Microwave interrogation cavity for the rubidium space cold atom clock*

Wei Ren(任伟)[1], Yuan-Ci Gao(高源慈)[2], Tang Li(李唐)[1],
De-Sheng Lü(吕德胜)[1,†], and Liang Liu(刘亮)[1,‡]

[1] *Key Laboratory of Quantum Optics and Center of Cold Atom Physics, Shanghai Institute of Optics and Fine Mechanics, Chinese Academy of Sciences, Shanghai* 201800, *China*

[2] *School of Electronic Engineering, University of Electronic Science and Technology of China, Chengdu* 611731, *China*

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The performance of space cold atom clocks (SCACs) should be improved thanks to the microgravity environment in space. The microwave interrogation cavity is a key element in a SCAC. In this paper, we develop a microwave interrogation cavity especially for the rubidium SCAC. The interrogation cavity has two microwave interaction zones with a single feed-in source, which is located at the center of the cavity for symmetric coupling excitation and to ensure that the two interaction zones are in phase. The interrogation cavity has a measured resonance frequency of 6.835056471 GHz with a loaded quality factor of nearly 4200, which shows good agreement with simulation results. We measure the Rabi frequency of the clock transition of the rubidium atom in each microwave interaction zone, and subsequently demonstrate that the distributions of the magnetic field in the two interaction zones are the same and meet all requirements of the rubidium SCAC.

Keywords: interrogation cavity, microwave interaction, phase, quality factor

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1. Introduction

Space cold atom clocks (SCACs) operate in space using laser cooling technique. Benefiting from the space microgravity environment, the cold atoms can be launched more slowly than on-earth and travel at a constant speed through the microwave cavity. Therefore, the interaction time between cold atoms and microwave in space can be largely increased, which leads to improvement in the performance improvement of the atom clock beyond that achieved on-earth. There have been several proposals related to the SCAC project, for example, the ACES (Atomic Clock Ensemble in Space) mission will be launched in 2017.[1–5] During the SCAC operation, cold atoms interact with microwave twice during the flight through a microwave interrogation cavity.[6,7] The population of the cold atoms on the atomic ground state hyperfine levels is then detected by laser-induced fluorescence.[8,9] If the phase and the quality factor or resonance frequency of the interrogation cavity are not appropriate, the apparent resonance frequency of the clock will be shifted from the frequency of the ground state hyperfine transition, and then an inaccurate frequency or time will be obtained.[10–13] Therefore, the microwave interrogation cavity is a critical element for the SCAC. However, the microwave interrogation cavity of the SCAC is different from that of atomic fountain clocks, since the cold atoms can only fly forward in space. Consequently, the twice interactions between cold atoms and the microwave pulse can only be realized by two microwave interaction zones.[14] As a result, it is necessary to design a new type microwave interrogation cavity especially for the SCAC. There are several choices for the SCAC interrogation cavity, such as U-type cavity,[15] ring cavity,[16] and two separated cylindrical cavities.[17]

In our proposal,[4] the rubidium atom is chosen because of its lower cold collision frequency shift compared with that of the cesium atom.[18] The clock transition for rubidium is the ground state hyperfine transition $\left({}^{87}\mathrm{Rb}, 5^2S_{1/2}F=1 \rightarrow 5^2S_{1/2}F=2\right)$. The following design and measurements focus on the rubidium cold atoms interrogation, and the requirements listed below should be fulfilled in the microwave interrogation cavity of the rubidium SCAC.

(i) There should be two interaction zones in the microwave cavity of the SCAC for twice interactions with cold atoms successively because the cold atoms can only flight forward in the microgravity environment.

(ii) Since the clock transition of the rubidium SCAC is the ground state hyperfine transition of ^{87}Rb atoms, the interrogation cavity should have an eigen frequency near the transition frequency of ^{87}Rb atom, that is, 6.834682610 GHz.

(iii) To reduce the cavity pulling effect in the rubidium SCAC, the loaded quality factor Q_{L} of the interrogation cavity

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†Corresponding author. E-mail: dslv@siom.ac.cn

‡Corresponding author. E-mail: liang.liu@siom.ac.cn

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should be reduced, because the corresponding frequency shift is proportional to the square of Q_L.[19–21]

(iv) The phase difference of the two microwave interaction zones must be lower than 2.2×10^{-5} rad because the frequency errors arising from the phase shift must be lower than 2.0×10^{-16} in our rubidium SCAC.[4]

(v) The microwave leakage effect should be avoided as far as possible by adding cutoff waveguides out of the interaction zones.

This paper presents the design and simulation of a microwave interrogation cavity for the rubidium SCAC. Some measurements are also performed, including the measurement of the Rabi oscillation curve of the rubidium cold atoms in each interaction zone. We proved that this microwave interrogation cavity is appropriate for the rubidium SCAC.

2. Design and simulation

Considering the requirements of the rubidium SCAC, we designed a microwave interrogation cavity and simulated it with a finite element analysis software.

2.1. Design of the microwave interrogation cavity

The microwave interrogation cavity is designed based on the rectangular waveguide cavity, according to the U-type interrogation cavity[15] and the ring cavity.[16] The cavity structure is displayed in Fig. 1. The structure of the cavity consists of three parts, namely bracket, cover, and coupled waveguide. The bracket and the cover form the main structure of the microwave cavity. The main structure can be divided into three kinds of sub-structure: microwave interaction zone, guided wave zone and cutoff waveguide zone. Two guided wave zones are located at each side of the cavity and two microwave interaction zones are connected with them at their each end. Therefore, the main cavity can be seen as a ring cavity with four rectangular waveguide cavities connected end-to-end. The microwave is injected into the coupled waveguide through the feed-in hole. The feed-in microwave propagates symmetrically along the two guided wave zones and finally forms a standing wave field at each microwave interaction zone. Consequently, the one-way flight rubidium cold atoms can interact with the microwave pulse twice at the two interaction zones successively. Four cutoff waveguides are present along the trajectory of cold atoms in this cavity. Two of them are located at the two cavity ends, and the other two are placed at the center of the cavity but out of both interaction zones. All the cutoff waveguides suppress the microwave oscillations out of the interaction zones and reduce the microwave leakage effect during the free evolution of the rubidium cold atoms.

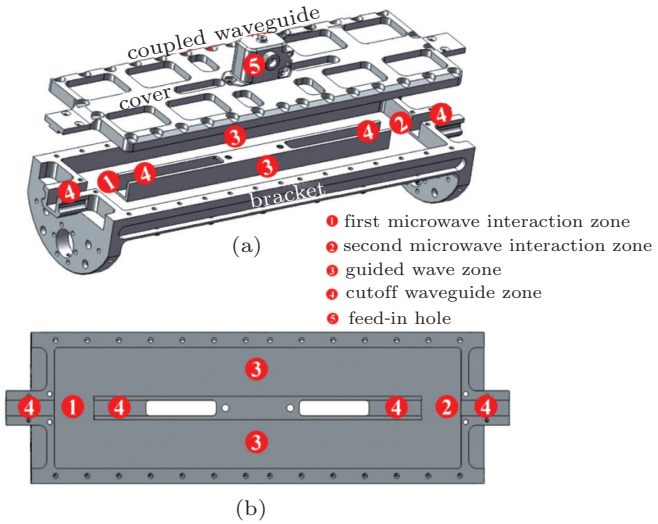


Fig. 1. (color online) The structure of our microwave interrogation cavity for the rubidium SCAC. (a) Tee isometric side view. The cover and the bracket are moved apart for a clear view of the inner structure. (b) The *xy*-plane cross-sectional view.

To determine the dimension of the two interaction zones and the two guided wave zones, we model the interrogation cavity as four rectangular waveguide cavities connected end-to-end as described above and apply the principle of rectangular waveguide cavity in our calculations.[22,23] For single-mode oscillation, each rectangular waveguide cavity must work in transverse electric (TE) mode. We choose the guided wave zones working in TE_{107} mode and the interaction zones working in TE_{201} mode according to the SCAC's overall architecture and its operating characteristics. To ensure that only the TE_{10p} mode propagates in the interrogation cavity, the width a and the height b of the rectangular waveguide should fulfill the following conditions[22,23]

$$\frac{\lambda_0}{2} \le a \le \lambda_0 \quad \text{and} \quad 0 \le b \le \lambda_0, \tag{1}$$

where $\lambda_0 = 43.863$ mm is the target resonance wavelength of the microwave interrogation cavity, derived from clock resonance frequency $\nu_0 = 6.834682610$ GHz. To keep the resonance frequency of the cavity always near the target frequency when the temperature changes slightly due to the temperature controller accuracy of ± 0.01 °C, we make a compromise which implies that the width and height of the guided wave zone are

$$a = \frac{\sqrt{2}}{2}\lambda_0 = \frac{\lambda_c}{2} \quad \text{and} \quad b \approx \frac{a}{2}, \tag{2}$$

where λ_c is the waveguide or cutoff wavelength. We determined the width and height of each guided wave zone to be 31.016 mm and 15 mm, respectively. The length of the guided wave zone can be deduced from the TE_{107} working mode as $l_g = 7a = 217.11$ mm.

In this work, the dimension of the interaction zones is determined by the rectangular waveguides and the cutoff waveguides. The rubidium cold atoms fly along the y-axis and, thus, the dimension of the atom cloud limits the width of the cutoff waveguide a_c. Consequently, we set a_c to be 10 mm in our rubidium SCAC, while the thickness of the cutoff waveguide is $a_t = 2.5$ mm which is the mechanical strength limit. We then derive the width of each microwave interaction zone working in TE_{201} mode as $a_{201} = 2a + a_c + 2a_t$. Accordingly, we determine the length of each microwave interaction zone to be $l_{201} = 26.684$ mm from the following equation

$$l_{201} = \left[\left(\frac{2}{\lambda_0}\right)^2 - \left(\frac{2}{a_{201}}\right)^2\right]^{-1/2}. \tag{3}$$

To efficiently suppress the microwave leakage, the attenuation of each cutoff waveguide should be as large as possible to near 77 dB. We calculate the attenuation coefficient for the TE_{10} mode in the rectangular cutoff waveguide as

$$\alpha_{10} = \frac{\pi}{a_c}\sqrt{1 - \left(\frac{2a_c}{\lambda_0}\right)^2}\ \mathrm{dB/mm}. \tag{4}$$

The attenuation of each cutoff waveguide can be derived from α_{10} with

$$\alpha_c = 8.68\alpha_{10}l_c, \tag{5}$$

where l_c is the length of each cutoff waveguide. Hence, l_c should be larger than 31.727 mm.

With the above calculations, we determined the dimension of each part in the microwave interrogation cavity's main structure and summarized them in Table 1, considering the machining accuracy limit.

Table 1. Dimension of each part in the microwave cavity's main structure.

Parameter	Dimension
The width of each wave guided zone a	31.00 mm
The inner diameter of cold atoms' flight pipeline $a_c \times a_c$	10 mm×10 mm
The outer diameter of cold atoms' flight pipeline	15 mm×14 mm
The distance between two microwave interaction zones l_g	217.00 mm
The length of each microwave interaction l_{201}	26.68 mm
The width of each microwave interaction zone a_{201}	77.00 mm
The length of each cutoff waveguide l_c	32.00 mm

2.2. Simulation of the microwave interrogation cavity

According to Table 1, we have established a structural model for the cavity and simulated it with finite element analysis software. We calculate the eigenmodes and show the results in Table 2. The cavity has an eigenmode near the resonance frequency of the rubidium cold atom ν_0, and because of the thermal expansion and contraction effects, we can adjust the eigen frequency to the atom's resonance frequency by controlling the working temperature of the cavity. Furthermore, the intrinsic quality factor Q_0 of this eigenmode is 12046 which is large enough for the rubidium SCAC.

Table 2. The results of eigenmode solving in the microwave interrogation cavity model with finite element analysis software.

Eigenmode	Frequency/GHz	Q_0
Mode 1	$6.8371 + j0.00028378$	12046
Mode 2	$6.9186 + j0.00028299$	12224
Mode 3	$7.1705 + j0.00028295$	12671

The microwave interrogation cavity works in the eigenmode named mode 1 while other eigenmodes are suppressed effectively. We then set a port with power of 1 W at the feed-in hole and simulated the electromagnetic field distribution in this cavity. From the modal solution date report, we get the electromagnetic filed distribution as shown in Fig. 2. Evidently, wave guided zones work in the TE_{107} mode, while the microwave interaction zones work in the TE_{201} mode. Moreover, microwave oscillation along the trajectory of cold atoms is cut off out of the microwave interaction zones.

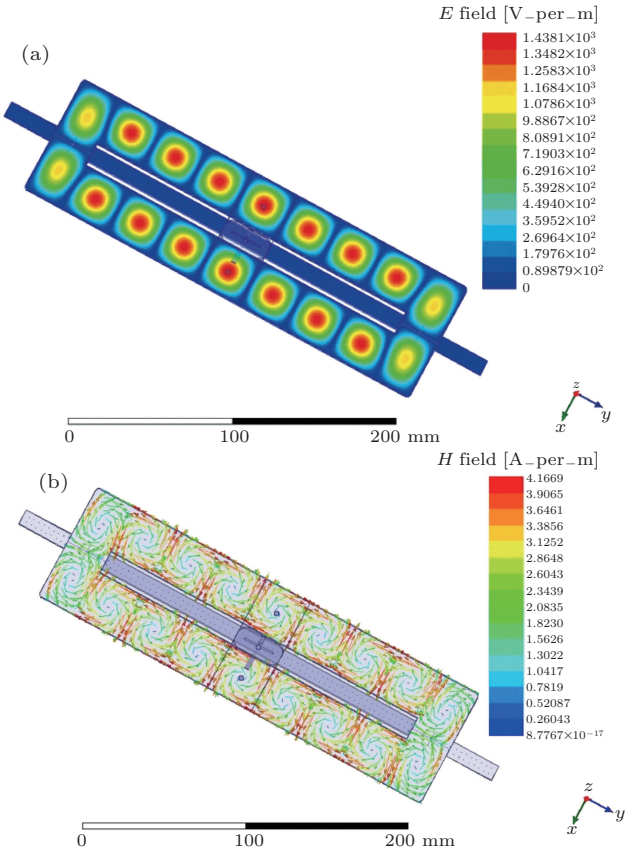


Fig. 2. (color online) The distribution of the electromagnetic field in xy-plane of the new type microwave interrogation cavity. Different colors represent different amplitudes of $\boldsymbol{E}$ or $\boldsymbol{H}$, while the arrow represents the direction of $\boldsymbol{H}$. (a) The electric field strength. (b) The magnetic field.

As shown in Fig. 2, in the rectangular coordinate system, the electric field $\boldsymbol{E}$ and the magnetic field $\boldsymbol{H}$ at a point (x, y, z)

can be written as follows:

$$\boldsymbol{E}(x,y,z) = (E_x, E_y, E_z), \tag{6}$$

$$\boldsymbol{H}(x,y,z) = (H_x, H_y, H_z), \tag{7}$$

where

$$E_i \ (i = x,y,z) \ \text{ and } \ H_i \ (i = x,y,z)$$

are the i-direction components of $\boldsymbol{E}$ and $\boldsymbol{H}$, respectively. Since cold atoms only interact with the magnetic field in the microwave pulse and the quantization axis of the cold atoms is along the y direction, we focus on the y-direction component of the magnetic field $\boldsymbol{H}$ in the two microwave interaction zones. The phase of H_y can be written as[12]

$$\text{Phase}_{H_y} = \arctan\left(\text{Im}\left(H_y\right)/\text{Re}\left(H_y\right)\right). \tag{8}$$

We extracted the distributions of the amplitude H and the Phase_{H_y} in each microwave interaction zone and displayed the transverse distributions in Fig. 3(a) while the longitudinal distribution in Fig. 3(b). The transverse solving lines are along the x-direction and located in the center of each interaction zone, while the longitudinal solving lines are located at the middle of each interaction zone along the y direction.

In the transverse distribution, as shown in Fig. 3(a), the variation of H is very small in both microwave interaction zone because each of them can be seen as a rectangular waveguide cavity working in TE_{201} mode. Furthermore, the fluctuation of Phase_{H_y} is almost zero in each zone, which keeps the phases of cold atoms at different transverse positions approximately the same. These characteristics are advantageous to improve the frequency accuracy of the SCAC.

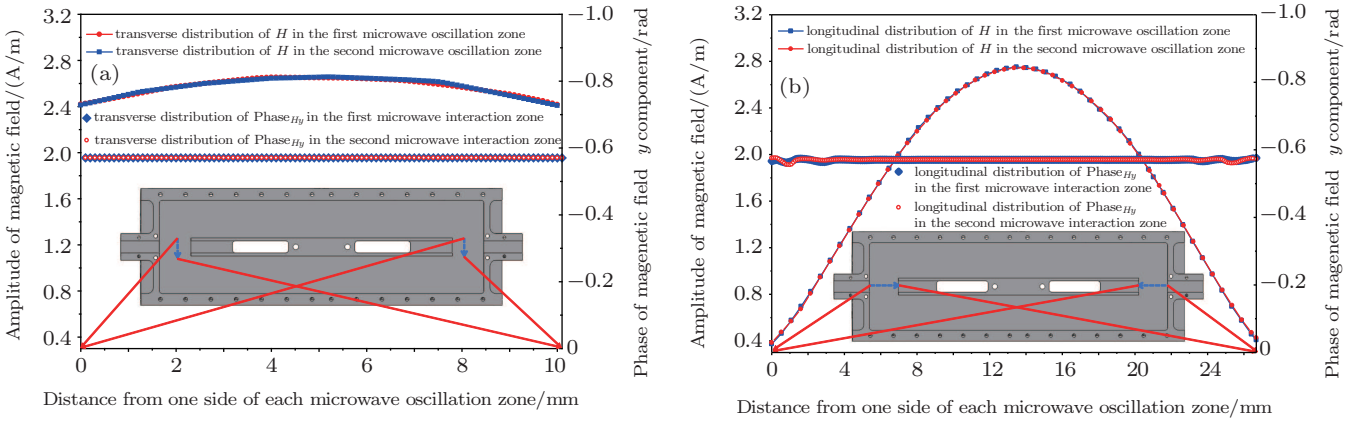


Fig. 3. (color online) The transverse distribution (a) and the longitudinal distribution (b) of H and Phase_{H_y} in each microwave interaction zone.

In the longitudinal distributions, as shown in Fig. 3(b), the amplitude H are perfectly symmetrical because of the structural symmetry of the interrogation cavity. Moreover, the Phase_{H_y} fluctuates near the edge positions of each interaction zone, which is caused by the losses of the flight holes. The average of the Phase_{H_y} can be derived from equation

$$\overline{\text{Phase}_{H_y}} = \frac{\int_0^{l_{201}} \text{Phase}_{H_y}\,\mathrm{d}l}{l_{201}} = \frac{\int_0^{26.68} \text{Phase}_{H_y}\,\mathrm{d}l}{26.68}, \tag{9}$$

from which we obtained the differences of the longitudinal average Phase_{H_y} between the two interaction zones. The difference of the longitudinal average Phase_{H_y} is 8.6×10^{-6} rad and the frequency error originated by this phase shift is lower than 1.0×10^{-16}[15] which is below the target frequency error of 2.0×10^{-16} in our rubidium SCAC. However, the asymmetry of the interrogation cavity will enlarge this difference,[15] and if the center of this cavity shifts by 1 mm, the average phase difference will increase about 1.0×10^{-3} rad. Consequently, the machining tolerance should be kept smaller than ±7 μm in order to guarantee the phase shift smaller than 2.2×10^{-5} rad.

The simulation results indicate that the interrogation cavity only works in the eigenmode named mode 1, in which the resonance frequency of the cavity is near ν_0 and the intrinsic quality factor is large enough for the rubidium SCAC. The electromagnetic field distribution is consistent with our design. What's more, the differences of the average Phase_{H_y} between different microwave interaction zones are small enough and meet the need of our rubidium SCAC.

3. Measurements and results

In this work, we have manufactured an interrogation cavity with machining tolerance ±5 μm. A photograph of the cavity is displayed in Fig. 4. The cavity is made of TC4 (Ti-6Al-4V) coating with silver, and the thermal expansion coefficient of TC4 is 8.8×10^{-6} m/°C which leads to a temperature coefficient about -0.06 MHz/°C. The microwave cavity is integrated with two mounts at each end and these mounts are the facility for mounting the cavity in the vacuum cavity. In what follows, some measurements have been taken to verify the applicability of this microwave interrogation cavity.

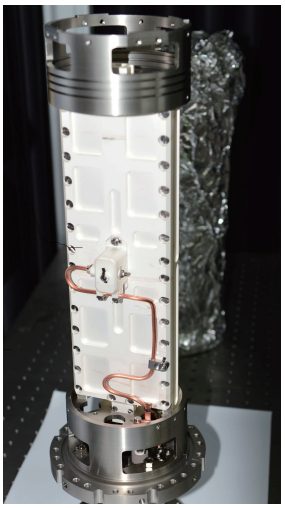


Fig. 4. (color online) Photograph of the microwave interrogation cavity before being assembled in our rubidium SCAC.

3.1. Measurement of S parameter

In our SCAC, the interrogation cavity has been integrated in the ultra-high vacuum tube whose vacuum is better than 1.0×10^{-8} Pa.[24] We have measured the S parameter and the resonance frequency with a vector network analyzer from 17 °C to 22 °C on-earth. The temperature coefficient of the resonance frequency of the cavity is -0.051 MHz/°C which is consistent with the design basically. The results of the measurement at 22 °C are shown in Fig. 5. The resonance frequency of this cavity is 6.835056471 GHz and the loaded quality factor is about 4200. The frequency is near the ground state hyperfine transition frequency of the cold rubidium atoms and the relative resonance frequency offset with respect to ν_0 is lower than 3×10^{-4} which is small enough for the interrogation of the rubidium atoms. Moreover, the loaded quality factor is much smaller than the intrinsic quality factor Q_0 of the design, and thus the cavity pulling effect is suppressed sufficiently in the rubidium SCAC.

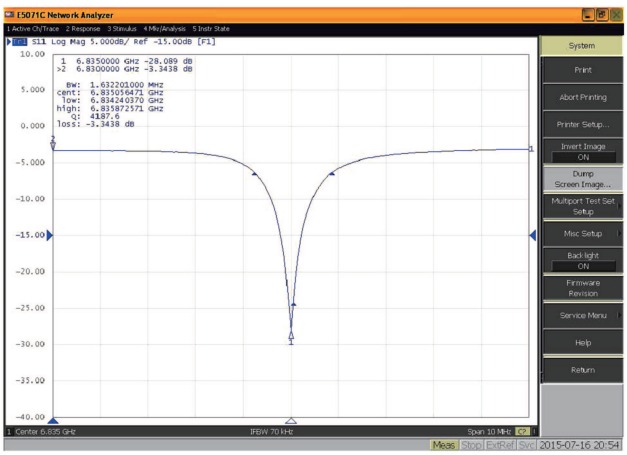


Fig. 5. (color online) The S parameter of the microwave interrogation cavity measured in the vacuum tube of our rubidium SCAC at 22 °C.

3.2. Measurement of the Rabi frequency

As Rabi *et al.* mentioned, in addition to the influence of the C-field, the Rabi frequency Ω of the cold atoms' ground state hyperfine transition in a microwave cavity at the resonance frequency is defined as $\Omega = \mu_B \mu H_{MW_0}/\hbar$, where H_{MW_0} is the intensity of the magnetic field in the microwave field, μ_B is the Bohr magneton, and μ is the dielectric permeability.[25] Additionally, the Rabi resonance magnetic field intensity H_{MW_0} corresponds to a specific value of the microwave feeding in power P_Ω, which is proportional to the square of the magnetic field intensity of the microwave field. Hence, the Rabi frequency Ω is proportional to the square root of P_Ω. In order to obtain the information of the magnetic field in each microwave interaction zone, we have measured P_Ω of the rubidium cold atoms in each interaction zone with our rubidium SCAC on-earth, in which the C-field has always been set to be 120 nT.

In this experiment, we treated each microwave interaction zone as a single microwave cavity and have measured them respectively. The rubidium cold atoms were launched down from the cooling zone[26] of our rubidium SCAC with a same velocity, 4 m/s, and microwave power was feed in the interrogation cavity with a specific sequence. Finally, the transition probability of the rubidium cold atoms was derived from time-of-flight (TOF) signals[8,9] during the rubidium cold atoms passing through the detection zone. The microwave frequency was fixed at ν_0, but its power was changed step by step by 0.25 dBm. When the first interaction zone was being measured, the microwave power was shut down during the flight of the rubidium cold atoms over the second interaction zone, and *vice versa*.

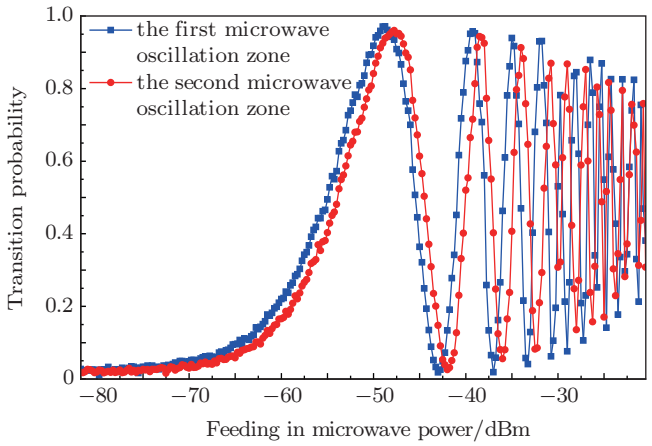


Fig. 6. (color online) The result of the Rabi frequency measurements. The vertical axis indicates the transition probability of cold atoms and the horizontal axis indicates the microwave feeding power.

As results, we obtained the Rabi oscillation curves of each microwave interaction zone in which the transition probability of the rubidium cold atoms changes with the feed-in

power as shown in Fig. 6. Because the resolution of the microwave feed-in power becomes insufficient for P_Ω, the envelopes of the Rabi oscillation curves shrink as the power increases. However, the Rabi oscillation curves are good enough for our SCAC, since we always take the microwave feed-in power at first peak as P_Ω. Consequently, the P_Ω of the first and second microwave interaction zone are -48.75 dBm and -47.75 dBm, respectively. The Rabi frequency ratio of the two microwave interaction zones on ground is

$$\frac{\Omega_2}{\Omega_1} = \frac{\sqrt{P_{\Omega 2}}}{\sqrt{P_{\Omega 1}}}, \tag{10}$$

where $\Omega_i\,(i=1,2)$ is the Rabi frequency in the i-th microwave interaction zone with different interaction time τ, while $P_{\Omega i}$ $(i=1,2)$ is the microwave feed-in power at Ω_i, taking mW as power unit. Bringing the results of this measurement into Eq. (10), we obtain Ω_2/Ω_1, which is just the ratio of the flight time of the cold atoms across each microwave interaction zone, τ_1/τ_2. We then have

$$\Omega_1\tau_1 = \Omega_2\tau_2 = \pi. \tag{11}$$

If the flight times of rubidium cold atoms over the two different interaction zones are the same just as the situation in space, $\tau_1 = \tau_2 = \tau_0$, we then obtain that

$$\Omega_{10} = \Omega_{20}, \tag{12}$$

which proves that the magnetic field of the two microwave interaction zones is distributed in the same manner.[27] Consequently, the electromagnetic fields in the two microwave interaction zones maintain a high consistency in this microwave interrogation cavity, which satisfies the demands of the rubidium SCAC.

4. Conclusion

In this paper, we have presented a microwave interrogation cavity which was designed for the rubidium SCAC. This cavity can be considered as four small cavities connected end-to-end, with two of them as guided wave zones and the other two as microwave interaction zones. The guided wave zones work in TE_{107} mode and the microwave interaction zones work in TE_{201} mode, ensuring that the electromagnetic field distribution is suitable for the interrogation of the rubidium cold atoms. There are four cutoff waveguides which guarantee the microwave oscillations are terminated effectively during the rubidium atoms' free evolution. Accordingly, in this interrogation cavity, forward flying rubidium cold atoms can successively interact with the two microwave pulses separated by a zero electromagnetic field area.

Furthermore, we have simulated this interrogation cavity with finite element analysis software and presented its characteristics. The magnetic fields in the two microwave interaction zones distribute in a highly consistent manner along the trajectory of the cold atoms. The longitudinal average phase differences of the y component of the magnetic field is 8.6×10^{-6} rad, which is small enough for the rubidium SCAC.

Finally, we have manufactured a microwave interrogation cavity and measured its S parameter. The loaded quality factor is about 4200 and the resonance frequency is 6.835056471 GHz which is near the rubidium cold atom's resonance frequency 6.834682610 GHz in our SCAC. Moreover, we have measured the Rabi frequency in each microwave interaction zone and derived that $\Omega_{10} = \Omega_{20}$ when the rubidium SCAC works in space, which indicates that the magnetic field is distributed in the same manner in the two microwave interaction zones.

In all, this interrogation cavity is designed based on the rectangular waveguide cavity principle with a compromise of $a = (\sqrt{2}/2)\lambda_0$. The electromagnetic field distribution can provide two microwave interaction zones, which is suitable for the rubidium SCAC. To a great extent, the microwave interrogation cavity presented in this work is the first to applied in the rubidium SCAC. Furthermore, since the design principle is a universal one, this type microwave cavity can be generalized to interrogate other atoms in the SCAC with other medium.

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