Robust H_∞ control for uncertain Markovian jump systems with mixed delays*

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(Received 11 September 2015; revised manuscript received 21 March 2016; published online 9 June 2016)

We scrutinize the problem of robust H_∞ control for a class of Markovian jump uncertain systems with interval time-varying and distributed delays. The Markovian jumping parameters are modeled as a continuous-time finite-state Markov chain. The main aim is to design a delay-dependent robust H_∞ control synthesis which ensures the mean-square asymptotic stability of the equilibrium point. By constructing a suitable Lyapunov–Krasovskii functional (LKF), sufficient conditions for delay-dependent robust H_∞ control criteria are obtained in terms of linear matrix inequalities (LMIs). The advantage of the proposed method is illustrated by numerical examples. The results are also compared with the existing results to show the less conservativeness.

Keywords: linear matrix inequality, Lyapunov method, Markovian jumping parameters, robust H_∞ control

PACS: 02.10.Yn, 02.30.Hq, 02.30.Ks, 02.30.Yy **DOI:** 10.1088/1674-1056/25/7/070201

1. Introduction

Time delays are frequently encountered in many practical systems. They are common in many industrial and engineering systems such as communication networks, manufacturing devices, and biology. Since the system stability is an essential requirement in many applications, much effort has been devoted to investigate the stability criteria for various time-delay systems during the last few decades.[1–4] It is well recognized that the presence of time delay in a dynamical system is often a primary source of instability and performance degradation. Moreover, the delay-dependent[5] results are generally less conservative than the delay-independent ones,[6] especially when the delay is small. Recently, much attention has been paid to the robust stability problem for uncertain systems with time delays.[7,8]

Markovian jump systems are a special sort of hybrid system, which are driven by a Markov chain, where they may undergo unexpected changes in their structures and parameters, e.g., aerospace systems, power systems, etc.[9–11] Such systems can be described by a set of time-delayed linear systems with the transitions between models determined by a Markov chain in a finite mode set. Applications of this class of systems may be found in processes including those in production systems and economic problems, etc. In the last few years, some quite significant results on the stability analysis and controller synthesis for Markovian jumping systems with time-delay have been reported.[11,12]

The control design for uncertain systems with time-varying delays has been a difficult problem of control theory.[13,14] Theoretically, this issue could be solved by the Lyapunov method. The H_∞ performance is usually analyzed in the control theory to synthesize controllers achieving stabilization with guaranteed performance. Stability is one of the most important problems in the synthesis of control systems. Therefore, the problems of delay-dependent stability analysis and H_∞ control for delayed systems have received substantial attention among the control community for the last few decades.[15]

The H_∞ control concept was proposed to reduce the effect of the disturbance input on the regulated output within a prescribed level and guarantee that the closed-loop system is stable. In Refs. [16] and [17], the H_∞ technology was proposed to solve the disturbance attenuation problem in various difficult systems. In this paper, we provide a delay-dependent H_∞ control for Markovian jump uncertain systems with interval time-varying and distributed delays. A linear matrix inequality (LMI) optimization approach is used to achieve the best effect of disturbance attenuation. The obtained results in this paper are generally less conservative than those of some existing methods.

The following notations are used. Throughout this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times n}$ denote the n-dimensional Euclidean space and the set of all $n\times n$ real matrices, respectively. For a matrix $\boldsymbol{B}$ and two symmetric matrices $\boldsymbol{A}$ and $\boldsymbol{C}$, $\begin{bmatrix} \boldsymbol{A} & \boldsymbol{B} \\ * & \boldsymbol{C} \end{bmatrix}$ denotes a symmetric matrix, where the notation * represents the entries implied by symmetry. $\boldsymbol{A}^{\mathrm{T}}$ and $\boldsymbol{A}^{-1}$ are the matrix transpose and inverse of $\boldsymbol{A}$, respectively. $\boldsymbol{X} > 0$ means that matrix $\boldsymbol{X}$ is a symmetric positive definite matrix with appropriate dimensions. $\boldsymbol{I}$ denotes the identity matrix with appropriate dimensions. $\mathcal{E}$ denotes the expectation operator with respect to some

*Project supported by Department of Science and Technology (DST) under research project No. SR/FTP/MS-039/2011.

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probability measure $\mathcal{P}$. Let $(\Omega,\mathcal{F},\mathcal{P})$ be a complete probability space which relates to an increasing family $(\mathcal{F}_t)_{t>0}$ of σ algebras $(\mathcal{F}_t)_{t>0}\subset\mathcal{F}$, where Ω is the sample space, $\mathcal{F}$ is the σ algebra of subsets of the sample space, and $\mathcal{P}$ is the probability measure on $\mathcal{F}$. Let $||\boldsymbol{f}||_2=\sqrt{\int_0^\infty||\boldsymbol{f}||^2\mathrm{d}t}$, $\boldsymbol{f}(t)\in L_2[0,\infty)$, $\|\boldsymbol{f}\|$ refers to the Euclidean norm of function $\boldsymbol{f}(t)$ at time t. $\mathbb{L}_2[0,\infty)$ is the space of square integrable vectors on $[0,\infty)$.

2. Problem statement

Let $\{r_t,t\geq 0\}$ be a right-continuous Markov chain on a probability space $(\Omega,\mathcal{F},\mathcal{P})$ taking values in a finite state space $\mathcal{N}=\{1,2,\ldots,m\}$ with generator $\Pi=\{\pi_{pq}\}$ given by

$$P\{r_{t+\Delta}=q|r_t=p\}=\begin{cases}\pi_{pq}\Delta+o(\Delta), & p\neq q,\\ 1+\pi_{pp}\Delta+o(\Delta), & p=q,\end{cases}$$

where $\Delta>0$, $\lim_{t\to+\infty}\frac{o(\Delta)}{\Delta}=0$, and $\pi_{pq}\geq 0$ is the transition rate from p to q if $q\neq p$, while $\pi_{pp}=-\sum_{q=1,q\neq p}^{m}\pi_{pq}$.

Consider the following uncertain Markovian jump system with interval time-varying and distributed delays:

$$\begin{aligned}\dot{\boldsymbol{x}}(t)&=\boldsymbol{A}(r_t,t)\boldsymbol{x}(t)+\boldsymbol{A}_{\mathrm{d}}(r_t,t)\boldsymbol{x}(t-\tau(t))\\&\quad+\boldsymbol{A}_{\mathrm{f}}(r_t,t)\int_{t-h}^{t}\boldsymbol{x}(s)\mathrm{d}s+\boldsymbol{B}(r_t,t)\boldsymbol{u}(t)+\boldsymbol{B}_w(r_t)\boldsymbol{w}(t),\\ \boldsymbol{z}(t)&=\boldsymbol{C}(r_t)\boldsymbol{x}(t)+\boldsymbol{C}_{\mathrm{d}}(r_t)\boldsymbol{x}(t-\tau(t))\\&\quad+\boldsymbol{D}(r_t)u(r_t)+\boldsymbol{D}_w(r_t)\boldsymbol{w}(r_t),\\ \boldsymbol{x}(s)&=\boldsymbol{\phi}(s),\quad s\in[-\max(\tau_{\mathrm{M}},h),0],\end{aligned}\tag{1}$$

where $\boldsymbol{x}(t)\in\mathbb{R}^n$ is the state, $\boldsymbol{u}(t)\in\mathbb{R}^m$ is the control input, $\boldsymbol{w}(t)\in\mathbb{R}^p$ is the disturbance input which belongs to $L_2[0,\infty)$, and $\boldsymbol{z}(t)\in\mathbb{R}^q$ is the controlled output of the system. The initial vector $\boldsymbol{\phi}\in\boldsymbol{C}_0$, where $\boldsymbol{C}_0$ is the set of continuous functions from $[-\tau_{\mathrm{M}},0]$ to $\mathbb{R}^n$. $\tau(t)$ and h denote the time-varying delay and the distributed delay, respectively, and $\tau(t)$ is assumed to satisfy

$$\tau_{\mathrm{m}}\leq\tau(t)\leq\tau_{\mathrm{M}},\ \dot{\tau}(t)\leq\mu,\tag{2}$$

where τ_{m}, τ_{M}, h, and μ are constants. The matrices $\boldsymbol{A}(r_t,t)=\boldsymbol{A}(r_t)+\Delta\boldsymbol{A}(r_t,t)$, $\boldsymbol{A}_{\mathrm{d}}(r_t,t)=\boldsymbol{A}_{\mathrm{d}}(r_t)+\Delta\boldsymbol{A}_{\mathrm{d}}(r_t,t)$, $\boldsymbol{A}_{\mathrm{f}}(r_t,t)=\boldsymbol{A}_{\mathrm{f}}(r_t)+\Delta\boldsymbol{A}_{\mathrm{f}}(r_t,t)$, $\boldsymbol{B}(r_t,t)=\boldsymbol{B}(r_t)+\Delta\boldsymbol{B}(r_t,t)$, where $\boldsymbol{B}_w(r_t),\boldsymbol{C}(r_t),\boldsymbol{C}_{\mathrm{d}}(r_t),\boldsymbol{D}(r_t)$ are known real constant matrices with appropriate dimensions, and $\Delta\boldsymbol{A}(r_t,t),\Delta\boldsymbol{A}_{\mathrm{d}}(r_t,t),\Delta\boldsymbol{A}_{\mathrm{f}}(r_t,t),\Delta\boldsymbol{B}(r_t,t)$ are real-valued unknown matrices representing the time-varying parameter uncertainties which are assumed to be of the following form:

$$\begin{aligned}&\left[\Delta\boldsymbol{A}(r_t,t)\ \Delta\boldsymbol{A}_{\mathrm{d}}(r_t,t)\ \Delta\boldsymbol{A}_{\mathrm{f}}(r_t,t)\ \Delta\boldsymbol{B}(r_t,t)\right]\\&=\boldsymbol{L}(r_t)\boldsymbol{F}(r_t,t)\left[\boldsymbol{E}_1(r_t)\ \boldsymbol{E}_2(r_t)\ \boldsymbol{E}_3(r_t)\ \boldsymbol{E}_4(r_t)\right],\end{aligned}\tag{3}$$

where $\boldsymbol{L}(r_t)$ and $\boldsymbol{E}_i(r_t)$ $(i=1,2,3,4)$ are known real constant matrices and $\boldsymbol{F}(t)$ is an unknown time-varying matrix function satisfying $\boldsymbol{F}^{\mathrm{T}}(t)\boldsymbol{F}(t)\leq\boldsymbol{I}$, $\forall t$. Since almost every sample path of the above Markov chain is a right-continuous step function with a finite number of simple jumps, the global existence results on the solutions of the Markovian jump system (1) can be found in Section 4 of Ref. [20].

Lemma 1 (Schur complement[18]) Let $\boldsymbol{M}$, $\boldsymbol{P}$, $\boldsymbol{Q}$ be given matrices and $\boldsymbol{Q}>0$, then

$$\begin{bmatrix}\boldsymbol{P}&\boldsymbol{M}^{\mathrm{T}}\\ \boldsymbol{M}&-\boldsymbol{Q}\end{bmatrix}<0\Leftrightarrow\boldsymbol{P}+\boldsymbol{M}^{\mathrm{T}}\boldsymbol{Q}^{-1}\boldsymbol{M}<0.$$

Lemma 2[18] For any constant matrix $\boldsymbol{M}\in\mathbb{R}^{n\times n}$, $\boldsymbol{M}=\boldsymbol{M}^{\mathrm{T}}>0$, scalar $\eta>0$, vector function $\boldsymbol{w}:[0,\eta]\to\mathbb{R}^n$ such that the integrations concerned are well defined,

$$\left[\int_0^\eta\boldsymbol{w}(s)\mathrm{d}s\right]^{\mathrm{T}}\boldsymbol{M}\left[\int_0^\eta\boldsymbol{w}(s)\mathrm{d}s\right]\leq\eta\int_0^\eta\boldsymbol{w}^{\mathrm{T}}(s)\boldsymbol{M}\boldsymbol{w}(s)\mathrm{d}s.$$

Lemma 3[19] Let $\boldsymbol{D}$, $\boldsymbol{E}$, and $\boldsymbol{F}(t)$ be real matrices of appropriate dimensions, and $\boldsymbol{F}(t)$ satisfies $\boldsymbol{F}^{\mathrm{T}}(t)\boldsymbol{F}(t)\leq\boldsymbol{I}$, then the following inequality holds for any constant $\varepsilon>0$:

$$\boldsymbol{D}\boldsymbol{F}(t)\boldsymbol{E}+\boldsymbol{E}^{\mathrm{T}}\boldsymbol{F}^{\mathrm{T}}(t)\boldsymbol{D}^{\mathrm{T}}\leq\varepsilon\boldsymbol{D}\boldsymbol{D}^{\mathrm{T}}+\varepsilon^{-1}\boldsymbol{E}^{\mathrm{T}}\boldsymbol{E}.$$

3. Main results

For the sake of notation simplification, in the following, for each possible $r_t=p$ $(p=1,2,\ldots,m)$, we simply write $\boldsymbol{L}(r_t)$ as $\boldsymbol{L}_i$; for instance, $\boldsymbol{A}(r_t,t)$ is denoted by $\boldsymbol{A}_p(t)$, and $\boldsymbol{A}_{\mathrm{d}}(r_t,t)$ by $\boldsymbol{A}_{\mathrm{d}p}$, and so on. On the other hand, we have $\boldsymbol{u}(r_t)=u_p(t)$. We now rewrite system (1) by using state feedback control $\boldsymbol{u}_p(t)=\boldsymbol{K}_p\boldsymbol{x}(t)$ as follows:

$$\begin{aligned}\dot{\boldsymbol{x}}(t)&=(\boldsymbol{A}_p+\boldsymbol{B}_p\boldsymbol{K}_p)\boldsymbol{x}(t)+\boldsymbol{A}_{\mathrm{d}p}\boldsymbol{x}(t-\tau(t))\\&\quad+\boldsymbol{A}_{\mathrm{f}p}\int_{t-h}^{t}\boldsymbol{x}(s)\mathrm{d}s+\boldsymbol{B}_{wp}\boldsymbol{w}(t),\\ \boldsymbol{z}(t)&=(\boldsymbol{C}_p+\boldsymbol{D}_p\boldsymbol{K}_p)\boldsymbol{x}(t)+\boldsymbol{C}_{\mathrm{d}p}\boldsymbol{x}(t-\tau(t))+\boldsymbol{D}_{wp}\boldsymbol{w}(t),\\ \boldsymbol{x}(s)&=\boldsymbol{\phi}(s),\ s\in[-\max(\tau_{\mathrm{M}},h),0].\end{aligned}\tag{4}$$

Theorem 1 For a prescribed scalar $\gamma>0$ and some given positive scalars τ_{m}, τ_{M}, h, μ and matrix $\boldsymbol{K}$, system (4) with condition (2) is asymptotically stable and satisfies $\|\boldsymbol{z}(t)\|_2\leq\gamma\|\boldsymbol{w}(t)\|_2$ for all nonzero $\boldsymbol{w}(t)\in\mathbb{L}_2[0,\infty)$ if there exist matrices $\boldsymbol{P}_p>0$, $\boldsymbol{Q}_i>0$ $(i=1,2,3)$, $\boldsymbol{R}>0$, $\boldsymbol{S}_j>0$ $(j=1,2)$, and appropriate dimension matrices $\boldsymbol{N}_l$, $\boldsymbol{O}_l$ $(l=1,2,3,4,5)$ such that the following LMI is satisfied:

$$\begin{bmatrix}\overline{\boldsymbol{\Gamma}}&\boldsymbol{\Upsilon}_{12}&\widehat{\boldsymbol{C}}^{\mathrm{T}}\\ *&\boldsymbol{\Upsilon}_{22}&0\\ *&*&-\boldsymbol{I}\end{bmatrix}<0,\tag{5}$$

where

$$\begin{aligned}\bar{\boldsymbol{\Gamma}}&=\begin{bmatrix}\boldsymbol{\Gamma}_{5\times5}&P\boldsymbol{B}_w\\ *&-\gamma^2\boldsymbol{I}\end{bmatrix}+\boldsymbol{\Sigma}_N+\boldsymbol{\Sigma}_N^{\mathrm{T}}+\boldsymbol{\Sigma}_O+\boldsymbol{\Sigma}_O^{\mathrm{T}},\\ \boldsymbol{\Upsilon}_{12}&=\begin{bmatrix}\tau_{\mathrm{m}}\hat{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{S}_1&\sqrt{\tau_{21}}\hat{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{S}_2&\sqrt{\tau_{21}}\boldsymbol{N}&\sqrt{\tau_{21}}\boldsymbol{O}\end{bmatrix},\\ \hat{\boldsymbol{A}}&=\begin{bmatrix}(\boldsymbol{A}_p+\boldsymbol{B}_p\boldsymbol{K}_p)&0&\boldsymbol{A}_{\mathrm{d}p}&0&\boldsymbol{A}_{\mathrm{f}p}&0\end{bmatrix},\end{aligned}$$

$$\hat{C} = \begin{bmatrix} (C_p + D_p K_p) & 0 & C_{dp} & 0 & 0 & D_{wp} & 0 \end{bmatrix},$$
$$N^{\mathrm{T}} = \begin{bmatrix} N_{1p}^{\mathrm{T}} & N_{2p}^{\mathrm{T}} & N_{3p}^{\mathrm{T}} & N_{4p}^{\mathrm{T}} & N_{5p}^{\mathrm{T}} & 0 \end{bmatrix},$$
$$O^{\mathrm{T}} = \begin{bmatrix} O_{1p}^{\mathrm{T}} & O_{2p}^{\mathrm{T}} & O_{3p}^{\mathrm{T}} & O_{4p}^{\mathrm{T}} & O_{5p}^{\mathrm{T}} & 0 \end{bmatrix},$$

$$\Upsilon_{22} = \mathrm{diag}\{-S_1, -S_2, -S_2, -S_2\},$$
$$\Sigma_N = \begin{bmatrix} 0 & N & -N & 0 & 0 & 0 \end{bmatrix},$$
$$\Sigma_O = \begin{bmatrix} 0 & 0 & O & -O & 0 & 0 \end{bmatrix},$$
$$\Gamma_{11} = \sum_{q=1}^{N} \pi_{pq} P_q + P_p(A_p + B_p K_p) + (A_p + B_p K_p)^{\mathrm{T}} P_p + Q_1 + Q_2 + Q_3 + h^2 R - S_1,$$
$$\Gamma_{12} = S_1, \quad \Gamma_{13} = P_p A_{dp}, \quad \Gamma_{15} = P_p A_{fp},$$
$$\Gamma_{22} = -Q_1 - S_1, \quad \Gamma_{33} = -(1-\mu)Q_3, \quad \Gamma_{44} = -Q_2,$$
$$\Gamma_{55} = -R, \quad \tau_{21} = \tau_{\mathrm{M}} - \tau_{\mathrm{m}}.$$

Proof Define the Lyapunov functional candidate as

$$V(x_t) = V_1(x_t) + V_2(x_t) + V_3(x_t), \tag{6}$$

where

$$V_1(x_t) = x^{\mathrm{T}}(t) P_p x(t),$$

$$V_2(x_t) = \int_{t-\tau_{\mathrm{m}}}^{t} x^{\mathrm{T}}(s) Q_1 x(s)\,\mathrm{d}s + \int_{t-\tau_{\mathrm{M}}}^{t} x^{\mathrm{T}}(s) Q_2 x(s)\,\mathrm{d}s + \int_{t-\tau(t)}^{t} x^{\mathrm{T}}(s) Q_3 x(s)\,\mathrm{d}s,$$

$$V_3(x_t) = h\int_{t-h}^{t} [s-(t-h)] x^{\mathrm{T}}(s) R x(s)\,\mathrm{d}s + \tau_{\mathrm{m}} \int_{-\tau_{\mathrm{m}}}^{0} \int_{t+\theta}^{t} \dot{x}^{\mathrm{T}}(s) S_1 \dot{x}(s)\,\mathrm{d}s\,\mathrm{d}\theta + \int_{-\tau_{\mathrm{M}}}^{-\tau_{\mathrm{m}}} \int_{t+\theta}^{t} \dot{x}^{\mathrm{T}}(s) S_2 \dot{x}(s)\,\mathrm{d}s\,\mathrm{d}\theta.$$

By calculating the time derivative of $V(x_t)$ along the trajectories of system (4), we obtain

$$\dot{V}(x_t) = \dot{V}_1(x_t) + \dot{V}_2(x_t) + \dot{V}_3(x_t) + \alpha + \beta, \tag{7}$$

where

$$\alpha = 2\xi^{\mathrm{T}}(t)[\Sigma_N + \Sigma_N^{\mathrm{T}}]\xi(t) - 2\xi^{\mathrm{T}}(t) N \int_{t-\tau(t)}^{t-\tau_{\mathrm{m}}} \dot{x}(s)\,\mathrm{d}s = 0,$$

$$\beta = 2\xi^{\mathrm{T}}(t)[\Sigma_O + \Sigma_O^{\mathrm{T}}]\xi(t) - 2\xi^{\mathrm{T}}(t) O \int_{t-\tau_{\mathrm{M}}}^{t-\tau(t)} \dot{x}(s)\,\mathrm{d}s = 0,$$

$$\xi^{\mathrm{T}}(t) = \begin{bmatrix} x^{\mathrm{T}}(t) & x^{\mathrm{T}}(t-\tau_{\mathrm{m}}) & x^{\mathrm{T}}(t-\tau(t)) & x^{\mathrm{T}}(t-\tau_{\mathrm{M}}) & \left(\int_{t-h}^{t} x(s)\,\mathrm{d}s\right)^{\mathrm{T}} & w^{\mathrm{T}}(t) \end{bmatrix},$$

$$-2\xi^{\mathrm{T}}(t) N \int_{t-\tau(t)}^{t-\tau_{\mathrm{m}}} \dot{x}(s)\,\mathrm{d}s \le \int_{t-\tau(t)}^{t-\tau_{\mathrm{m}}} \dot{x}(s)^{\mathrm{T}} S_2 \dot{x}(s)\,\mathrm{d}s + (\tau(t) - \tau_{\mathrm{m}}) \xi^{\mathrm{T}}(t) N S_2^{-1} N^{\mathrm{T}} \xi(t), \tag{8}$$

$$-2\xi^{\mathrm{T}}(t) O \int_{t-\tau_{\mathrm{M}}}^{t-\tau(t)} \dot{x}(s)\,\mathrm{d}s \le \int_{t-\tau_{\mathrm{M}}}^{t-\tau(t)} \dot{x}(s)^{\mathrm{T}} S_2 \dot{x}(s)\,\mathrm{d}s + (\tau_{\mathrm{M}} - \tau(t)) \xi^{\mathrm{T}}(t) O S_2^{-1} O^{\mathrm{T}} \xi(t). \tag{9}$$

From Eq. (6), $\dot{V}_1(x_t)$, $\dot{V}_2(x_t)$, and $\dot{V}_3(x_t)$ are calculated as follows:

$$\begin{aligned} \dot{V}_1(x_t) &= 2x^{\mathrm{T}}(t) P_p [(A_p + B_p K_p) x(t) + A_{dp} x(t-\tau(t)) + A_{fp} \int_{t-h}^{t} x(s)\,\mathrm{d}s + B_{wp} w(t)], \\ \dot{V}_2(x_t) &= x^{\mathrm{T}}(t)[Q_1 + Q_2 + Q_3] x(t) - x^{\mathrm{T}}(t-\tau_{\mathrm{m}}) Q_1 x(t-\tau_{\mathrm{m}}) - x^{\mathrm{T}}(t-\tau_{\mathrm{M}}) Q_2 x(t-\tau_{\mathrm{M}}) \\ &\quad - (1-\mu) x^{\mathrm{T}}(t-\tau(t)) Q_3 x(t-\tau(t)), \\ \dot{V}_3(x_t) &= h^2 x^{\mathrm{T}}(t) R x(t) - h\int_{t-h}^{t} x^{\mathrm{T}}(s) R x(s)\,\mathrm{d}s + \tau_{\mathrm{m}}^2 \dot{x}^{\mathrm{T}}(t) S_1 \dot{x}(t) - \tau_{\mathrm{m}} \int_{t-\tau_{\mathrm{m}}}^{t} \dot{x}^{\mathrm{T}}(s) S_1 \dot{x}(s)\,\mathrm{d}s \\ &\quad + \tau_{21} \dot{x}^{\mathrm{T}}(t) S_2 \dot{x}(t) - \int_{t-\tau_{\mathrm{M}}}^{t-\tau_{\mathrm{m}}} \dot{x}^{\mathrm{T}}(s) S_2 \dot{x}(s)\,\mathrm{d}s. \end{aligned}$$

By applying Jensen's inequality, we obtain

$$-h\int_{t-h}^{t} x^{\mathrm{T}}(s) R x(s)\,\mathrm{d}s \le -\left(\int_{t-h}^{t} x(s)\,\mathrm{d}s\right)^{\mathrm{T}} R \left(\int_{t-h}^{t} x(s)\,\mathrm{d}s\right), \tag{10}$$

$$-\tau_{\mathrm{m}} \int_{t-\tau_{\mathrm{m}}}^{t} \dot{x}^{\mathrm{T}}(s) S_1 \dot{x}(s)\,\mathrm{d}s \le -[x(t) - x(t-\tau_{\mathrm{m}})]^{\mathrm{T}} S_1 [x(t) - x(t-\tau_{\mathrm{m}})]. \tag{11}$$

Substituting Eqs. (8)–(11) into Eq. (7), we obtain

$$\begin{aligned} \dot{V}(x_t) \le\ & \xi^{\mathrm{T}}(t) \Gamma \xi(t) + \tau_{\mathrm{m}}^2 \dot{x}^{\mathrm{T}}(t) S_1 \dot{x}(t) + \tau_{21} \dot{x}^{\mathrm{T}}(t) S_2 \dot{x}(t) \\ & + (\tau(t) - \tau_{\mathrm{m}}) \xi^{\mathrm{T}}(t) N S_2^{-1} N^{\mathrm{T}} \xi(t) \\ & + (\tau_{\mathrm{M}} - \tau(t)) \xi^{\mathrm{T}}(t) O S_2^{-1} O^{\mathrm{T}} \xi(t), \end{aligned} \tag{12}$$

where

$$\Gamma = \begin{bmatrix} \Gamma_{5\times 5} & P B_{wp} \\ * & 0 \end{bmatrix} + \Sigma_N + \Sigma_N^{\mathrm{T}} + \Sigma_O + \Sigma_O^{\mathrm{T}}.$$

Under the zero initial conditions and $V(x(t))|_{t=\infty} \to 0$ for system (4), we obtain

$$\begin{aligned} J_\infty &= \int_0^\infty [z^{\mathrm{T}}(t) z(t) - \gamma^2 w^{\mathrm{T}}(t) w(t)]\,\mathrm{d}t \\ &\le \int_0^\infty [z^{\mathrm{T}}(t) z(t) - \gamma^2 w^{\mathrm{T}}(t) w(t) + \dot{V}(x(t))]\,\mathrm{d}t, \end{aligned}$$

$$J_\infty \le \int_0^\infty \boldsymbol{\xi}^{\mathrm{T}}(t)\widetilde{\boldsymbol{\Gamma}}\boldsymbol{\xi}(t)\mathrm{d}t. \tag{13}$$

By using Lemma 2, $\widetilde{\boldsymbol{\Gamma}}$ is equivalent to inequality (5), which means that $||\boldsymbol{z}||_2 \le \gamma||\boldsymbol{w}||_2$ is satisfied for any nonzero $\boldsymbol{w}(t) \in L_2[0,\infty)$, which ensures the asymptotical stability of system (4).

Theorem 2 For a prescribed scalar $\gamma > 0$ and some given positive scalars τ_{m}, τ_{M}, u, h and matrix K, system (4) with condition (2) is robustly asymptotically stable and satisfies $\|\boldsymbol{z}(t)\|_2 \le \gamma\|\boldsymbol{w}(t)\|_2$ for all nonzero $\boldsymbol{w}(t) \in \mathbb{L}_2[0,\infty)$ if there are matrices $\boldsymbol{P}_p > 0$, $\boldsymbol{Q}_i > 0$ $(i = 1,2,3)$, $\boldsymbol{R} > 0$, $\boldsymbol{S}_j > 0$ $(j = 1,2)$, and appropriate dimension matrices $\boldsymbol{N}_l$, $\boldsymbol{O}_l$ $(l = 1,2,3,4,5)$, and a scalar $\varepsilon_p > 0$ such that the following LMI is satisfied:

$$\begin{bmatrix} \overline{\boldsymbol{\Gamma}} & \boldsymbol{\Upsilon}_{12} & \boldsymbol{\Upsilon}_{13} \\ * & \boldsymbol{\Upsilon}_{22} & 0 \\ * & * & \boldsymbol{\Upsilon}_{33} \end{bmatrix} < 0, \tag{14}$$

where

$$\begin{aligned} \boldsymbol{\Upsilon}_{13} &= \left[\, \overline{\boldsymbol{P}\boldsymbol{L}} \;\; \boldsymbol{E}^{\mathrm{T}} \;\; \hat{\boldsymbol{C}}^{\mathrm{T}} \,\right], \\ \boldsymbol{\Upsilon}_{22} &= \mathrm{diag}\{-\boldsymbol{S}_1, -\boldsymbol{S}_2, -\boldsymbol{S}_2, -\boldsymbol{S}_2\}, \\ \boldsymbol{\Upsilon}_{33} &= \mathrm{diag}\{-\varepsilon_1\boldsymbol{I}, -\varepsilon^{-1}\boldsymbol{I}, -\boldsymbol{I}\}. \end{aligned}$$

Proof Replace $\boldsymbol{A}_p$, $\boldsymbol{A}_{\mathrm{d}p}$, $\boldsymbol{A}_{\mathrm{f}p}$, and $\boldsymbol{B}_p$ in LMI (5) by $\boldsymbol{A}_p + \Delta\boldsymbol{A}_p(t)$, $\boldsymbol{A}_{\mathrm{d}p} + \Delta\boldsymbol{A}_{\mathrm{d}p}(t)$, $\boldsymbol{A}_{\mathrm{f}p} + \Delta\boldsymbol{A}_{\mathrm{f}p}(t)$, and $\boldsymbol{B}_p + \Delta\boldsymbol{B}_p(t)$, respectively, we obtain

$$\overline{\boldsymbol{\Gamma}} + \overline{\boldsymbol{P}\boldsymbol{L}}\boldsymbol{F}(t)\boldsymbol{E} + (\overline{\boldsymbol{P}\boldsymbol{L}}\boldsymbol{F}(t)\boldsymbol{E})^{\mathrm{T}} < 0, \tag{15}$$

where

$$\begin{aligned} \overline{\boldsymbol{P}\boldsymbol{L}} &= \left[\, \boldsymbol{L}_p^{\mathrm{T}}\boldsymbol{P}_p \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; \tau_{\mathrm{m}}\boldsymbol{L}_p^{\mathrm{T}}\boldsymbol{S}_1 \;\; \sqrt{\tau_{21}}\boldsymbol{L}_p^{\mathrm{T}}\boldsymbol{S}_2 \;\; 0 \;\; 0 \,\right]^{\mathrm{T}}, \\ \boldsymbol{E} &= \left[\, \boldsymbol{E}_{1p} + \boldsymbol{E}_{4p}\boldsymbol{K}_p \;\; 0 \;\; \boldsymbol{E}_{2p} \;\; 0 \;\; \boldsymbol{E}_{3p} \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \,\right]. \end{aligned}$$

By Lemma 3, we know that LMI (15) is equivalent to

$$\widetilde{\boldsymbol{\Gamma}} + \varepsilon^{-1}(\overline{\boldsymbol{P}\boldsymbol{L}})(\overline{\boldsymbol{P}\boldsymbol{L}})^{\mathrm{T}} + \varepsilon\boldsymbol{E}^{\mathrm{T}}\boldsymbol{E} < 0. \tag{16}$$

Obviously, by applying Schur complement, LMI (16) is equivalent to LMI (14).

Theorem 3 For a prescribed scalar $\gamma > 0$ and some given positive scalars $\tau_{\mathrm{m}}, \tau_{\mathrm{M}}, u, h$, system (4) with condition (2) is robustly asymptotically stable and satisfies $\|\boldsymbol{z}(t)\|_2 \le \gamma\|\boldsymbol{w}(t)\|_2$ for all nonzero $\boldsymbol{w}(t) \in \mathbb{L}_2[0,\infty)$ if there are matrices $\bar{\boldsymbol{Q}}_i > 0$ $(i = 1,2,3)$, $\overline{\boldsymbol{R}} > 0$, $\overline{\boldsymbol{S}}_j > 0$ $(j = 1,2)$, $\boldsymbol{X}_p > 0$, $\boldsymbol{Y}_p$, appropriate dimension matrices $\overline{\boldsymbol{N}}_l$, $\overline{\boldsymbol{O}}_l$ $(l = 1,2,3,4,5)$, and a scalar $\varepsilon_p > 0$ such that the following LMI is satisfied:

$$\begin{bmatrix} \boldsymbol{\Theta}_{11} & \boldsymbol{\Theta}_{12} & \boldsymbol{\Theta}_{13} \\ * & \boldsymbol{\Theta}_{22} & 0 \\ * & * & \boldsymbol{\Theta}_{33} \end{bmatrix} < 0, \tag{17}$$

where

$$\boldsymbol{\Theta}_{11} = \begin{bmatrix} \Omega_{5\times5} & \boldsymbol{B}_w \\ * & -\gamma^2\boldsymbol{I} \end{bmatrix} + \boldsymbol{\Pi}_N + \boldsymbol{\Pi}_N^{\mathrm{T}} + \boldsymbol{\Pi}_O + \boldsymbol{\Pi}_O^{\mathrm{T}},$$

$$\begin{aligned} \boldsymbol{\Theta}_{12}^{\mathrm{T}} &= \left[\, \tau_{\mathrm{m}}\widetilde{\boldsymbol{A}} \;\; \sqrt{\tau_{21}}\widetilde{\boldsymbol{A}} \;\; \sqrt{\tau_{21}}\overline{\boldsymbol{N}} \;\; \sqrt{\tau_{21}}\overline{\boldsymbol{O}} \,\right], \\ \boldsymbol{\Theta}_{13} &= \left[\, \overline{\boldsymbol{L}} \;\; \overline{\boldsymbol{E}}^{\mathrm{T}} \;\; \overline{\boldsymbol{C}}^{\mathrm{T}} \;\; \boldsymbol{Z}(p) \,\right], \\ \boldsymbol{\Theta}_{22} &= \mathrm{diag}\{-\boldsymbol{I}, -\boldsymbol{I}, -\overline{\boldsymbol{S}}_2, -\overline{\boldsymbol{S}}_2\}, \\ \boldsymbol{\Theta}_{33} &= \mathrm{diag}\{-\varepsilon_p\boldsymbol{I}, -\varepsilon_p^{-1}\boldsymbol{I}, -\boldsymbol{I}, -\boldsymbol{\Lambda}_p\}, \\ \boldsymbol{\Pi}_N &= \left[\, 0 \;\; \overline{\boldsymbol{N}} \;\; -\overline{\boldsymbol{N}} \;\; 0 \;\; 0 \;\; 0 \,\right], \\ \boldsymbol{\Pi}_O &= \left[\, 0 \;\; 0 \;\; \overline{\boldsymbol{O}} \;\; -\overline{\boldsymbol{O}} \;\; 0 \;\; 0 \,\right], \\ \overline{\boldsymbol{L}} &= \left[\, \boldsymbol{L}^{\mathrm{T}} \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; \tau_{\mathrm{m}}\boldsymbol{L}^{\mathrm{T}} \;\; \sqrt{\tau_{21}}\boldsymbol{L}^{\mathrm{T}} \;\; 0 \;\; 0 \,\right]^{\mathrm{T}}, \\ \overline{\boldsymbol{N}}^{\mathrm{T}} &= \left[\, \overline{\boldsymbol{N}}_{1p}^{\mathrm{T}} \;\; \overline{\boldsymbol{N}}_{2p}^{\mathrm{T}} \;\; \overline{\boldsymbol{N}}_{3p}^{\mathrm{T}} \;\; \overline{\boldsymbol{N}}_{4p}^{\mathrm{T}} \;\; \overline{\boldsymbol{N}}_{5p}^{\mathrm{T}} \;\; 0 \,\right], \\ \overline{\boldsymbol{O}}^{\mathrm{T}} &= \left[\, \overline{\boldsymbol{O}}_{1p}^{\mathrm{T}} \;\; \overline{\boldsymbol{O}}_{2p}^{\mathrm{T}} \;\; \overline{\boldsymbol{O}}_{3p}^{\mathrm{T}} \;\; \overline{\boldsymbol{O}}_{4p}^{\mathrm{T}} \;\; \overline{\boldsymbol{O}}_{5p}^{\mathrm{T}} \;\; 0 \,\right], \\ \widetilde{\boldsymbol{A}} &= \left[\, (\boldsymbol{A}_p\boldsymbol{X}_p + \boldsymbol{B}_p\boldsymbol{Y}_p) \;\; 0 \;\; \boldsymbol{A}_{\mathrm{d}p}\boldsymbol{X}_p \;\; 0 \;\; \boldsymbol{A}_{\mathrm{f}p}\boldsymbol{X}_p \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \,\right], \\ \overline{\boldsymbol{C}} &= \left[\, (\boldsymbol{C}_p\boldsymbol{X}_p + \boldsymbol{D}_p\boldsymbol{Y}_p) \;\; 0 \;\; \boldsymbol{C}_{\mathrm{d}p}\boldsymbol{X}_p \;\; 0 \;\; 0 \;\; \boldsymbol{D}_{wp}\boldsymbol{X}_p \;\; 0 \;\; 0 \;\; 0 \;\; 0 \,\right], \\ \boldsymbol{Z}(p) &= [\sqrt{\pi_{p,1}}\boldsymbol{X}_p, \ldots, \sqrt{\pi_{p,p-1}}\boldsymbol{X}_p, \ldots, \\ &\qquad \sqrt{\pi_{p,p+1}}\boldsymbol{X}_p, \ldots, \sqrt{\pi_{p,\mathcal{N}}}\boldsymbol{X}_p], \\ \boldsymbol{\Lambda}_p &= \mathrm{diag}\{\boldsymbol{X}_1, \ldots, \boldsymbol{X}_{p-1}, \ldots, \boldsymbol{X}_{p+1}, \ldots, \boldsymbol{X}_{\mathcal{N}}\}, \\ \boldsymbol{\Omega}_{11} &= \pi_{pp}\boldsymbol{X}_p + \boldsymbol{A}_p\boldsymbol{X}_p + \boldsymbol{B}_p\boldsymbol{Y}_p + \boldsymbol{X}_p\boldsymbol{A}_p^{\mathrm{T}} + \boldsymbol{Y}_p^{\mathrm{T}}\boldsymbol{B}_p \\ &\quad + \bar{\boldsymbol{Q}}_1 + \bar{\boldsymbol{Q}}_2 + \bar{\boldsymbol{Q}}_3 + h^2\overline{\boldsymbol{R}} - \overline{\boldsymbol{S}}_1, \; \boldsymbol{\Omega}_{12} = \overline{\boldsymbol{S}}_1, \\ \boldsymbol{\Omega}_{13} &= \boldsymbol{A}_{\mathrm{d}p}\boldsymbol{X}_p, \quad \boldsymbol{\Omega}_{15} = \boldsymbol{A}_{\mathrm{f}p}\boldsymbol{X}_p, \\ \boldsymbol{\Omega}_{22} &= -\bar{\boldsymbol{Q}}_1 - \overline{\boldsymbol{S}}_1, \quad \boldsymbol{\Omega}_{33} = -(1-\mu)\bar{\boldsymbol{Q}}_3, \\ \boldsymbol{\Omega}_{44} &= -\bar{\boldsymbol{Q}}_2, \quad \boldsymbol{\Omega}_{55} = -\overline{\boldsymbol{R}}. \end{aligned}$$

The feedback gain matrix is given by $\boldsymbol{K}_p = \boldsymbol{Y}_p\boldsymbol{X}_p^{-1}$.

Proof We define

$$\begin{aligned} &\boldsymbol{P}_p = \boldsymbol{X}_p^{-1}, \;\; \bar{\boldsymbol{Q}}_i = \boldsymbol{X}_p\boldsymbol{Q}_i\boldsymbol{X}_p^{\mathrm{T}}, \quad (i = 1,2,3), \\ &\overline{\boldsymbol{R}} = \boldsymbol{X}_p\boldsymbol{R}\boldsymbol{X}_p^{\mathrm{T}}, \;\; \overline{\boldsymbol{S}}_j = \boldsymbol{X}_p\boldsymbol{S}_j\boldsymbol{X}_p^{\mathrm{T}}, \quad (j = 1,2), \\ &\overline{\boldsymbol{N}}_{lp} = \boldsymbol{X}\boldsymbol{N}_{lp}\boldsymbol{X}^{\mathrm{T}}, \;\; \overline{\boldsymbol{O}}_{lp} = \boldsymbol{X}\boldsymbol{O}_{lp}\boldsymbol{X}^{\mathrm{T}}, \quad (l = 1,2,3,4,5). \end{aligned}$$

Then pre and post multiplying LMI (14) by

$$\mathrm{diag}(\boldsymbol{X}_p, \boldsymbol{X}_p, \boldsymbol{X}_p, \boldsymbol{X}_p, \boldsymbol{X}_p, \boldsymbol{I}, \boldsymbol{I}, \boldsymbol{I}, \boldsymbol{X}_p, \boldsymbol{X}_p, \boldsymbol{I}, \boldsymbol{I}, \boldsymbol{I})$$

and

$$\mathrm{diag}(\boldsymbol{X}_p, \boldsymbol{X}_p, \boldsymbol{X}_p, \boldsymbol{X}_p, \boldsymbol{X}_p, \boldsymbol{I}, \boldsymbol{S}_1^{-1}, \boldsymbol{S}_2^{-1}, \boldsymbol{X}_p, \boldsymbol{X}_p, \boldsymbol{I}, \boldsymbol{I}, \boldsymbol{I})^{\mathrm{T}},$$

we can obtain LMI (17).

4. Numerical examples

In this section, some numerical examples are provided to demonstrate that the method proposed in this paper is effective and less conservative than some existing methods.

Example 1 Consider system (1) with the following matrix parameters.

Mode 1:

$$\boldsymbol{A}_1 = \begin{bmatrix} -0.2 & 0 \\ 0 & -0.1 \end{bmatrix}, \quad \boldsymbol{A}_{\mathrm{d1}} = \begin{bmatrix} -0.1 & 0 \\ -0.1 & 0.1 \end{bmatrix},$$
$$\boldsymbol{A}_{\mathrm{f1}} = \begin{bmatrix} 0 & 1 \\ -0.1 & -0.2 \end{bmatrix}, \quad \boldsymbol{B}_1 = \begin{bmatrix} 0.2 & 0.5 \\ -0.5 & 1 \end{bmatrix},$$
$$\boldsymbol{B}_{w1} = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}, \quad \boldsymbol{C}_1 = \begin{bmatrix} 0.5 & 0.5 \end{bmatrix}, \quad \boldsymbol{C}_{\mathrm{d1}} = \begin{bmatrix} 0.1 & 0.2 \end{bmatrix},$$
$$\boldsymbol{D}_1 = \begin{bmatrix} -0.1 & 2 \end{bmatrix}, \quad \boldsymbol{D}_{w1} = 0_2,$$
$$\boldsymbol{L}_p = 0.1\boldsymbol{I}_2, \quad \boldsymbol{E}_{11} = \boldsymbol{E}_{21} = \boldsymbol{E}_{31} = \boldsymbol{E}_{41} = \boldsymbol{E}_{51} = 0.5\boldsymbol{I}_2.$$

Mode 2:

$$\boldsymbol{A}_2 = \begin{bmatrix} 0.5 & 0 \\ 0 & -0.4 \end{bmatrix}, \quad \boldsymbol{A}_{\mathrm{d2}} = \begin{bmatrix} 0.5 & 0.3 \\ 0.2 & 0.3 \end{bmatrix},$$
$$\boldsymbol{A}_{\mathrm{f2}} = \begin{bmatrix} 0.5 & 0.7 \\ 0.8 & 0.4 \end{bmatrix}, \quad \boldsymbol{B}_2 = \begin{bmatrix} 0.6 & 0.5 \\ 0.3 & 0.5 \end{bmatrix},$$
$$\boldsymbol{B}_{w2} = \begin{bmatrix} 0.4 \\ 0.5 \end{bmatrix}, \quad \boldsymbol{C}_2 = \begin{bmatrix} 0.5 & 0.3 \end{bmatrix}, \quad \boldsymbol{C}_{\mathrm{d2}} = \begin{bmatrix} 0.3 & 0.5 \end{bmatrix},$$
$$\boldsymbol{D}_2 = \begin{bmatrix} 0.5 & 0.6 \end{bmatrix}, \quad \boldsymbol{D}_{w2} = 0_2, \quad \boldsymbol{L}_p = 0.1\boldsymbol{I}_2,$$
$$\boldsymbol{E}_{11} = \boldsymbol{E}_{21} = \boldsymbol{E}_{31} = \boldsymbol{E}_{41} = \boldsymbol{E}_{51} = 0.5\boldsymbol{I}_2.$$

In order to design a robust delay-dependent H_∞ state-feedback control, taking $\tau_{\mathrm{m}} = 0.5, \tau_{\mathrm{M}} = 1.5, h = 0.3$, and $\mu = 0.5$, applying Theorem 3 in MATLAB LMI control toolbox, we obtain $\gamma = 0.9845$, $\varepsilon_1 = 1.2354$, $\varepsilon_2 = 1.6524$, and the desired state feedback controller as follows:

$$\boldsymbol{K}_1 = \begin{bmatrix} -0.9284 & -0.2900 \\ -0.2082 & -0.3495 \end{bmatrix}, \quad \boldsymbol{K}_2 = \begin{bmatrix} -1.8288 & -0.1778 \\ 0.2217 & -0.6472 \end{bmatrix}.$$

Therefore, the system of interest is robustly asymptotically stable.

Example 2 Consider the following time delay system with system mode $p = 1$:

$$\begin{aligned} \dot{\boldsymbol{x}}(t) &= \boldsymbol{A}_p\boldsymbol{x}(t) + \boldsymbol{A}_{\mathrm{d}p}\boldsymbol{x}(t - d(t)) + \boldsymbol{B}_{wp}\boldsymbol{w}(t), \\ \boldsymbol{z}(t) &= \boldsymbol{C}_p\boldsymbol{x}(t) + \boldsymbol{C}_{\mathrm{d}p}\boldsymbol{x}(t - d(t)) + \boldsymbol{D}_{wp}\boldsymbol{w}(t), \\ \boldsymbol{x}(s) &= \varphi(s),\ s \in [-\tau_{\mathrm{M}}, 0], \end{aligned} \tag{18}$$

with matrix parameters in Ref. [7]

$$\boldsymbol{A}_1 = \begin{bmatrix} 0 & 1 \\ 0 & -0.1 \end{bmatrix}, \quad \boldsymbol{A}_{\mathrm{d1}} = \begin{bmatrix} 0 & 1 \\ -0.375 & -1.15 \end{bmatrix},$$
$$\boldsymbol{B}_{w1} = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, \quad \boldsymbol{C}_1 = \begin{bmatrix} 0 & 1 \end{bmatrix},$$
$$\boldsymbol{C}_{\mathrm{d1}} = \begin{bmatrix} -0.375 & 1.15 \end{bmatrix}, \ D_{w1} = 0, \ \boldsymbol{L}_1 = \boldsymbol{E}_{11} = \boldsymbol{E}_{21} = 0_2.$$

Taking parameters $h = 0$ and unknown μ, and using LMI in Theorem 1, we obtain the minimum allowable H_∞ performance γ for different values of τ_{M} and τ_{m} as shown in Table 1, which clearly shows that the results of the proposed method in this paper are significantly better than those in Refs. [7] and [8].

Table 1. Example 4: minimized H_∞ performance for different τ_{M} and τ_{m}.

Method	$\tau_{\mathrm{M}} = 0.8695$		$\tau_{\mathrm{M}} = 1$	
	$\tau_{\mathrm{m}} = 0$	$\tau_{\mathrm{m}} = 0.5695$	$\tau_{\mathrm{m}} = 0$	$\tau_{\mathrm{m}} = 0.5$
Ref. [7]	0.87	0.81	4.05	3.27
Ref. [8]	0.81	0.72	2.95	2.13
Thm 3	0.5614	0.5305	1.7458	1.1254

Example 3 Consider system (18) with the following matrix parameters in Ref. [9]:

$$\boldsymbol{A}_1 = \begin{bmatrix} -2.2460 & -1.4410 \\ -1.5937 & -2.9289 \end{bmatrix}, \quad \boldsymbol{A}_{\mathrm{d1}} = \begin{bmatrix} -0.7098 & 1.1908 \\ 0.6686 & -3.2025 \end{bmatrix},$$
$$\boldsymbol{A}_2 = \begin{bmatrix} -1.8999 & 0.8156 \\ 0.6900 & -0.7881 \end{bmatrix}, \quad \boldsymbol{A}_{\mathrm{d2}} = \begin{bmatrix} -1.5198 & -1.6041 \\ -0.1567 & -1.2427 \end{bmatrix},$$
$$\boldsymbol{B}_{w1} = \begin{bmatrix} 0.0403 \\ 0.6771 \end{bmatrix}, \quad \boldsymbol{B}_{w2} = \begin{bmatrix} 0.5689 \\ -0.2556 \end{bmatrix},$$
$$\boldsymbol{C}_1 = \begin{bmatrix} -0.3775 & -0.2959 \end{bmatrix}, \quad C_{\mathrm{d1}} = 0, \quad D_1 = 0,$$
$$D_{w1} = 0.1184, \quad \boldsymbol{C}_2 = \begin{bmatrix} -1.4751 & -0.2340 \end{bmatrix}, \quad C_{\mathrm{d2}} = 0,$$
$$D_2 = 0, \quad D_{w2} = 0.3148.$$

Taking parameters $\tau_{\mathrm{m}} = h = 0$ and using LMI in Theorem 1, we obtain the maximum admissible delay upper bound τ_{M} or different values of γ as shown in Table 2, which clearly shows that the result of the proposed method in this paper are significantly better than those in Refs. [9] and [10].

Table 2. Example 4: τ_{M} and γ.

Method	μ	0.5	1.0	1.5	2.0
Ref. [9]	0	0.47	0.52	0.55	0.56
Thm 3		0.60	0.64	0.67	0.69
Ref. [10]	0.5	0.3666	0.4293	0.4450	0.4524
Thm 3		0.5355	0.5754	0.5863	0.5898

5. Conclusion

We have studied the H_∞ control problem for systems with interval time-varying and distributed delays. On the basis of the Lyapunov functional, a delay-dependent H_∞ control scheme is presented in terms of LMIs. It is shown that a desired state feedback controller can be constructed when the given LMIs are feasible, and it leads to less conservative results than the existing methods. Finally, numerical examples are given to illustrate the effectiveness of the proposed method. We are interested to extend this work to the discrete-time uncertain stochastic systems with discrete interval and distributed time-varying delays. The results will be obtained in the future.

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