A novel single-order diffraction grating: Random position rectangle grating*

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Spectral diagnosis of radiation from laser plasma interaction and monochromation of radiation source are hot and important topics recently. Grating is one of the primary optical elements to do this job. Although easy to fabricate, traditional diffraction grating suffers from multi-order diffraction contamination. On the other hand, sinusoidal amplitude grating has the nonharmonic diffraction property, but it is too difficult to fabricate, especially for x-ray application. A novel nonharmonic diffraction grating named random position rectangle grating (RPRG) is proposed in this paper. Theoretical analysis and experiment results show that the RPRG is both higher order diffraction suppressing and not difficult to fabricate. Additionally, it is highly efficient; its first order absolute theoretical diffraction efficiency reaches 4.1%. Our result shows that RPRG is a novel tool for radiation diagnosis and monochromation.

Keywords: diffraction grating, single order, x-ray

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1. Introduction

As a dispersion optical element, diffraction grating is widely used in radiation diagnoses and source monochromation, and it plays an important role in studying inertial confinement fusion (ICF).[1–3] It is well known that ordinary diffraction grating has inevitable multi-order diffraction and this leads to harmonic spectral measurement error.[4] Ideal sinusoidal transmission grating (STG) is an excellent option to solve the multi-order diffraction contamination, which has only 0th and 1st order diffractions, since its transmittance function is an ideal sinusoidal function.[5,6] However, the ideal STG is too difficult to fabricate for its sinusoidal structure, especially for x-ray regime.

Based on the STG theory, Cao *et al*. have proposed several types of binary sinusoidal transmission gratings[7–15] since 2007. All these binary sinusoidal gratings with nonharmonic diffraction are composed of lots of small size units or complicated structures. The transmission functions of these nonharmonic diffraction gratings are all quasi-sine functions to realize the single-order diffraction. The structures of these nonharmonic diffraction gratings are complicated or the number of the structure units is very large. Therefore, they are difficult in fabricating the high line density grating. Here, we report random position rectangle grating (RPRG) which is designed to overcome these disadvantages. The fabrication of RPRG is much easier than the binary sinusoidal gratings mentioned above[7–15] while the property of high-order diffraction restraint is kept. Its 1st-order theoretical absolute diffraction efficiency is as high as 4.1%, far above all binary sinusoidal transmission gratings.

2. Model of RPRG

The basic structure of RPRG is similar to traditional grating, as shown in Fig. 1, the upper panel is the traditional grating and the lower one RPRG, in which black parts are opaque and the white parts are transparent. In both cases, the grating period is d, and the duty ratio of the white bar is 0.5. The difference between RPRG and traditional grating is that the white bars (or the black bars) are shifted forward for a quasi-random distance r_n which falls in the range of $[d/4,\ 3d/4)$ in each grating period. In the visible light or soft x-ray regime grating, photoetching and electron-beam lithography are matured technologies to fabricate the gratings.

According to the Fraunhofer diffraction theory, the far-field diffraction pattern is the Fourier transform of the transmission function. Hence, the Fourier series coefficients of the transmission function can give all the diffraction order efficiencies.

Along the grating period direction, the nth grating bar coordinate can be written as

$$x_n = r_n + (n-1)d, \tag{1}$$

where r_n is a random number, r_n satisfies the uniform random distribution and its probability density function is

$$P(r_n) = \frac{2}{d}, \quad r_n \in \left[\frac{d}{4}, \frac{3d}{4}\right). \tag{2}$$

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Finally, the transmission function of the RPRG along the grating period direction is

$$t(x) = \sum_{n=1}^{N} \mathrm{rect}\left(\frac{x - x_n}{d/2}\right), \tag{3}$$

where N is the number of illuminated grating periods.

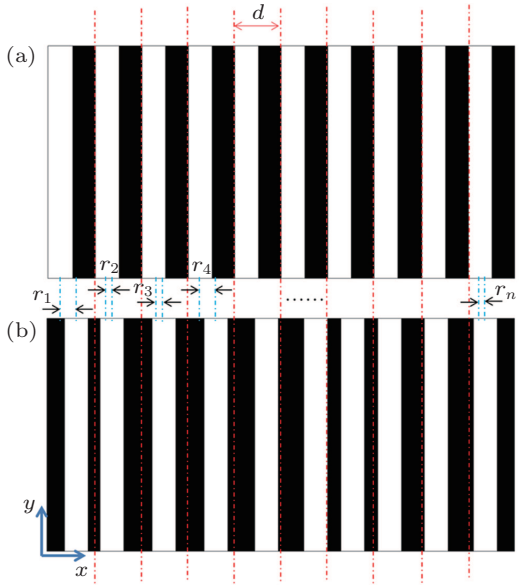


Fig. 1. (color online) Structures of traditional grating (a) and RPRG (b).

The Fourier series coefficients of the transmission function $t(x)$ are

$$\begin{aligned} F_m &= \frac{1}{d}\int_{-d/2}^{d/2} t(x)\,\mathrm{e}^{\mathrm{j}m\Omega x}\,\mathrm{d}x \\ &= \frac{1}{d}\sum_{n=1}^{N}\int_{-d/2}^{d/2} \mathrm{rect}\left(\frac{x-x_n}{d/2}\right)\mathrm{e}^{-\mathrm{j}m\Omega x}\,\mathrm{d}x \\ &= \frac{1}{2}\mathrm{sinc}\left(\frac{m\Omega d}{4}\right)\sum_{n=1}^{N}\mathrm{e}^{-\mathrm{j}m\Omega x} \\ &= \frac{1}{2}\mathrm{sinc}\left(\frac{m\Omega d}{4}\right)\sum_{n=1}^{N}\mathrm{e}^{-\mathrm{j}m\Omega[r_n+(n-1)d]}, \end{aligned} \tag{4}$$

where $\Omega = 2\pi/d$ and N is a large number (such as $N = 1000$), then F_m is

$$\begin{aligned} F_m &= \frac{1}{2}\mathrm{sinc}\left(\frac{m\Omega d}{4}\right)\sum_{n=1}^{N}\mathrm{e}^{-\mathrm{j}m\Omega r_n x}\mathrm{e}^{-\mathrm{j}m2\pi(n-1)d/d} \\ &= \frac{1}{2}\mathrm{sinc}\left(\frac{m\Omega d}{4}\right)\sum_{n=1}^{N}\mathrm{e}^{-\mathrm{j}m\Omega r} \\ &= \frac{1}{2}\mathrm{sinc}\left(\frac{\pi m}{2}\right)\int_{d/4}^{3d/4}\frac{2}{d}\mathrm{e}^{-\mathrm{j}m2\pi x/d}\,\mathrm{d}x \\ &= \frac{1}{2}\mathrm{sinc}\left(\frac{\pi m}{2}\right)\frac{2}{d}\frac{d}{\mathrm{j}m2\pi}(\mathrm{e}^{-\mathrm{j}\pi m/2} - \mathrm{e}^{-\mathrm{j}3\pi m/2}) \\ &= \frac{1}{2}\mathrm{sinc}\left(\frac{\pi m}{2}\right)^2. \end{aligned} \tag{5}$$

The diffraction efficiency of mth diffraction order is $\eta_m = |F_m|^2$, and

$$\eta_m = \begin{cases} \dfrac{1}{4}, & m = 0, \\ 0, & m = 2,4,6,\ldots, \\ \dfrac{4}{m^4\pi^4}, & m = 1,3,5,\ldots \end{cases} \tag{6}$$

The theoretical analysis shows that the diffraction efficiencies of the 0th, 1st, 3rd, 5th diffraction orders of RPRG are 25%, 4.1064%, 0.0507%, and 0.0066%, respectively, and all even-order diffractions vanish. The diffraction efficiency ratios of the 1st to the 3rd and 5th diffraction are 81 and 625, which are much higher than the ordinary grating diffraction efficiency ratios which are 9.0 and 25.0. It shows that the RPRG can restrain the high-order diffraction.

Then, we change the duty ratio of RPRG to optimize the diffraction efficiency of the 1st order. The diffraction property can be analyzed through the process and the r_n and the x_n are the same as above. The transmission function of RPRG with duty ratio t in range (0, 1] is

$$t(x) = \sum_{n=1}^{N} \mathrm{rect}\left(\frac{x - x_n}{dt}\right). \tag{7}$$

The Fourier series coefficients of the transmission function $t(x)$, or in other words, F_m (for large N) are

$$F_m = \begin{cases} 0, & m = 2,4,6,\ldots, \\ t\,\mathrm{sinc}(\pi m t)\,\mathrm{sinc}(\pi m/2), & m = \text{others}. \end{cases} \tag{8}$$

Then, the mth-order diffraction efficiency is $\eta_m = |F_m|^2$.

Duty ratios can be chosen as $t = 1/2$, 1/3, 1/4, and so on. The diffraction efficiencies of the 0th, 1st, 3rd, 5th diffraction orders for t=1/2, 1/3, 1/4, respectively, are listed in Table 1.

Table 1. The efficiencies of the 0th, 1st, 3rd, 5th diffraction orders for $t = 1/2$, 1/3, 1/4.

m	t			
	1/2	1/3	1/4	1/5
0	25%	11.1111%	6.25%	4%
1	4.1%	3.0798 %	2.0532%	1.4187%
2	0	0	0	0
3	0.0507%	0	0.0253%	0.0459%
4	0	0	0	0
5	0.0066%	0.0049%	0.0033%	0

According to Table 1, to give the optimal 1st-order diffraction efficiency and restrain other diffraction orders, the optimal RPRG is $t = 1/2$. However, for restraining the other high diffraction orders and for easier fabrication of the RPRG, the optimal RPRG is $t = 1/3$.

3. Simulation and experiment

In this paper, for the maximum diffraction efficiency, the RPRG with duty ratio $t = 1/2$ was verified by simulation and experiment.

From the theoretical analysis above, as long as the number of illuminated grating periods N is a large number, our RPRG model should agree well with the theoretical result that was obtained at large N limit. To estimate the effect of N in the performance of RPRG, we carried out a simulation which includes two cases, one case has $N = 150$ and the other $N = 1000$, the former case has an N that is not really large compared to the latter case. The grating period $d = 10$ μm and the duty ratio of 0.5 are the same in both cases. Our simulation model of RPRG is shown in Fig. 2. Using the fast Fourier transform algorithm, the far-field diffraction intensity distributions along the diffracted angle of the two cases are shown in Fig. 3. The background noise of the $N = 150$ case is much larger than that of the $N = 1000$ case, the 3rd diffraction of the N=150 case is almost overwhelmed by the background noise. In comparison, the 3rd diffraction of the $N = 1000$ case is obvious and the relative efficiencies (normalized by the 0th order) of the 1st and the 3rd order diffraction are about 16.7% and 0.1644%, respectively, which agree well with the theoretical prediction.

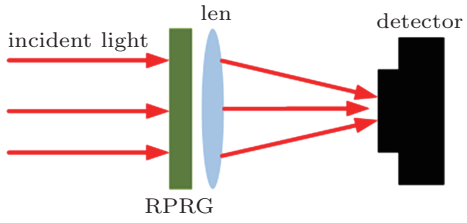


Fig. 2. (color online) Simulation model of RPRG.

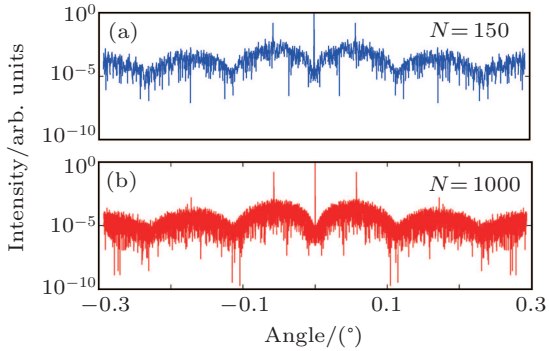


Fig. 3. (color online) Diffraction result comparison. Here, $N = 150$ (a) and $N = 1000$ (b).

Considering balancing the visible RPRG manufacture and the RPRG design, the tolerance interval of the adjacent grating ridges with 500 nm and the variance of grating period with 500 nm have been simulated. The number of the simulation RPRG periods is $N = 1000$. The simulation results (as shown in Fig. 4) indicate that all this variance does not influence the performance of the RPRG.

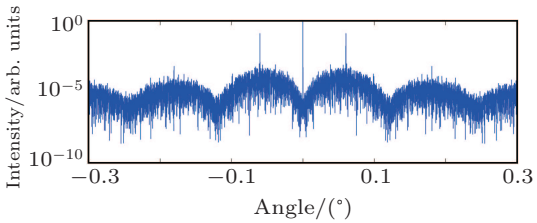


Fig. 4. (color online) Diffraction result with the tolerance interval 500 nm and the grating period variance 500 nm, and $N = 1000$.

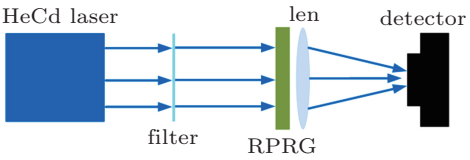


Fig. 5. (color online) Visible light experimental layout.



Fig. 6. (color online) RPRG cross section.

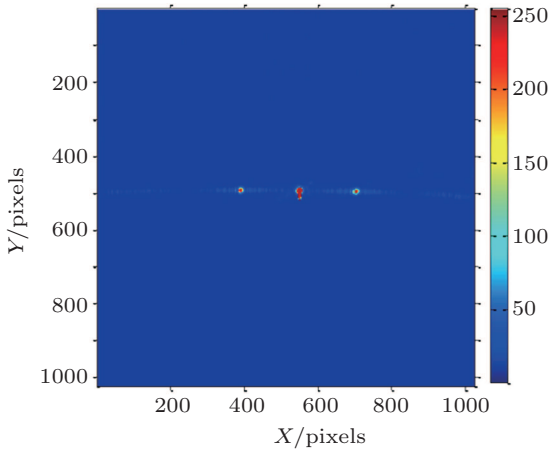


Fig. 7. (color online) Diffraction pattern on CCD.

To verify our theoretical analysis and simulation results, a visible light RPRG experiment was performed. The adopted RPRG was fabricated through lithograph technology in the Institute of Microelectronics, Chinese Academy of Sciences. The period of tested RPRG d is 10 μm and its duty ratio is 0.5. The experimental layout is shown schematically in Fig. 5 and the RPRG is composed of quartz substrate base and metal chrome shown in Fig. 6. Monochromatic helium cadmium (HeCd) laser with which wavelength 325 nm or 442nm can be chosen was adopted and we chose the 432-nm wavelength

in this experiment. The diameter of the laser beam is about 1.5 mm. Consequently, the number of illuminated RPRG periods is 150. We used a visible light CCD which has 1024×1024 pixels as the detector, the pixel size is 13.5 μm. Our experimental result is shown in Fig. 7. A comparison of the experimental 1D intensity distribution and the corresponding simulation result are shown Fig. 8. As shown in Fig. 7, there is no obvious 3rd diffraction pattern and there are lots of side lobes which agree with our theoretical analysis. Furthermore, the relative diffraction efficiency of 1st to 0th is about 37% which is much larger than the theoretical prediction. The reason is that the detected 0th diffraction intensity reached the CCD saturation level.

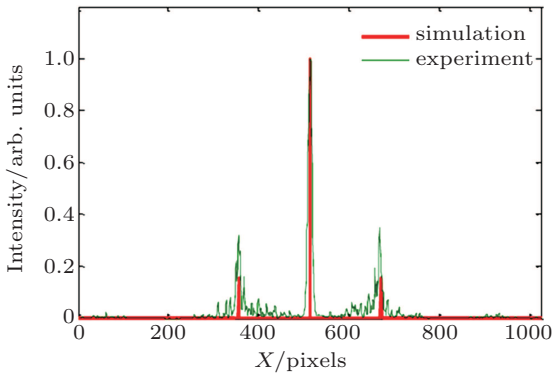


Fig. 8. (color online) The 1D intensity distribution comparison of the experiment and simulation.

4. Conclusion and perspectives

In this paper, a novel nonharmonic diffraction grating named RPRG is proposed. Through theoretical analysis, numerical simulation and the visible wavelength experiment, we have demonstrated that the RPRG with duty ratio 1/2 can restrain high-order diffraction and keep a high diffraction efficiency of the ±1 order diffraction of 4.1%. The designed grating overcomes the pre-proposed single-order grating fabrication difficulty. This novel grating is a potential key optical element for spectral measurement and x-ray, visible light monochromatization.

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References

[1] Lei Y H, L X, Guo J C, Zhao Z G and Niu H B 2011 *Chin. Phys. B* **20** 042901
[2] Wang J, Zhang H and Cheng X L 2013 *Chin. Phys. B* **22** 085201
[3] Hogan W J, Bangerter R and Kulcinski G L 1992 *Phys. Today* **45** 42
[4] Born M and Wolf E 1999 *Principles of Optics* (London: Cambridge University Press) pp. 40–50
[5] Wang C, Kuang L, Wang Z, Liu S, Ding Y, Cao L, Förster E, Wang D, Xie C and Ye T 2007 *Rev. Sci. Instrum.* **78** 053503
[6] Cao L F, Förster E F A, Wang C K, Kuang L Y, Liu S Y and Ding Y K 2007 *Appl. Phys. Lett.* **90** 053501
[7] Gao Y, Cao L, Zhou W, Wei L, Zang H, Gu Y and Zhang B 2011 *High Power Laser and Particle Beams* **23** 0
[8] Kuang L, Wang C, Wang Z, Cao L, Zhu X, Xie C, Liu S and Ding Y 2010 *Rev. Sci. Instrum.* **81** 073508
[9] Wei L, Cao L, Fan W, Zang H, Gao Y, Zhu X L, Xie C, Gu Y, Zhang B and Wang X 2011 *High Power Laser and Particle Beams* **23** 387
[10] Weng Y C, Kuang L Y, Gao N, Cao L F, Zhu X L, Wang X H and Xie C Q 2012 *Acta Phys. Sin.* **15** 154203 (in Chinese)
[11] Zang H, Cao L, Wang C, Jiang G, Wei L, Fan W, Zhou W and Gu Y 2011 *Acta Phys. Sin.* **60** 34215 (in Chinese)
[12] Zang H P, Wang C K, Gao Y L, Zhou W M, Kuang L Y, Wei L, Fan W, Zhang W H, Zhao Z Q, Cao L F, Gu Y Q, Zhang B H, Jiang G, Zhu X L, Xie C Q, Zhao Y D and Cui M Q 2012 *Appl. Phys. Lett.* **100** 111904
[13] Zhang Q Q, Wei L, Yang Z H, Qian F, Fan Q P, Zhang B, Gu Y Q and Cao L F 2014 *Chin. Phys. B* **23** 044215
[14] Xie C, Zhu X, Shi L and Liu M 2010 *Opt. Lett.* **35** 1765
[15] Gao Y, Zhou W, Wei L, Cao L, Zhu X, Zhao Z, Gu Y, Zhang B and Xie C 2012 *Laser and Particle Beams* **30** 313