Multi-hop teleportation based on W state and EPR pairs*

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Multi-hop teleportation has significant value due to long-distance delivery of quantum information. Many studies about multi-hop teleportation are based on Bell pairs, partially entangled pairs or W state. The possibility of multi-hop teleportation constituted by partially entangled pairs relates to the number of nodes. The possibility of multi-hop teleportation constituted by double W states is $\left(\frac{4}{9}\right)^n$ after n-hop teleportation. In this paper, a multi-hop teleportation scheme based on W state and EPR pairs is presented and proved. The successful possibility of quantum information transmitted hop by hop through intermediate nodes is deduced. The possibility of successful transmission is $\left(\frac{2}{3}\right)^n$ after n-hop teleportation.

Keywords: multi-hop teleportation, W state, Bell pairs

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1. Introduction

Quantum teleportation plays a significant role in the quantum communication and has drawn more and more attention in recent decades due to its potential applications.[1–6] Quantum teleportation, first proposed by Bennett *et al.* in 1993,[7] could transmit quantum information from a source particle to a destination particle by quantum entanglement. Shared entanglement can be achieved by generating an EPR pair and distributing this pair to the source and destination in advance.[8] The first experiment was realized in 1997, and a quantum teleportation distance of 143 kilometers has been achieved by Ma *et al.*[9]

Long-distance delivery of quantum information is an essential challenge in the future quantum network. Considering the quantum entanglement state can be generated and distributed between smaller segments, quantum repeaters are used as a router to share and distribute quantum entanglement.[10] In recent decades, lots of innovative approaches have been proposed in the quantum teleportation domain.[11–15] In recent years, some significant research works have been done in the domain of multi-hop quantum teleportation. Wang *et al.*[16] analyzed the multi-hop teleportation based on arbitrary Bell pairs using EPR-pair bridging. The possibility of multi-hop teleportation based on partially entangled state $|\varphi\rangle = \frac{1}{\sqrt{1+n^2}}(|00\rangle + n|11\rangle)$ has a relationship with the number of the nodes N. If N is odd, the possibility of successful transmission $P = \left(\frac{\sqrt{2}n}{1+n^2}\right)^K$, where K is the hops of the transmission.[17] If N is even, the destination node will add another teleportation to itself.[18] Dur *et al.*[19] studied the use of N pairs Werner states for quantum communication over long distances, but they focused on purification. Su *et al.*[20] proposed a quantum teleportation scheme using double W states and its successful possibility is $\left(\frac{4}{9}\right)$ after one hop teleportation. Shi *et al.*[21] proposed a quantum wireless multi-hop network and the information is transmitted from the source to the destination using Werner states as the quantum channel directly.

In this paper, we proposed a multi-hop quantum transmission scheme using the combination of W state and EPR pairs and proved its feasibility. The quantum information is transmitted hop by hop via intermediate nodes through the quantum channel established by W state and EPR pairs. We assume that the sender intends to teleport an unknown two-level quantum state: $|\psi\rangle = \alpha|00\rangle + \beta|11\rangle$ to the receiver. Firstly, we deduce the possibility of successful transmission after one hop teleportation. The recovered qubits are used as the teleported states for the next-hop teleportation. Based on the results of one-hop and two-hop teleportation, we make an assumption about multi-hop possibility and use mathematical induction to prove this assumption. Finally, we obtain a higher possibility of $\left(\frac{2}{3}\right)^n$ after n-hop teleportation compared to the schemes only using double W states.

The rest of the paper is organized as follows. In Sections 2 and 3, the details of the multi-hop teleportation are discussed and proved, including one-hop, two-hop, and multi-hop teleportation. The conclusions are presented in Section 4. Operations of the Pauli operator used in this paper are introduced in Appendix A.

2. One-hop teleportation

The fidelity of the quantum entanglement decreases as the distance increases in the long-distance quantum transmission.

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It is practical to transmit quantum states from source to destination in a multi-hop way with more than one intermediate state. In the following two sections, a multi-hop transmission method based on W state and EPR pairs is presented and its validity is verified.

There are two nodes in Fig. 1. Alice is the source node and Bob is the destination node. Alice holds particles 1, 2, 3, and 6. Bob holds particles 4, 5, 7, and 8. Particle 8 is an auxiliary particle. The quantum state to be transmitted is $|\psi_{12}\rangle = \alpha|00\rangle_{12} + \beta|11\rangle_{12}$, where α and β satisfy $\alpha^2 + \beta^2 = 1$. Particles 6 and 7 share an EPR pair represented as $|\psi\rangle_{67} = \frac{1}{\sqrt{2}}(|00\rangle_{67} + |11\rangle_{67})$. Particles 3–5 are in a W state represented as: $|\psi\rangle_{345} = \frac{1}{\sqrt{3}}(|001\rangle_{345} + |010\rangle_{345} + |100\rangle_{345})$. The W state and EPR pair constitute the quantum channel.

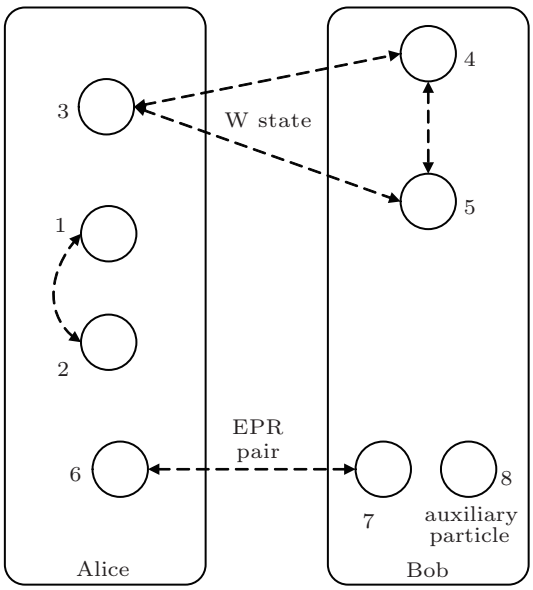


Fig. 1. One-hop teleportation.

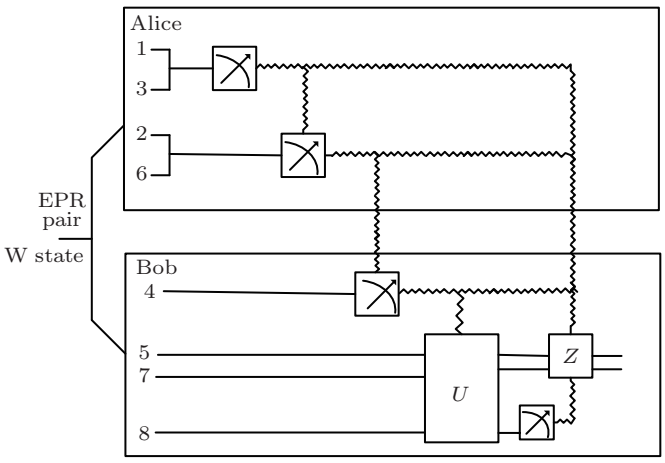


Fig. 2. One-hop quantum circuit.

The process of quantum state transmission is described as follows.

(i) Bell measurement on particles 1 and 3.

(ii) Bell measurement on particles 2 and 6.

(iii) Do the projection measurement on particle 4 according to the measurement results of steps (i) and (ii).

(iv) Perform a Pauli operation on particles 5 and 7 according to the result of step (iii), then introduce an auxiliary particle 8 and recover the transmitted state according to the measurement result of the auxiliary particle 8.

According to this process, the entire quantum circuit is given in Fig. 2. The solid line represents quantum data and the wavy line represents classical measurement results.

The entire system can be represented as

$$\begin{aligned} &|\psi\rangle_{12} \otimes |W\rangle_{345} \otimes |\psi\rangle_{67} \\ &\quad = (\alpha|00\rangle_{12} + \beta|11\rangle_{12}) \otimes \frac{1}{\sqrt{3}}(|001\rangle_{345} \\ &\qquad + |010\rangle_{345} + |100\rangle_{345}) \otimes |\psi\rangle_{67}. \end{aligned} \tag{1}$$

Considering the following equation:

$$|\phi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle), \quad |\psi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle),$$

expression (1) can be rewritten as

$$\begin{aligned} &|\psi\rangle_{12} \otimes |W\rangle_{345} \otimes |\psi\rangle_{67} \\ &\quad = \frac{1}{\sqrt{6}}\big(|\phi_{13}^{+}\rangle(\alpha|001\rangle_{245} + \alpha|010\rangle_{245} + \beta|100\rangle_{245}) \\ &\qquad + |\phi_{13}^{-}\rangle(\alpha|001\rangle_{245} + \alpha|010\rangle_{245} - \beta|100\rangle_{245}) \\ &\qquad + |\psi_{13}^{+}\rangle(\alpha|000\rangle_{245} + \alpha|101\rangle_{245} + \beta|110\rangle_{245}) \\ &\qquad + |\psi_{13}^{-}\rangle(\alpha|000\rangle_{245} - \alpha|101\rangle_{245} - \beta|110\rangle_{245})\big) \\ &\qquad \otimes |\psi\rangle_{67}. \end{aligned} \tag{2}$$

After the Bell measurement on particles 1 and 3, the possibility of getting $|\phi_{13}^{+}\rangle$ or $|\phi_{13}^{-}\rangle$ is

$$\frac{|\alpha|^2 + |\alpha|^2 + |\beta|^2}{2\left[2|\alpha|^2 + |\beta|^2 + |\alpha|^2 + 2|\beta|^2\right]} = \frac{2 - |\beta|^2}{6}. \tag{3}$$

The possibility of getting $|\psi_{13}^{+}\rangle$ or $|\psi_{13}^{-}\rangle$ is

$$\frac{|\alpha|^2 + |\beta|^2 + |\beta|^2}{2\left[2|\alpha|^2 + |\beta|^2 + |\alpha|^2 + 2|\beta|^2\right]} = \frac{1 + |\beta|^2}{6}. \tag{4}$$

Assuming the measurement result is $|\phi_{13}^{+}\rangle$, ignoring the coefficient, then the system state becomes

$$\begin{aligned} &(\alpha|001\rangle_{245} + \alpha|010\rangle_{245} + \beta|100\rangle_{245}) \otimes (|00\rangle_{67} + |11\rangle_{67}) \\ &= \frac{1}{\sqrt{2}}\big(|\phi_{26}^{+}\rangle(\alpha|010\rangle_{457} + \alpha|100\rangle_{457} + \beta|001\rangle_{457}) \\ &\quad + |\phi_{26}^{-}\rangle(\alpha|010\rangle_{457} + \alpha|100\rangle_{457} - \beta|001\rangle_{457}) \\ &\quad + |\psi_{26}^{+}\rangle(\alpha|001\rangle_{457} + \alpha|101\rangle_{457} + \beta|000\rangle_{457}) \\ &\quad + |\psi_{26}^{-}\rangle(\alpha|001\rangle_{457} - \alpha|101\rangle_{457} - \beta|000\rangle_{457})\big). \end{aligned} \tag{5}$$

The possibility of getting $|\phi_{26}^{+}\rangle$, $|\phi_{26}^{-}\rangle$, $|\psi_{26}^{+}\rangle$, and $|\psi_{26}^{-}\rangle$ is

$$\frac{|\alpha|^2+|\alpha|^2+|\beta|^2}{2\left[2|\alpha|^2+|\beta|^2+2|\alpha|^2+|\beta|^2\right]}=\frac{1}{4}. \tag{6}$$

Assuming the measurement result is $|\phi_{26}^{+}\rangle$, then the system state is

$$\alpha|010\rangle_{457}+\alpha|100\rangle_{457}+\beta|001\rangle_{457}. \tag{7}$$

At last, after the measurement, if the $|0\rangle_4$ is obtained with the possibility of $\frac{1}{2-|\beta|^2}$, the system state would turn into: $\alpha|10\rangle_{57}+\beta|01\rangle_{57}$ and the possibility is

$$P=\frac{2-|\beta|^2}{6}\times\frac{1}{4}\times\frac{1}{2-|\beta|^2}=\frac{1}{24}. \tag{8}$$

Every destination state can be turned into

$$|\psi\rangle_{12}=\alpha|00\rangle_{12}+\beta|11\rangle_{12}$$

by introducing the auxiliary particle and performing a Pauli operator, so the entire possibility of the one-hop teleportation is $P_{\text{one-hop}}=\frac{2}{3}$. The related Pauli operators are given in Appendix A. With the same method, we obtain Table 1.

Table 1. Results of one-hop teleportation for $|\psi\rangle_{12}\otimes|W\rangle_{345}\otimes|\psi\rangle_{67}$.

Bell measurement result of particles 1,3 and 2,6	Result of particle 4	Destination state	Possibility
$\|\phi\rangle_{13}^{+},\|\phi\rangle_{26}^{+}or\|\phi\rangle_{13}^{-},\|\phi\rangle_{26}^{-}$	$\|0\rangle_4$	$\alpha\|10\rangle_{57}+\beta\|01\rangle_{57}$	$\frac{1}{12}$
$\|\phi\rangle_{13}^{+},\|\phi\rangle_{26}^{-}or\|\phi\rangle_{13}^{-},\|\phi\rangle_{26}^{+}$	$\|0\rangle_4$	$\alpha\|10\rangle_{57}-\beta\|01\rangle_{57}$	$\frac{1}{12}$
$\|\phi\rangle_{13}^{+},\|\psi\rangle_{26}^{+}or\|\phi\rangle_{13}^{-},\|\psi\rangle_{26}^{-}$	$\|0\rangle_4$	$\alpha\|10\rangle_{57}+\beta\|01\rangle_{57}$	$\frac{1}{12}$
$\|\phi\rangle_{13}^{+},\|\psi\rangle_{26}^{-}or\|\phi\rangle_{13}^{-},\|\psi\rangle_{26}^{+}$	$\|0\rangle_4$	$\alpha\|10\rangle_{57}+\beta\|01\rangle_{57}$	$\frac{1}{12}$
$\|\psi\rangle_{13}^{+},\|\phi\rangle_{26}^{+}or\|\psi\rangle_{13}^{-},\|\phi\rangle_{26}^{-}$	$\|0\rangle_4$	$\alpha\|00\rangle_{57}+\beta\|11\rangle_{57}$	$\frac{1}{12}$
$\|\psi\rangle_{13}^{+},\|\phi\rangle_{26}^{-}or\|\psi\rangle_{13}^{-},\|\phi\rangle_{26}^{+}$	$\|0\rangle_4$	$\alpha\|00\rangle_{57}-\beta\|11\rangle_{57}$	$\frac{1}{12}$
$\|\psi\rangle_{13}^{+},\|\psi\rangle_{26}^{+}or\|\psi\rangle_{13}^{-},\|\psi\rangle_{26}^{-}$	$\|0\rangle_4$	$\alpha\|01\rangle_{57}+\beta\|10\rangle_{57}$	$\frac{1}{12}$
$\|\psi\rangle_{13}^{+},\|\psi\rangle_{26}^{-}or\|\psi\rangle_{13}^{-},\|\psi\rangle_{26}^{+}$	$\|0\rangle_4$	$\alpha\|01\rangle_{57}-\beta\|10\rangle_{57}$	$\frac{1}{12}$

3. Multi-hop teleportation

3.1. two-hop case

As shown in Fig. 3, Alice is the source node, Cindy is the destination node, and Bob is the intermediate node. The states of particles 1 and 2 will be transmitted through the quantum channel constituted by W state and EPR pairs.

From the deduction of one-hop teleportation, after the first teleportation, the four possible states of particles 5 and 7 are: $\alpha|10\rangle_{57}\pm\beta|01\rangle_{57}$, $\alpha|01\rangle_{57}\pm\beta|00\rangle_{57}$, $\alpha|00\rangle_{57}\pm\beta|11\rangle_{57}$, and $\alpha|01\rangle_{57}\pm\beta|10\rangle_{57}$. Assuming that $\alpha|00\rangle_{57}+\beta|11\rangle_{57}$ (same as $\alpha|00\rangle_{57}-\beta|11\rangle_{57}$) is achieved at node Bob after one-hop teleportation, the system state can be represented as

$$\begin{aligned}&|\psi\rangle_{57}\otimes|W\rangle_{9,10,11}\otimes|\psi\rangle_{12,13}\\&=(\alpha|10\rangle_{57}+\beta|01\rangle_{57})\otimes\frac{1}{\sqrt{3}}(|001\rangle_{9,10,11}\\&\quad+|010\rangle_{9,10,11}+|100\rangle_{9,10,11})\otimes|\psi\rangle_{12,13}.\end{aligned} \tag{9}$$

Considering the following equation:

$$|\phi^{\pm}\rangle=\frac{1}{\sqrt{2}}(|00\rangle\pm|11\rangle),\quad|\psi^{\pm}\rangle=\frac{1}{\sqrt{2}}(|01\rangle\pm|10\rangle),$$

expression (9) can be rewritten as

$$\begin{aligned}&|\psi\rangle_{57}\otimes|W\rangle_{9,10,11}\otimes|\psi\rangle_{12,13}\\&=\frac{1}{\sqrt{6}}\big(|\phi_{59}^{+}\rangle(\alpha|000\rangle_{7,10,11}+\beta|101\rangle_{7,10,11}\\&\quad+\beta|110\rangle_{7,10,11})+|\phi_{59}^{-}\rangle(-\alpha|000\rangle_{7,10,11}\\&\quad+\beta|101\rangle_{7,10,11}+\beta|110\rangle_{7,10,11})\\&\quad+|\psi_{59}^{+}\rangle(\alpha|001\rangle_{7,10,11}+\alpha|010\rangle_{7,10,11}\\&\quad+\beta|100\rangle_{7,10,11})+|\psi_{59}^{-}\rangle(-\alpha|001\rangle_{7,10,11}\\&\quad-\alpha|010\rangle_{7,10,11}+\beta|100\rangle_{7,10,11})\big)\\&\quad\otimes|\psi\rangle_{12,13}.\end{aligned} \tag{10}$$

The expression (10) is similar to expression (2). By carrying out the same calculation as expression (2), we obtain the same four-type results with the same possibility. Our calculation also indicates that $\alpha|01\rangle_{57}\pm\beta|00\rangle_{57}$ and $\alpha|01\rangle_{57}\pm\beta|10\rangle_{57}$ cannot reach the same four-type results as in the one-hop case. When we get $\alpha|01\rangle_{57}\pm\beta|00\rangle_{57}$ and $\alpha|01\rangle_{57}\pm\beta|10\rangle_{57}$ in the intermediate nodes, we introduce an auxiliary particle and perform the corresponding Pauli operator to get the state $\alpha|00\rangle_{57}+\beta|11\rangle_{57}$. Therefore, after two-hop teleportation we can finally obtain the four-type results with the same possibility. The possibility of each type is $P_{\text{each-type}}=\frac{1}{6}\times\frac{1}{6}\times4=\frac{1}{9}$. The possibility of the entire two-hop teleportation system is $P_{\text{two-hop}}=P_{\text{each-type}}\times4=\frac{1}{9}\times4=\frac{2^2}{3^2}$. The flow charts are shown in Table 2.

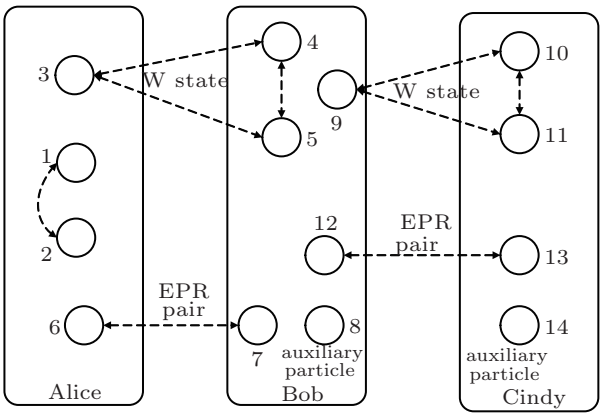


Fig. 3. Two-hop teleportation.

Table 2. Two-hop teleportation flow charts.

Alice	Bob	Cindy
$\alpha\|00\rangle+\beta\|11\rangle$	$\alpha\|10\rangle\pm\beta\|01\rangle$	$\alpha\|10\rangle\pm\beta\|01\rangle$
		$\alpha\|01\rangle\pm\beta\|00\rangle$
		$\alpha\|00\rangle\pm\beta\|11\rangle$
		$\alpha\|01\rangle\pm\beta\|10\rangle$
	$\alpha\|01\rangle\pm\beta\|00\rangle$ $(\alpha\|00\rangle+\beta\|11\rangle)$	$\alpha\|10\rangle\pm\beta\|01\rangle$
		$\alpha\|01\rangle\pm\beta\|00\rangle$
		$\alpha\|00\rangle\pm\beta\|11\rangle$
		$\alpha\|01\rangle\pm\beta\|10\rangle$
	$\alpha\|00\rangle\pm\beta\|11\rangle$	$\alpha\|10\rangle\pm\beta\|01\rangle$
		$\alpha\|01\rangle\pm\beta\|00\rangle$
		$\alpha\|00\rangle\pm\beta\|11\rangle$
		$\alpha\|01\rangle\pm\beta\|10\rangle$
	$\alpha\|01\rangle\pm\beta\|10\rangle$ $(\alpha\|00\rangle+\beta\|11\rangle)$	$\alpha\|10\rangle\pm\beta\|01\rangle$
		$\alpha\|01\rangle\pm\beta\|00\rangle$
		$\alpha\|00\rangle\pm\beta\|11\rangle$
		$\alpha\|01\rangle\pm\beta\|10\rangle$

3.2. n-hop case

According to the results of the 1-hop and 2-hop cases, we consider that the system gets the four-type results: $\alpha|10\rangle\pm\beta|01\rangle$, $\alpha|01\rangle\pm\beta|00\rangle$, $\alpha|00\rangle\pm\beta|11\rangle$, and $\alpha|01\rangle\pm\beta|10\rangle$ after $(n-1)$-hop teleportation and we assume that the possibility of each type is $p_1=\frac{2^{n-3}}{3^{n-1}}$, so the entire possibility of $(n-1)$-hop teleportation is

$$p_{(n-1)\text{hop}}=\frac{2^{n-3}}{3^{n-1}}\times 4=\left(\frac{2}{3}\right)^{n-1}.$$

$\alpha|01\rangle\pm\beta|00\rangle$ and $\alpha|01\rangle\pm\beta|10\rangle$ can be converted to $\alpha|00\rangle+\beta|11\rangle$ by introducing the auxiliary particle and performing the Pauli operator. Therefore, after n-hop teleportation we can gain the four-type results and the possibility of each type is $p_2=p_1\times\frac{1}{6}\times 4=\frac{2^{n-2}}{3^n}$. At the destination, we can recover the state $\alpha|00\rangle+\beta|11\rangle$ by introducing the auxiliary particle and performing the Pauli operator. The entire possibility is $p_{n-\text{hop}}=P_2\times 4=\left(\frac{2}{3}\right)^n$. The process can be seen in Fig. 4.

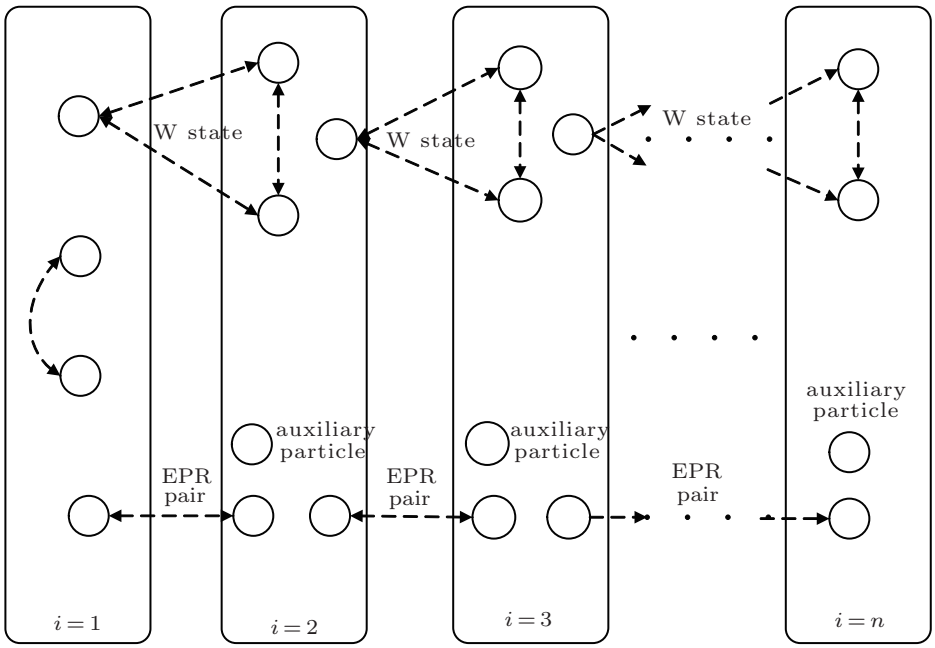


Fig. 4. n-hop teleportation.

4. Conclusion

We assume that quantum information at the source node is teleported through a multi-hop quantum channel based on W state and EPR pairs and we deduce the expression of each type of the results at the destination node and its possibility. We also introduce an auxiliary particle and give the corresponding operation of the Pauli operator to recover the message. The possibility of each type is $p=\frac{2^{n-2}}{3^n}$ and we can recover the transmitted message with the possibility of $\left(\frac{2}{3}\right)^n$ after n-hop teleportation. There are still some issues worthy of study. In this paper, quantum channels are established by maximally entangled states. In practice, the maximally entangled state usually degrades and changes to a partially entangled state. A scheme using partially entangled states is worth studying. Meanwhile, a more efficient approach by introducing more auxiliary particles is also worth studying.

Appendix A: Pauli Operator

(i) $\alpha|01\rangle_{\text{AB}}\pm\beta|00\rangle_{\text{AB}}$

The system state can be represented as

$$|\psi\rangle = (\alpha|01\rangle_{AB} + \beta|00\rangle_{AB})|0\rangle_{aux} = (\beta,0,\alpha,0,0,0,0,0)^{T},$$

after introducing auxiliary particle $|0\rangle_{aux}$. The unitary matrix which is performed on particles A and B is

$$U = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\begin{aligned} U|\psi\rangle &= U\{(\alpha|10\rangle_{AB} + \beta|01\rangle_{AB})|0\rangle_{aux}\} \\ &= (\alpha,0,0,0,0,0,\beta,0)^{T} \\ &= (\alpha|00\rangle_{AB} + \beta|11\rangle_{AB})|0\rangle_{aux}, \end{aligned}$$

if the measured state is $|0\rangle_{aux}$, the message is recovered successfully.

(ii) $\alpha|01\rangle_{AB} \pm \beta|10\rangle_{AB}$

The system state can be represented as

$$|\psi\rangle = (\alpha|01\rangle + \beta|10\rangle)|0\rangle_{aux} = (0,0,\alpha,0,\beta,0,0,0)^{T},$$

after introducing auxiliary particle $|0\rangle_{aux}$. The unitary matrix which is performed on particles A and B is

$$U = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\begin{aligned} U|\psi\rangle &= U\{(\alpha|01\rangle + \beta|10\rangle)|0\rangle_{aux}\} \\ &= (\alpha,0,0,0,0,0,\beta,0)^{T} \\ &= (\alpha|00\rangle + \beta|11\rangle)|0\rangle_{aux}, \end{aligned}$$

if the measured state is $|0\rangle_{aux}$, the message is recovered successfully.

(iii) $\alpha|10\rangle_{AB} \pm \beta|01\rangle_{AB}$

The system state can be represented as

$$|\psi\rangle = (\alpha|10\rangle + \beta|01\rangle)|0\rangle_{aux} = (0,0,\beta,0,\alpha,0,0,0)^{T},$$

after introducing auxiliary particle $|0\rangle_{aux}$. The unitary matrix which is performed on particles A and B is

$$U = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\begin{aligned} U|\psi\rangle &= U\{(\alpha|10\rangle + \beta|01\rangle)|0\rangle_{aux}\} \\ &= (\alpha,0,0,0,0,0,\beta,0)^{T} \\ &= (\alpha|00\rangle + \beta|11\rangle)|0\rangle_{aux}, \end{aligned}$$

if the measured state is $|0\rangle_{aux}$, the message is recovered successfully.

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