Biphasic behavior of energy in a stepped chain*

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The impact energy decay in a step-up chain containing two sections is numerically studied. There is a marked biphasic behavior of energy decay in the first section. Two sections close to the interface are in compression state. The degree of compression of the first section first decreases and becomes weakest at "crossing" time of biphasic behavior of energy, then increases. The further calculations provide the dependence of the character time on mass ratio (m_1/m_2), where m_1 and m_2 are the particle mass in the first and second section respectively. The bigger the α ($\alpha = [(\Omega m_1 - m_2)/(\Omega m_1 + m_2)]^2$ with $\Omega = 1.345$), the bigger the energy ratio is. The multipulse structure restricts the transport of energy.

Keywords: granular chain, solitary, biphasic behavior

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1. Introduction

The energy transportation in three-dimensional (3D) granular matter is an extremely complex problem due to many parameters and strong nonlinearity of interaction between particles. The simplest and most widely used model of granular matter consists of a one-dimensional (1D) granular chain. The transportation of mechanical energy in an alignment of grains has attracted significant attention.[1–37]

As is well known, the elastic impulse propagates with soliton-like features in a granular chain. The propagation of the elastic impulse depends on the structure of the granular chain.[1,7] The "granular container" using a series of sections with particles was proposed to study the leakage of trapped energy.[7,14,16] The trapped energy decays according to a power law. We[14,16] found the crossover in power-law behavior of energy decay. The power-law exponent depends on the ratio of granular mass in the compression states. The "granular container" consists of stepped chains. The decomposition, reflection and transmission of waves occur during the transportation of a soliton-like wave in the composite granular chain due to the different interfaces of particles. For example, a strong soliton-like wave is decomposed into many weaker solitary waves during the soliton-like wave propagating from heavy particle section to light particle section.[9] The maturing process of solitary wave trains was studied.[20] The soliton-like wave is reflected to the light particle section and transmitted to the heavy particle section during the soliton-like wave propagating from the light particle section to the heavy particle section.[9]

In this paper, we revisit the leakage of impulse energy inside the light section in the stepped granular chain. There is a marked biphasic behavior with the energy.

2. Computer method

In the present work, we model the granular chain as a collection of spherical particles, which contains two sets of particles and is divided into one light section and one heavy section as shown in Fig. 1. The numbers of first and second granular sections are 30 and 170. Both ends of the chain could move freely.

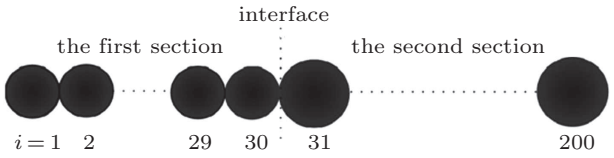


Fig. 1. Schematic setup of stepped chain.

The mass of particle in the first section is m_1, and the mass of particle in the second section is m_2. The particles repel one another when they are in touch by nonlinear Hertz force: $F(\delta_{i,i+1}) = k_{i,i+1}(\delta_{i,i+1})^{3/2}$ if $\delta_{i,i+1} > 0$, and $F(\delta_{i,i+1}) = 0$ if $\delta_{i,i+1} < 0$. The exponent (n) of Hertz law is 1.5. The overlap between two adjacent particles is written as $\delta_{i,i+1} = R_i + R_{i+1} - (x_{i+1} - x_i)$, where R_i and x_i are radius and position of particle i respectively. The elastic constant $k_{i,i+1}$ is quantified as follows:

$$k_{i,i+1} = \frac{\sqrt{R_i R_{i+1}/(R_i + R_{i+1})}}{2\theta},$$

with $\theta = 3(1-\nu^2)/4Y$, where ν is the Poisson ratio and Y is Young modulus of particle material. Without static precom-

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pression and in the frictionless medium, the equation of motion for i-th particle reads

$$m_i \frac{\mathrm{d}^2 x_i}{\mathrm{d}t^2} = F(\delta_{i-1,i}) - F(\delta_{i,i+1}).$$

It is assumed that all particles are made of the same material. In the equation of motion, the governing parameter is the ratio of particle radius. At the initial time, the first heavy (light) particle obtains an initial velocity, then the impulse transfers along the stepped chain. The fifth-order Gear predictor–corrector method[3] is used to integrate the equation of particle motion and the time step is 1.25×10^{-5}.

3. Simulation results and discussion

In Fig. 2(a), we present the plots of the remaining energy inside the first section of the step-up granular chain versus elapsed time. The mass ratio of particles in the first and second section is 0.125 and the initial velocity of the first particle in the first section is set to be 2. We observe the biphasic behavior of the remaining energy in the first section.

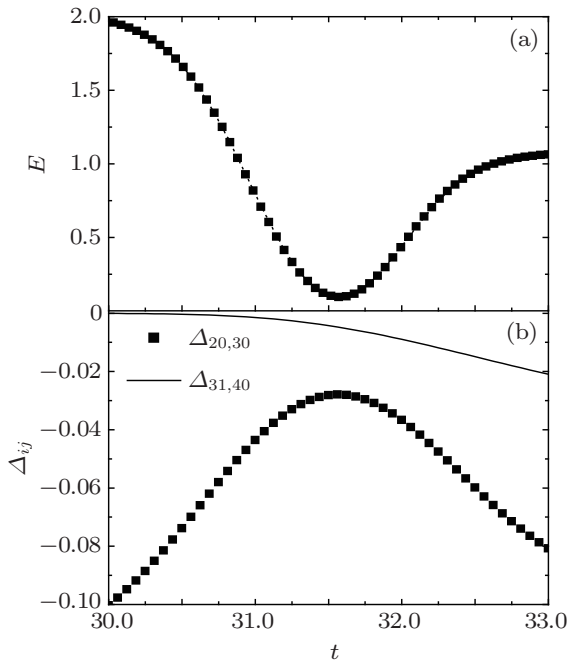


Fig. 2. Plots of (a) energy in first section versus time and (b) Δ_{ij} versus time.

The energy in the first section reads

$$E = \frac{1}{2}\sum_{i=1}^{30} m_i \left(\frac{\mathrm{d}x_i}{\mathrm{d}t}\right)^2 + \frac{2}{5}\sum_{i=1}^{29} k_{i,i+1}(\delta_{i,i+1})^{5/2}.$$

The parameter Δ_{ij}, introduced as in previous work,[14,16] can be expressed as follows:

$$\Delta_{ij} = \frac{x_j - x_i - [R_i + 2(R_{i+1} + \cdots + R_{j-1}) + R_j]}{R_i + 2(R_{i+1} + \cdots + R_{j-1}) + R_j},$$

which represents the degree of compression or dilation of the granular chain between particles i and j. The granular chain between particles i and j is in compression state if $\Delta_{ij} < 0$, which means there is no gap between particles. At initial state, the particles just touch each other, which means $\Delta_{ij}(0) = 0$ for any i and j.

The biphasic behavior of energy can be inferred from the collision of two particles. The impulse first propagates with soliton-like features in the light section, the energy in the light section keeps constant. At the time $t \approx 30$, the impulse propagates to the interface between light and heavy particles. The energy of the first section is transferred to the second section and the energy decreases. Then the energy of the second section is partly reflected to the first section and the energy increases, owing to the heavy particle in the second section. The biphasic behavior may be the transmission and reflection of the wave from the perspective of wave propagation. At the "crossing" time, that is, the moment of smallest energy, there is a balance between transmission and reflection of the wave. The degree of compression or dilation of the granular chain near the interface is shown in Fig. 2(b). For particles close to the interface, $\Delta_{31,40}$ keeps a negative value and decreases with time, indicating that during the decay of energy the particles in the heavy section are maintained in the compression state and compression is enhanced with time increasing. For particles close to the interface, $\Delta_{20,30}$ keeps a negative value, indicating that during the decay of energy the particles in the light section are maintained in the compression state too. But at the early time it increases with time increasing and arrives at the maximum at "crossing" time then decreases with time increasing. As is well known, the remaining energy in the light section has the biphasic behavior. We fit it using the function as below.

$$E(t) = E_{\min} + \frac{E_{\max 1} - E_{\min}}{1 + 10^{-(t-t_{01})/\tau_1}} + \frac{E_{\max 2} - E_{\min}}{1 + 10^{-(t-t_{02})/\tau_2}}.$$

We extend our calculations about the dependence of character time on the mass ratio of light to heavy particles (m_1/m_2). There are many waves in the composite granular chain. The number of impulses is considered. We calculate the characteristic times under three situations. (i) The initial velocity of the first particle is set to be 2. There is one impulse. The τ_{11} and τ_{12} are the characteristic times. (ii) The initial velocities of first and second particles are both set to be 2. There are two impulses. The τ_{21} and τ_{22} are the characteristic times. (iii) The initial velocities of first, second and third particles are all set to be 2. There are three impulses. The τ_{31} and τ_{32} are the characteristic times. In the step-up chain, the number of biphasic behaviors equals the number of impulses. Because of the similar biphasic behaviors, the first biphasic behavior is considered. In Fig. 3, we display characteristic time as a function of mass ratio. From this figure we can clearly see the influence of mass ratio. The τ_{11} (τ_{21} and τ_{31}) increases and

τ_{12} (τ_{22} and τ_{32}) decreases with the increase of mass ratio, which indicates that the biphasic effect of energy decay decreases with the increase of mass ratio; while τ_{11} (τ_{21} and τ_{31}) and τ_{12} (τ_{22} and τ_{32}) have the same value if mass ratio tends to zero, that is, the wave is reflected completely. The mass ratio has greater influences on τ_{11} (τ_{21} and τ_{31}) than τ_{12} (τ_{22} and τ_{32}). When τ_{11} (τ_{21} and τ_{31}) and τ_{12} (τ_{22} and τ_{32}) are plotted against $(m_1/m_2)^{0.8}$, we find that the linear fits are in good agreement with the data. So the approximate expressions of τ_{11} (τ_{21} and τ_{31}) and τ_{12} (τ_{22} and τ_{32}) could be obtained. The interaction between touching particles follows the Hertz's law, and the value of Hertz law exponent (n) is 1.5. We find that the exponent (0.8) of mass ratio equals $2/(n+1)$. The exponent (0.8) depends on the Hertz law exponent. The bigger the Hertz-law exponent, the faster the velocity of impulse.[6] With the increase of Hertz law exponent, the exponent of mass ratio becomes smaller, τ_{11} (τ_{21} and τ_{31}) becomes smaller too. The energy in the first section decays faster. With the number of wave increasing, the difference in adjacent characteristic time becomes smaller. We could infer that the characteristic time becomes constant when the number of wave is very large. If the value of mass ratio is greater than one, the biphasic behavior of energy disappears and the function of time-dependent energy follows the expression as follows:

$$E(t) = E_{\min} + \frac{E_{\max} - E_{\min}}{1 + 10^{-(t-t_0)/\tau}}.$$

Therefore we could obtain $\tau \sim (m_1/m_2)^{0.8}$ too.

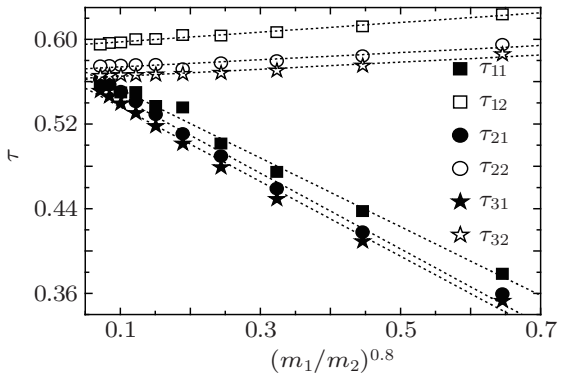


Fig. 3. Plots of character time versus mass ratio.

As is well known, the larger the mass ratio of light to heavy particles, the more the energy remaining in the light particle is, when a moving light particle impacts on a static heavy particle. The functional relationship between energy ratio and mass is shown in Fig. 4. The energy ratio refers to the ratio of initial energy to final energy in the first biphasic behavior. The functional expression may be obtained through quasi-particle model.[15]

In Fig. 4 $\alpha = [(\Omega m_1 - m_2)/(\Omega m_1 + m_2)]^2$, where $\Omega = 1.345$.[15] With increasing the number of impulse, the energy ratio increases.

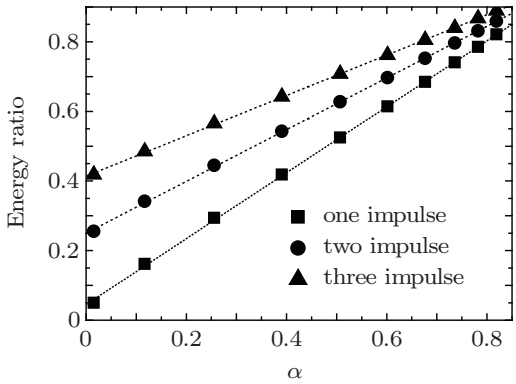


Fig. 4. Plots of energy ratio versus α.

4. Conclusions

In this paper, we analyze the biphasic behavior in the stepped granular chain. The energy in the light section first increases and then decreases, while the degree of compression first decreases then increases. That is, when the degree of compression in the light section becomes smallest, the energy begins to be reflected to the light section. There is the biphasic behavior of energy decay in the light section. The function of biphasic behavior is obtained, and scaling relation between the characteristic time of biphasic behavior and mass ratio is obtained too. With the increase of α, energy ratio also increases, which could be explained with the quasi-particle mode. The multipulse structure is against the transport of energy.

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