Quantum and semiclassical studies on photodetachment cross sections of H^- in a harmonic potential*

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The photodetachment cross section of H^- in a linear harmonic oscillator potential is investigated. This system provides a rare example that can be studied analytically by both quantum and semiclassical methods with some approximations. The formulas of the cross section for different laser polarization directions are explicitly derived by both the traditional quantum approach and closed-orbit theory. In the traditional quantum approach, we calculate the cross sections in coordinate representation and momentum representation, and get the same formulas. We compare the quantum formulas with closed-orbit theory formulas, and find that when the detachment electron energy is larger than $\frac{3}{2}\hbar\omega$, where ω is the frequency of the oscillator potential, the quantum results are shown to be in good agreement with the semiclassical results.

Keywords: photodetachment, harmonic oscillator potential, closed-orbit theory

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1. Introduction

Recently, much experimental and theoretical work has been conducted in effort to understand photodetachment of electrons from negative ions. Bryant observed oscillations in the cross sections of H^- [1] in a static electric field. Rau and Wong[2] held that the oscillations came from the interference between the detached electron going up and down the electric potential hill. They used quantum method to calculate the cross section in the coordinate representation. Meanwhile, Du and Delos[3] presented another quantum approach based on wave functions in momentum space in the same year.

To give a clear physical picture for the oscillations and the cross section, Du applied closed-orbit theory (COT) to explain the oscillations as an interference of the initial wave and the returning wave caused by the electric field.[4–7] Inspired by the studies of the photodetachment of H^- in an electric field, intensive researches in the photodetachment of H^- in different environments or external fields have also been done, such as in electric and magnetic fields with any orientation,[8,9] in an inhomogeneous electric field,[10,11] near a metal surface,[12,13] inside a square microcavity,[14] inside a circular microcavity,[15] near a deform sphere,[16] and near a dielectric surface in a magnetic field.[17] The photodetachment of H^- in these systems have been studied theoretically using either quantum method or COT or both. There was a good agreement in the comparison between quantum and COT results for the cross sections in some systems.[3,7–11,18,19] In another development, in recent years Zhang and Lin *et al.* have done lots of interesting work in self-similarity of artificial atom and atom in fields.[20–25]

A linear harmonic oscillator is a basic model to solve problems in classical and quantum physics. The equations of motion of a harmonic oscillator are resoluble, which makes the mathematical analysis easy. In the present work, we apply quantum method and COT to calculate the photodetachment cross sections of H^- in a linear harmonic oscillator potential. We find that the quantum results in different representation agree well with each other. We also make a comparison between quantum and COT results.

The paper is organized as follows. In Section 2, we derive the quantum formulas for the photodetachment cross section of H^-, while the formulas for COT are presented in Section 3. Then the numerical results and discussion are provided in Section 4.

2. Quantum formulations for photodetachment cross section

Let us consider that H^- is placed in a linear harmonic potential $V_z = \frac{1}{2}\omega^2 z^2$ and assume that the frequency of ω is positive, then the Hamiltonian describing the motion of the escaping electron in the linear harmonic potential is given by

$$H = \frac{1}{2}p_x^2 + \frac{1}{2}p_y^2 + \frac{1}{2}p_z^2 + \frac{1}{2}\omega^2 z^2 + V_b(r). \quad (1)$$

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When the electron is far away from the nucleus, the binding potential V_b can be neglected. The cross section is given by the following expression in atomic units:[19]

$$\sigma^q_{\rm quant} = \frac{2\pi^2}{c}\int {\rm d}f 2E_{\rm p}|\langle\Psi_{\rm f}|\boldsymbol{q}|\Psi_{\rm i}\rangle|^2\delta(E_{\rm f}-E), \tag{2}$$

where $\boldsymbol{q}$ represents either x, y, or z for different laser polarizations, c is the speed of light and its value is approximately 137 a.u.; the photon energy $E_{\rm p}$ is equal to the sum of the binding energy $E_{\rm b} = k_{\rm b}^2/2$, which is approximately 0.754 eV, and the final energy $E = k^2/2$ of the photodetached electron; $\Psi_{\rm f}$ is the final wave function and is normalized according to $\langle\Psi_{\rm f}|\Psi_{\rm f'}\rangle = \delta(f-f')$. A good approximation to the initial wave function of H^- is

$$\Psi_{\rm i}(\boldsymbol{r}) = B_0\frac{{\rm e}^{-k_{\rm b}r}}{r}. \tag{3}$$

In order to obtain the cross sections in Eq. (2), the module square of the dipole matrix element $\langle\Psi_{\rm f}|\boldsymbol{q}|\Psi_{\rm i}\rangle$, which can be derived in either momentum or coordinate representation, will be computed firstly.

2.1. Derivation of photodetachment cross section in momentum representation

The initial wave function in momentum space is given by[3]

$$\tilde{\Psi}_{\rm i}(\boldsymbol{p}) = \left(\frac{2}{\pi}\right)^{1/2} B_0\frac{1}{(k_{\rm b}^2+p^2)}. \tag{4}$$

The final wave functions for the detached electron in the linear harmonic oscillator potential are obtained as the solution of the Schrödinger equation in momentum space,

$$\left(\frac{1}{2}p_x^2+\frac{1}{2}p_y^2+\frac{1}{2}p_z^2-\frac{1}{2}\omega^2\frac{\partial^2}{\partial p_z^2}\right)\tilde{\Psi}_{\rm f}(\boldsymbol{p}) = E_{\rm f}\tilde{\Psi}_{\rm f}(\boldsymbol{p}). \tag{5}$$

The final states $\tilde{\Psi}_{\rm f}(\boldsymbol{p})$ are

$$\tilde{\Psi}_{\rm f}(\boldsymbol{p}) = \delta(p_x-p_{x_0})\delta(p_y-p_{y_0})\tilde{\Psi}_n(p_z), \tag{6}$$

and the total energy of the final state is

$$E_{\rm f} = \frac{1}{2}p_{x_0}^2+\frac{1}{2}p_{y_0}^2+(n+\frac{1}{2})\omega, \tag{7}$$

where

$$\tilde{\Psi}_n(p_z) = \left(\frac{1}{\sqrt{\omega}\sqrt{\pi}2^n n!}\right)^{\frac{1}{2}}{\rm e}^{-\frac{1}{2\omega}p_z{}^2}H_n\left(\frac{p_z}{\sqrt{\omega}}\right)(-{\rm i})^n, \tag{8}$$

with $H_n(x)$ being the Hermite polynomials. The wave functions are normalized according to

$$\int\tilde{\Psi}_{\rm f}(\boldsymbol{p})\,\tilde{\Psi}_{\rm f'}(\boldsymbol{p})\,{\rm d}\boldsymbol{p} = \delta\left(p_{x_0}-p'_{x_0}\right)\delta\left(p_{y_0}-p'_{y_0}\right)\delta_{nn'}. \tag{9}$$

Close to the nucleus, the small effects of the external potential are ignored. Similar to the WKB approximation used by Peters,[19] we project the final state $\tilde{\Psi}_n(p_z)$ onto free states in opposite directions, which gives

$$\tilde{\Psi}_n(p_z) = A_n\delta(p_z-p_{z_0})+B_n\delta(p_z+p_{z_0}), \tag{10}$$

where

$$A_n = \sqrt{\frac{\pi}{2}}\left(\frac{\sqrt{\omega}}{\sqrt{\pi}2^n n!}\right)^{1/2}\left[H_n(0)-{\rm i}\frac{1}{p_{z_0}}\sqrt{\omega}H'_n(0)\right], \tag{11}$$

$$B_n = \sqrt{\frac{\pi}{2}}\left(\frac{\sqrt{\omega}}{\sqrt{\pi}2^n n!}\right)^{1/2}\left[H_n(0)+{\rm i}\frac{1}{p_{z_0}}\sqrt{\omega}H'_n(0)\right]. \tag{12}$$

Then the approximation to the final wave function is then re-expressed by

$$\begin{aligned}\tilde{\Psi}_{\rm f}(\boldsymbol{p}) \;=\; &\left[A_n\delta(p_z-p_{z_0})+B_n\delta(p_z+p_{z0})\right]\\ &\times\delta(p_x-p_{x_0})\delta(p_y-p_{y_0}).\end{aligned} \tag{13}$$

The module square of the dipole matrix element for an arbitrary linear polarization is give by

$$|\langle\Psi_{\rm f}|\boldsymbol{q}|\Psi_{\rm i}\rangle|^2 = \left|\langle\tilde{\Psi}_{\rm f}|{\rm i}\nabla_{\boldsymbol{p}}|\tilde{\Psi}_{\rm i}\rangle\right|^2. \tag{14}$$

Substituting Eqs. (4) and (13) into Eq. (14), then we obtain

$$\left|\left\langle\tilde{\Psi}_{\rm f}\left|{\rm i}\frac{\rm d}{{\rm d}p_x}\right|\tilde{\Psi}_{\rm i}\right\rangle\right|^2 = p_{x0}^2\frac{16{B_0}^2}{\left({k_{\rm b}}^2+k^2\right)^4}\frac{\omega^{1/2}}{\sqrt{\pi}2^n n!}H_n^2(0), \tag{15}$$

$$\left|\left\langle\tilde{\Psi}_{\rm f}\left|{\rm i}\frac{\rm d}{{\rm d}p_y}\right|\tilde{\Psi}_{\rm i}\right\rangle\right|^2 = p_{y0}^2\frac{16{B_0}^2}{\left({k_{\rm b}}^2+k^2\right)^4}\frac{\omega^{1/2}}{\sqrt{\pi}2^n n!}H_n^2(0), \tag{16}$$

$$\left|\left\langle\tilde{\Psi}_{\rm f}\left|{\rm i}\frac{\rm d}{{\rm d}p_z}\right|\tilde{\Psi}_{\rm i}\right\rangle\right|^2 = \frac{16B_0^2}{\left({k_{\rm b}}^2+k_0^2\right)^4}\frac{\omega^{1/2}}{\sqrt{\pi}2^n n!}\omega{H_n}'^2(0). \tag{17}$$

With the help of Eqs. (2), (15), (16), and (17), the photodetachment cross section for every direction can be calculated as follows:

$$\begin{aligned}\sigma^x_{\rm quant} &= \sigma^y_{\rm quant}\\ &= \sigma_0\frac{3\sqrt{\pi\omega}}{(2E)^{3/2}}\sum_{n=0}^{n_{\rm max}}{H_n}^2(0)[E-(n+\frac{1}{2})\omega]\frac{1}{2^n n!},\end{aligned} \tag{18}$$

$$\sigma^z_{\rm quant} = \sigma_0\frac{3\sqrt{\pi}\omega^{3/2}}{(2E)^{3/2}}\sum_{n=0}^{n_{\rm max}}\frac{[H_n'(0)]^2}{2^n n!}, \tag{19}$$

where

$$\sigma_0 = \frac{16\pi^2 2^{1/2}{B_0}^2}{3c}\frac{E^{3/2}}{(E_{\rm b}+E)^3}$$

is the cross section in the free field; n satisfies $E-E_z>0$, then $n_{\rm max}$ can be obtained,

$$n_{\rm max} = {\rm Int}\left[\frac{E}{\omega}-\frac{1}{2}\right], \tag{20}$$

where Int denotes the integer part of the expression.

In Fig. 1, the cross sections show oscillations about the free field cross section. The cross section displays sawtooth structure in slope for the parallel to the z axis. For the x and y polarization case, oscillations also present, but their amplitudes are small and the cross section does not deviate much from the free field cross section.

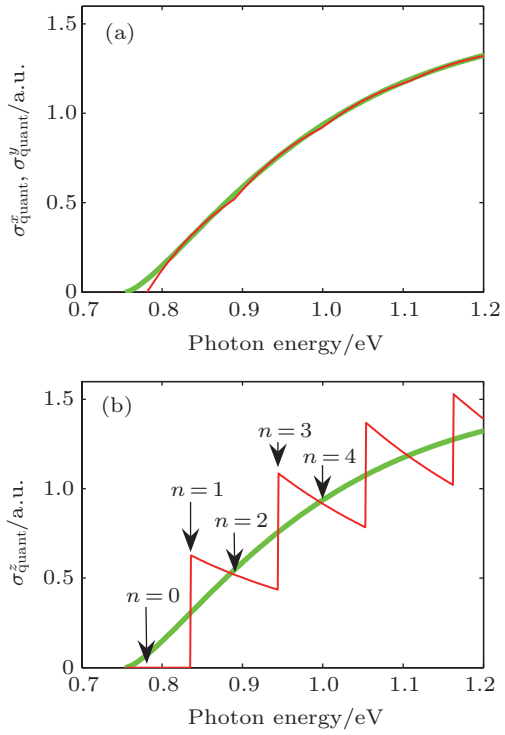


Fig. 1. (color online) Photodetachment cross section of H^- calculated from the quantum method with a linear harmonic oscillator potential (red lines), compared with the cross section in the absence of fields (green dashed lines). We choose the angular frequency ω of the oscillator to be 0.002. (a) x and y-polarized light. (b) z-polarized light. The arrows point to the conersponding harmonic oscillator energy levels.

2.2. Derivation of photodetachment cross section in coordinate representation

A more familiar quantum method, which is frame transformation,[2] can be used to compute the cross sections. The Hamiltonian can be rewritten in another form in the cylindrical coordinate (ρ,ϕ,z),

$$H=\frac{1}{2}p_\rho^2+\frac{1}{2}p_z^2+\frac{1}{2}\omega^2 z^2. \tag{21}$$

After simple calculation, we can obtain the wave functions

$$\begin{aligned}\psi_{E_{\rm f},n,m}(\rho,\varphi,z) &= (2\pi)^{-1/2}\,{\rm e}^{{\rm i}m\varphi}J_m[(k_{\rm f}^2-q_n^2)^{1/2}\rho]\\ &\times\left(\frac{\omega^{1/2}}{\pi^{1/2}2^n n!}\right)^{1/2}{\rm e}^{-\omega z^2/2}H_n(\omega^{1/2}z),\end{aligned} \tag{22}$$

where $q_n^2/2$ is the energy of z motion; J_m is the Bessel function; the wave functions $\Psi_{E_{\rm f},n,m}$ are normalized according to

$$\left\langle\Psi_{E_{\rm f},n,m}\middle|\Psi_{E'_{\rm f},n',m'}\right\rangle=\delta_{mm'}\delta_{nn'}\delta\left(E_{\rm f}-E'_{\rm f}\right). \tag{23}$$

For the purpose of calculation, we transform the final wave function from cylindrical coordinate to spherical coordinate. The first step in transforming is to expand $\psi_{E_{\rm f},n,m}$ in Eq. (23) as a linear combination of free-field wave functions in the cylindrical coordinates[26]

$$\psi_{E_{\rm f},n,m}(\rho,\varphi,z)=A\psi^+_{E_{\rm f},n,m}(\rho,\varphi,z)+B\psi^-_{E_{\rm f},n,m}(\rho,\varphi,z), \tag{24}$$

where the energy-normalized wave functions for the free field in cylindrical coordinates is[26]

$$\begin{aligned}\psi^\pm_{E_{\rm f},n,m}(\rho,z,\varphi) &= (\pi q_n)^{-1/2}(2\pi)^{-1/2}\,{\rm e}^{{\rm i}m\varphi}J_m[(k_{\rm f}^2-{q_n}^2)^{1/2}\rho]\\ &\times\begin{cases}\cos(q_n z), & \prod\limits_z=+,\\ \sin(q_n z), & \prod\limits_z=-.\end{cases}\end{aligned} \tag{25}$$

Equations (22) and (24) satisfy the following conditions:

$$\begin{aligned}&\psi_{E_{\rm f},n,m}(\rho,\varphi,z)\,|_{z=0}\\ &=\left[A\psi^+_{E_{\rm f},n,m}(\rho,\varphi,z)+B\psi^-_{E_{\rm f},n,m}(\rho,\varphi,z)\right]|_{z=0}\,,\end{aligned} \tag{26}$$

$$\begin{aligned}&\frac{\rm d}{{\rm d}z}\psi_{E_{\rm f},n,m}(\rho,\varphi,z)\,|_{z=0}\\ &=\frac{\rm d}{{\rm d}z}\left[A\psi^+_{E_{\rm f},n,m}(\rho,\varphi,z)+B\psi^-_{E_{\rm f},n,m}(\rho,\varphi,z)\right]|_{z=0}\,.\end{aligned} \tag{27}$$

From the above two equations, we can obtain

$$A=(\pi q_n)^{1/2}\left(\frac{\sqrt{\omega}}{\sqrt{\pi}2^n n!}\right)^{1/2}H_n(0), \tag{28}$$

$$B=\left(\frac{\pi}{q_n}\right)^{1/2}\left(\frac{\sqrt{\omega}}{\sqrt{\pi}2^n n!}\right)^{1/2}\omega^{1/2}H'_n(0). \tag{29}$$

The free-field wave functions in cylindrical can also be written in spherical coordinates[3,18]

$$\begin{aligned}&\psi^\pm_{E_{\rm f},n,m}(r,\theta,\varphi)\\ &=\begin{cases}\sum\limits'_l U_{p_z,l}(n,m)f_{lm}(\boldsymbol{r}), & (-1)^l=(-1)^m,\\ \sum\limits''_l U_{p_z,l}(n,m)f_{lm}(\boldsymbol{r}), & (-1)^l=(-1)^{m+1},\end{cases}\end{aligned} \tag{30}$$

where

$$\begin{aligned}&U_{q_n,l}(n,m)\\ &=\left(\frac{2l+1}{k_{\rm f}q_n}\right)^{1/2}\left(\frac{(l-m)!}{(l+m)!}\right)^{1/2}(-1)^{\left[\frac{l-1-m}{2}\right]}P_{lm}\left(\frac{q_n}{k_{\rm f}}\right),\end{aligned} \tag{31}$$

$$f_{lm}(\boldsymbol{r})=Y_{lm}(\theta,\varphi)\left(\frac{2k_{\rm f}}{\pi}\right)^{1/2}j_l(k_{\rm f}r). \tag{32}$$

With the help of Eqs. (24), (28), (29), and (30), the final wave functions in spherical coordinates can be obtained.

The dipole operators in spherical coordinates are given by

$$q_x=r\sin\theta\cos\varphi, \tag{33}$$

$$q_y=r\sin\theta\sin\varphi, \tag{34}$$

$$q_z=r\cos\theta. \tag{35}$$

Substituting the dipole operators, the final wave functions in spherical coordinates, and Eq. (3) into the module square of the dipole matrix elements, we obtain

$$\begin{aligned}&\left|\left\langle\Psi_{E_{\rm f},n,m}\middle|q_x\middle|\Psi_{\rm i}\right\rangle\right|^2\\ &=\left|\left\langle\Psi_{E_{\rm f},n,m}\middle|q_y\middle|\Psi_{\rm i}\right\rangle\right|^2\\ &=\pi\left(k_{\rm f}^2-q_n^2\right)\frac{16B_0^2}{\left(k_{\rm f}^2+{k_{\rm b}}^2\right)^4}\left(\frac{\omega^{1/2}}{\pi^{1/2}2^n n!}\right)H_n^2(0),\\ &\left|\left\langle\Psi_{E_{\rm f},n,m}\middle|q_z\middle|\Psi_{\rm i}\right\rangle\right|^2\end{aligned} \tag{36}$$

$$= 2\pi \frac{16B_0^2}{\left(k_f^2 + k_b^2\right)^4} \left(\frac{\omega^{3/2}}{\pi^{1/2} 2^n n!} \right) H_n'^2(0). \quad (37)$$

Substituting the module square of the dipole matrix elements into Eq. (2), we obtain the same computational forms for the cross sections as that calculated in momentum representation. We come to a conclusion that the two different quantum methods for computing the cross sections in the linear harmonic oscillator potential are equivalent, which is supposed to be suitable for other fields.

3. Formulations of photodetachment cross section based on closed orbit theory

The most well-known method to compute the cross sections is the closed orbit theory, based on which, the photodetachment cross section of H^- in a linear harmonic oscillator potential can be divided into two parts,

$$\sigma_{cot}(E) = \sigma_{dir}(E) + \sigma_{ret}(E), \quad (38)$$

where $\sigma_{dir}(E)$ is equal to the free field cross section $\sigma_0(E)$, and $\sigma_{ret}(E)$ is the contribution of the returning wave

$$\sigma_{ret} = -\frac{4\pi (E + E_b)}{c} \mathrm{Im} \langle D\Psi_i | \Psi_{ret} \rangle, \quad (39)$$

where Ψ_i is the initial bound state wave function and is given by Eq. (3), Ψ_{ret} is the returning wave, which propagates outward into the external area and gets back to the vicinity of the nucleus by the outer field, and is given by[12]

$$\Psi_{ret}(\rho, z, \phi) = -\sum_j \frac{4Bk^2 i}{\left(k_b^2 + k^2\right)^2} h_1^{(1)}(kr) \cos\theta A_j e^{i\left[S_j - \mu_j \pi/2\right]}, \quad (40)$$

where A_j is the amplitude, S_j is the action, μ_j is the Maslov index for orbit j, and the summation is over all closed orbits.

As stated earlier, the Hamiltonian of the electron is given by Eq. (1). Then the solution to equations of motion is obtained after some manipulations,

$$\rho(t) = R\sin\theta + (k\sin\theta)t, \quad (41)$$

$$z(t) = R\cos\theta + \frac{k\cos\theta}{\omega}\sin\omega t, \quad (42)$$

where R is the radius of sphere that surrounds the H^-. The initial position of the electron is on the surface of the sphere. Returning orbits satisfy

$$\rho(t_{ret}) = R\sin\theta, \quad (43)$$

$$z(t_{ret}) = R\cos\theta. \quad (44)$$

From Eqs. (41)–(44), it is easy to show that the orbits along the z direction can be drawn back to the origin by the harmonic oscillator potential. The returning time t_{ret} is given by

$$t_{ret} = T_j = j\frac{\pi}{\omega}, \quad (45)$$

where $j = 1, 2, \ldots$, and $T_1 = \pi/\omega$ is the basic time. We find that two returning orbits correspond to the same returning time. The two orbits have opposite emanative direction, which is the $+z$ direction or the $-z$ direction. For a given value of j, there are two closed orbits. We show some closed orbits in Fig. 2.

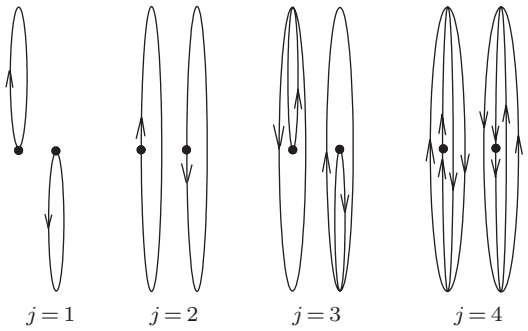


Fig. 2. The first few closed orbits involved.

After simple manipulation , the action, the amplitude, and the Maslov index can be obtained as

$$S_j = \int_0^{T_j} \dot{z}\mathrm{d}z = k^2 \frac{j\pi}{2\omega}, \quad (46)$$

$$A_j(\rho, z, \phi) = \sqrt{\frac{R}{R + kT_j}}, \quad (47)$$

$$\mu_j = j. \quad (48)$$

Substituting Eqs. (3), (39), and (40) into Eq. (38), the cross sections for x, y, and z polarizations can be obtained by

$$\sigma_{cot}^x = \sigma_{cot}^y = \sigma_0, \quad (49)$$

$$\sigma_{cot}^z = \sigma_0 + \sigma_0 \frac{3\omega}{\pi E} \sum_{j=1}^{\infty} \frac{(-1)^j}{j} \sin\left(E\frac{j\pi}{\omega} + j\pi/2\right). \quad (50)$$

For x or y linear polarization, the returning orbits have no influence on the cross section, and therefore, the photodetachment cross section is just the same as that in free field. The photodetachment cross section for the z polarization is shown in Fig. 3. It also displays sawtooth structure. However, there are some difference between Fig. 3 and Fig. 1(b). For the convenience of comparison, we further simplify the formulas obtained from the COT and quantum method. Accordingly, the formulas can be presented as the form of depending on the scaled energy.

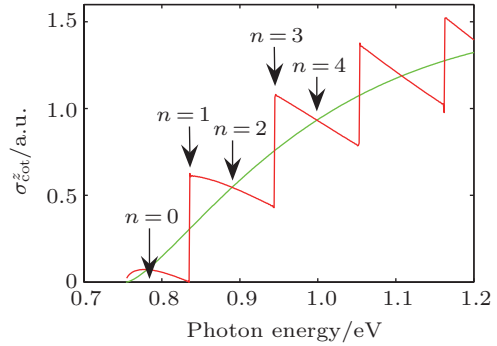


Fig. 3. (color online) Photodetachment cross section for z polarization (red line) derived from closed orbit theory, compared with the cross sections for x and y polarization (green dashed line), which is also free field cross section. The angular frequency ω of the oscillator is selected to be $\omega = 0.002$.

4. Comparisons the COT formulas with quantum formulas

Using Eqs. (18), (19), (49), and (50), the cross sections can be calculated for different values of ω and E. In fact, we can see that ω and E are not independent. We can define the scaled energy as

$$\varepsilon = \frac{E}{\omega}, \tag{51}$$

where ω is a given constant for every figure, then ε is proportional to E. Then the cross sections can be written in the following form:

$$\begin{aligned}
\sigma^x_{\rm quant} &= \sigma^y_{\rm quant} \\
&= \sigma_0 \frac{3\sqrt{\pi}}{(2\varepsilon)^{3/2}} \sum_{n=0}^{n_{\max}} {H_n}^2(0) \left(\varepsilon - \left(n+\frac{1}{2}\right)\right) \left[\frac{1}{2^n n!}\right], \quad (52)\\
\sigma^z_{\rm quant} &= \sigma_0 \frac{3\sqrt{\pi}}{(2\varepsilon)^{3/2}} \sum_{n=0}^{n_{\max}} \frac{H_n'^2(0)}{2^n n!}, \quad (53)\\
\sigma^x_{\rm cot} &= \sigma^y_{\rm cot} = \sigma_0, \quad (54)\\
\sigma^z_{\rm cot} &= \sigma_0 + \sigma_0 \frac{3}{\pi\varepsilon} \sum_{j=1}^{\infty} \frac{1}{j} \sin(\varepsilon j\pi - j\pi/2), \quad (55)
\end{aligned}$$

where $n_{\max} = \mathrm{Int}(\varepsilon - 1/2)$.

In Fig. 4, we compare the cross sections between the quantum results and the COT results. For large ε, the quantum results and COT results agree with each other very well, but for small ε (especially for $\varepsilon < 1.5$), the quantum results deviate obviously from COT results. For x or y linear polarization, when $\varepsilon < 1/2$, the quantum cross section is zero; when $\varepsilon = n + 1/2$ and $n = 0, 2, 4, 6, \ldots$, the cross section has slight changes. It can be explained by the parity selection rules of quantum transition. $x\Psi_i$ or $y\Psi_i$ is even parity under $z \to -z$, so the even parity energy eigenstates open canales.

According to the physical picture of COT, when the active electron is detached by a laser, the active electron and the associated wave propagates out from the negative ion outward in all directions. The electron may be turned back by the potential fields to the region of the negative ion. Then, the returning electron wave will interfere with the initial outgoing electron and induce oscillations in the photodetachment cross sections. It is noted that there is no closed orbit in the xy plane. Therefore, the cross section for perpendicular polarization is the same as the cross section without fields.

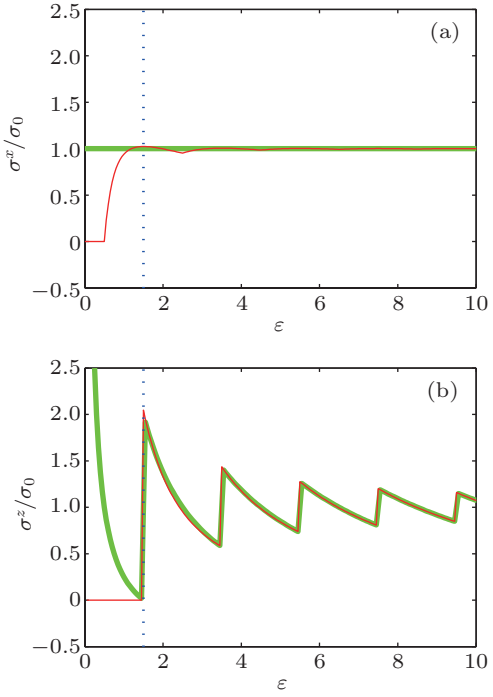


Fig. 4. (color online) Comparison of photodetachment cross section of H^- in a linear harmonic oscillator potential: quantum results (red lines) and COT results (green dashed lines). The laser is linearly polarized in the x-axis direction (a) and z-axis direction. The vertical dashed lines are at $\varepsilon = 3/2$. In the region above $3/2$, quantum results and COT results agree with each other very well.

For z linear polarization, when $\varepsilon < 1 + 1/2$, the quantum cross section is zero, because $z\Psi_i$ has a different parity with the lowest-energy state of harmonic oscillator under $z \to -z$. When $\varepsilon = n + 1/2$ and $n = 1, 3, 5, \ldots$, the cross section has abrupt changes. It can also be explained by the parity selection rules of quantum transition.

According to closed-orbit theory, each closed-orbit orbit does not correspond to an energy level, it is the interference of the many closed-orbits that gives the information on the energy levels of the system. For example, the sudden jumps in Figs. 1(b) and 3 correspond to the energy levels of the harmonic oscillator in the potential in Eq. (1). Because the initial state of the negative ion has an S symmetry, the sudden jumps occur only at energy levels with odd indices.

5. Conclusion

In summary, we applied closed-orbit theory and derived formulas describing the photodetachment cross sections of H^- in the presence of a a linear harmonic oscillator potential. We also derived fully quantum-mechanical formulas of the same system and compared them with those of the closed-orbit theory formulas. The photodetachment cross section shows step-

like structure or smooth structure depending on the photon polarization. We also find that quantum and semiclassical cross sections agree quite well with each other for higher energy.

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