Entanglement properties between two atoms in the binomial optical field interacting with two entangled atoms*

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The temporal evolution of the degree of entanglement between two atoms in a system of the binomial optical field interacting with two arbitrary entangled atoms is investigated. The influence of the strength of the dipole–dipole interaction between two atoms, probabilities of the Bernoulli trial, and particle number of the binomial optical field on the temporal evolution of the atomic entanglement are discussed. The result shows that the two atoms are always in the entanglement state. Moreover, if and only if the two atoms are initially in the maximally entangled state, the entanglement evolution is not affected by the parameters, and the degree of entanglement is always kept as 1.

Keywords: quantum optics, quantum entanglement, binomial optical field, negative eigenvalue

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1. Introduction

In quantum physics, the entanglement states of two or more particles have not only played a vital role in the discussion of nonlocal quantum correlations,[1] but also formed the basis of quantum communication and information processing, such as quantum computation,[2] quantum cryptography,[3] quantum teleportation,[4] and quantum dense coding.[5] For a bipartite pure state it is straightforward, that there has been a lot of work on bipartite entanglement. For example, with the von Neumann entropy,[6–15] distance between density operators,[16,17] fidelity,[18,19] negative eigenvalue,[20–23] and concurrence.[24] Recently, more and more researchers are paying attention to the non-orthogonality entangled coherent state.[25–28] Researching the entanglement properties of two two-level atoms interacting with the two-mode non-orthogonality entangled coherent state which has symmetry in phase space may be the vital practical significance and application.[29]

In this paper, based on the negative eigenvalue, which can be used to measure the quantum entanglement between two particles, we investigated the system which consists of two identical two-level atoms resonantly interacting with the binomial optical field. Consider that two atoms A, B are initially in an arbitrary Bell state and coupled to a cavity, simultaneously. The influence of the strength of the dipole–dipole interaction between two atoms, probabilities of the Bernoulli trial and the particle number of the binomial optical field are discussed by using full quantum theory and a numerical processing method.

Our paper is organized as follows. In Section 2, the theory model and state-vector equations solution are given. In Section 3, the negative eigenvalue of two two-level atoms in an entangled state interacting with the binomial optical field is investigated. Finally, the discussion and a brief summary are shown in Section 4.

2. Physical model and state-vector equations solution

We consider the case that two identical two-level atoms A and B are simultaneously injected into a binomial optical cavity. Assuming that the distance between atoms is smaller than the wavelength of the cavity field, made by exchanging between atoms produced by the virtual photons, the dipole–dipole interaction should not be neglected, and there are the same couplings of the two atoms as interacting with a binomial optical field. Under these conditions, the effective Hamiltonian in the rotating wave approximation can be written as ($\hbar = 1$)

$$H = H_0 + H_I, \tag{1}$$

$$H_0 = \Omega a^+ a + \frac{1}{2}\omega \sum_{i=A}^{B} \sigma_i^z, \tag{2}$$

$$H_I = g \sum_{i=A}^{B} (a^+ \sigma_i^- + a\sigma_i^+) + g_a \sum_{i\neq j=A}^{B} (\sigma_i^- \sigma_j^+), \tag{3}$$

where a^+ and a are the creation and annihilation operators of the field mode of frequency Ω, ω is the atomic transition frequency, $\sigma_i^z = |e_i\rangle\langle e_i|$, $\sigma_i^+ = |e_i\rangle\langle g_i|$, $\sigma_i^- = |g_i\rangle\langle e_i|$ are the inversion, rise, and drop operators of the ith atom ($i = A, B$), respectively, $|e\rangle$ denotes an excited state of atom, $|g\rangle$ denotes a ground state of atom, g is the atom-field coupling constant,

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and g_a is the atomic dipole–dipole coupling constant. For simplicity, we consider the resonant case ($\Omega = \omega$).

We consider that at $t = 0$, two two-level atoms are in the entangled state

$$|\varphi_a(0)\rangle = \cos\frac{\theta}{2}|e_A, g_B\rangle - \sin\frac{\theta}{2}\,e^{i\phi}|g_A, e_B\rangle,$$
$$\theta \in [0,\pi], \quad \phi \in [-\pi,\pi], \tag{4}$$

the field is in a single-mode binomial state

$$|\varphi_f(0)\rangle = |\sigma, M\rangle = \sum_{n=0}^{M} \beta_M^n |n\rangle,$$
$$(0 \le \sigma \le 1, \ \ M \ge n, \ \ n = 0,1,2,\ldots). \tag{5}$$

Here,

$$\beta_M^n = \left[\binom{M}{n}(\sigma)^n(1-\sigma)^{M-n}\right]^{1/2} = \left[\frac{M!}{n!\,(M-n)!}\sigma^n(1-\sigma)^{M-n}\right]^{1/2}, \tag{6}$$

where $|n\rangle$ is the number state of the field mode, σ and $(1-\sigma)$ are the probabilities of the two possible outcomes of the Bernoulli trial, and M is the largest number of photons in the Fock state. In the interacting picture, at any time $t > 0$, the evolution of the state vector of the system obeys the Schrödinger equation

$$i\frac{\partial}{\partial t}|\varphi_s(t)\rangle = H_I|\varphi_s(t)\rangle. \tag{7}$$

It can be obtained by solving the Schrödinger equation

$$\begin{aligned}|\varphi_s(t)\rangle = &\sum_{n=0}^{M} C_1(t,n)|e_A, e_B, n-1\rangle \\ &+ \sum_{n=0}^{M} C_2(t,n)|e_A, g_B, n\rangle \\ &+ \sum_{n=0}^{M} C_3(t,n)|g_A, e_B, n\rangle \\ &+ \sum_{n=0}^{M} C_4(t,n)|g_A, g_B, n+1\rangle.\end{aligned} \tag{8}$$

Here, the coefficients

$$C_1(t,n) = \frac{\left(\cos\frac{\theta}{2} - \sin\frac{\theta}{2}\,e^{i\phi}\right)\beta_M^n\sqrt{n}(1-e^{i\Delta gt})\,e^{-iagt}}{\Delta},$$

$$C_2(t,n) = \frac{1}{4}\beta_M^n e^{-iagt}\left[\left(c + \frac{(1-e^{i\Delta gt})G}{\Delta}\right)\cos\frac{\theta}{2} - \left(d + \frac{(1-e^{i\Delta gt})G}{\Delta}\right)\sin\frac{\theta}{2}\,e^{i\phi}\right],$$

$$C_3(t,n) = \frac{1}{4}\beta_M^n e^{-iagt}\left[\left(d + \frac{(1-e^{i\Delta gt})G}{\Delta}\right)\cos\frac{\theta}{2} - \left(c + \frac{(1-e^{i\Delta gt})G}{\Delta}\right)\sin\frac{\theta}{2}\,e^{i\phi}\right],$$

$$C_4(t,n) = \frac{\left(\cos\frac{\theta}{2} - \sin\frac{\theta}{2}\,e^{i\phi}\right)\beta_M^n\sqrt{n+1}(1-e^{i\Delta gt})\,e^{-iagt}}{\Delta}, \tag{9}$$

with

$$G = \frac{g_a}{g}, \quad a = \frac{G+\Delta}{2}, \quad \Delta = \sqrt{8(1+2n)+G^2},$$
$$c = 1 + e^{i\Delta gt} + 2e^{i\frac{1}{2}(3G+\Delta)gt},$$
$$d = 1 + e^{i\Delta gt} - 2e^{i\frac{1}{2}(3G+\Delta)gt}. \tag{10}$$

The density matrix of the system is $\rho(t) = |\varphi_s(t)\rangle\langle\varphi_s(t)|$.

3. Entanglement between the two entangled atoms

For two subsystems in the mixed state, we may use the negative eigenvalues of partial transposition of density matrix as a measurement of the degree of entanglement between the two subsystems. Consider a density matrix $\rho(t)$ and its partial transposition $\rho^T(t)$ for a system of two spin-1/2. The measure of entanglement $E_{AB}(t)$ is then defined by[30]

$$E_{AB}(t) = -2\sum_{i=1}^{n}\mu_i^-. \tag{11}$$

Here, μ_i^- is a negative eigenvalue of $\rho^T(t)$. Then $E_{AB}(t) = 0$ denotes that two atoms are separated. Then $E_{AB}(t) = 1$ denotes that two atoms are in the maximum entangled state. Then $0 < E_{AB}(t) < 1$ denotes that two atoms are in the entanglement state. In the following, we only consider the time behavior of the degree of entanglement between the two atoms $E_{AB}(t)$. In order to derive a calculation formalism of the degree of entanglement between the two atoms, we must obtain the eigenvalues of the reduced atom density operator. In the atom–atom bases $|e_A, e_B\rangle$, $|e_A, g_B\rangle$, $|g_A, e_B\rangle$, and $|g_A, g_B\rangle$, from Eq. (8), we have partial transposition $\rho^T(t)$ of the reduced atom density operator

$$\rho^T(t) = \begin{pmatrix} \rho_{11} & \rho_{21} & \rho_{13} & \rho_{23} \\ \rho_{12} & \rho_{22} & \rho_{14} & \rho_{24} \\ \rho_{31} & \rho_{41} & \rho_{33} & \rho_{43} \\ \rho_{32} & \rho_{42} & \rho_{34} & \rho_{44} \end{pmatrix}, \tag{12}$$

where

$$\rho_{11} = \sum_{n=1}^{M}|C_1(t,n)|^2, \quad \rho_{12} = \sum_{n=0}^{M}C_1(t,n+1)C_2^*(t,n),$$
$$\rho_{13} = \sum_{n=0}^{M}C_1(t,n+1)C_3^*(t,n), \quad \rho_{14} = \sum_{n=0}^{M}C_1(t,n+2)C_4^*(t,n),$$
$$\rho_{21} = \sum_{n=0}^{M}C_2(t,n)C_1^*(t,n+1), \quad \rho_{22} = \sum_{n=0}^{M}|C_2(t,n)|^2,$$
$$\rho_{23} = \sum_{n=0}^{M}C_2(t,n)C_3^*(t,n), \quad \rho_{24} = \sum_{n=0}^{M}C_2(t,n+1)C_4^*(t,n),$$
$$\rho_{31} = \sum_{n=0}^{M}C_3(t,n)C_1^*(t,n+1), \quad \rho_{32} = \sum_{n=0}^{M}C_3(t,n)C_2^*(t,n),$$

$$\rho_{33} = \sum_{n=0}^{M} |C_3(t,n)|^2, \quad \rho_{34} = \sum_{n=0}^{M} C_3(t,n+1)C_4^*(t,n),$$
$$\rho_{41} = \sum_{n=0}^{M} C_4(t,n)C_1^*(t,n+2), \quad \rho_{42} = \sum_{n=0}^{M} C_4(t,n)C_2^*(t,n+1),$$
$$\rho_{43} = \sum_{n=0}^{M} C_4(t,n)C_3^*(t,n+1), \quad \rho_{44} = \sum_{n=0}^{M} |C_4(t,n)|^2. \tag{13}$$

Consequently, according to the eigenvalue equation of the density operator $\rho^{\mathrm{T}}(t)$, the eigenvalues of the partial transposition of the density matrix of the atom are given by

$$\mu_1(t) = \frac{-k+\sqrt{\frac{2q}{k}-2p-k^2}}{2} - \frac{\chi_1}{4}, \tag{14}$$
$$\mu_2(t) = \frac{-k-\sqrt{\frac{2q}{k}-2p-k^2}}{2} - \frac{\chi_1}{4}, \tag{15}$$
$$\mu_3(t) = \frac{k+\sqrt{\frac{-2q}{k}-2p-k^2}}{2} - \frac{\chi_1}{4}, \tag{16}$$
$$\mu_4(t) = \frac{k-\sqrt{\frac{-2q}{k}-2p-k^2}}{2} - \frac{\chi_1}{4}, \tag{17}$$

where

$$p = \frac{8\chi_2 - 3\chi_1^2}{8}, \quad q = \frac{\chi_1^3 - 4\chi_1\chi_2 + 8\chi_3}{8},$$
$$k = \sqrt{\xi_1 + \xi_2 - \frac{2p}{3}}, \quad \xi_1 = \sqrt[3]{-\frac{h}{2} + \sqrt{\frac{h^2}{4} + \frac{f^3}{27}}},$$
$$\xi_2 = \sqrt[3]{-\frac{h}{2} - \sqrt{\frac{h^2}{4} + \frac{f^3}{27}}}, \quad f = -\frac{p^2+12r}{3},$$
$$h = -\frac{2p^3 - 72pr + 27q^2}{27},$$
$$r = -\frac{3\chi_1^4}{256} + \frac{\chi_1^2\chi_2}{16} - \frac{\chi_1\chi_3}{4} + \chi_4, \tag{18}$$

with

$$\begin{aligned}
\chi_1 &= -(\rho_{11}+\rho_{22}+\rho_{33}+\rho_{44}) = -1,\\
\chi_2 &= \rho_{11}\rho_{22}+\rho_{11}\rho_{33}+\rho_{11}\rho_{44}+\rho_{22}\rho_{33}+\rho_{22}\rho_{44}+\rho_{33}\rho_{44}\\
&\quad -\rho_{14}\rho_{41}-\rho_{24}\rho_{42}-\rho_{34}\rho_{43}-\rho_{12}\rho_{21}-\rho_{13}\rho_{31}-\rho_{23}\rho_{32},\\
\chi_3 &= -\rho_{11}\rho_{22}\rho_{33}-\rho_{11}\rho_{22}\rho_{44}-\rho_{11}\rho_{33}\rho_{44}-\rho_{22}\rho_{33}\rho_{44}\\
&\quad -\rho_{14}\rho_{42}\rho_{43}-\rho_{24}\rho_{41}\rho_{34}-\rho_{12}\rho_{23}\rho_{42}-\rho_{12}\rho_{13}\rho_{41}\\
&\quad -\rho_{31}\rho_{21}\rho_{14}-\rho_{31}\rho_{23}\rho_{34}-\rho_{21}\rho_{24}\rho_{32}-\rho_{13}\rho_{32}\rho_{43}\\
&\quad +\rho_{11}\rho_{14}\rho_{41}+\rho_{11}\rho_{24}\rho_{42}+\rho_{11}\rho_{34}\rho_{43}+\rho_{12}\rho_{21}\rho_{33}\\
&\quad +\rho_{12}\rho_{21}\rho_{44}+\rho_{22}\rho_{34}\rho_{43}+\rho_{22}\rho_{23}\rho_{32}+\rho_{33}\rho_{24}\rho_{42}\\
&\quad +\rho_{31}\rho_{13}\rho_{44}+\rho_{31}\rho_{13}\rho_{22}+\rho_{23}\rho_{32}\rho_{33}+\rho_{14}\rho_{41}\rho_{44},\\
\chi_4 &= \rho_{11}\rho_{22}\rho_{33}\rho_{44}+\rho_{11}\rho_{14}\rho_{42}\rho_{43}+\rho_{11}\rho_{24}\rho_{34}\rho_{41}\\
&\quad +\rho_{12}\rho_{21}\rho_{34}\rho_{43}+\rho_{12}\rho_{23}\rho_{33}\rho_{42}+\rho_{12}\rho_{13}\rho_{41}\rho_{44}\\
&\quad +\rho_{22}\rho_{23}\rho_{31}\rho_{34}+\rho_{13}\rho_{24}\rho_{31}\rho_{42}+\rho_{21}\rho_{24}\rho_{32}\rho_{33}\\
&\quad +\rho_{13}\rho_{22}\rho_{32}\rho_{43}+\rho_{14}\rho_{23}\rho_{32}\rho_{43}+\rho_{14}\rho_{21}\rho_{31}\rho_{44}\\
&\quad -\rho_{11}\rho_{24}\rho_{33}\rho_{42}-\rho_{11}\rho_{22}\rho_{34}\rho_{43}-\rho_{11}\rho_{14}\rho_{41}\rho_{44}\\
&\quad -\rho_{12}\rho_{21}\rho_{33}\rho_{44}-\rho_{12}\rho_{13}\rho_{43}\rho_{42}-\rho_{12}\rho_{23}\rho_{41}\rho_{34}\\
&\quad -\rho_{21}\rho_{24}\rho_{31}\rho_{34}-\rho_{22}\rho_{23}\rho_{32}\rho_{33}-\rho_{13}\rho_{24}\rho_{32}\rho_{43}\\
&\quad -\rho_{14}\rho_{23}\rho_{31}\rho_{42}-\rho_{14}\rho_{21}\rho_{32}\rho_{43}-\rho_{13}\rho_{22}\rho_{31}\rho_{44}.
\end{aligned} \tag{19}$$

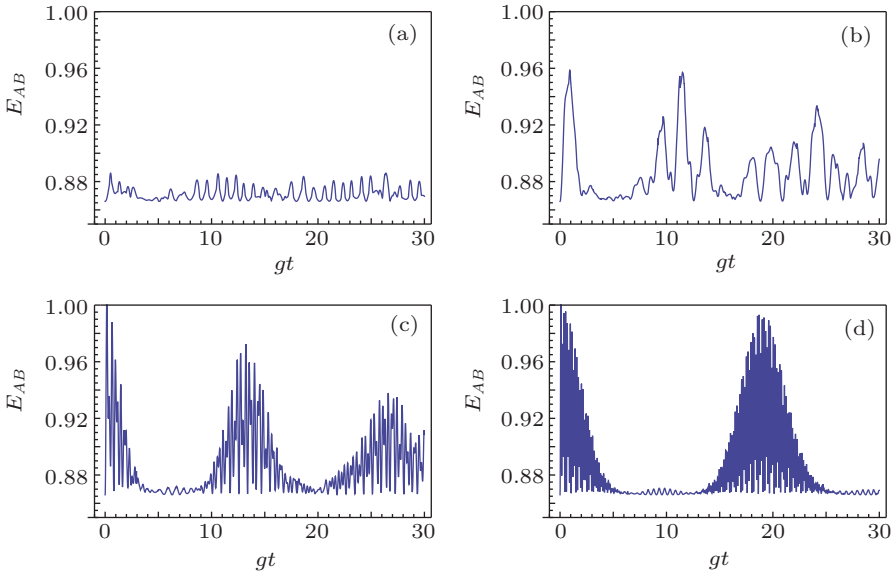


Fig. 1. (color online) Time evolution of E_{AB} versus $\theta = \pi/3$, or $\theta = 2\pi/3$ for different G. The parameters are $\phi = 0$, $M = 5$, and $\sigma = 0.5$. From panel (a) to panel (d): $G = 0$; $G = 1$; $G = 5$; $G = 10$.

According to the above theory, we can conveniently obtain quantum entanglement between two atoms, corresponding to three different atomic initial states and parameters respectively, shown in Figs. 1–7.

Case 1 We assume $\theta = \pi/3$ or $\theta = 2\pi/3$, and $\phi = 0$, the two atomic initial states are

$$|\varphi_{\mathrm{a}}(0)\rangle = \frac{\sqrt{3}}{2}|e_A, g_B\rangle - \frac{1}{2}|g_A, e_B\rangle, \tag{20}$$

and

$$|\varphi_{\rm a}(0)\rangle = \frac{1}{2}|e_A, g_B\rangle - \frac{\sqrt{3}}{2}|g_A, e_B\rangle, \tag{21}$$

respectively. The temporal evolution curves of the quantum entanglement between two atoms are much alike, with the above two atomic initial states, shown in Figs. 1–3.

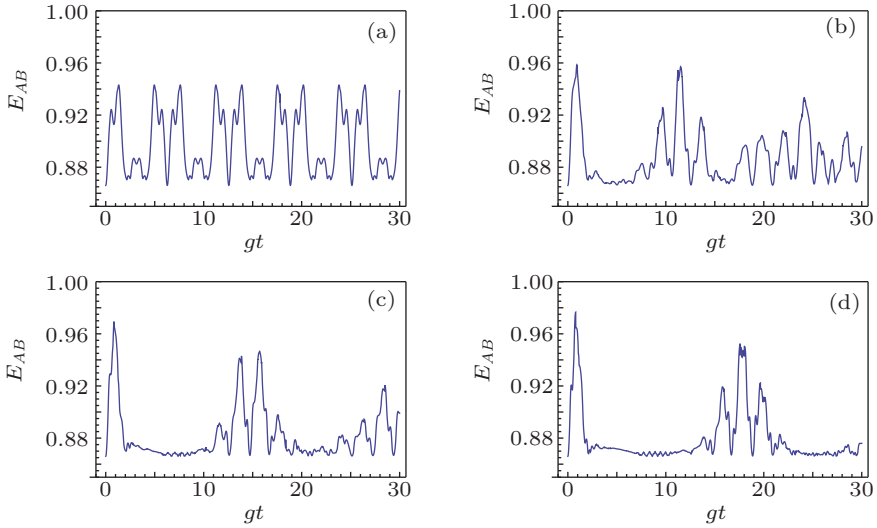


Fig. 2. (color online) Time evolution of E_{AB} versus $\theta = \pi/3$, or $\theta = 2\pi/3$ for different M. The parameters are $\phi = 0$, $G = 1$, and $\sigma = 0.5$. From panel (a) to panel (d): $M = 1$; $M = 5$; $M = 10$; $M = 15$.

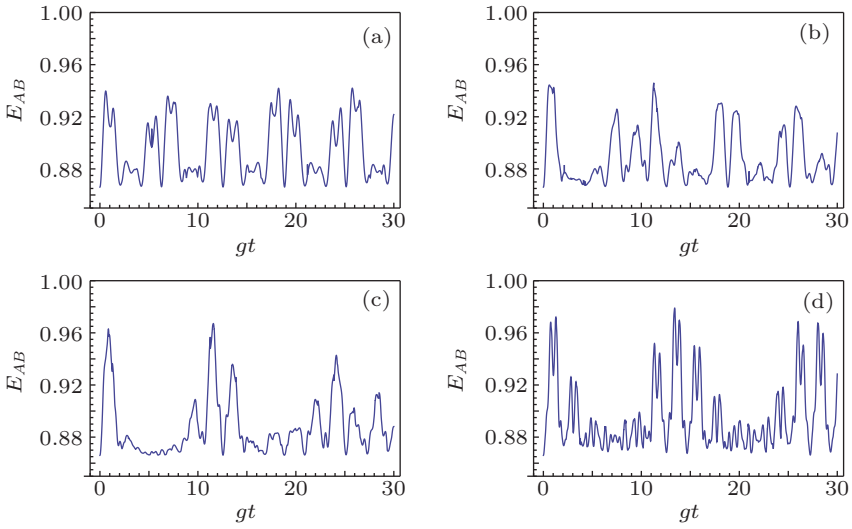


Fig. 3. (color online) Time evolution of E_{AB} versus $\theta = \pi/3$, or $\theta = 2\pi/3$ for different σ. The parameters are $\phi = 0$, $M = 5$, and $G = 1$. From panel (a) to panel (d): $\sigma = 0.1$; $\sigma = 0.3$; $\sigma = 0.6$; $\sigma = 0.9$.

In Fig. 1, we choose that M and σ are fixed values respectively. In Fig. 1(a), we have $G = 0$, corresponding to the temporal evolution of E_{AB} in the absence of the dipole–dipole interaction between two atoms, while with atomic coupling enhancing shown in Figs. 1(b)–1(d). Figure 1(a) shows that the oscillation of the temporal evolution has a smaller range and irregularity compared with that in Figs. 1(b)–1(d), and the initial value of E_{AB} is not maximal, namely 0.865. With atomic coupling enhancing (see Figs. 1(b)–1(d)), the temporal evolution of E_{AB} is in the range between 0.865 and 1, and that the oscillation frequency accelerates. The greater atomic coupling makes the oscillation of collapse-and-revival more obvious (see Figs. 1(c) and 1(d)).

In Fig. 2, we choose G and σ are fixed values respectively. Figure 2(a) shows that the oscillation of the temporal evolution of EAB appears cyclical, in the single photon process, that is $M = 1$. With the particle number enhancing (see Figs. 2(b)–2(d)), the oscillation of the temporal evolution of EAB is ruleless.

In Fig. 3, we select that G and M are fixed values respec-

tively. With probabilities of a Bernoulli trial enhancing (see Figs. 3(a)–3(d)), the oscillation of the temporal evolution of EAB has irregularity.

Case 2 In the case of $\theta = \pi/3$ or $\theta = 2\pi/3$, and $\phi = \pi$, the initial states of the two atoms are

$$|\varphi_{\rm a}(0)\rangle = \frac{\sqrt{3}}{2}|e_A, g_B\rangle + \frac{1}{2}|g_A, e_B\rangle, \tag{22}$$

$$|\varphi_{\rm a}(0)\rangle = \frac{1}{2}|e_A, g_B\rangle + \frac{\sqrt{3}}{2}|g_A, e_B\rangle, \tag{23}$$

respectively. The temporal evolution curves of quantum entanglement between two atoms are very similar, with the condition of the above two atomic initial states, shown in Figs. 4–6.

In Fig. 4, the chosen parameters M and σ are the same as those in Fig. 1. Compared with Fig. 1, we can find the temporal evolution of EAB has the same initial value and the oscillation of collapse-and-revival occurs, meanwhile, the oscillation frequency accelerates; but the evolution curve is different and its range is between 0 and 1.

Homologous, the chosen parameters G and σ in Fig. 5 are the same as those in Fig. 2, the chosen parameters G and M in Fig. 6 are the same as those in Fig. 3, respectively. Compared with Figs. 2 and 3, separately, it is observed that the initial evolution of EAB has the same value, while the evolution curve is different and its range is between 0 and 1.

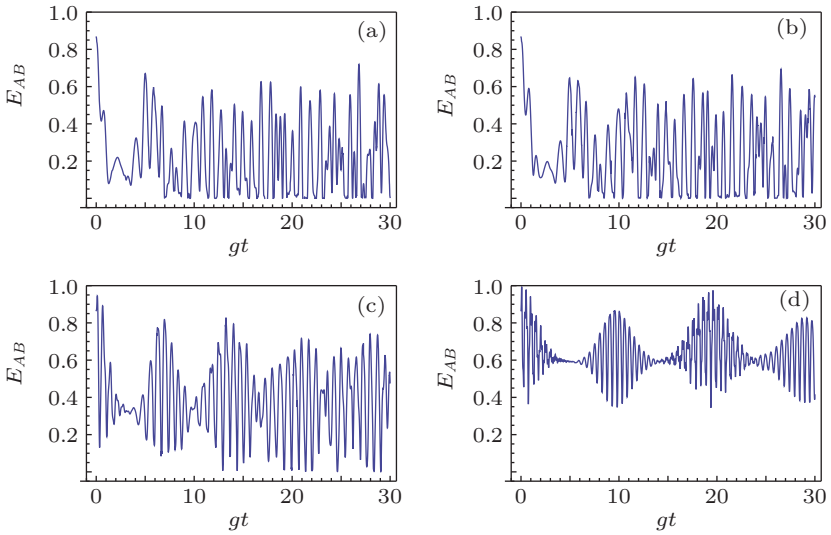


Fig. 4. (color online) Time evolution of E_{AB} versus $\theta = \pi/3$, or $\theta = 2\pi/3$ for different G. The parameters are $\phi = \pi$, $M = 5$, and $\sigma = 0.5$. From panel (a) to panel (d): $G = 0$; $G = 1$; $G = 5$; $G = 10$.

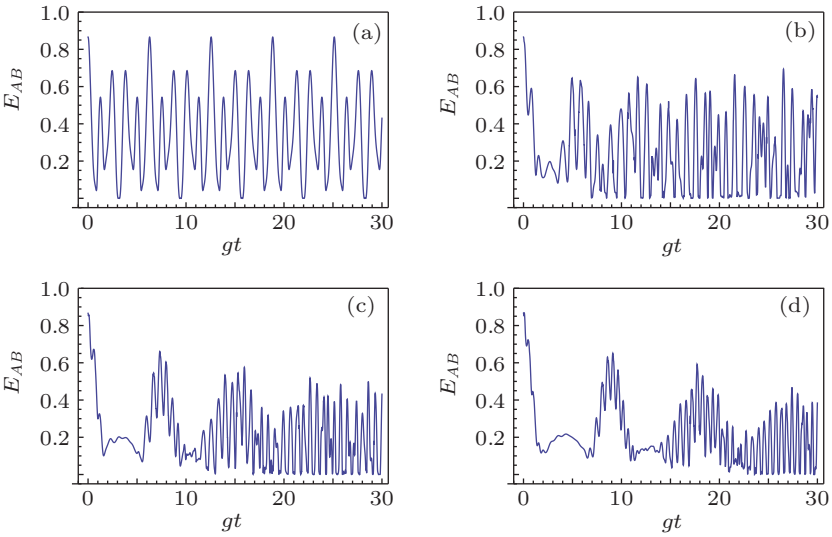


Fig. 5. (color online) Time evolution of E_{AB} versus $\theta = \pi/3$, or $\theta = 2\pi/3$ for different M. The parameters are $\phi = \pi$, $G = 1$, and $\sigma = 0.5$. From panel (a) to panel (d): $M = 1$; $M = 5$; $M = 10$; $M = 15$.

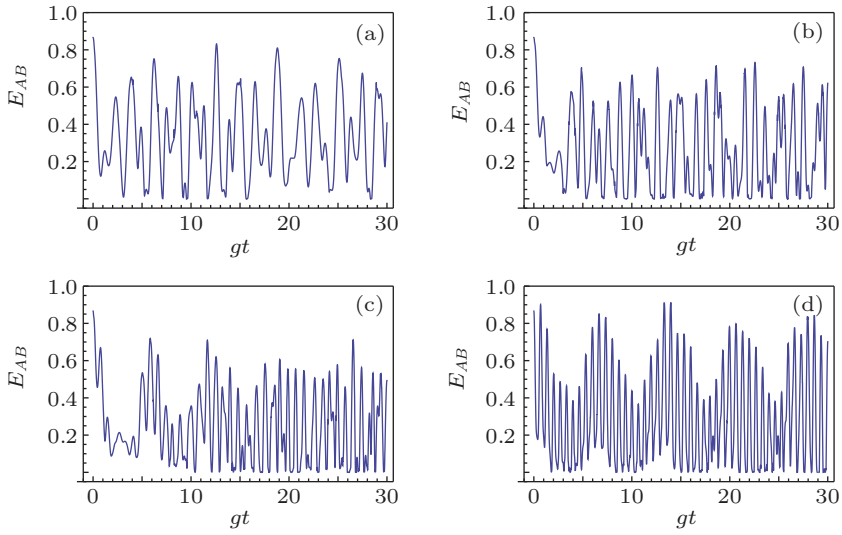


Fig. 6. (color online) Time evolution of E_{AB} versus $\theta = \pi/3$, or $\theta = 2\pi/3$ for different σ. The parameters are $\phi = \pi$, $M = 5$, and $G = 1$. From panel (a) to panel (d): $\sigma = 0.1$; $\sigma = 0.3$; $\sigma = 0.6$; $\sigma = 0.9$.

Case 3 Suppose $\theta = \pi/2$, $\phi = 0$ or $\phi = \pi$, the initial states of the two atoms are

$$|\varphi_{\rm a}(0)\rangle = \frac{1}{\sqrt{2}}(|e_A, g_B\rangle - |g_A, e_B\rangle), \tag{24}$$

$$|\varphi_{\rm a}(0)\rangle = \frac{1}{\sqrt{2}}(|e_A, g_B\rangle + |g_A, e_B\rangle), \tag{25}$$

respectively. They are two atoms in the maximal entangled state initially. The temporal evolution curves of the quantum entanglement between two atoms are the same, with the condition of the above two atomic initial states, shown in Fig. 7.

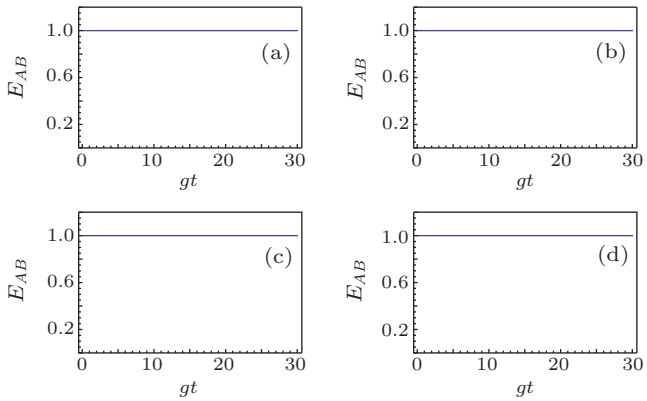


Fig. 7. (color online) Time evolution of E_{AB} versus $\theta = \pi/2$ for different σ. The parameters are $\phi = 0$ or $\phi = \pi$, $M = 5$, and $G = 1$. From panel (a) to panel (d): $\sigma = 0.1$; $\sigma = 0.3$; $\sigma = 0.6$; $\sigma = 0.9$.

In Case 3, with certain parameters, we obtained the temporal evolution of EAB in Fig. 7. Under the condition that two two-level atoms are initially in the maximum entangled state, the entanglement between the two atoms is always kept as 1.

4. Summary

In conclusion, the temporal evolution of the degree of entanglement between two atoms in a system of two entangled atoms interacting with the binomial optical field has been studied. The influence of the strength of the dipole–dipole interaction between two atoms, probabilities of the Bernoulli trial, and particle number of the binomial optical field on the time evolution of the atomic entanglement are discussed. The result shows that the two atoms are always in the entanglement state. Moreover, an interesting phenomenon is that when and only when the two atoms are initially in the maximally entangled state, the entanglement evolution between two atoms is not affected by the parameters, and the degree of entanglement is always kept as 1. This result will be helpful for the understanding of entanglement by further investigation of the entangled atoms interacting with all kinds of fields.

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