on the atomic squeezing dynamics in the Markovian regime ($\lambda = 5\gamma_0$). (a) The dynamics revolution of the squeezing factor F_2 as functions of φ and $\gamma_0 t$ when $\theta = \pi/4$. (b) The dynamics revolution of the squeezing factor F_2 as functions of θ and $\gamma_0 t$ when $\varphi = 0$. Other parameters are $\Omega = \gamma_0$ and $\omega_0 = 20\gamma_0$.

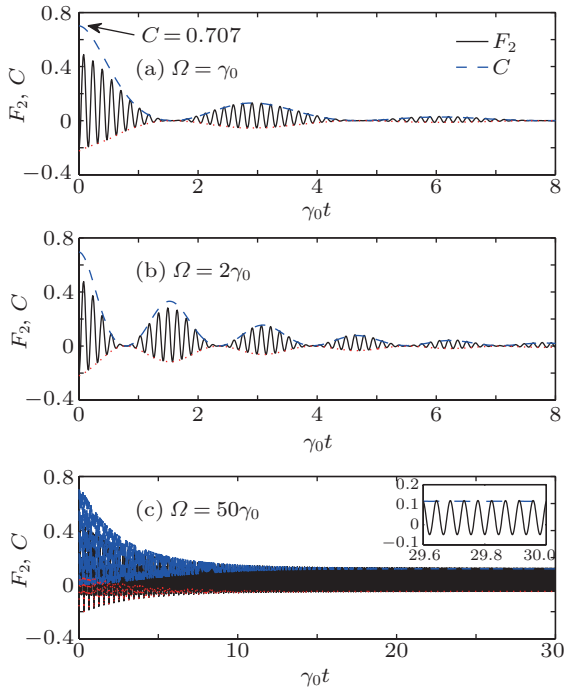


Fig. 2. (color online) The influence of the atom–cavity coupling Ω on the dynamics of atomic squeezing factor F_2 (black solid line) and the concurrence C (blue dashed line) versus $\gamma_0 t$ in the Markovian regime ($\lambda = 5\gamma_0$). (a) $\Omega = \gamma_0$; (b) $\Omega = 2\gamma_0$; (c) $\Omega = 50\gamma_0$. Other parameters are $\theta = \pi/4$, $\varphi = 0$, and $\omega_0 = 20\gamma_0$. The red dotted lines indicate the envelope lines of the F_2-oscillations.

The physical interpretations of the above results are as follows. Due to the interaction between the atom and its cavity, the quantum information will be exchanged between the atom and its cavity so that the C oscillates and the collapse–revival phenomenon of F_2-oscillation will occur in a short time. On the other hand, due to the coupling of the cavity with its bosonic environment, the quantum information exchanged to the cavity will continuously reduce thus the C and the collapse–revival phenomenon of F_2-oscillation disappears in a long time, as shown in Figs. 2(a) and 2(b). When Ω is very large, the quantum information can be exchanged very rapidly between the atom–cavity and can be partly trapped in the atom–cavity so that C tends to a stable value 0.12 and F_2 stably oscillates between -0.052 to 0.124 in a long time, shown as Fig. 2(c).

Figure 3 exhibits the influence of the non-Markovian effect on the atomic squeezing when $\Omega = \gamma_0$. From Fig. 3, we can find that, for the same Ω, the collapse–revival frequencies of F_2-oscillations are the same whether the Markovian or the non-Markovian regimes. The influence of the non-Markovian effect on the F_2 is embodied in the revived effect of F_2-oscillation. In the Markovian regime ($\lambda = 5\gamma_0$), the maximum atomic squeezing is $F_2 = -0.2$ at $\gamma_0 t = 0$ then collapses to zero at $\gamma_0 t = 1.57$. Afterwards, the maximum atomic squeezing is recovered to $F_2 = -0.05$ at $\gamma_0 t = 3.14$, which is much smaller than $F_2 = -0.2$. In particular, the second revival of F_2-oscillation is very close to zero due to the cavity dissipation, as shown in Fig. 3(a). For Fig. 3(b), when

$\lambda = 0.5\gamma_0$, due to the memory and feedback effect of the non-Markovian environment, the maximum atomic squeezing revived for the first time may reach to $F_2 = -0.18$ and the maximum atomic squeezing revived for the second time may also reach to $F_2 = -0.15$. This shows that the revived effect of F_2-oscillation in the non-Markovian regime is obviously better than the Markovian regime. In particular, when $\lambda = 0.05\gamma_0$, the collapse–revival phenomenon of F_2-oscillation is very remarkable in a short time and the F_2 will tend to a stably periodic oscillation between -0.052 to 0.124 in a long time, as shown in Fig. 3(c). Hence, the smaller the value of λ is, the stronger the non-Markovian effect is, the more remarkable the collapse–revival phenomenon of F_2-oscillation is.

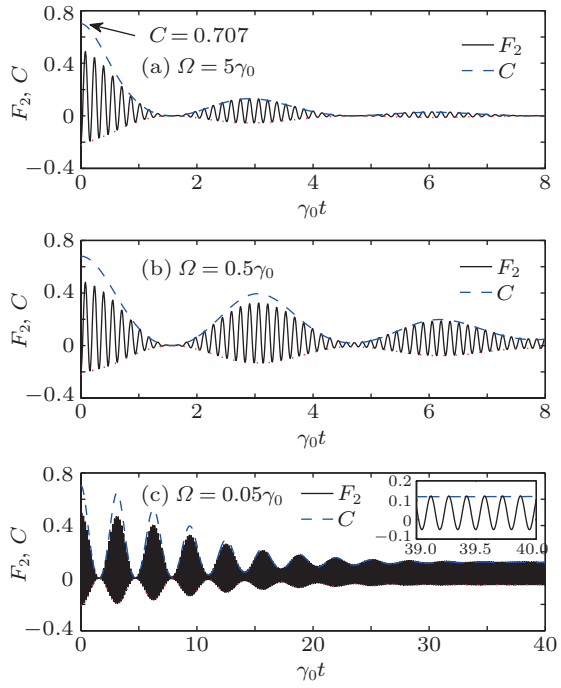


Fig. 3. (color online) The influence of the non-Markovian effect on the dynamics of atomic squeezing factor F_2 (black solid line) and the concurrence C (blue dashed line) versus $\gamma_0 t$ when $\Omega = \gamma_0$. (a) $\lambda = 5\gamma_0$; (b) $\lambda = 0.5\gamma_0$; (c) $\lambda = 0.05\gamma_0$. Other parameters are $\theta = \pi/4$, $\varphi = 0$, and $\omega_0 = 20\gamma_0$. The red dotted lines indicate the envelope lines of the F_2-oscillations.

The physical explanation is that the quantum information dissipated to the environments can be partly returned to the cavities due to the memory and feedback effect of the non-Markovian environments, and the stronger the non-Markovian effect is, the more the quantum information returned is. Therefore, in the Markovian or the weak non-Markovian regimes, the C and the collapse–revival of F_2-oscillation will change with a small amplitude and then decay to zero (see Figs. 3(a) and 3(b)). If the non-Markovian effect is very strong, the quantum information dissipated to the environment can be effectively fed back to the cavities so that the C and the F_2-oscillation can be well revived after they decrease to zero (see Fig. 3(c)).

Figure 4 displays the influence of the resonant frequency ω_0 of the atom–cavity on the atomic squeezing dynamics in the non-Markovian regime ($\lambda = 0.01\gamma_0$) and with $\Omega = \gamma_0$. From Fig. 4, we know that the frequency of the F_2-oscillations is obviously dependent on the resonant frequency ω_0, which arises from the resonant coupling of the atoms with its cavity model. Comparing Figs. 4(a) and 4(b), we find that the frequency of the F_2-oscillations in the latter case is twice the former because the resonant frequency in the latter case is twice the former. Thus, the more the value of ω_0 is, the more violent the Rabi oscillating of the atom–cavity is.

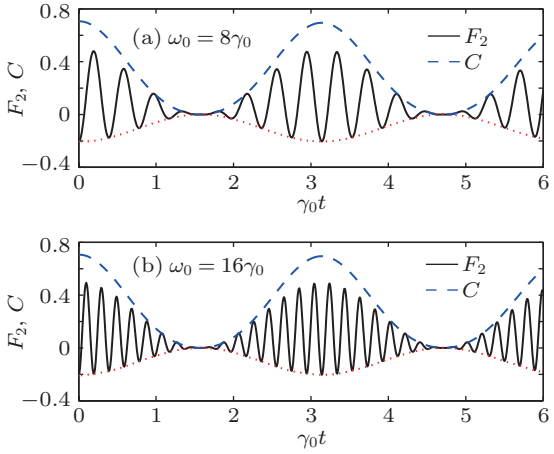


Fig. 4. (color online) The influence of the atomic Bohr frequency ω_0 on the dynamics of atomic squeezing factor F_2 (black solid line) and the concurrence C (blue dashed line) versus $\gamma_0 t$ when $\Omega = \gamma_0$. (a) $\omega_0 = 8\gamma_0$; (b) $\omega_0 = 16\gamma_0$. Other parameters are $\theta = \pi/4$, $\varphi = 0$, and $\lambda = 0.01\gamma_0$. The red dotted lines indicate the envelope lines of the F_2-oscillations.

5. Conclusion

In conclusion, we have investigated the squeezing dynamics of two atoms in dissipative cavities by the time-convolutionless master-equation approach. We discuss in detail the influences of the initially atomic state, the atom–cavity coupling, the non-Markovian effect, and the resonant frequency on the atomic squeezing dynamics when

$$|\psi(0)\rangle = \left[\cos\left(\frac{\theta}{2}\right)|gg\rangle + \mathrm{e}^{\mathrm{i}\varphi}\sin\left(\frac{\theta}{2}\right)|ee\rangle\right]_{AB}|00\rangle_{ab}.$$

The results show that the collapse–revival of the F_2-oscillations can occur whether the Markovian or the non-Markovian regimes. The squeezing factor F_2 is obviously θ dependent but is φ independent for different initial atomic states and the maximum atomic squeezing is $F_2 = -0.2$ when $\theta = \pi/4$. The frequency of the F_2-oscillations is obviously dependent on the resonant frequency ω_0, the more the value of ω_0 is, the larger the oscillating frequency of the atomic squeezing is. Strengthening the atom–cavity coupling can increase the collapse–revival frequency of F_2-oscillation and reduce the decay of F_2. The collapse–revival phenomenon of F_2-oscillation lies on the non-Markovian effect, the smaller the value of λ is, the stronger the non-Markovian effect is, the more remarkable the collapse–revival phenomenon of F_2-oscillation is. In particular, if the atom–cavity coupling or the non-Markovian effect is very strong, the atomic squeezing will

tend to a stably periodic oscillation in a long time. These results may offer interesting perspectives for future applications of open quantum systems in quantum optical, microwave cavity QED implementations, quantum communication, and quantum information processing.

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