H_∞ synchronization of the coronary artery system with input time-varying delay*

Xiao-Meng Li(李晓蒙)[1], Zhan-Shan Zhao(赵占山)[1,†], Jing Zhang(张静)[2,3], and Lian-Kun Sun(孙连坤)[1]

[1] *School of Computer Science & Software Engineering, Tianjin Polytechnic University, Tianjin* 300387, *China*

[2] *School of Textiles, Tianjin Polytechnic University, Tianjin* 300387, *China*

[3] *Tianjin Vocational Institute, Tianjin* 300410, *China*

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This paper investigates the H_∞ synchronization of the coronary artery system with input delay and disturbance. We focus on reducing the conservatism of existing synchronization strategies. Base on the triple integral forms of the Lyapunov–Krasovskii functional (LKF), we utilize single and double integral forms of Wirtinger-based inequality to guarantee that the synchronization feedback controller has good performance against time-varying delay and external disturbance. The effectiveness of our strategy can be exhibited by simulations under the different time-varying delays and different disturbances.

Keywords: coronary artery system, chaotic system, H_∞ synchronization, input delay

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1. Introduction

In recent decades, chaos has combined with biological engineering due to it being widely encountered in the nervous system, brain activity, epidemic diseases and coronary artery system.[1–3] The coronary artery system is a complex system which conveys oxygen and nutrients to the myocardium day and night. Healthy coronary artery vessels are important to maintain our life. In clinical practice, myocardial infarction caused by obstructing or spasm of the coronary artery will threaten our life. Due to its important effect on our health, this system deserves considerable attention in many areas. With the development of nonlinear science, researchers made a lot of effort in the investigation of the coronary artery system. Based on the muscle mechanics model, Guanggui Bao proposed the mechanics equation of the coronary artery vessel. Utilizing this equation, the mathematical model of the coronary artery vessel is derived in Ref. [4]. We find that this model is a chaotic system.

Since the coronary artery system can be described as a chaos system, many studies of the coronary artery system are focusing on synchronization of a chaotic system. From a medical point of view, when myocardial infarction occurs, the coronary artery vessel must be thrown into convulsion. It will cause degraded performance of oxygen supply to the myocardium. In order to alleviate this symptom, some emergency medicine should be adopted to expand the coronary artery vessel, such as glyceryl trinitrate. From the perspective of mathematics, glyceryl trinitrate can make the diseased coronary artery vessel synchronize with the healthy one by changing the inner diameter and pressure of the vessel. Therefore, chaotic synchronization of the coronary artery system is an essential work. Reference [5] utilized a backstepping method to achieve chaos synchronization. Reference [6] used a nonlinear state feedback approach to design a controller for the coronary artery system. Considering the structure uncertainties, reference [7] proposed a sliding mode control based chaos synchronization strategy for the coronary artery system. By applying high-order sliding mode adaptive control approach, Zhao *et al.* studied the finite-time chaos synchronization problem; this approach can obtain a good effect even though the information of perturbation bound is unknown.[8]

However, there are some things worth noting. First, between the paroxysm of myocardial infarction and taking medicine there is always some time lag. Not only that, the effect of medication on the coronary artery will have some delay because the absorbing of medication will cost some time. It is not difficult to understand that the time delays mentioned above are not fixed. That is to say, considering the actual situation the input time-varying delay should not be ignored. Unfortunately, aforementioned literature did not consider that. It is not hard to imagine that if we fail to input time-varying delay, the performance of the existing control strategy will decline even failure.

Time delay is widely encountered in communication systems, neural networks, economic systems, biological systems, and networked control systems.[9–12] Since time delay will cause serious degradation of system performance, in the last few decades a lot of researchers have been making consider-

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†Corresponding author. E-mail: zhzhsh127@163.com

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able effort to reduce the influence that comes from time delay. Jensen's inequality plays an important role in a lot of existing literature. However, it will induce some conservativeness hard to overcome. In order to reduce the conservatism, Wirtinger-based integral inequality,[13] which can be used to obtain even tighter lower bound of single integral terms was proposed. Very recently, a Wirtinger-based double integral inequality was proposed to get even tighter lower bound of double integral terms.[14]

As well as the input of time-varying delay, as a practical system, the coronary artery system must have many nonlinear disturbances which come from the external environment or emotion. These disturbances will make the performance of control strategy decrease seriously. To counter these nonlinear disturbances, lots of techniques have been proposed, such as adaptive tracking control,[15] model predictive control,[16,17] fuzzy control,[18–21] sliding mode control.[22–26] H_∞ control is a valid approach to reduce the effect caused by disturbances. As the application of H_∞ control, H_∞ synchronization for a chaotic system is studied in Ref. [27]. A criterion for existence of the controller can be given in terms of LMIs. Using the LMI box, the feasible solutions of LMIs can be obtained.

Motivated by the previous discussions, we research H_∞ synchronization of the coronary artery system with input time-varying delay and external disturbances. Based on LKF approach and LMI method, we propose a new control strategy. In this strategy, the triple integral forms of LKF, Wirtinger-based inequality and double integral forms of Wirtinger-based inequality will be used to reduce the conservative. It means that the new strategy can ensure the effectiveness of the controller under the appropriate input time-varying delay. Moreover, H_∞ control approach has been introduced to conflict with the disturbances. Some simulations show that this control strategy still has better robustness under the input time-varying delay.

2. System description and problem analysis

The coronary artery vessels supply oxygen and nutrients for the myocardium. The mathematical model of the coronary artery vessel is derived in Ref. [4], which can be written as a Duffing equation:

$$\begin{aligned}\dot{x}_1(t) &= -bx_1(t) - cx_2(t),\\ \dot{x}_2(t) &= -(b+1)\lambda x_1(t) - (c+1)\lambda x_2(t)\\ &\quad + \lambda x_1^3(t) + E\cos\sigma t,\end{aligned} \tag{1}$$

where $x_1(t)$ and $x_2(t)$ represent inner diameter changes and inner pressure changes of the coronary artery vessel, respectively. $E\cos\sigma t$ is the perturbation with a certain period. With the parameters $b = 0.15$, $c = -1.7$, $\lambda = -0.5$, $E = 0.3$, $\sigma = 1$, and initial condition $x_1(0) = 0.5$, $x_2(0) = 0$, trajectory of system (1) can be shown in Fig. 1.

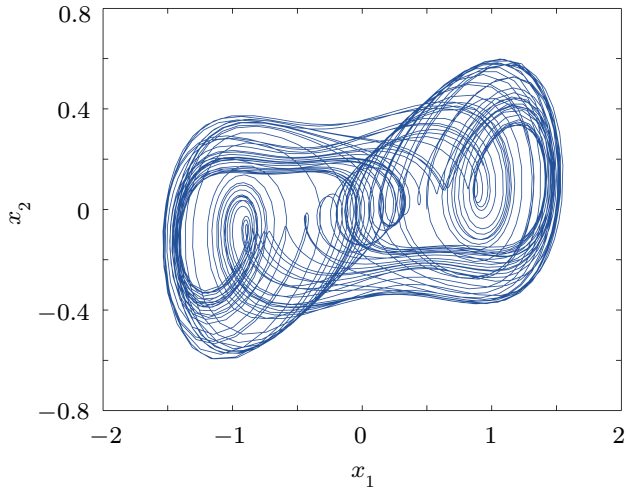


Fig. 1. (color online) Phase portrait of the coronary artery system.

In order to facilitate expression, system (1) can be written as follows:

$$\dot{x}(t) = Ax(t) + Cg(x(t)) + h(t), \tag{2}$$

the above system shows a healthy coronary artery system, where A and C are the matrices determined by the value of b, c, and λ, $x(t) = (x_1(t), x_2(t))$, $g(x(t)) = (0, x_1^3(t))$, $h(t) = (0, E\cos\sigma t)$. The corresponding system which shows a coronary artery system suffering from myocardial infarction can be written as follows:

$$\dot{y}(t) = Ay(t) + Cg(y(t)) + h(t) + \omega(t) + u(t), \tag{3}$$

where $y(t) = (y_1(t), y_2(t))$, $g(y(t)) = (0, y_1^3(t))$, $\omega(t) = (\omega_1(t), \omega_2(t))$ is the external disturbance, $u(t) = (u_1(t), u_2(t))$ can be considered as drug treatment. We want to synchronize the diseased coronary artery system (3) with the healthy one (2) through an appropriate $u(t)$. Taking time delay of taking medicine and drug absorption into consideration, we can design an input time-varying-delay feedback controller $u(t) = Ke(t - d(t))$ $(d(t) \in [d_m, d_M], d_d = \dot{d}(t))$ to achieve this goal. Define $e(t) = y(t) - x(t)$, we can obtain an error system by Eqs. (2) and (3):

$$\dot{e}(t) = Ae(t) + Ke(t - d(t)) + Cg(t) + D\omega(t), \tag{4}$$

where $g(t) = g(y(t)) - g(x(t))$ satisfies the following Lipschitz condition

$$|g(y(t)) - g(x(t))| \le |L(y(t) - x(t))|, \tag{5}$$

where L is a constant matrix.

When $\omega(t) = 0$ and the initial conditions of systems (2) and (3) are $(x_1(0), x_2(0)) = (0.5, 0)$, $(y_1(0), y_2(0)) = (-1, 1.5)$, we can draw the trajectory of $e(t)$ before taking control.

From Fig. 2, we can see error system $e(t)$ cannot approach zero asymptotically. That is, systems (2) and (3) are not synchronized. The rest of this paper will design an input time-varying delay feedback control strategy to synchronize system (3) with (2) under the disturbance $\omega(t) \neq 0$.

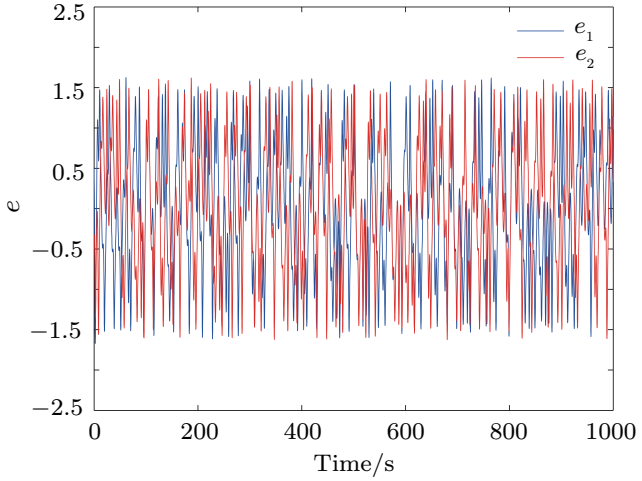


Fig. 2. (color online) Synchronization error between the coronary artery systems (2) and (3).

In order to derive the new control strategy, the following lemmas will be used.

Lemma 1[13] R is a symmetric positive-definite matrix, for differentiable function $x \in [d_1, d_2] \to \mathbb{R}^n$, we can obtain

$$\int_{d_1}^{d_2} \dot{x}^{\mathrm{T}}(s) Z \dot{x}(s)\,\mathrm{d}s \geq \frac{1}{d_2-d_1} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} Z & 0 \\ * & Z \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix},$$

where

$$\varepsilon_1 = x(d_1) - x(d_2),$$
$$\varepsilon_2 = \sqrt{3}x(d_1) + \sqrt{3}x(d_2) - \frac{2\sqrt{3}}{d_2-d_1}\int_{d_1}^{d_2} x(s)\,\mathrm{d}s.$$

Lemma 2[28] For positive definite $d_v : \mathbb{R}^n \to \mathbb{R}$, $d_v \in \mathscr{G} \subseteq \mathbb{R}^n (v = 1 \cdots N)$, reciprocally convex combination of d_v can be written as follows:

$$\min_{\{\rho_m | \rho_m > 0, \sum_m \rho_m = 1\}} \sum_m \frac{1}{\rho_m} k_m(t) = \sum_m k_m(t) + \max_{l_{m,w}(t)} \sum_{m \neq w} l_{m,w}(t),$$

where

$$\left\{ l_{m,w} : \mathbb{R}^n \to \mathbb{R}, l_{w,m}(t) \triangleq l_{m,w}(t), \begin{bmatrix} k_m(t) & l_{m,w}(t) \\ l_{m,w}(t) & k_w(t) \end{bmatrix} \geq 0 \right\}.$$

Lemma 3[14] R is a symmetric positive-definite matrix, for differentiable function $x \in [h_a, h_b] \to \mathbb{R}^n$, we have

$$\frac{(d_2-d_1)^2}{2}\int_{d_1}^{d_2}\int_{s}^{d_2} \dot{x}^{\mathrm{T}}(u) R \dot{x}(u)\,\mathrm{d}u\,\mathrm{d}s \geq \begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} R & 0 \\ * & R \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix},$$

where

$$\omega_1 = (d_2-d_1)x(d_2) - \int_{d_1}^{d_2} x(s)\,\mathrm{d}s,$$
$$\omega_2 = \frac{\sqrt{2}(d_2-d_1)}{2}x(d_2) + \sqrt{2}\int_{d_1}^{d_2} x(s)\,\mathrm{d}s - \frac{3\sqrt{2}}{d_2-d_1}\int_{d_1}^{d_2}\int_{s}^{d_2} x(u)\,\mathrm{d}u\,\mathrm{d}s.$$

3. H_∞ synchronization of coronary artery system

In this section, we get a new synchronization method by using the aforementioned Lemmas.

Theorem 1 Consider the drive and response systems (2) and (3) under the disturbance $\omega(t)$, by application of the input time-varying delay feedback controller, for given scalars $d_m, d_M, d_d, \chi_{\kappa-1} (\kappa = 1,2,3,4)$, if there are positive symmetric matrices

$$P = \begin{bmatrix} \bar{P}_{11} & \chi_1\bar{P}_{11} & \chi_2\bar{P}_{11} & \chi_3\bar{P}_{11} & \chi_4\bar{P}_{11} \\ * & \tilde{P}_{22} & \tilde{P}_{23} & \tilde{P}_{24} & \tilde{P}_{25} \\ * & * & \tilde{P}_{33} & \tilde{P}_{34} & \tilde{P}_{35} \\ * & * & * & \tilde{P}_{44} & \tilde{P}_{45} \\ * & * & * & * & \tilde{P}_{55} \end{bmatrix} \in \mathbb{R}^{5n*5n},$$

$\tilde{W}_s \in \mathbb{R}^{n*n}$ $(s = 1,2,3)$, $\tilde{Z}_v \in \mathbb{R}^{n*n}$ $(v = 1,2,\ldots,4)$, and appropriate dimensions matrices $\tilde{S}_m$, satisfying the following LMIs

$$\begin{bmatrix} \Phi & d_m^2\Delta^T & d_{Mm}\Delta^{\mathrm{T}} & (d_m^3/2)\Delta^{\mathrm{T}} & (d_{Mm}^3/2)\Delta^{\mathrm{T}} & \bar{P}_{11} & \bar{P}_{11}L^{\mathrm{T}} \\ * & -\bar{P}_{11}\tilde{Z}_1^{-1}\bar{P}_{11} & 0 & 0 & 0 & 0 & 0 \\ * & * & -\bar{P}_{11}\tilde{Z}_2^{-1}\bar{P}_{11} & 0 & 0 & 0 & 0 \\ * & * & * & -\bar{P}_{11}\tilde{Z}_3^{-1}\bar{P}_{11} & 0 & 0 & 0 \\ * & * & * & * & -\bar{P}_{11}\tilde{Z}_4^{-1}\bar{P}_{11} & 0 & 0 \\ * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & -I \end{bmatrix} < 0, \tag{6}$$

$$\begin{bmatrix} \tilde{Z}_2 & \tilde{S}_m \\ * & \tilde{Z}_2 \end{bmatrix} \geq 0, \quad (m = 1,2), \tag{7}$$

where

$$\Phi = \sum_{i=1}^{5}\left[\tilde{\Omega}_i^{\mathrm{T}}\tilde{G}_i + \tilde{G}_i^{\mathrm{T}}\tilde{\Omega}_i\right] + \tilde{W} + \tilde{M} - d_m^2\sum_{i=1}^{2}\Gamma_i^{\mathrm{T}}\tilde{Z}_1\Gamma_i - \sum_{i=3}^{6}\Gamma_i^{\mathrm{T}}\tilde{Z}_2\Gamma_i - \Gamma_3^{\mathrm{T}}\tilde{S}_1\Gamma_5$$

$$-\Gamma_5^{\mathrm{T}}\tilde{S}_1^{\mathrm{T}}\Gamma_3-\Gamma_4^{\mathrm{T}}\tilde{S}_2\Gamma_6-\Gamma_6^{\mathrm{T}}\tilde{S}_2^{\mathrm{T}}\Gamma_4-d_m^2\sum_{i=7}^{8}\Gamma_i^{\mathrm{T}}\tilde{Z}_3\Gamma_i-d_{Mm}^2\sum_{i=9}^{10}\Gamma_i^{\mathrm{T}}\tilde{Z}_4\Gamma_i,$$

$$\bar{P}_{11}=P_{11}^{-1},\quad \tilde{\Omega}_1=\Delta=A\bar{P}_{11}h_1+K\bar{P}_{11}h_3+Ch_{11}+Dh_{12},\quad \tilde{\Omega}_2=h_1-h_2,$$

$$\tilde{\Omega}_3=h_2-h_4,\quad \tilde{\Omega}_4=d_mh_1-h_5,\quad \tilde{\Omega}_5=(d_M-d_m)h_2-h_6,$$

$$\tilde{G}_1=h_1I+\chi_1h_5I+\chi_2h_6I+\chi_3h_7I+\chi_4h_8I,\qquad \tilde{G}_2=\chi_1\bar{P}_{11}h_1+\tilde{P}_{22}h_5+\tilde{P}_{23}h_6+\tilde{P}_{24}h_7+\tilde{P}_{25}h_8,$$

$$\tilde{G}_3=\chi_2\bar{P}_{11}h_1+\tilde{P}_{23}^{\mathrm{T}}h_5+\tilde{P}_{33}h_6+\tilde{P}_{34}h_7+\tilde{P}_{35}h_8,\qquad \tilde{G}_4=\chi_3\bar{P}_{11}h_1+\tilde{P}_{24}^{\mathrm{T}}h_5+\tilde{P}_{34}^{T}h_6+\tilde{P}_{44}h_7+\tilde{P}_{45}h_8,$$

$$\tilde{G}_5=\chi_4\bar{P}_{11}h_1+\tilde{P}_{25}^{\mathrm{T}}h_5+\tilde{P}_{35}^{\mathrm{T}}h_6+\tilde{P}_{45}^{T}h_7+\tilde{P}_{55}h_8,$$

$$\tilde{W}=\mathrm{diag}\{\tilde{W}_1+\tilde{W}_2+\tilde{W}_3\quad -\tilde{W}_2\quad -(1-d_d)\tilde{W}_1\quad -\tilde{W}_3\quad 0\quad 0\quad 0\quad 0\quad 0\quad 0\quad 0\quad 0\},$$

$$\tilde{M}=\mathrm{diag}\{0\quad 0\quad 0\quad 0\quad 0\quad 0\quad 0\quad 0\quad 0\quad 0\quad -I\quad -\gamma^2I\},$$

$$\Gamma_1=h_1-h_2,\quad \Gamma_2=\sqrt{3}h_1+\sqrt{3}h_2-\frac{2\sqrt{3}}{d_m}h_5,\quad \Gamma_3=h_2-h_3,\quad \Gamma_4=\sqrt{3}h_2+\sqrt{3}h_3-2\sqrt{3}h_9,$$

$$\Gamma_5=h_3-h_4,\quad \Gamma_6=\sqrt{3}h_3+\sqrt{3}h_4-2\sqrt{3}h_{10},\quad \Gamma_7=d_mh_1-h_5,$$

$$\Gamma_8=\frac{\sqrt{2}}{2}d_mh_1+\sqrt{2}h_5-\frac{3\sqrt{2}}{d_m}h_7,\quad \Gamma_9=d_{Mm}h_2-h_6,\quad \Gamma_{10}=\frac{\sqrt{2}d_{Mm}}{2}h_2+\sqrt{2}h_6-\frac{3\sqrt{2}}{d_{Mm}}h_8,$$

$$d_{Mm}=d_M-d_m,\quad h_k=[0_{n\times(k-1)n}\ I_{n\times n}\ 0_{n\times(12-k)n}],\ \ k=1,2,\ldots,12.$$

Then, H_∞ synchronization with disturbance attenuation γ is achieved.

Proof Construct the following LKF:

$$V(t)=\sum_{i=1}^{4}V_i(t), \tag{8}$$

where

$$V_1(t)=\nu^T(t)P\nu(t), \tag{9}$$

$$V_2(t)=\int_{t-d(t)}^{t}e^{\mathrm{T}}(s)W_1e(s)\mathrm{d}s+\int_{t-d_m}^{t}e^{\mathrm{T}}(s)W_2e(s)\mathrm{d}s+\int_{t-d_M}^{t}e^{\mathrm{T}}(s)W_3e(s)\mathrm{d}s, \tag{10}$$

$$V_3(t)=d_m^3\int_{-d_m}^{0}\int_{t+s}^{t}\dot{e}^{\mathrm{T}}(u)Z_1\dot{e}(u)\mathrm{d}u\mathrm{d}s+d_{Mm}\int_{-d_M}^{-d_m}\int_{t+s}^{t}\dot{e}^{\mathrm{T}}(u)Z_2\dot{e}(u)\mathrm{d}u\mathrm{d}s, \tag{11}$$

$$V_4(t)=\frac{d_m^4}{2}\int_{t-d_m}^{t}\int_{s}^{t}\int_{u}^{t}\dot{e}^{\mathrm{T}}(v)Z_3\dot{e}(v)\mathrm{d}v\mathrm{d}u\mathrm{d}s+\frac{d_{Mm}^4}{2}\int_{t-d_M}^{t-d_m}\int_{s}^{t-d_m}\int_{u}^{t}\dot{e}^{\mathrm{T}}(v)Z_4\dot{e}(v)\mathrm{d}v\mathrm{d}u\mathrm{d}s, \tag{12}$$

$$u_1(t)=\int_{t-d_m}^{t}e^{\mathrm{T}}(s)\mathrm{d}s,$$

$$u_2(t)=\int_{t-d_M}^{t-d_m}e^{\mathrm{T}}(s)\mathrm{d}s,$$

$$u_3(t)=\int_{t-d_m}^{t}\int_{s}^{t}e^{\mathrm{T}}(u)\mathrm{d}u\mathrm{d}s,$$

$$u_4(t)=\int_{t-d_M}^{t-d_m}\int_{s}^{t-d_m}e^{\mathrm{T}}(u)\mathrm{d}u\mathrm{d}s,$$

$$\nu^T(t)=\left[e^{\mathrm{T}}(t)\quad u_1(t)\quad u_2(t)\quad u_3(t)\quad u_4(t)\right].$$

The time derivative of $V(t)$:

$$\dot{V}(t)=\sum_{i=1}^{4}\dot{V}_i(t), \tag{13}$$

where

$$\begin{aligned}\dot{V}_1(t)&=2\begin{bmatrix}\dot{e}(t)\\ \varpi_1(t)\\ \varpi_2(t)\\ \varpi_3(t)\\ \varpi_4(t)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}P_{11}&\chi_1P_{11}&\chi_2P_{11}&\chi_3P_{11}&\chi_4P_{11}\\ *&P_{22}&P_{23}&P_{24}&P_{25}\\ *&*&P_{33}&P_{34}&P_{35}\\ *&*&*&P_{44}&P_{45}\\ *&*&*&*&P_{55}\end{bmatrix}\begin{bmatrix}e(t)\\ u_1(t)\\ u_2(t)\\ u_3(t)\\ u_4(t)\end{bmatrix}\\ &=2\dot{e}^{\mathrm{T}}(t)[P_{11}e(t)+\chi_1P_{11}u_1(t)+\chi_2P_{11}u_2(t)+\chi_3P_{11}u_3(t)+\chi_4P_{11}u_4(t)]\\ &\quad+2\varpi_1^{\mathrm{T}}(t)[\chi_1P_{11}^{\mathrm{T}}e(t)+P_{22}u_1(t)+P_{23}u_2(t)+P_{24}u_3(t)+P_{25}u_4(t)]\\ &\quad+2\varpi_2^{\mathrm{T}}(t)[\chi_2P_{11}^{\mathrm{T}}e(t)+P_{23}^{\mathrm{T}}u_1(t)+P_{33}u_2(t)+P_{34}u_3(t)+P_{35}u_4(t)]\\ &\quad+2\varpi_3^{\mathrm{T}}(t)[\chi_3P_{11}^{\mathrm{T}}e(t)+P_{24}^{\mathrm{T}}u_1(t)+P_{34}^{T}u_2(t)+P_{44}u_3(t)+P_{45}u_4(t)]\\ &\quad+2\varpi_4^{\mathrm{T}}(t)[\chi_4P_{11}^{\mathrm{T}}e(t)+P_{25}^{\mathrm{T}}u_1(t)+P_{35}^{\mathrm{T}}u_2(t)+P_{45}^{\mathrm{T}}u_3(t)+P_{55}u_4(t)]\\ &=\xi^{\mathrm{T}}(t)\sum_{i=1}^{5}\left[\Omega_i^{\mathrm{T}}G_i+G_i^{\mathrm{T}}\Omega_i\right]\xi(t),\end{aligned} \tag{14}$$

$$\dot{V}_2(t) = \sum_{i=1}^{3} e^{\mathrm{T}}(t)W_i e(t) - (1-\dot{d}(t))e^{\mathrm{T}}(t-d(t))W_1 e(t-d(t))$$
$$- e^{\mathrm{T}}(t-d_m)W_2 e(t-d_m) - e^{\mathrm{T}}(t-d_M)W_3 e(t-d_M) \le \xi^{T}(t)Q\xi(t), \tag{15}$$

$$\dot{V}_3(t) = \dot{e}^{\mathrm{T}}(t)[d_m^4 Z_1 + d_{Mm}^2 Z_2]\dot{e}(t) + \sum_{i=1}^{3}\zeta_i = \xi^{\mathrm{T}}(t)\Delta^{\mathrm{T}}N_1\Delta\xi(t) + \sum_{i=1}^{3}\zeta_i, \tag{16}$$

$$\dot{V}_4(t) = \frac{d_m^4}{2}\int_{t-d_m}^{t}\int_{s}^{t}[\dot{e}^{\mathrm{T}}(t)Z_3\dot{e}(t) - \dot{e}^{\mathrm{T}}(u)Z_3\dot{e}(u)]\mathrm{d}u\mathrm{d}s$$
$$+ \frac{d_{Mm}^4}{2}\int_{t-d_M}^{t-d_m}\int_{s}^{t-d_m}[\dot{e}^{\mathrm{T}}(t)Z_4\dot{e}(t) - \dot{e}^{\mathrm{T}}(u)Z_4\dot{e}(u)]\mathrm{d}u\mathrm{d}s = \xi^{\mathrm{T}}(t)\Delta^{\mathrm{T}}N_2\Delta\xi(t) + \sum_{i=4}^{5}\zeta_i, \tag{17}$$

with

$$N_1 = d_m^4 Z_1 + d_{Mm}^2 Z_2, \quad N_2 = \frac{d_m^6}{4}Z_3 + \frac{d_{Mm}^6}{4}Z_4,$$
$$\zeta_1(t) = -d_m^3\int_{t-d_m}^{t}\dot{e}^{\mathrm{T}}(s)Z_1\dot{e}(s)\mathrm{d}s,$$
$$\zeta_2(t) = -d_{Mm}\int_{t-d(t)}^{t-d_m}\dot{e}^{\mathrm{T}}(s)Z_2\dot{e}(s)\mathrm{d}s,$$
$$\zeta_3(t) = -d_{Mm}\int_{t-d_M}^{t-d(t)}\dot{e}^{\mathrm{T}}(s)Z_2\dot{e}(s)\mathrm{d}s,$$
$$\zeta_4(t) = -\frac{d_m^4}{2}\int_{t-d_m}^{t}\int_{s}^{t}\dot{e}^{\mathrm{T}}(u)Z_3\dot{e}(u)\mathrm{d}u\mathrm{d}s,$$
$$\zeta_5(t) = -\frac{d_{Mm}^4}{2}\int_{t-d_M}^{t-d_m}\int_{s}^{t-d_m}\dot{e}^{\mathrm{T}}(u)Z_4\dot{e}(u)\mathrm{d}u\mathrm{d}s,$$
$$u_5(t) = \frac{1}{d(t)-d_m}\int_{t-d(t)}^{t-d_m}e^{\mathrm{T}}(s)\mathrm{d}s,$$
$$u_6(t) = \frac{1}{d_M-d(t)}\int_{t-d_M}^{t-d(t)}e^{\mathrm{T}}(s)\mathrm{d}s,$$
$$\varpi_1(t) = e(t) - e(t-d_m),$$
$$\varpi_2(t) = e(t-d_m) - e(t-d_M),$$
$$\varpi_3(t) = d_m e(t) - u_1(t),$$
$$\varpi_4(t) = (d_M-d_m)e(t-d_m) - u_2(t),$$

$$\xi^{\mathrm{T}}(t) = \left[e^{\mathrm{T}}(t) \quad e^{\mathrm{T}}(t-d_m), \quad e^{\mathrm{T}}(t-d(t)) \quad e^{\mathrm{T}}(t-d_M) \quad u_1(t)\cdots u_6(t) \quad g^{\mathrm{T}}(t) \quad \omega^{\mathrm{T}}(t)\right],$$

$$\Omega_1 = Ah_1 + Kh_3 + Ch_{11} + Dh_{12},$$
$$\Omega_2 = h_1 - h_2, \quad \Omega_3 = h_2 - h_4, \quad \Omega_4 = d_m h_1 - h_5,$$
$$\Omega_5 = (d_M - d_m)h_2 - h_6,$$
$$G_1 = P_{11}h_1 + P_{12}h_5 + P_{13}h_6 + P_{14}h_7 + P_{15}h_8,$$
$$G_2 = P_{12}^{T}h_1 + P_{22}h_5 + P_{23}h_6 + P_{24}h_7 + P_{25}h_8,$$
$$G_3 = P_{13}^{\mathrm{T}}h_1 + P_{23}^{\mathrm{T}}h_5 + P_{33}h_6 + P_{34}h_7 + P_{35}h_8,$$
$$G_4 = P_{14}^{T}h_1 + P_{24}^{T}h_5 + P_{34}^{T}h_6 + P_{44}h_7 + P_{45}h_8,$$
$$G_5 = P_{15}^{\mathrm{T}}h_1 + P_{25}^{\mathrm{T}}h_5 + P_{35}^{\mathrm{T}}h_6 + P_{45}^{\mathrm{T}}h_7 + P_{55}h_8.$$

According to Lemma 1, we obtain

$$\zeta_1(t) \le -d_m^2\begin{bmatrix}\varepsilon_1(t)\\ \varepsilon_2(t)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}Z_1 & 0\\ * & Z_1\end{bmatrix}\begin{bmatrix}\varepsilon_1(t)\\ \varepsilon_2(t)\end{bmatrix}, \tag{18}$$

$$\zeta_2(t) \le -\frac{1}{\rho_1}\begin{bmatrix}\varepsilon_3(t)\\ \varepsilon_4(t)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}Z_2 & 0\\ * & Z_2\end{bmatrix}\begin{bmatrix}\varepsilon_3(t)\\ \varepsilon_4(t)\end{bmatrix}, \tag{19}$$

$$\zeta_3(t) \le -\frac{1}{\rho_2}\begin{bmatrix}\varepsilon_5(t)\\ \varepsilon_6(t)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}Z_2 & 0\\ * & Z_2\end{bmatrix}\begin{bmatrix}\varepsilon_5(t)\\ \varepsilon_6(t)\end{bmatrix}, \tag{20}$$

where

$$\rho_1 = \frac{d(t)-d_m}{d_{Mm}}, \quad \rho_2 = \frac{d_M - d(t)}{d_{Mm}},$$

$$\varepsilon_1(t) = e(t) - e(t-d_m),$$
$$\varepsilon_2(t) = \sqrt{3}e(t) + \sqrt{3}e(t-d_m) - \frac{2\sqrt{3}}{d_m}u_1(t),$$
$$\varepsilon_3(t) = e(t-d_m) - e(t-d(t)),$$
$$\varepsilon_4(t) = \sqrt{3}e(t-d_m) + \sqrt{3}e(t-d(t)) - 2\sqrt{3}u_5(t),$$
$$\varepsilon_5(t) = e(t-d(t)) - e(t-d_M),$$
$$\varepsilon_6(t) = \sqrt{3}e(t-d(t)) + \sqrt{3}e(t-d_M) - 2\sqrt{3}u_6(t),$$

inequality (18) can be denoted as

$$\zeta_1(t) \le -\xi^{\mathrm{T}}(t)d_m^2[\Gamma_1^{\mathrm{T}}Z_1\Gamma_1 + \Gamma_2^{\mathrm{T}}Z_1\Gamma_2]\xi(t). \tag{21}$$

It is clear that the real numbers ρ_1 and ρ_2 satisfy $\rho_1 > 0$, $\rho_2 > 0$, and $\rho_1 + \rho_2 = 1$. Then introduce appropriate dimensions matrices S_1 and S_2, such that

$$\begin{bmatrix}Z_2 & S_1\\ * & Z_2\end{bmatrix} \ge 0, \quad \begin{bmatrix}Z_2 & S_2\\ * & Z_2\end{bmatrix} \ge 0. \tag{22}$$

Applying Lemma 2 to inequalities (19) and (20), we have

$$\zeta_2(t) + \zeta_3(t) \le -\left(\frac{1}{\rho_1}\varepsilon_3^{\mathrm{T}}(t)Z_2\varepsilon_3(t) + \frac{1}{\rho_2}\varepsilon_5^{\mathrm{T}}(t)Z_2\varepsilon_5(t)\right) - \left(\frac{1}{\rho_1}\varepsilon_4^{\mathrm{T}}(t)Z_2\varepsilon_4(t) + \frac{1}{\rho_2}\varepsilon_6^{\mathrm{T}}(t)Z_2\varepsilon_6(t)\right)$$

$$\leq -\begin{bmatrix}\varepsilon_3(t)\\ \varepsilon_5(t)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}Z_2 & S_1\\ * & Z_2\end{bmatrix}\begin{bmatrix}\varepsilon_3(t)\\ \varepsilon_5(t)\end{bmatrix}-\begin{bmatrix}\varepsilon_4(t)\\ \varepsilon_6(t)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}Z_2 & S_2\\ * & Z_2\end{bmatrix}\begin{bmatrix}\varepsilon_4(t)\\ \varepsilon_6(t)\end{bmatrix}$$

$$= -\xi^{\mathrm{T}}(t)\Big[\sum_{i=3}^{6}\Gamma_i^{\mathrm{T}}Z_2\Gamma_i+\Gamma_3^{\mathrm{T}}S_1\Gamma_5+\Gamma_5^{\mathrm{T}}S_1^{\mathrm{T}}\Gamma_3+\Gamma_4^{\mathrm{T}}S_2\Gamma_6+\Gamma_6^{\mathrm{T}}S_2^{\mathrm{T}}\Gamma_4\Big]\xi(t). \tag{23}$$

Utilizing Lemma 3, $\zeta_4(t)$ and $\zeta_5(t)$ can be written as

$$\zeta_4(t)\leq -d_m^2\begin{bmatrix}\omega_1(t)\\ \omega_2(t)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}Z_3 & 0\\ * & Z_3\end{bmatrix}\begin{bmatrix}\omega_1(t)\\ \omega_2(t)\end{bmatrix}=-\xi^{\mathrm{T}}(t)d_m^2\sum_{i=7}^{8}\Gamma_i^{\mathrm{T}}Z_3\Gamma_i\xi(t), \tag{24}$$

$$\zeta_5(t)\leq -d_{Mm}^2\begin{bmatrix}\omega_3(t)\\ \omega_4(t)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}Z_4 & 0\\ * & Z_4\end{bmatrix}\begin{bmatrix}\omega_3(t)\\ \omega_4(t)\end{bmatrix}=-\xi^{\mathrm{T}}(t)d_{Mm}^2\sum_{i=9}^{10}\Gamma_i^{\mathrm{T}}Z_4\Gamma_i\xi(t), \tag{25}$$

where

$$\begin{aligned}\omega_1(t)&=d_m e(t)-u_1(t),\\ \omega_2(t)&=\frac{\sqrt{2}d_m}{2}e(t)+\sqrt{2}u_1(t)-\frac{3\sqrt{2}}{d_m}u_3(t),\\ \omega_3(t)&=d_{Mm}e(t-d_m)-u_2(t),\\ \omega_4(t)&=\frac{\sqrt{2}d_{Mm}}{2}e(t-d_m)+\sqrt{2}u_2(t)-\frac{3\sqrt{2}}{d_{Mm}}u_4(t).\end{aligned}$$

From Eq. (5), we have

$$g^{\mathrm{T}}(t)g(t)-e^{\mathrm{T}}(t)L^{\mathrm{T}}Le(t)=\xi^{\mathrm{T}}(t)M_1\xi(t)\leq 0, \tag{26}$$

where $M_1=\mathrm{diag}\{L^{\mathrm{T}}L\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ -I\ 0\}$. Combining expressions (14)–(17), (21), and (24)–(26), we obtain the following equation

$$\dot{V}(t)\leq\xi^{\mathrm{T}}(t)[\Xi+\Delta^{\mathrm{T}}(N_1+N_2)\Delta]\xi(t), \tag{27}$$

where

$$\begin{aligned}\Xi=\ & G_1^{\mathrm{T}}PG_2+G_2^{\mathrm{T}}PG_1+W+M_1\\ & -d_m^2\sum_{i=1}^{2}\Gamma_i^{\mathrm{T}}Z_1\Gamma_i-\sum_{i=3}^{6}\Gamma_i^{\mathrm{T}}Z_2\Gamma_i-\Gamma_3^{\mathrm{T}}S_1\Gamma_5\\ & -\Gamma_5^{\mathrm{T}}S_1^{\mathrm{T}}\Gamma_3-\Gamma_4^{\mathrm{T}}S_2\Gamma_6-\Gamma_6^{\mathrm{T}}S_2^{\mathrm{T}}\Gamma_4\\ & -d_m^2\sum_{i=7}^{8}\Gamma_i^{\mathrm{T}}Z_3\Gamma_i-d_{Mm}^2\sum_{i=9}^{10}\Gamma_i^{\mathrm{T}}Z_4\Gamma_i.\end{aligned} \tag{28}$$

To investigate H_∞ performance, we define $J(e(t),\omega(t))$ as follows:

$$J(e(t),\omega(t))=\int_0^\infty\left[e^{\mathrm{T}}(s)e(s)-\gamma^2\omega^{\mathrm{T}}(s)\omega(s)\right]\mathrm{d}s, \tag{29}$$

where $\gamma>0$ is the disturbance attention rate. With zero initial condition, we can introduce

$$\begin{aligned}J(e(t),\omega(t))&=\int_0^\infty\left[e^{\mathrm{T}}(s)e(s)-\gamma^2\omega^{\mathrm{T}}(s)\omega(s)+\dot{V}(t)\right]\mathrm{d}s-V(t)\Big|_{t\to\infty}\\ &\leq\int_0^\infty\left[e^{\mathrm{T}}(s)e(s)-\gamma^2\omega^{\mathrm{T}}(s)\omega(s)+\dot{V}(t)\right]ds=\int_0^\infty\xi^{\mathrm{T}}(t)[\Phi+\Delta^{\mathrm{T}}(N_1+N_2)\Delta]\xi(t),\end{aligned} \tag{30}$$

where $\Phi=\Xi+M_2$,

$$M_2=\mathrm{diag}\{I\quad 0\quad 0\quad 0\quad 0\quad 0\quad 0\quad 0\quad 0\quad 0\quad 0\quad -\gamma^2 I\}.$$

Multiplying both sides of Φ with

$$\mathrm{diag}\,(\underbrace{P_{11}^{-1},\ldots,P_{11}^{-1}}_{10},I,I)$$

and using Schur complement we can get inequality (6). Multiplying both sides of expression (22) with $\mathrm{diag}\,(P_{11}^{-1},P_{11}^{-1})$ we can get inequality (7). Where

$$\begin{aligned}&\tilde{Z}_v=P_{11}^{-1}Z_vP_{11}^{-1}\quad(v=1,2,3,4),\\ &\tilde{W}_s=P_{11}^{-1}W_sP_{11}^{-1}\quad(s=1,2,3),\\ &\tilde{S}_m=P_{11}^{-1}S_mP_{11}^{-1},\\ &\tilde{M}_m=P_{11}^{-1}M_mP_{11}^{-1},\quad(m=1,2),\\ &\tilde{P}_{ij}=P_{11}^{-1}P_{ij}P_{11}^{-1}\quad(i,j=1,2,\ldots,5),\\ &F=K\bar{P}_{11}^{-1}.\end{aligned}$$

This completes the proof.

We still cannot obtain a feasible solution by LMI tool box according to Theorem 1 because there are nonlinear terms in the form of $\bar{P}_{11}Z_v^{-1}\bar{P}_{11}$ $(v=1,2,3,4)$. The authors in Ref. [29] proposed a cone complementary linearization algorithm (CCLA) to solve this problem.

First, define new variables U_v $(v=1,2,3,4)$, satisfying

$$\bar{P}_{11}\tilde{Z}_v^{-1}\bar{P}_{11}\geq U_v,\quad(v=1,2,3,4), \tag{31}$$

expression (31) can be written as follows:

$$\begin{bmatrix}-\tilde{Z}_v^{-1} & \bar{P}_{11}^{-1}\\ * & -U_v^{-1}\end{bmatrix}\leq 0,\quad(v=1,2,3,4). \tag{32}$$

Set $J_v = \tilde{Z}_v^{-1}$, $P_n = \bar{P}_{11}^{-1}$, $H_v = U_v^{-1}$, the LMI (32) can be replaced by

$$\begin{bmatrix} -J_v & P_n \\ * & -H_v \end{bmatrix} \leq 0, \quad (v = 1,2,3,4). \tag{33}$$

Then, according to CCLA, the following minimization problem can take the place of the original problem of Theorem 1.

Minimize

$$\mathrm{tr}\left(\bar{P}_{11}P_n + \sum_{v=1}^{4}(J_v\tilde{Z}_v + H_vU_v)\right)$$

subject to formulas (6), (7), (33) and

$$\begin{bmatrix} \Phi & d_m^2\Delta^{\mathrm{T}} & d_{Mm}\Delta^{\mathrm{T}} & (d_m^3/2)\Delta^{\mathrm{T}} & (d_{Mm}^3/2)\Delta^{\mathrm{T}} & \bar{P}_{11} & \bar{P}_{11}L^{\mathrm{T}} \\ * & -U_1 & 0 & 0 & 0 & 0 & 0 \\ * & * & -U_2 & 0 & 0 & 0 & 0 \\ * & * & * & -U_3 & 0 & 0 & 0 \\ * & * & * & * & -U_4 & 0 & 0 \\ * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & -I \end{bmatrix} < 0,$$

$$\begin{bmatrix} \bar{P}_{11} & I \\ * & P_n \end{bmatrix} \geq 0, \quad \begin{bmatrix} J_v & I \\ * & \tilde{Z}_v \end{bmatrix} \geq 0, \quad \begin{bmatrix} H_v & I \\ * & U_v \end{bmatrix} \geq 0, \quad (v = 1,2,3,4). \tag{34}$$

Algorithm:

(i) For given scalars $d_m, d_M, d_d, \chi_{\kappa-1}$ $(\kappa = 1,2,3,4)$, find a feasible set $(\bar{P}_{11}^0, P_n^0, J_v^0, \tilde{Z}_v^0, H_v^0, U_v^0, F^0)$.

(ii) Solve minimization problem with

$$(\bar{P}, P_n, J_v, \tilde{Z}_v, H_v, U_v, F).$$

Minimize:

$$\mathrm{tr}\left(\bar{P}_{11}^c P_n + P_n^c\bar{P} + \sum_{v=1}^{4}(J_v^c\tilde{Z}_v + \tilde{Z}_v^c J_v + H_v^c U_v + U_v^c H_v)\right) \tag{35}$$

subject to LMI (34). Set $\bar{P}_{11}^{c+1} = \bar{P}$, $P_n^{c+1} = P_n$, $J_v^{c+1} = J_v$, $\tilde{Z}_v^{c+1} = \tilde{Z}_v$, $H_v^{c+1} = H_v$, $U_v^{c+1} = U_v$, and $F^{c+1} = F$.

(iii) If linear matrix inequalities (6) and (7) are feasible for $K = F\bar{P}_{11}^{-1}$, then drop out. Otherwise, set $c = c+1$ and go to item (ii).

Remark 1 In much of the existing literature,[5–8] synchronization of the coronary artery system can be achieved. However, this literature assumes that patients can take medicines in time and the medicines can be quickly absorbed. In fact, taking medicine and drug absorption must cost some time. In Theorem 1, we regard these lags as input delays and get an effective synchronization strategy.

4. Simulation

In this section, numerical examples are given to demonstrate the effectiveness of the proposed strategy.

Considering coronary artery systems (2) and (3) with the different time-varying delays and different disturbances

$$\omega_1(t) = (0.3\sin(40t), 0.1\sin(30t)),$$

$$\omega_2(t) = (0.09\sin(4t), 0.03\sin(5t)).$$

The common parameters are listed as follows:

$$A = \begin{bmatrix} -0.15 & 1.7 \\ 0.575 & -0.35 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 \\ 0 & -0.5 \end{bmatrix},$$

$$D = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.1 \end{bmatrix}, \quad L = \begin{bmatrix} 0 & 0 \\ 0 & 0.36 \end{bmatrix},$$

initial conditions are $x(0) = (0.5,0)y(0) = (-1,1.5)$. In this simulation, we set $\chi_1, \chi_2, \ldots, \chi_4 = 0$. The performance of our strategy can be displayed in Figs. 3 and 4.

(I) When $d(t) = 0.15 + 0.05\sin(t)$, $\gamma^2 = 0.1$, applying Theorem 1, we can obtain the gain matrices

$$K_1 = \begin{bmatrix} -1.5052 & -2.2067 \\ -2.0589 & -3.0028 \end{bmatrix}.$$

With different disturbances $\omega_1(t)$, $\omega_2(t)$, the time response of healthy coronary artery system (2) (x_1, x_2), the time response of the convulsing coronary artery system (3) $(y_{a1}, y_{a2}, y_{b1}, y_{b2})$, and the error system (e_a, e_b) are illustrated in Figs. 3(a)–3(c) and Figs. 4(a)–4(c), respectively.

(II) When $d(t) = 0.3 + 0.1\sin(t)$, $\gamma^2 = 0.5$, applying Theorem 1, we can obtain the gain matrices

$$K_2 = \begin{bmatrix} -0.9457 & -1.4564 \\ -1.1506 & -1.7866 \end{bmatrix}.$$

With different disturbances $\omega_1(t)$, $\omega_2(t)$, the time response of healthy coronary artery system (2) (x_1, x_2), the time response of the convulsing coronary artery system (3) $(y_{c1}, y_{c2}, y_{d1}, y_{d2})$, and the error system (e_c, e_d) are illustrated in Figs. 3(a)–3(c) and Figs. 4(a)–4(c), respectively.

(III) When

$$d(t) = 0.425 + 0.025\sin(t), \quad \gamma^2 = 0.8,$$

applying Theorem 1, we can obtain the gain matrices

$$K_3 = \begin{bmatrix} -0.8806 & -1.3016 \\ -0.9644 & -1.5001 \end{bmatrix}.$$

With different disturbances $\omega_1(t)$, $\omega_2(t)$, the time response of healthy coronary artery system (2) (x_1, x_2), the time response of the convulsing coronary artery system (3) $(y_{e1}, y_{e2}, y_{f1}, y_{f2})$, and the error system (e_e, e_f) are illustrated in 3(a)–3(c) and Figs. 4(a)–4(c), respectively.

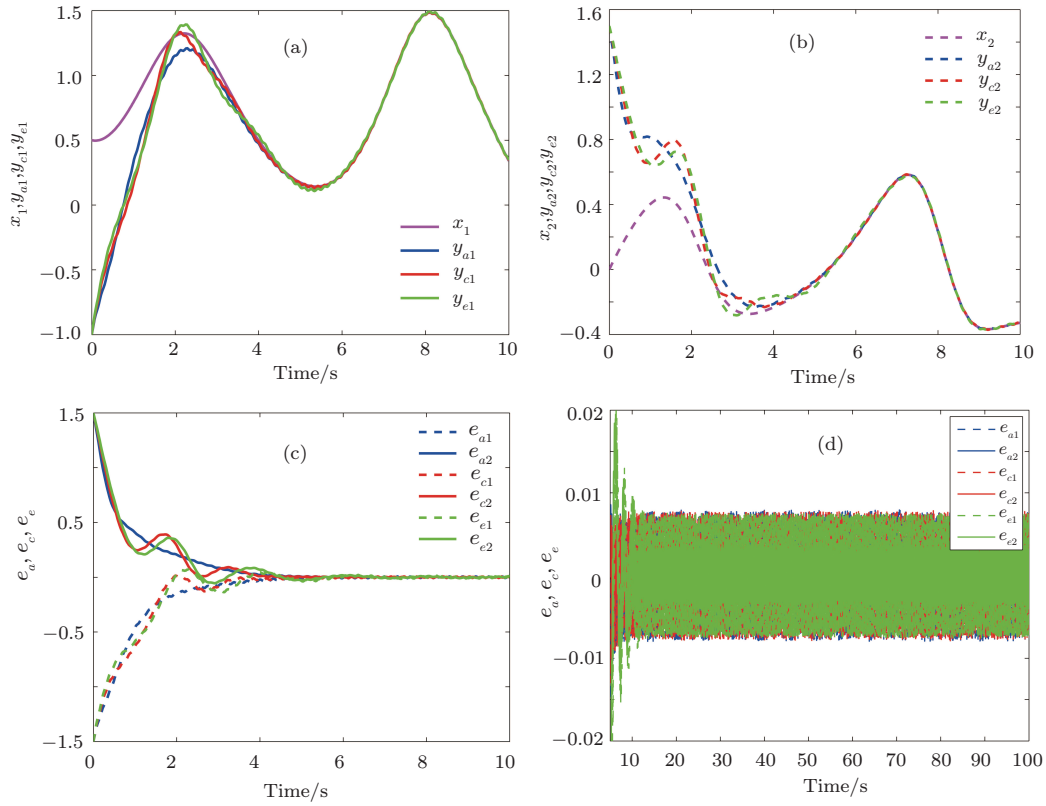


Fig. 3. (color online) The behavior of the coronary artery systems (2) and (3) with different time-varying delays and disturbance $\omega_1(t) = (0.3\sin(40t), 0.1\sin(30t))$: (a) time response of $x_1, y_{a1}, y_{c1}, y_{e1}$, (b) time response of $x_2, y_{a2}, y_{c2}, y_{e2}$, (c) error state of e_a, e_c, e_e when $0 \le t \le 10$, and (d) error state of e_a, e_c, e_e when $5 \le t \le 100$.

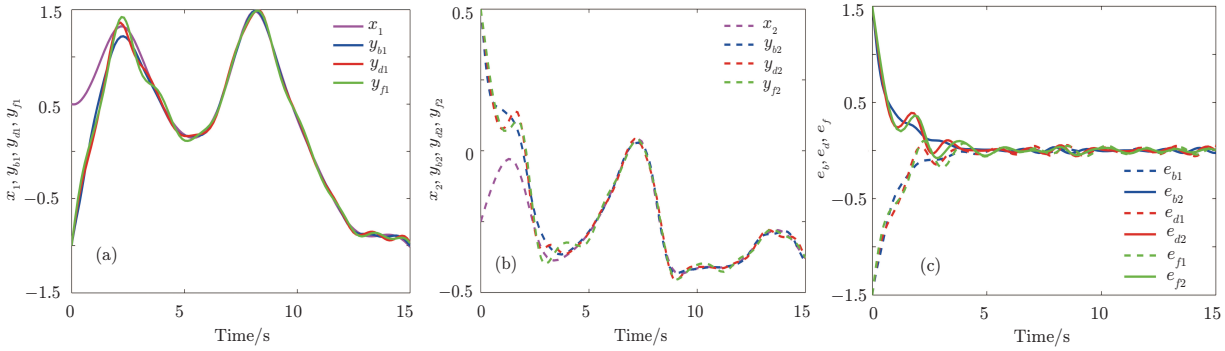


Fig. 4. (color online) The behavior of the coronary artery systems (2) and (3) with different time-varying delays and disturbance $\omega_2(t) = (0.09\sin(4t), 0.03\sin(5t))$: (a) time response of $x_1, y_{b1}, y_{d1}, y_{f1}$, (b) time response of $x_2, y_{b2}, y_{d2}, y_{f2}$, and (c) error state of e_b, e_d, e_f when $0 \le t \le 15$.

The above three cases illustrate the performance of Theorem 1. Figures 3(a) and 3(b) and figures 4(a) and 4(b) show that the coronary artery system (3) can follow the tracks of system (2) under the different time-vary delays and disturbances. From Figs. 3(c) and 3(d) and Fig.4(c), we can clearly see that error systems are converging to a small region around to zero by utilizing our control strategy. That is, our method can make the convulsing coronary artery system synchronize with the healthy one through an input time-varying delay feedback controller.

5. Conclusions

In this paper, considering the disturbance comes from the external environment or emotion and the time delay caused by taking medicine and drug absorption, we research the chaotic synchronization for the coronary artery system. We utilize LKF approach, LMI method, and some advanced inequality technology to reduce the conservatism. From the simula-

tions, we can see our synchronization strategy is valid to make the convulsing coronary artery system synchronize with the healthy coronary artery system under the input delay and external disturbance. As a part of our body, the coronary artery system is an extremely complicated system. Therefore, it is difficult to obtain a precise model of this system. With the development of active shape models[30] and the computer control technology,[31] we can derive accurate models of the coronary artery system in our future work.

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