Dynamics of spinor Bose–Einstein condensate subject to dissipation*

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We investigate the internal dynamics of the spinor Bose–Einstein condensates subject to dissipation by solving the Lindblad master equation. It is shown that for the condensates without dissipation its dynamics always evolve along a specific orbital in the phase space of (n_0, θ) and display three kinds of dynamical properties including Josephson-like oscillation, self-trapping-like oscillation, and 'running phase'. In contrast, the condensates subject to dissipation will not evolve along the specific dynamical orbital. If component-1 and component-(-1) dissipate at different rates, the magnetization m will not conserve and the system transits between different dynamical regions. The dynamical properties can be exhibited in the phase space of (n_0, θ, m).

Keywords: spinor Bose–Einstein condensates, dissipation, master equation

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1. Introduction

As Bose–Einstein condensates (BECs) are confined in an optical trap regardless of the hyperfine state,[1,2] the atomic spin degree of freedom is liberated and the spinor BECs are realized. This allows us to explore the properties related to spin of ultracold quantum gas.[3,4] Magnetism of the condensates has ever been extensively investigated due to its importance in traditional condensed matter physics.[5–8] Ryan *et al.* have found that many body states of spinor atoms can be classified into several kinds of novel phases according to its spin symmetry.[9] In addition, the realization of spinor BECs stimulated a great many theories and experimental study on the dynamical properties of spin-dependent interaction. The researchers have investigated the irregular many-body spin-mixing dynamics of $F=1$ spinor condensates in the absence[10–13] and in the presence of an external magnetic field.[14,15] The magnetic properties of a spinor BEC of high spin[8] are also the present popular topic.

In experiments, one non-negligible problem is the coupling of condensates with the environment. For example, the interaction between the condensed atom and noncondensed thermal atoms results in unavoidable atom loss.[16] In some situations the dissipation effects,[17–20] the thermal fluctuations[21] and the dephasing[22] will play crucial roles. Diehl has suggested that with the quantum optics method we can drive an open ultracold atomic system into a given pure quantum state, which provides a route towards preparing many-body states and non-equilibrium quantum phases.[23] The proposal to prepare a spin squeezed state, phase- and number-squeezed state has been given.[24,25]

In this paper, we investigate the spin-mixing dynamics of spin-1 antiferromagnetic spinor BECs subject to dissipation. As the dissipation rates are different for each component, the magnetization will not be conserved and the pseudo-angular momentum operator defined in Refs. [26] and [27] cannot describe the dynamics of spinor BECs. By constructing a set of operators, the Lindblad master equation,[28,29] which governs the dynamics of the open, quantum system, can be formulated as a group of nonlinear dynamical evolution equations under the mean-field approximation. Therefore, we can obtain the dynamics of open spinor BECs by numerically solving the set of nonlinear equations.

This paper is organized as follows. In Section 2 we introduce our model and method and show the evolution equations of our system. In Section 3 we investigate diverse types of oscillations of spinor BECs without dissipation and demonstrate that the oscillation period sensitively depends on the initial state. In Section 4 we explore the oscillations of spinor BECs under the effects of dissipation and the transition caused by dissipation. Finally, a brief summary is presented in Section 5.

2. Model and method

We consider spin-1 spinor BECs with N atoms of mass M trapped in a spin-independent external potential $V(\boldsymbol{r})$, the second quantized Hamiltonian is formulated as

$$\begin{aligned}\mathscr{H} = &\int \mathrm{d}\boldsymbol{r}\hat{\Psi}_\alpha^\dagger(\boldsymbol{r})\left[-\frac{\hbar^2\nabla^2}{2M}+V(\boldsymbol{r})-\mu\right]\hat{\Psi}_\alpha(\boldsymbol{r})\\ &+\frac{c_0}{2}\hat{\Psi}_\alpha^\dagger(\boldsymbol{r})\hat{\Psi}_\beta^\dagger(\boldsymbol{r})\hat{\Psi}_\beta(\boldsymbol{r})\hat{\Psi}_\alpha(\boldsymbol{r})\\ &+\frac{c_2}{2}\hat{\Psi}_\alpha^\dagger(\boldsymbol{r})\hat{\Psi}_{\alpha'}^\dagger(\boldsymbol{r})\boldsymbol{F}_{\alpha\beta}\cdot\boldsymbol{F}_{\alpha'\beta'}\hat{\Psi}_{\beta'}(\boldsymbol{r})\hat{\Psi}_\beta(\boldsymbol{r}),\end{aligned}\quad(1)$$

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where $\hat{\Psi}_\alpha(\boldsymbol{r})\,(\alpha=0,\pm1)$ is the field operator associated with the annihilation of an atom in the hyperfine spin state $|F=1,m_F=\alpha\rangle$. The interaction parameters $c_0=4\pi\hbar^2(a_0+2a_2)/(3M)$ and $c_2=4\pi\hbar^2(a_2-a_0)/(3M)$ correspond to the spin-independent and spin-dependent interactions, respectively. Here a_f $(f=0,2)$ is the s-wave scattering length for atoms in the channel of total spin f. Because of $|c_2|\ll c_0$ ($a_0=50a_{\rm B}$ and $a_2=55a_{\rm B}$ for ^{23}Na with $a_{\rm B}$ being Bohr radius), the single-mode approximation (SMA)[30] was utilized effectively to study spinor BECs. In the approximation the spatial wave functions for each spin component $\phi_\alpha(\boldsymbol{r})$ $(\alpha=0,\pm1)$ are described by the same wave function $\phi(\boldsymbol{r})$ determined by the ground-state solution of the Gross–Pitaevskii equation[31,32] rather than the coupled Gross–Pitaevskii equations:

$$\left(-\frac{\hbar^2\nabla^2}{2M}+V(\boldsymbol{r})+c_0N|\phi|^2\right)\phi=\mu\phi \tag{2}$$

with μ being the chemical potential. Here N is the total particle number, which is a classical number. Therefore we have $\hat{\Psi}_\alpha\approx\hat{a}_\alpha\phi(\boldsymbol{r})$ with $\hat{a}_\alpha$ being the annihilation operator for the α component, and the Hamiltonian is given by

$$\begin{aligned}\hat{H}=&\,\mu\hat{N}-\lambda_0'\hat{N}(\hat{N}-1)+\lambda_2'(\hat{a}_1^\dagger\hat{a}_1^\dagger\hat{a}_1\hat{a}_1+\hat{a}_{-1}^\dagger\hat{a}_{-1}^\dagger\hat{a}_{-1}\hat{a}_{-1}+2\hat{a}_1^\dagger\hat{a}_0^\dagger\hat{a}_1\hat{a}_0+2\hat{a}_{-1}^\dagger\hat{a}_0^\dagger\hat{a}_{-1}\hat{a}_0-2\hat{a}_1^\dagger\hat{a}_{-1}^\dagger\hat{a}_1\hat{a}_{-1}\\&+2\hat{a}_0^\dagger\hat{a}_0^\dagger\hat{a}_1\hat{a}_{-1}+2\hat{a}_1^\dagger\hat{a}_{-1}^\dagger\hat{a}_0\hat{a}_0),\end{aligned} \tag{3}$$

where $\hat{N}$ is the total atom number operator in the condensate and

$$2\lambda_i'\equiv c_i\int(|\phi(\boldsymbol{r})|^4)\mathrm{d}\boldsymbol{r},\quad(i=0,2).$$

For a closed system, the total atom number $\hat{N}$ and magnetization $\hat{m}=\hat{a}_1^\dagger\hat{a}_1-\hat{a}_{-1}^\dagger\hat{a}_{-1}$ are conserved quantities. Following the algebra in Refs. [26] and [27], we can define the operators

$$\begin{aligned}\hat{L}_-&=\sqrt{2}(\hat{a}_1^\dagger\hat{a}_0+\hat{a}_0^\dagger\hat{a}_{-1}),\\\hat{L}_+&=\sqrt{2}(\hat{a}_0^\dagger\hat{a}_1+\hat{a}_{-1}^\dagger\hat{a}_0),\\\hat{L}_z&=\sqrt{2}(\hat{a}_{-1}^\dagger\hat{a}_{-1}-\hat{a}_1^\dagger\hat{a}_1),\end{aligned}$$

which satisfy the usual angular momentum commutation relation. They can completely describe the properties of the spin-1 BECs if the system is closed. In the present paper we will investigate the dynamics of spinor BECs subject to dissipation for the coupling with the environment. In this case both the total atom number and magnetization will not be conserved and the new algebraic structure has to be employed. We construct the operators $\hat{Y}_k=\mathrm{i}(\hat{a}_\alpha^\dagger\hat{a}_\beta-\hat{a}_\beta^\dagger\hat{a}_\alpha)$ and $\hat{Z}_k=(\hat{a}_\alpha^\dagger\hat{a}_\beta+\hat{a}_\beta^\dagger\hat{a}_\alpha)$ ((α,β) is $(1,-1)$, $(-1,0)$, and $(0,1)$ for $k=1,2,3$, respectively). Combing $\hat{Y}_k$ and $\hat{Z}_k$ with $\hat{N}$, $\hat{m}$, and atom number in 0-component $\hat{N}_0$, the open system can be described. With the help of the above operators, $\hat{H}$ takes the following form

$$\begin{aligned}\hat{H}=&\,\mu\hat{N}-\lambda_0'\hat{N}(\hat{N}-1)\\&+\frac{1}{2}\lambda_2'[(\hat{Z}_2+\hat{Z}_3)^2+(\hat{Y}_2+\hat{Y}_3)^2+2\hat{m}^2-4\hat{N}].\end{aligned} \tag{4}$$

For the system coupled with environment inducing non-equilibrium dynamics, its time evolution is determined by the Lindblad master equation[29] for the reduced system density operator $\hat{\rho}$

$$\begin{aligned}\dot{\hat{\rho}}=&\frac{1}{\mathrm{i}\hbar}[\hat{H},\hat{\rho}]\\&-\frac{1}{2}\sum_{\alpha=0,\pm1}\gamma_{a_\alpha}(\hat{a}_\alpha^\dagger\hat{a}_\alpha\hat{\rho}+\hat{\rho}\hat{a}_\alpha^\dagger\hat{a}_\alpha-2\hat{a}_\alpha\hat{\rho}\hat{a}_\alpha^\dagger).\end{aligned} \tag{5}$$

Here, $\hat{H}$ is the system Hamiltonian, and γ_{a_α} is the dissipation rate of component α determined by the interaction between the cold atoms and thermal atoms. The one-time average of arbitrary operator $\hat{A}$ can be calculated via $\langle A\rangle=\mathrm{tr}\,[\hat{A}\hat{\rho}(t)]$ and its time derivative $\langle\dot{\hat{A}}\rangle=\mathrm{tr}\,[\hat{A}\dot{\hat{\rho}}(t)]$. We define $n(t)=\mathrm{tr}\,[\hat{N}\hat{\rho}(t)]/N$, $m(t)=\mathrm{tr}\,[\hat{m}\hat{\rho}(t)]/N$, $n_0(t)=\mathrm{tr}\,[\hat{N}_0\hat{\rho}(t)]/N$, $y_i(t)=\mathrm{tr}\,[\hat{Y}_i\hat{\rho}(t)]/N$, $z_i(t)=\mathrm{tr}\,[\hat{Z}_i\hat{\rho}(t)]/N$. Inserting the Hamiltonian $\hat{H}$ into Eq. (5) and taking the first-order approximation, i.e., $\langle\hat{A}\hat{B}\rangle=\langle\hat{A}\rangle\langle\hat{B}\rangle$, we can obtain a set of closed equations

$$\begin{aligned}\dot{n}&=-\frac{1}{2}(\gamma_{a_1}+\gamma_{a_{-1}})n-\frac{1}{2}(\gamma_{a_1}-\gamma_{a_{-1}})m+\frac{1}{2}(\gamma_{a_1}+\gamma_{a_{-1}}-2\gamma_{a_0})n_0,\\\dot{m}&=-\frac{1}{2}(\gamma_{a_1}-\gamma_{a_{-1}})n-\frac{1}{2}(\gamma_{a_1}+\gamma_{a_{-1}})m+\frac{1}{2}(\gamma_{a_1}-\gamma_{a_{-1}})n_0,\\\dot{n}_0&=2\lambda_2'(y_2z_3-y_3z_2)-\gamma_{a_0}n_0,\\\dot{y}_1&=\lambda_2'\left(y_2^2-y_3^2-z_2^2+z_3^2-4mz_1\right)-\frac{1}{2}\left(\gamma_{a_1}+\gamma_{a_{-1}}\right)y_1,\\\dot{y}_2&=\lambda_2'[-y_1y_3-y_1y_2+z_1z_2+z_1z_3+z_2(n-m-3n_0)+z_3(n-m-3n_0)+2mz_2]-\frac{1}{2}(\gamma_{a_{-1}}+\gamma_{a_0})y_2,\\\dot{y}_3&=\lambda_2'[y_1y_3+y_1y_2-z_1z_2-z_1z_3-z_2(n+m-3n_0)-z_3(n+m-3n_0)+2mz_3]-\frac{1}{2}(\gamma_{a_0}+\gamma_{a_1})y_3,\end{aligned}$$

$$\dot{z}_1 = \lambda_2'(-2y_2z_2 + 2y_3z_3 + 4my_1) - \frac{1}{2}(\gamma_{a_1} + \gamma_{a_{-1}})z_1,$$
$$\dot{z}_2 = \lambda_2'[y_1z_2 + y_1z_3 + y_2z_1 - y_2(n-m-3n_0) + y_3z_1 - y_3(n-m-3n_0) - 2my_2] - \frac{1}{2}(\gamma_{a_0} + \gamma_{a_{-1}})z_2,$$
$$\dot{z}_3 = \lambda_2'[-y_1z_2 - y_1z_3 - y_2z_1 + y_2(n+m-3n_0) - y_3z_1 + y_3(n+m-3n_0) - 2my_3] - \frac{1}{2}(\gamma_{a_0} + \gamma_{a_1})z_3. \quad (6)$$

For simplicity, we use the dimensionless formula and in the following evaluation the time and energy are in units of the trapping frequency ω^{-1} and $\hbar\omega$, respectively.

After solving the above equations, we can calculate the interested quantities. For comparison with the dynamical properties of closed spinor BECs, we will focus on the evolving dynamics in the phase space of (n_0, θ). θ is the relative phase of three components θ_α ($\theta = \theta_1 + \theta_{-1} - 2\theta_0$). The evolution of n_0 can be obtained by solving the equations. As we have time dependence of θ, the dynamical orbitals are determined in the phase space of (n_0, θ). According to Ref. [33], in the single-mode approximation we have $\theta = \arctan(y_3z_2 - y_2z_3)/(y_2y_3 + z_2z_3)$. While the population in component 1 and -1 can be obtained by $n_1 = (n+m-n_0)/2$ and $n_{-1} = (n-m-n_0)/2$.

3. Dynamics of open spinor BECs

In this section we first show the dynamical properties of spinor BECs decoupled with the environment, i.e., there is no particle dissipation, and then we exhibit the dynamics as each component is subject to dissipation. In the present paper we will display our results by taking the antiferromagnetic spinor BECs as the example and the spin-dependent interaction constant $\lambda_2' = 0.2$.

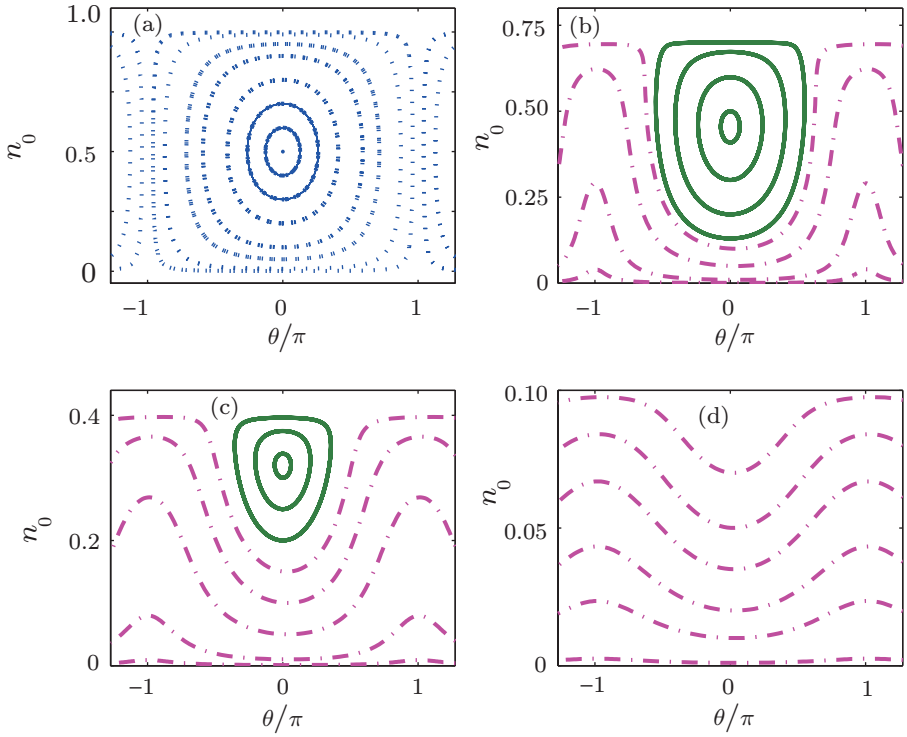


Fig. 1. (color online) Dynamical trajectories in $n_0 - \theta$ plane for $\gamma_{a_1} = \gamma_{a_{-1}} = \gamma_{a_0} = 0$: (a) m=0; (b) m=0.3; (c) m=0.6; (d) m=0.9.

In Fig. 1 the dynamical orbitals of close spinor BECs are shown for different magnetization m. For a given initial magnetization $m(0)$, it is conserved and will not change in the full dynamics. This is consistent with the fact that magnetization is a conserved quantity in the spinor BECs. For different magnetization the dynamical orbitals display specific characteristics even for the same initial condition. In Fig. 1(a) ($m = 0$), the dynamical trajectories evolve in the center of $n_0 = 0.5$ and $\theta = 0.0$ and all of them are closed. Corresponding to the two-component BECs or BECs in a double well,[34] we define the particle number difference between the component 0 and the others, $z = (n_1 + n_{-1}) - n_0$. The mean value of z in an evolving period is 0.0 ($\bar{n}_0 = 0.5$), which is similar to the Josephson oscillation in two component BECs. In Fig. 1(b) and Fig. 1(c) (m =0.3 and 0.6) the open orbitals appear. In these situations, with the time evolution the phase θ will be

always 'running' rather than be periodical. The orbitals have the characteristic of the 'running phase'. For the closed dynamical orbitals here, $\bar{n}_0$ deviates from 0.5 ($\bar{z} \neq 0.0$), which is much like the 'self-trapping' in two-component BECs. Similar to the case of two component BECs, three kinds of dynamical orbitals are shown for different initial population ($n_0(0)$ and $m(0)$) in spinor BECs: Josephson-like oscillation, self-trapping-like, and the 'running phase'. For large magnetization the dynamical orbitals show the 'running phase' properties (m=0.9 in Fig. 1(d)).

For the given magnetization m the spinor BECs are always in the specified dynamical region in the phase space of (n_0, θ). The system will evolve in one special dynamical orbital and keep up evolving with m being constant. This is what we have shown so far in Fig. 1, all of which are internal spin-mixing dynamics of spinor BECs. As the spinor BECs are subject to dissipation, the internal dynamical properties will be affected by the external environment. The magnetization m will change if the dissipation rates of component 1 and component -1 are distinct. Even if the dissipation rates are the same for components 1 and -1, the dynamical properties will be extremely different from the spinor BECs without coupling with the environment.

In Fig. 2 the dynamical orbitals are shown for the case of $\gamma_{a_1} = \gamma_{a_{-1}}$ and $m = 0$. Since the atoms in component 1 and component -1 dissipate at the same rate as the internal dynamical properties of spinor BECs, the magnetization will be conserved as a constant 0. We can exhibit the dynamical orbitals of closed spinor BECs in the phase space of (n_0, θ) of $m = 0$. It is shown that the system still evolves in the Josephson-like oscillation region, but it will transit between different orbitals rather than evolve in one specific close orbital. In addition, the external coupling with the environment can be utilized to control the evolving orbital and the arrived final state. As only component 0 dissipates ($\gamma_{a_1} = \gamma_{a_{-1}} = 0$ and $\gamma_{a_0} = 1/3$), the atoms of component 0 will completely disappear and some atoms of component 1 and component -1 remain in the system ($n_1 = n_{-1} = 0.05$ finally). As only component 1 and component -1 dissipate ($\gamma_{a_1} = \gamma_{a_{-1}} = 1/3$ and $\gamma_{a_0} = 0$), the contrary situation will take place ($n_0 = 0.2$). In this case most atoms of component 0 remain in the system after the complete loss of the other two components.

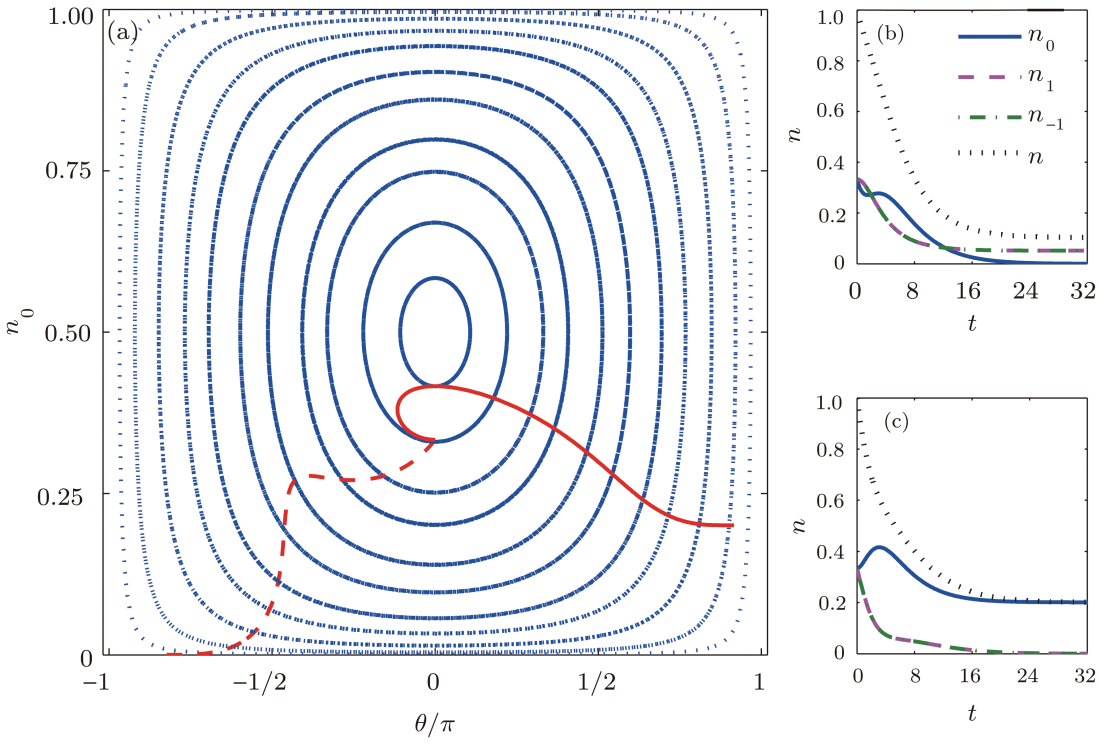


Fig. 2. (color online) (a) Dynamical orbitals of open spinor BECs of m=0. Dashed lines: $\gamma_{a_1} = \gamma_{a_{-1}} = 0, \gamma_{a_0} = 1/3$; Solid line: $\gamma_{a_1} = \gamma_{a_{-1}} = 1/3, \gamma_{a_0} = 0$. For comparison with the close system, the dynamical orbitals of the system without dissipation are shown (dotted lines). (b) The dynamical evolution of the particle number of each component for $\gamma_{a_1} = \gamma_{a_{-1}} = 0, \gamma_{a_0} = 1/3$. (c) The dynamical evolution of the particle number of each component for $\gamma_{a_1} = \gamma_{a_{-1}} = 1/3, \gamma_{a_0} = 0$.

In Fig. 3a the dynamical orbitals are shown for the case of $\gamma_{a_1} \neq \gamma_{a_{-1}}$ and $m(0) = 0$. As the dissipation rate of component 1 and component -1 are different, the dynamical orbital will evolve in the phase space of (n_0, θ, m) rather than (n_0, θ) because the magnetization will change in this case. The dynamics evolution starts at the Josephson-like oscillation regime. If the spinor BECs are not subject to dissipation, the system maintains the stable Josephson-like oscilla-

tion along the specific close orbital of $m = 0$. Since component 1 dissipates at the rate $\gamma_{a_1} = 1/3$, the magnetization m deviates from zero and the system gradually evolves into a self-trapping-like region at $|m|$=0.1. With the loss of spin-1 atoms, the magnetization becomes larger and the system evolves into the 'running phase' region at $|m| = 0.4$. Therefore by controlling the component-dependent dissipation rate the system can evolve from the stable Josephson-like oscillation into the self-trapping-like region and ultimately into the 'running phase' region.

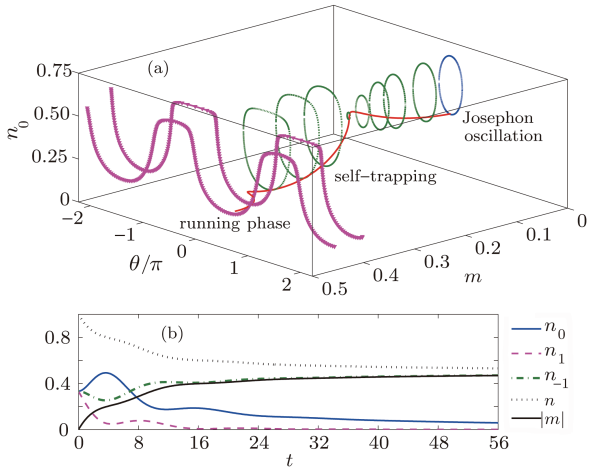


Fig. 3. (color online) (a) Dynamical orbitals of open spinor BECs. For comparison with the close system, the dynamical orbitals of the system without dissipation are shown (dotted lines). $\gamma_{a_1} = 1/3$, $\gamma_{a_{-1}} = \gamma_{a_0} = 0$. (b) The dynamical evolution of the particle number of each component.

4. Conclusion

In summary, we investigated the spin-mixing dynamics of spinor BECs subjected to dissipations by constructing a set of operators and solving the Lindblad master equation. As the system is closed, for different initial conditions the dynamical orbitals show the properties of Josephson-like oscillation, self-trapping, and the 'running phase'. The spinor BECs evolve along the specific dynamical orbital. When the system is coupled with the environment and atom dissipation exists, the system will transit between different dynamical orbitals. For the spinor BECs with the same dissipation rates for components ±1, the magnetization conservation and the system evolves in the phase space of (n_0, θ). For the spinor BECs with different dissipation rates for components ±1, the system can evolve from Josephson-like oscillation into the self-trapping-like region and 'running phase' region. In this situation, the dynamical orbitals evolve in the phase space of (n_0, θ, m).

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