Hybrid entanglement concentration assisted with single coherent state*

Rui Guo(郭锐)[1], Lan Zhou(周澜)[1,2], Shi-Pu Gu(顾世浦)[3], Xing-Fu Wang(王兴福)[2], and Yu-Bo Sheng(盛宇波)[1,†]

[1] *Key Laboratory of Broadband Wireless Communication and Sensor Network Technology (Ministry of Education), Nanjing University of Posts and Telecommunications, Nanjing* 210003, *China*

[2] *College of Mathematics & Physics, Nanjing University of Posts and Telecommunications, Nanjing* 210003, *China*

[3] *College of Electronic Science and Engineering, Nanjing University of Posts and Telecommunications, Nanjing* 210003, *China*

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Hybrid entangled state (HES) is a new type of entanglement, which combines the advantages of an entangled polarization state and an entangled coherent state. HES is widely discussed in the applications of quantum communication and computation. In this paper, we propose three entanglement concentration protocols (ECPs) for Bell-type HES, W-type HES, and cluster-type HES, respectively. After performing these ECPs, we can obtain the maximally entangled HES with some success probability. All the ECPs exploit the single coherent state to complete the concentration. These protocols are based on the linear optics, which are feasible in future experiments.

Keywords: hybrid entangled state, quantum communication and computation, entanglement concentration

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1. Introduction

Quantum entanglement is a significant physical resource which plays a fundamental role in many meaningful applications of quantum information processing.[1] For example, quantum entanglement can be used in quantum teleportation,[2] quantum key distribution,[3] quantum secure direct communication,[4,5] and some other important quantum communication protocols.[6–18] Hence, entanglement is considered to be the information carrier in quantum information and should be created first.

Typically, there are two types of entanglements. The first one is encoded in discrete variables, such as horizontal and vertical polarization states of photons.[19] The second one is encoded in continuous variables, such as the entangled coherent state (ECS).[20] In recent years, there is another type of entanglement which has been widely studied. It is the hybrid entanglement.[21–29] The hybrid entangled state (HES) means that the entanglement is generated between different degrees of freedom of a particle pair. In 2006, van Loock *et al.* proposed the first hybrid quantum repeater protocol using bright coherent light.[21] In 2013, Sheng *et al.* first discussed the entanglement purification for the hybrid entangled state.[26] Recently, an important work about the generation of a hybrid entangled state was reported.[27] Interestingly, Lee and Jeong put forward a new type of hybrid entanglement.[28] Based on the HES, they proposed near-deterministic quantum teleportation and resource-efficient quantum computation. In their protocol, a qubit is encoded in the combination of polarization states and coherent states. The orthogonal basis to define an optical hybrid qubit is $\{|0_{\rm L}\rangle = |+\rangle|\alpha\rangle, |1_{\rm L}\rangle = |-\rangle|-\alpha\rangle\}$, where $|\pm\rangle = (|H\rangle \pm |V\rangle)/\sqrt{2}$ and $|\pm\alpha\rangle$ are coherent states with amplitudes $\pm\alpha$. In particular, they showed that near-deterministic quantum teleportation can be performed using linear optics and hybrid qubits.

In quantum communication protocols, the quantum channel must be maximally entangled. Unfortunately, the maximally entangled states usually collapse into nonmaximally entangled states or even mixed states because of the imperfect channel in the practical process of storage and transmission. In order to obtain the high quality and secure quantum communication, we should first repair the degraded quantum entanglement channel. Entanglement concentration is an important method that we can distill the maximally entangled states from the less-entangled states.[30] In 1996, Bennett *et al.* proposed the first entanglement concentration protocol (ECP) with collective measurement, which is based on the Schimidt decomposition.[30] Until now, many ECPs have been proposed.[30–80] For example, Zhao *et al.* proposed ECPs based on the linear optical element.[33] In 2008, Sheng *et al.* presented a recyclable ECP based on cross-Kerr nonlinearities[34] and the efficiency of their protocol was improved obviously. In 2012, Sheng and Deng first presented the basic principle of efficient single-photon-assisted ECP for

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†Corresponding author. E-mail: shengyb@njupt.edu.cn

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partially entangled photon pairs.[35,36] Subsequently, with the help of the single qubit, the ECPs for arbitrary W states,[51–62] cluster states,[63–69] cat states,[71] χ type entangled states,[72] hyper-entangled states,[74–77] and hybrid entangled states[78] were also proposed.

In this paper, we will focus on the entanglement of the HES, which has the different form of Ref. [78]. We propose three ECPs for different types of HESs. The first ECP is to concentrate the Bell-type HES. The second and the third ECPs are to concentrate the W-type and the cluster-type HES, respectively. In the first ECP, we exploit an ancillary single coherent state and use the beam splitter (BS) to complete the task. In the second ECP, we require two coherent states, and in the third ECP, we require a single coherent state and a pair of ECS to realize the concentration. All protocols are based on the linear optics, which makes them feasible in future experiments.

This paper is organized as follows. In Section 2, we will describe the ECP for an arbitrary less-entangled Bell-type HES. In Section 3, we will propose the ECP for an arbitrary less-entangled W-type HES. In Section 4, we will discuss the ECP for an arbitrary less-entangled cluster-type HES. In Section 5, we will present a discussion.

2. Single coherent state assisted entanglement concentration for arbitrary Bell-type HES

As shown in Fig. 1, Alice and Bob share the less-entangled HES of the form

$$\begin{aligned}|\psi\rangle_{ab} &= \gamma|0_{\rm L}0_{\rm L}\rangle_{ab}+\beta|1_{\rm L}1_{\rm L}\rangle_{ab}\\ &= \gamma|+\rangle_a|+\rangle_b|\alpha\rangle_a|\alpha\rangle_b+\beta|-\rangle_a|-\rangle_b|-\alpha\rangle_a|-\alpha\rangle_b. \quad (1)\end{aligned}$$

In the ECP, Bob prepares an auxiliary single coherent state

$$|\psi\rangle_c = N_0(\gamma|-\alpha\rangle_c+\beta|\alpha\rangle_c). \quad (2)$$

Here, $N_0=[\gamma^2+\beta^2+2\gamma\beta\,{\rm e}^{-2|\alpha|^2}]^{-1/2}$ and $|\gamma|^2+|\beta|^2=1$.

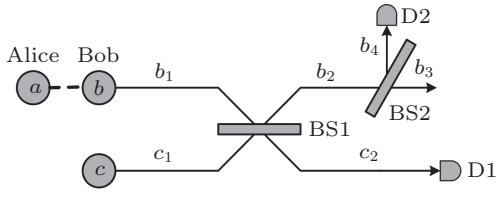


Fig. 1. Schematic diagram of concentrating an arbitrary less-entangled Bell-type HES. Alice and Bob share the HES, and own the hybrid states denoted as a and b, respectively. Bob prepares the ancillary single coherent state c. BS1 is used to perform a parity check measurement. BS2 is used to obtain the Bell-type maximally HES.

Therefore, the whole system can be written as

$$\begin{aligned}|\psi\rangle_{abc} &= |\psi\rangle_{ab}\otimes|\psi\rangle_c\\ &= (\gamma|0_{\rm L}0_{\rm L}\rangle_{ab}+\beta|1_{\rm L}1_{\rm L}\rangle_{ab})\\ &\quad\otimes N_0(\gamma|-\alpha\rangle_c+\beta|\alpha\rangle_c). \quad (3)\end{aligned}$$

It also can be written as

$$\begin{aligned}|\psi\rangle_{ab_1c_1} &= N_0(\gamma^2|+\rangle_a|+\rangle_b|\alpha\rangle_a|\alpha\rangle_{b_1}|-\alpha\rangle_{c_1}\\ &\quad+\beta^2|-\rangle_a|-\rangle_b|-\alpha\rangle_a|-\alpha\rangle_{b_1}|\alpha\rangle_{c_1}\\ &\quad+\gamma\beta(|+\rangle_a|+\rangle_b|\alpha\rangle_a|\alpha\rangle_{b_1}|\alpha\rangle_{c_1}\\ &\quad+|-\rangle_a|-\rangle_b|-\alpha\rangle_a|-\alpha\rangle_{b_1}|-\alpha\rangle_{c_1})). \quad (4)\end{aligned}$$

As shown in Fig. 1, Alice and Bob both own one single photon and one coherent state. The subscript a denotes the state possessed by Alice and b denotes the state possessed by Bob. In this protocol, the single photons do not participate in the concentration. Then, they let the coherent states in Bob's location in spatial modes b_1 and c_1 pass through the 50:50 beam splitter (BS). The BS changes the two different coherent states $|\alpha\rangle$ and $|\beta\rangle$ as

$$\mathrm{BS}(|\alpha\rangle|\beta\rangle)\rightarrow\left|\frac{\alpha+\beta}{\sqrt{2}}\right\rangle\left|\frac{\alpha-\beta}{\sqrt{2}}\right\rangle. \quad (5)$$

After the coherent states passing through BS1, the state can be rewritten as

$$\begin{aligned}|\psi\rangle_{ab_2c_2} &= N_0(\gamma^2|+\rangle_a|+\rangle_b|\alpha\rangle_a|0\rangle_{b_2}|\sqrt{2}\alpha\rangle_{c_2}\\ &\quad+\beta^2|-\rangle_a|-\rangle_b|-\alpha\rangle_a|0\rangle_{b_2}|-\sqrt{2}\alpha\rangle_{c_2}\\ &\quad+\gamma\beta(|+\rangle_a|+\rangle_b|\alpha\rangle_a|\sqrt{2}\alpha\rangle_{b_2}|0\rangle_{c_2}\\ &\quad+|-\rangle_a|-\rangle_b|-\alpha\rangle_a|-\sqrt{2}\alpha\rangle_{b_2}|0\rangle_{c_2})). \quad (6)\end{aligned}$$

From Eq. (6), we find that the items $|+\rangle_a|+\rangle_b|\alpha\rangle_a|\sqrt{2}\alpha\rangle_{b_2}|0\rangle_{c_2}$ and $|-\rangle_a|-\rangle_b|-\alpha\rangle_a|-\sqrt{2}\alpha\rangle_{b_2}|0\rangle_{c_2}$ will make the spatial mode c_2 have no photon. Meanwhile, the other items will make the spatial mode b_2 have no photon. Therefore, by choosing the items which make the spatial mode c_2 have no photon, they can obtain a maximally entangled state

$$\begin{aligned}|\psi\rangle_{ab_2} &= \frac{1}{\sqrt{2}}(|+\rangle_a|+\rangle_b|\alpha\rangle_a|\sqrt{2}\alpha\rangle_{b_2}\\ &\quad+|-\rangle_a|-\rangle_b|-\alpha\rangle_a|-\sqrt{2}\alpha\rangle_{b_2}). \quad (7)\end{aligned}$$

The success probability is $P_1=2|N_0\gamma\beta|^2$. From Eq. (7), we find that they have obtained the maximally entangled state but the amplitude of the coherent state increases. In order to obtain the maximally entangled state with the same form of the original state, they should decrease the amplitude of the coherent state. Subsequently, they let the coherent state in spatial mode b_2 pass through BS2. The state in Eq. (7) evolves to

$$\begin{aligned}|\psi\rangle_{ab_3b_4} &= \frac{1}{\sqrt{2}}(|+\rangle_a|+\rangle_b|\alpha\rangle_a|\alpha\rangle_{b_3}|\alpha\rangle_{b_4}\\ &\quad+|-\rangle_a|-\rangle_b|-\alpha\rangle_a|-\alpha\rangle_{b_3}|-\alpha\rangle_{b_4}). \quad (8)\end{aligned}$$

Next, they can obtain the maximally Bell-type HES by undistinguishing $|\alpha\rangle$ from $|-\alpha\rangle$ when detecting the coherent state

in b_4 mode. It means that they should use the quantum nondemolition (QND) measurement to complete this task.[81,82] After implementing the QND measurement, they can ultimately obtain the Bell-type maximally entangled HES

$$\begin{aligned}|\psi\rangle_{ab_3} &= \frac{1}{\sqrt{2}}(|+\rangle_a|+\rangle_b|\alpha\rangle_a|\alpha\rangle_{b_3}\\ &\quad +|-\rangle_a|-\rangle_b|-\alpha\rangle_a|-\alpha\rangle_{b_3})\\ &= \frac{1}{\sqrt{2}}(|0_{\rm L}0_{\rm L}\rangle_{ab}+|1_{\rm L}1_{\rm L}\rangle_{ab}).\end{aligned} \tag{9}$$

3. Entanglement concentration for arbitrary W-type HES

In this section, we will extend the above protocol to the case of arbitrary W-type HES of the form

$$\begin{aligned}|\phi\rangle_{abc} &= \gamma|0_{\rm L}0_{\rm L}1_{\rm L}\rangle_{abc}+\beta|0_{\rm L}1_{\rm L}0_{\rm L}\rangle_{abc}+\xi|1_{\rm L}0_{\rm L}0_{\rm L}\rangle_{abc}\\ &= \gamma|+\rangle_a|+\rangle_b|-\rangle_c|\alpha\rangle_a|\alpha\rangle_b|-\alpha\rangle_c\\ &\quad +\beta|+\rangle_a|-\rangle_b|+\rangle_c|\alpha\rangle_a|-\alpha\rangle_b|\alpha\rangle_c\\ &\quad +\xi|-\rangle_a|+\rangle_b|+\rangle_c|-\alpha\rangle_a|\alpha\rangle_b|\alpha\rangle_c.\end{aligned} \tag{10}$$

Here, $|\gamma|^2+|\beta|^2+|\xi|^2=1$. As shown in Fig. 2, Alice, Bob, and Charlie share the less-entangled HES. Alice and Charlie prepare the auxiliary single coherent states in the spatial modes d and e, respectively. The two auxiliary single coherent states are given as

$$|\phi\rangle_d = N_1(\xi|\alpha\rangle_d+\beta|-\alpha\rangle_d), \tag{11}$$

$$|\phi\rangle_e = N_2(\gamma|\alpha\rangle_e+\beta|-\alpha\rangle_e). \tag{12}$$

Here, $N_1=(\xi^2+\beta^2+2\xi\beta\,{\rm e}^{-2|\alpha|^2})^{-1/2}$ and $N_2=(\gamma^2+\beta^2+2\gamma\beta\,{\rm e}^{-2|\alpha|^2})^{-1/2}$.

From Fig. 2, the whole less-entangled HES combined with the two single coherent states can be written as

$$\begin{aligned}|\phi\rangle_{abcde} &= |\phi\rangle_{abc}\otimes|\phi\rangle_d\otimes|\phi\rangle_e\\ &= (\gamma|0_{\rm L}0_{\rm L}1_{\rm L}\rangle_{abc}+\beta|0_{\rm L}1_{\rm L}0_{\rm L}\rangle_{abc}+\xi|1_{\rm L}0_{\rm L}0_{\rm L}\rangle_{abc})\\ &\quad \otimes(N_1(\xi|\alpha\rangle_d+\beta|-\alpha\rangle_d))\\ &\quad \otimes(N_2(\gamma|\alpha\rangle_e+\beta|-\alpha\rangle_e)).\end{aligned} \tag{13}$$

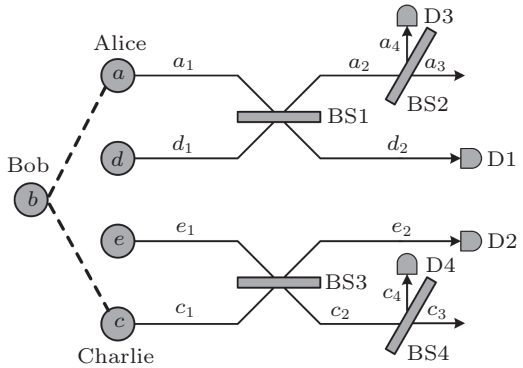


Fig. 2. Schematic diagram showing the principle of the proposed ECP for arbitrary W-type HES. Alice, Bob, and Charlie share the HES, and own the hybrid qubits a, b, and c, respectively. Alice and Charlie prepare the ancillary single coherent states d and e, respectively.

Then they let the coherent states in spatial modes a_1, d_1, e_1, and c_1 pass through the BSs, respectively. Therefore, the whole system can be rewritten as

$$\begin{aligned}&|\phi\rangle_{a_1bc_1d_1e_1}\\ &= N_1N_2(\gamma^2\xi|+\rangle_a|+\rangle_b|-\rangle_c|\alpha\rangle_{a_1}|\alpha\rangle_b|-\alpha\rangle_{c_1}|\alpha\rangle_{d_1}|\alpha\rangle_{e_1}\\ &\quad +\gamma\xi\beta|+\rangle_a|+\rangle_b|-\rangle_c|\alpha\rangle_{a_1}|\alpha\rangle_b|-\alpha\rangle_{c_1}|\alpha\rangle_{d_1}|-\alpha\rangle_{e_1}\\ &\quad +\gamma^2\beta|+\rangle_a|+\rangle_b|-\rangle_c|\alpha\rangle_{a_1}|\alpha\rangle_b|-\alpha\rangle_{c_1}|-\alpha\rangle_{d_1}|\alpha\rangle_{e_1}\\ &\quad +\gamma\beta^2|+\rangle_a|+\rangle_b|-\rangle_c|\alpha\rangle_{a_1}|\alpha\rangle_b|-\alpha\rangle_{c_1}|-\alpha\rangle_{d_1}|-\alpha\rangle_{e_1}\\ &\quad +\beta\xi\gamma|+\rangle_a|-\rangle_b|+\rangle_c|\alpha\rangle_{a_1}|-\alpha\rangle_b|\alpha\rangle_{c_1}|\alpha\rangle_{d_1}|\alpha\rangle_{e_1}\\ &\quad +\xi\beta^2|+\rangle_a|-\rangle_b|+\rangle_c|\alpha\rangle_{a_1}|-\alpha\rangle_b|\alpha\rangle_{c_1}|\alpha\rangle_{d_1}|-\alpha\rangle_{e_1}\\ &\quad +\gamma\beta^2|+\rangle_a|-\rangle_b|+\rangle_c|\alpha\rangle_{a_1}|-\alpha\rangle_b|\alpha\rangle_{c_1}|-\alpha\rangle_{d_1}|\alpha\rangle_{e_1}\\ &\quad +\beta^3|+\rangle_a|-\rangle_b|+\rangle_c|\alpha\rangle_{a_1}|-\alpha\rangle_b|\alpha\rangle_{c_1}|-\alpha\rangle_{d_1}|-\alpha\rangle_{e_1}\\ &\quad +\xi^2\gamma|-\rangle_a|+\rangle_b|+\rangle_c|-\alpha\rangle_{a_1}|\alpha\rangle_b|\alpha\rangle_{c_1}|\alpha\rangle_{d_1}|\alpha\rangle_{e_1}\\ &\quad +\xi^2\beta|-\rangle_a|+\rangle_b|+\rangle_c|-\alpha\rangle_{a_1}|\alpha\rangle_b|\alpha\rangle_{c_1}|\alpha\rangle_{d_1}|-\alpha\rangle_{e_1}\\ &\quad +\xi\gamma\beta|-\rangle_a|+\rangle_b|+\rangle_c|-\alpha\rangle_{a_1}|\alpha\rangle_b|\alpha\rangle_{c_1}|-\alpha\rangle_{d_1}|\alpha\rangle_{e_1}\\ &\quad +\beta^2\xi|-\rangle_a|+\rangle_b|+\rangle_c|-\alpha\rangle_{a_1}|\alpha\rangle_b|\alpha\rangle_{c_1}|-\alpha\rangle_{d_1}|-\alpha\rangle_{e_1}).\end{aligned} \tag{14}$$

After passing through BS1 and BS3, respectively, the whole system becomes

$$\begin{aligned}&|\phi\rangle_{a_2bc_2d_2e_2}\\ &= N_1N_2(\gamma^2\xi|+\rangle_a|+\rangle_b|-\rangle_c|\sqrt{2}\alpha\rangle_{a_2}|\alpha\rangle_b|0\rangle_{c_2}|0\rangle_{d_2}|-\sqrt{2}\alpha\rangle_{e_2}\\ &\quad +\gamma\xi\beta|+\rangle_a|+\rangle_b|-\rangle_c|\sqrt{2}\alpha\rangle_{a_2}|\alpha\rangle_b|-\sqrt{2}\alpha\rangle_{c_2}|0\rangle_{d_2}|0\rangle_{e_2}\\ &\quad +\gamma^2\beta|+\rangle_a|+\rangle_b|-\rangle_c|0\rangle_{a_2}|\alpha\rangle_b|0\rangle_{c_2}|\sqrt{2}\alpha\rangle_{d_2}|-\sqrt{2}\alpha\rangle_{e_2}\\ &\quad +\gamma\beta^2|+\rangle_a|+\rangle_b|-\rangle_c|0\rangle_{a_2}|\alpha\rangle_b|-\sqrt{2}\alpha\rangle_{c_2}|\sqrt{2}\alpha\rangle_{d_2}|0\rangle_{e_2}\\ &\quad +\beta\xi\gamma|+\rangle_a|-\rangle_b|+\rangle_c|\sqrt{2}\alpha\rangle_{a_2}|-\alpha\rangle_b|\sqrt{2}\alpha\rangle_{c_2}|0\rangle_{d_2}|0\rangle_{e_2}\\ &\quad +\xi\beta^2|+\rangle_a|-\rangle_b|+\rangle_c|\sqrt{2}\alpha\rangle_{a_2}|-\alpha\rangle_b|0\rangle_{c_2}|0\rangle_{d_2}|\sqrt{2}\alpha\rangle_{e_2}\\ &\quad +\gamma\beta^2|+\rangle_a|-\rangle_b|+\rangle_c|0\rangle_{a_2}|-\alpha\rangle_b|\sqrt{2}\alpha\rangle_{c_2}|\sqrt{2}\alpha\rangle_{d_2}|0\rangle_{e_2}\\ &\quad +\beta^3|+\rangle_a|-\rangle_b|+\rangle_c|0\rangle_{a_2}|-\alpha\rangle_b|0\rangle_{c_2}|\sqrt{2}\alpha\rangle_{d_2}|\sqrt{2}\alpha\rangle_{e_2}\\ &\quad +\xi^2\gamma|-\rangle_a|+\rangle_b|+\rangle_c|0\rangle_{a_2}|\alpha\rangle_b|\sqrt{2}\alpha\rangle_{c_2}|-\sqrt{2}\alpha\rangle_{d_2}|0\rangle_{e_2}\\ &\quad +\xi^2\beta|-\rangle_a|+\rangle_b|+\rangle_c|0\rangle_{a_2}|\alpha\rangle_b|0\rangle_{c_2}|-\sqrt{2}\alpha\rangle_{d_2}|\sqrt{2}\alpha\rangle_{e_2}\\ &\quad +\xi\gamma\beta|-\rangle_a|+\rangle_b|+\rangle_c|-\sqrt{2}\alpha\rangle_{a_2}|\alpha\rangle_b|\sqrt{2}\alpha\rangle_{c_2}|0\rangle_{d_2}|0\rangle_{e_2}\\ &\quad +\beta^2\xi|-\rangle_a|+\rangle_b|+\rangle_c|-\sqrt{2}\alpha\rangle_{a_2}|\alpha\rangle_b|0\rangle_{c_2}|0\rangle_{d_2}|\sqrt{2}\alpha\rangle_{e_2}).\end{aligned} \tag{15}$$

From the above equation, by choosing the cases where no photon is in spatial modes d_2 and e_2, they obtain

$$\begin{aligned}&|\phi\rangle_{a_2bc_2}\\ &= N_1N_2(\gamma\xi\beta|+\rangle_a|+\rangle_b|-\rangle_c|\sqrt{2}\alpha\rangle_{a_2}|\alpha\rangle_b|-\sqrt{2}\alpha\rangle_{c_2}\\ &\quad +\beta\xi\gamma|+\rangle_a|-\rangle_b|+\rangle_c|\sqrt{2}\alpha\rangle_{a_2}|-\alpha\rangle_b|\sqrt{2}\alpha\rangle_{c_2}\\ &\quad +\xi\gamma\beta|-\rangle_a|+\rangle_b|+\rangle_c|-\sqrt{2}\alpha\rangle_{a_2}|\alpha\rangle_b|\sqrt{2}\alpha\rangle_{c_2}).\end{aligned} \tag{16}$$

Similar to the case in the previous section, they have to decrease the amplitude of the coherent state using QND. As shown in Fig. 2, they let the coherent states in spatial modes a_2 and c_2 pass through BS2 and BS4, respectively, which makes state $|\phi\rangle_{a_2bc_2}$ become

$$\begin{aligned}&|\phi\rangle_{a_3a_4bc_3c_4}\\ =\ &\frac{1}{\sqrt{3}}(|+\rangle_a|+\rangle_b|-\rangle_c|\alpha\rangle_{a_3}|\alpha\rangle_{a_4}|\alpha\rangle_b|-\alpha\rangle_{c_3}|-\alpha\rangle_{c_4}\\ &+|+\rangle_a|-\rangle_b|+\rangle_c|\alpha\rangle_{a_3}|\alpha\rangle_{a_4}|-\alpha\rangle_b|\alpha\rangle_{c_3}|\alpha\rangle_{c_4}\\ &+|-\rangle_a|+\rangle_b|+\rangle_c|-\alpha\rangle_{a_3}|-\alpha\rangle_{a_4}|\alpha\rangle_b|\alpha\rangle_{c_3}|\alpha\rangle_{c_4}).\end{aligned}\tag{17}$$

Finally, by measuring the states in a_4 and c_4 modes, they can obtain the maximally entangled HES

$$\begin{aligned}&|\phi\rangle_{a_3bc_3}\\ =\ &\frac{1}{\sqrt{3}}(|+\rangle_a|+\rangle_b|-\rangle_c|\alpha\rangle_{a_3}|\alpha\rangle_b|-\alpha\rangle_{c_3}\\ &+|+\rangle_a|-\rangle_b|+\rangle_c|\alpha\rangle_{a_3}|-\alpha\rangle_b|\alpha\rangle_{c_3}\\ &+|-\rangle_a|+\rangle_b|+\rangle_c|-\alpha\rangle_{a_3}|\alpha\rangle_b|\alpha\rangle_{c_3})\\ =\ &\frac{1}{\sqrt{3}}(|0_L0_L1_L\rangle_{abc}+|0_L1_L0_L\rangle_{abc}+|1_L0_L0_L\rangle_{abc}).\end{aligned}\tag{18}$$

The probability of obtaining the state in Eq. (18) is $P_2 = 3|N_1N_2\gamma\beta\xi|^2$.

4. Entanglement concentration for arbitrary cluster-type HES

In this section, we will discuss the ECP for the cluster-type HES, where the initial less-entangled HES can be written as

$$\begin{aligned}&|\varphi\rangle_{abcd}\\ =\ &\xi|0_L0_L0_L0_L\rangle_{abcd}+\beta|0_L0_L1_L1_L\rangle_{abcd}\\ &+\gamma|1_L1_L0_L0_L\rangle_{abcd}-\delta|1_L1_L1_L1_L\rangle_{abcd}\\ =\ &\xi|+\rangle_a|+\rangle_b|+\rangle_c|+\rangle_d|\alpha\rangle_a|\alpha\rangle_b|\alpha\rangle_c|\alpha\rangle_d\\ &+\beta|+\rangle_a|+\rangle_b|-\rangle_c|-\rangle_d|\alpha\rangle_a|\alpha\rangle_b|-\alpha\rangle_c|-\alpha\rangle_d\\ &+\gamma|-\rangle_a|-\rangle_b|+\rangle_c|+\rangle_d|-\alpha\rangle_a|-\alpha\rangle_b|\alpha\rangle_c|\alpha\rangle_d\\ &-\delta|-\rangle_a|-\rangle_b|-\rangle_c|-\rangle_d|-\alpha\rangle_a|-\alpha\rangle_b|-\alpha\rangle_c|-\alpha\rangle_d.\end{aligned}\tag{19}$$

Here, $|\xi|^2+|\beta|^2|+|\gamma|^2+|\delta|^2=1$. As shown in Fig. 3, Alice, Bob, Charlie, and Dick share the less-entangled cluster-type HES. The subscripts a, b, c, and d denote the states belonging to Alice, Bob, Charlie, and Dick, respectively. In order to realize the concentration, they require a pair of auxiliary entangled coherent states and an auxiliary single coherent state. The whole auxiliary ECS can be prepared by Bob or Charlie, and shared by Bob and Charlie. The ECS can be written as

$$\begin{aligned}|\varphi\rangle_{ef} =\ &N_3(\xi|-\alpha\rangle_e|\alpha\rangle_f+\beta|-\alpha\rangle_e|-\alpha\rangle_f\\ &+\gamma|\alpha\rangle_e|\alpha\rangle_f+\delta|\alpha\rangle_e|-\alpha\rangle_f).\end{aligned}\tag{20}$$

The normalized coefficient $N_3=[\xi^2+\beta^2+\gamma^2+\delta^2+2(\xi\beta+\xi\gamma+\xi\delta+\beta\gamma+\beta\delta+\gamma\delta)\,\mathrm{e}^{-8|\alpha|^2}]^{-1/2}$. The auxiliary single coherent state is prepared by Dick and can be written as

$$|\varphi\rangle_g = N_4(\xi\gamma|-\alpha\rangle_g+\beta\delta|\alpha\rangle_g).\tag{21}$$

Here, the normalized coefficient $N_4=[(\xi\gamma)^2+(\beta\delta)^2+2\xi\beta\gamma\delta\,\mathrm{e}^{-2|\alpha|^2}]^{-1/2}$. The ECP can be realized in two steps. In the first step, Alice and Charlie prepare and share the auxiliary ECS $|\varphi\rangle_{ef}$ in the spatial modes e and f, respectively. Therefore, the whole system can be described as

$$|\varphi\rangle_{abcdef} = |\psi\rangle_{abcd}\otimes|\psi\rangle_{ef}.\tag{22}$$

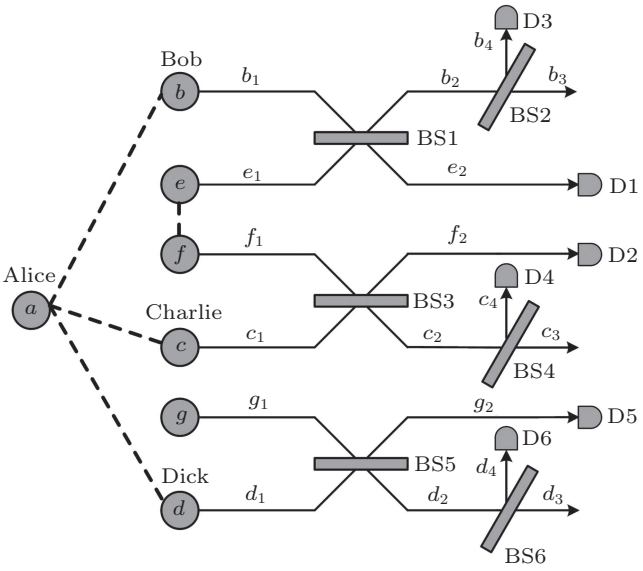


Fig. 3. (color online) Schematic diagram showing the principle of the ECP for cluster-type HES. Alice, Bob, Charlie, and Dick share the HES, and own the hybrid qubits a, b, c, and d, respectively.

It can be written as

$$\begin{aligned}&|\varphi\rangle_{ab_1c_1de_1f_1}\\ =\ &N_3(\xi^2|+\rangle_a|+\rangle_b|+\rangle_c|+\rangle_d|\alpha\rangle_a|\alpha\rangle_{b_1}|\alpha\rangle_{c_1}|\alpha\rangle_d|-\alpha\rangle_{e_1}|\alpha\rangle_{f_1}+\xi\beta|+\rangle_a|+\rangle_b|+\rangle_c|+\rangle_d|\alpha\rangle_a|\alpha\rangle_{b_1}|\alpha\rangle_{c_1}|\alpha\rangle_d|-\alpha\rangle_{e_1}|-\alpha\rangle_{f_1}\\ &+\xi\gamma|+\rangle_a|+\rangle_b|+\rangle_c|+\rangle_d|\alpha\rangle_a|\alpha\rangle_{b_1}|\alpha\rangle_{c_1}|\alpha\rangle_d|\alpha\rangle_{e_1}|\alpha\rangle_{f_1}+\xi\delta|+\rangle_a|+\rangle_b|+\rangle_c|+\rangle_d|\alpha\rangle_a|\alpha\rangle_{b_1}|\alpha\rangle_{c_1}|\alpha\rangle_d|\alpha\rangle_{e_1}|-\alpha\rangle_{f_1}\\ &+\beta\xi|+\rangle_a|+\rangle_b|-\rangle_c|-\rangle_d|\alpha\rangle_a|\alpha\rangle_{b_1}|-\alpha\rangle_{c_1}|-\alpha\rangle_d|-\alpha\rangle_{e_1}|\alpha\rangle_{f_1}+\beta^2|+\rangle_a|+\rangle_b|-\rangle_c|-\rangle_d|\alpha\rangle_a|\alpha\rangle_{b_1}|-\alpha\rangle_{c_1}|-\alpha\rangle_d|-\alpha\rangle_{e_1}|-\alpha\rangle_{f_1}\\ &+\beta\gamma|+\rangle_a|+\rangle_b|-\rangle_c|-\rangle_d|\alpha\rangle_a|\alpha\rangle_{b_1}|-\alpha\rangle_{c_1}|-\alpha\rangle_d|\alpha\rangle_{e_1}|\alpha\rangle_{f_1}+\beta\delta|+\rangle_a|+\rangle_b|-\rangle_c|-\rangle_d|\alpha\rangle_a|\alpha\rangle_{b_1}|-\alpha\rangle_{c_1}|-\alpha\rangle_d|\alpha\rangle_{e_1}|-\alpha\rangle_{f_1}\end{aligned}$$

$$+\gamma\xi|-\rangle_a|-\rangle_b|+\rangle_c|+\rangle_d|-\alpha\rangle_a|-\alpha\rangle_{b_1}|\alpha\rangle_{c_1}|\alpha\rangle_d|-\alpha\rangle_{e_1}|\alpha\rangle_{f_1}+\gamma\beta|-\rangle_a|-\rangle_b|+\rangle_c|+\rangle_d|-\alpha\rangle_a|-\alpha\rangle_{b_1}|\alpha\rangle_{c_1}|\alpha\rangle_d|-\alpha\rangle_{e_1}|-\alpha\rangle_{f_1}$$
$$+\gamma^2|-\rangle_a|-\rangle_b|+\rangle_c|+\rangle_d|-\alpha\rangle_a|-\alpha\rangle_{b_1}|\alpha\rangle_{c_1}|\alpha\rangle_d|\alpha\rangle_{e_1}|\alpha\rangle_{f_1}+\gamma\delta|-\rangle_a|-\rangle_b|+\rangle_c|+\rangle_d|-\alpha\rangle_a|-\alpha\rangle_{b_1}|\alpha\rangle_{c_1}|\alpha\rangle_d|\alpha\rangle_{e_1}|-\alpha\rangle_{f_1}$$
$$-\xi\delta|-\rangle_a|-\rangle_b|-\rangle_c|-\rangle_d|-\alpha\rangle_a|-\alpha\rangle_{b_1}|-\alpha\rangle_{c_1}|-\alpha\rangle_d|-\alpha\rangle_{e_1}|\alpha\rangle_{f_1}-\delta\beta|-\rangle_a|-\rangle_b|-\rangle_c|-\rangle_d|-\alpha\rangle_a|-\alpha\rangle_{b_1}|-\alpha\rangle_{c_1}|-\alpha\rangle_d|-\alpha\rangle_{e_1}|-\alpha\rangle_{f_1}$$
$$-\gamma\delta|-\rangle_a|-\rangle_b|-\rangle_c|-\rangle_d|-\alpha\rangle_a|-\alpha\rangle_{b_1}|-\alpha\rangle_{c_1}|-\alpha\rangle_d|\alpha\rangle_{e_1}|\alpha\rangle_{f_1}-\delta^2|-\rangle_a|-\rangle_b|-\rangle_c|-\rangle_d|-\alpha\rangle_a|-\alpha\rangle_{b_1}|-\alpha\rangle_{c_1}|-\alpha\rangle_d|\alpha\rangle_{e_1}|-\alpha\rangle_{f_1}). \tag{23}$$

As shown in Fig. 3, subsequently, they let the coherent state in spatial modes b_1 and e_1 pass through BS1. Meanwhile, they also let the coherent state in spatial modes c_1 and f_1 pass through BS3. The state $|\varphi\rangle_{ab_1c_1de_1f_1}$ evolves to

$$\begin{aligned}
&|\varphi\rangle_{ab_1c_1de_1f_1}\rightarrow|\varphi\rangle_{ab_2c_2de_2f_2}\\
&=N_3(\xi^2|+\rangle_a|+\rangle_b|+\rangle_c|+\rangle_d|\alpha\rangle_a|0\rangle_{b_2}|\sqrt{2}\alpha\rangle_{c_2}|\alpha\rangle_d|\sqrt{2}\alpha\rangle_{e_2}|0\rangle_{f_2}+\xi\beta|+\rangle_a|+\rangle_b|+\rangle_c|+\rangle_d|\alpha\rangle_a|0\rangle_{b_2}|0\rangle_{c_2}|\alpha\rangle_d|\sqrt{2}\alpha\rangle_{e_2}|\sqrt{2}\alpha\rangle_{f_2}\\
&+\xi\gamma|+\rangle_a|+\rangle_b|+\rangle_c|+\rangle_d|\alpha\rangle_a|\sqrt{2}\alpha\rangle_{b_2}|\sqrt{2}\alpha\rangle_{c_2}|\alpha\rangle_d|0\rangle_{e_2}|0\rangle_{f_2}+\xi\delta|+\rangle_a|+\rangle_b|+\rangle_c|+\rangle_d|\alpha\rangle_a|\sqrt{2}\alpha\rangle_{b_2}|0\rangle_{c_2}|\alpha\rangle_d|0\rangle_{e_2}|\sqrt{2}\alpha\rangle_{f_2}\\
&+\beta\xi|+\rangle_a|+\rangle_b|-\rangle_c|\rightarrow\rangle_d|\alpha\rangle_a|0\rangle_{b_2}|0\rangle_{c_2}|-\alpha\rangle_d|\sqrt{2}\alpha\rangle_{e_2}|-\sqrt{2}\alpha\rangle_{f_2}+\beta^2|+\rangle_a|+\rangle_b|-\rangle_c|-\rangle_d|\alpha\rangle_a|0\rangle_{b_2}|-\sqrt{2}\alpha\rangle_{c_2}|-\alpha\rangle_d|\sqrt{2}\alpha\rangle_{e_2}|0\rangle_{f_2}\\
&+\beta\gamma|+\rangle_a|+\rangle_b|-\rangle_c|-\rangle_d|\alpha\rangle_a|\sqrt{2}\alpha\rangle_{b_2}|0\rangle_{c_2}|-\alpha\rangle_d|0\rangle_{e_2}|-\sqrt{2}\alpha\rangle_{f_2}+\beta\delta|+\rangle_a|+\rangle_b|-\rangle_c|-\rangle_d|\alpha\rangle_a|\sqrt{2}\alpha\rangle_{b_2}|-\sqrt{2}\alpha\rangle_{c_2}|-\alpha\rangle_d|0\rangle_{e_2}|0\rangle_{f_2}\\
&+\gamma\xi|-\rangle_a|-\rangle_b|+\rangle_c|+\rangle_d|-\alpha\rangle_a|-\sqrt{2}\alpha\rangle_{b_2}|\sqrt{2}\alpha\rangle_{c_2}|\alpha\rangle_d|0\rangle_{e_2}|0\rangle_{f_2}+\gamma\beta|-\rangle_a|-\rangle_b|+\rangle_c|+\rangle_d|-\alpha\rangle_a|-\sqrt{2}\alpha\rangle_{b_2}|0\rangle_{c_2}|\alpha\rangle_d|0\rangle_{e_2}|\sqrt{2}\alpha\rangle_{f_2}\\
&+\gamma^2|-\rangle_a|-\rangle_b|+\rangle_c|+\rangle_d|-\alpha\rangle_a|0\rangle_{b_2}|\sqrt{2}\alpha\rangle_{c_2}|\alpha\rangle_d|-\sqrt{2}\alpha\rangle_{e_2}|0\rangle_{f_2}+\gamma\delta|-\rangle_a|-\rangle_b|+\rangle_c|+\rangle_d|-\alpha\rangle_a|0\rangle_{b_2}|0\rangle_{c_2}|\alpha\rangle_d|-\sqrt{2}\alpha\rangle_{e_2}|\sqrt{2}\alpha\rangle_{f_2}\\
&-\xi\delta|-\rangle_a|-\rangle_b|-\rangle_c|-\rangle_d|-\alpha\rangle_a|-\sqrt{2}\alpha\rangle_{b_2}|0\rangle_{c_2}|-\alpha\rangle_d|0\rangle_{e_2}|-\sqrt{2}\alpha\rangle_{f_2}\\
&-\delta\beta|-\rangle_a|-\rangle_b|-\rangle_c|-\rangle_d|-\alpha\rangle_a|-\sqrt{2}\alpha\rangle_{b_2}|-\sqrt{2}\alpha\rangle_{c_2}|-\alpha\rangle_d|0\rangle_{e_2}|0\rangle_{f_2}\\
&-\gamma\delta|-\rangle_a|-\rangle_b|-\rangle_c|-\rangle_d|-\alpha\rangle_a|0\rangle_{b_2}|0\rangle_{c_2}|-\alpha\rangle_d|-\sqrt{2}\alpha\rangle_{e_2}|-\sqrt{2}\alpha\rangle_{f_2}\\
&-\delta^2|-\rangle_a|-\rangle_b|-\rangle_c|-\rangle_d|-\alpha\rangle_a|0\rangle_{b_2}|-\sqrt{2}\alpha\rangle_{c_2}|-\alpha\rangle_d|-\sqrt{2}\alpha\rangle_{e_2}|0\rangle_{f_2}).
\end{aligned}\tag{24}$$

By choosing the cases that the spatial modes e_2 and f_2 both have no photon, the state becomes

$$\begin{aligned}
|\varphi\rangle_{ab_2c_2d}=&N_3(\xi\gamma(|+\rangle_a|+\rangle_b|+\rangle_c|+\rangle_d|\alpha\rangle_a|\sqrt{2}\alpha\rangle_{b_2}|\sqrt{2}\alpha\rangle_{c_2}|\alpha\rangle_d+|-\rangle_a|-\rangle_b|+\rangle_c|+\rangle_d|-\alpha\rangle_a|\\
&-\sqrt{2}\alpha\rangle_{b_2}|\sqrt{2}\alpha\rangle_{c_2}|\alpha\rangle_d)+\beta\delta(|+\rangle_a|+\rangle_b|-\rangle_c|-\rangle_d|\alpha\rangle_a|\sqrt{2}\alpha\rangle_{b_2}|\\
&-\sqrt{2}\alpha\rangle_{c_2}|-\alpha\rangle_d-|-\rangle_a|-\rangle_b|-\rangle_c|-\rangle_d|-\alpha\rangle_a|-\sqrt{2}\alpha\rangle_{b_2}|-\sqrt{2}\alpha\rangle_{c_2}|-\alpha\rangle_d)).
\end{aligned}\tag{25}$$

Then they let the coherent states in spatial mode b_2 and c_2 pass through BS2 and BS4, respectively, which makes the state $|\varphi\rangle_{ab_2c_2d}$ become

$$\begin{aligned}
&|\varphi\rangle_{ab_2c_2d}\rightarrow|\varphi\rangle_{ab_3b_4c_3c_4d}\\
&=N_3(\xi\gamma(|+\rangle_a|+\rangle_b|+\rangle_c|+\rangle_d|\alpha\rangle_a|\alpha\rangle_{b_3}|\alpha\rangle_{b_4}|\alpha\rangle_{c_3}|\alpha\rangle_{c_4}|\alpha\rangle_d+|-\rangle_a|-\rangle_b|+\rangle_c|+\rangle_d|-\alpha\rangle_a|-\alpha\rangle_{b_3}|-\alpha\rangle_{b_4}|\alpha\rangle_{c_3}|\alpha\rangle_{c_4}|\alpha\rangle_d)\\
&+\beta\delta(|+\rangle_a|+\rangle_b|-\rangle_c|-\rangle_d|\alpha\rangle_a|\alpha\rangle_{b_3}|\alpha\rangle_{b_4}|-\alpha\rangle_{c_3}|-\alpha\rangle_{c_4}|-\alpha\rangle_d-|-\rangle_a|-\rangle_b|-\rangle_c|-\rangle_d|-\alpha\rangle_a|-\alpha\rangle_{b_3}|-\alpha\rangle_{b_4}|-\alpha\rangle_{c_3}|-\alpha\rangle_{c_4}|-\alpha\rangle_d)).
\end{aligned}\tag{26}$$

Finally, by measuring the coherent states in b_4 and c_4 with QND, which makes $|\pm\alpha\rangle$ undistinguishable, state $|\varphi\rangle_{ab_2c_2d}$ becomes

$$\begin{aligned}
&|\varphi\rangle_{ab_2c_2d}\rightarrow|\varphi\rangle_{ab_3c_3d}\\
&=N_3(\xi\gamma(|+\rangle_a|+\rangle_b|+\rangle_c|+\rangle_d|\alpha\rangle_a|\alpha\rangle_{b_3}|\alpha\rangle_{c_3}|\alpha\rangle_d+|-\rangle_a|-\rangle_b|+\rangle_c|+\rangle_d|-\alpha\rangle_a|-\alpha\rangle_{b_3}|\alpha\rangle_{c_3}|\alpha\rangle_d)\\
&+\beta\delta(|+\rangle_a|+\rangle_b|-\rangle_c|-\rangle_d|\alpha\rangle_a|\alpha\rangle_{b_3}|-\alpha\rangle_{c_3}|-\alpha\rangle_d-|-\rangle_a|-\rangle_b|-\rangle_c|-\rangle_d|-\alpha\rangle_a|-\alpha\rangle_{b_3}|-\alpha\rangle_{c_3}|-\alpha\rangle_d)).
\end{aligned}\tag{27}$$

In the second step, they first prepare the ancillary single coherent state in spatial mode g as shown in Eq. (21). The state $|\varphi\rangle_{ab_3c_3d}$ combined with $|\varphi\rangle_g$ can be written as

$$\begin{aligned}
&|\varphi\rangle_{ab_3c_3d_1g_1}=|\varphi\rangle_{ab_3c_3d}\otimes|\varphi\rangle_g\\
&=N_3N_4(\xi^2\gamma^2(|+\rangle_a|+\rangle_b|+\rangle_c|+\rangle_d|\alpha\rangle_a|\alpha\rangle_{b_3}|\alpha\rangle_{c_3}|\alpha\rangle_{d_1}|-\alpha\rangle_{g_1}+|-\rangle_a|-\rangle_b|+\rangle_c|+\rangle_d|-\alpha\rangle_a|-\alpha\rangle_{b_3}|\alpha\rangle_{c_3}|\alpha\rangle_{d_1}|-\alpha\rangle_{g_1})\\
&+\beta^2\delta^2(|+\rangle_a|+\rangle_b|-\rangle_c|-\rangle_d|\alpha\rangle_a|\alpha\rangle_{b_3}|-\alpha\rangle_{c_3}|-\alpha\rangle_{d_1}|\alpha\rangle_{g_1}-|-\rangle_a|-\rangle_b|-\rangle_c|-\rangle_d|-\alpha\rangle_a|-\alpha\rangle_{b_3}|-\alpha\rangle_{c_3}|-\alpha\rangle_{d_1}|\alpha\rangle_{g_1})\\
&+\xi\gamma\beta\delta(|+\rangle_a|+\rangle_b|+\rangle_c|+\rangle_d|\alpha\rangle_a|\alpha\rangle_{b_3}|\alpha\rangle_{c_3}|\alpha\rangle_{d_1}|\alpha\rangle_{g_1}+|-\rangle_a|-\rangle_b|+\rangle_c|+\rangle_d|-\alpha\rangle_a|-\alpha\rangle_{b_3}|\alpha\rangle_{c_3}|\alpha\rangle_{d_1}|\alpha\rangle_{g_1}\\
&+|+\rangle_a|+\rangle_b|-\rangle_c|-\rangle_d|\alpha\rangle_a|\alpha\rangle_{b_3}|-\alpha\rangle_{c_3}|-\alpha\rangle_{d_1}|-\alpha\rangle_{g_1}-|-\rangle_a|-\rangle_b|-\rangle_c|-\rangle_d|-\alpha\rangle_a|-\alpha\rangle_{b_3}|-\alpha\rangle_{c_3}|-\alpha\rangle_{d_1}|-\alpha\rangle_{g_1})).
\end{aligned}\tag{28}$$

Then they let the coherent states in spatial modes g_1 and d_1 pass through BS5. The state in Eq. (28) evolves to

$$\begin{aligned}
&|\varphi\rangle_{ab_3c_3d_1g_1} \to |\varphi\rangle_{ab_3c_3d_2g_2}\\
&= N_3N_4(\xi^2\gamma^2(|+\rangle_a|+\rangle_b|+\rangle_c|+\rangle_d|\alpha\rangle_a|\alpha\rangle_{b_3}|\alpha\rangle_{c_3}|0\rangle_{d_2}|\sqrt{2}\alpha\rangle_{g_2} + |-\rangle_a|-\rangle_b|+\rangle_c|+\rangle_d|-\alpha\rangle_a|-\alpha\rangle_{b_3}|\alpha\rangle_{c_3}|0\rangle_{d_2}|\sqrt{2}\alpha\rangle_{g_2})\\
&\quad + \beta^2\delta^2(|+\rangle_a|+\rangle_b|-\rangle_c|-\rangle_d|\alpha\rangle_a|\alpha\rangle_{b_3}|-\alpha\rangle_{c_3}|0\rangle_{d_2}|-\sqrt{2}\alpha\rangle_{g_2} - |-\rangle_a|-\rangle_b|-\rangle_c|-\rangle_d|-\alpha\rangle_a|-\alpha\rangle_{b_3}|-\alpha\rangle_{c_3}|0\rangle_{d_2}|-\sqrt{2}\alpha\rangle_{g_2})\\
&\quad + \xi\gamma\beta\delta(|+\rangle_a|+\rangle_b|+\rangle_c|+\rangle_d|\alpha\rangle_a|\alpha\rangle_{b_3}|\alpha\rangle_{c_3}|\sqrt{2}\alpha\rangle_{d_2}|0\rangle_{g_2} + |-\rangle_a|-\rangle_b|+\rangle_c|+\rangle_d|-\alpha\rangle_a|-\alpha\rangle_{b_3}|\alpha\rangle_{c_3}|\sqrt{2}\alpha\rangle_{d_2}|0\rangle_{g_2}\\
&\quad + |+\rangle_a|+\rangle_b|-\rangle_c|-\rangle_d|\alpha\rangle_a|\alpha\rangle_{b_3}|-\alpha\rangle_{c_3}|-\sqrt{2}\alpha\rangle_{d_2}|0\rangle_{g_2} - |-\rangle_a|-\rangle_b|-\rangle_c|-\rangle_d|-\alpha\rangle_a|-\alpha\rangle_{b_3}|-\alpha\rangle_{c_3}|-\sqrt{2}\alpha\rangle_{d_2}|0\rangle_{g_2})).
\end{aligned} \tag{29}$$

They select the cases in which the spatial mode g_2 has no photon, which makes the state in Eq. (29) become

$$\begin{aligned}
&|\varphi\rangle ab_3c_3d_2\\
&= N_3N_4(\xi\gamma\beta\delta(|+\rangle_a|+\rangle_b|+\rangle_c|+\rangle_d|\alpha\rangle_a|\alpha\rangle_{b_3}|\alpha\rangle_{c_3}|\sqrt{2}\alpha\rangle_{d_2} + |-\rangle_a|-\rangle_b|+\rangle_c|+\rangle_d|-\alpha\rangle_a|-\alpha\rangle_{b_3}|\alpha\rangle_{c_3}|\sqrt{2}\alpha\rangle_{d_2}\\
&\quad + |+\rangle_a|+\rangle_b|-\rangle_c|-\rangle_d|\alpha\rangle_a|\alpha\rangle_{b_3}|-\alpha\rangle_{c_3}|-\sqrt{2}\alpha\rangle_{d_2} - |-\rangle_a|-\rangle_b|-\rangle_c|-\rangle_d|-\alpha\rangle_a|-\alpha\rangle_{b_3}|-\alpha\rangle_{c_3}|-\sqrt{2}\alpha\rangle_{d_2})).
\end{aligned} \tag{30}$$

For obtaining the maximally entangled state, they let the coherent state in spatial mode d_2 pass through BS6, which makes the state in Eq. (30) evolve to

$$\begin{aligned}
&|\varphi\rangle_{ab_3c_3d_3d_4}\\
&= N_3N_4(\xi\gamma\beta\delta(|+\rangle_a|+\rangle_b|+\rangle_c|+\rangle_d|\alpha\rangle_a|\alpha\rangle_{b_3}|\alpha\rangle_{c_3}|\alpha\rangle_{d_3}|\alpha\rangle_{d_4} + |-\rangle_a|-\rangle_b|+\rangle_c|+\rangle_d|-\alpha\rangle_a|-\alpha\rangle_{b_3}|\alpha\rangle_{c_3}|\alpha\rangle_{d_3}|\alpha\rangle_{d_4}\\
&\quad + |+\rangle_a|+\rangle_b|-\rangle_c|-\rangle_d|\alpha\rangle_a|\alpha\rangle_{b_3}|-\alpha\rangle_{c_3}|-\alpha\rangle_{d_3}|-\alpha\rangle_{d_4} - |-\rangle_a|-\rangle_b|-\rangle_c|-\rangle_d|-\alpha\rangle_a|-\alpha\rangle_{b_3}|-\alpha\rangle_{c_3}|-\alpha\rangle_{d_3}|-\alpha\rangle_{d_4})).
\end{aligned} \tag{31}$$

From Eq. (31), they also use the QND measurement to ensure the states $|\pm\alpha\rangle$ in spatial mode d_4 are undistinguishable, then they obtain the maximally entangled cluster-type HES

$$\begin{aligned}
|\varphi\rangle_{ab_3c_3d_3} &= \frac{1}{2}(|+\rangle_a|+\rangle_b|+\rangle_c|+\rangle_d|\alpha\rangle_a|\alpha\rangle_{b_3}|\alpha\rangle_{c_3}|\alpha\rangle_{d_3} + |-\rangle_a|-\rangle_b|+\rangle_c|+\rangle_d|-\alpha\rangle_a|-\alpha\rangle_{b_3}|\alpha\rangle_{c_3}|\alpha\rangle_{d_3}\\
&\quad + |+\rangle_a|+\rangle_b|-\rangle_c|-\rangle_d|\alpha\rangle_a|\alpha\rangle_{b_3}|-\alpha\rangle_{c_3}|-\alpha\rangle_{d_3} - |-\rangle_a|-\rangle_b|-\rangle_c|-\rangle_d|-\alpha\rangle_a|-\alpha\rangle_{b_3}|-\alpha\rangle_{c_3}|-\alpha\rangle_{d_3})\\
&= \frac{1}{2}(|0_L0_L0_L0_L\rangle_{abcd} + |0_L0_L1_L1_L\rangle_{abcd} + |1_L1_L0_L0_L\rangle_{abcd} - |1_L1_L1_L1_L\rangle_{abcd}).
\end{aligned} \tag{32}$$

The success probability to obtain the maximally entangled HES from the initial partially entangled state is $P_3 = 4|N_3N_4\xi\gamma\beta\delta|^2$.

5. Discussion

Thus far, we have described our protocols which mainly use BS and QND to complete the task. We described the ECPs for Bell-type HES, W-type HES, and cluster-type HES, respectively. With the post-section principle, we can ultimately obtain a maximally entangled HES with some probability. In Figs. 4–6, we show the success probability of each ECP.

In Fig. 4, we show the success probability P_1 altered with γ. We let $\alpha = 0.5$, 1, and 4 respectively. In Fig. 5, we show the success probability P_2 altered with γ and ξ. In this ECP, P_2 can reach the maximum value 1/4 with $\gamma = \xi = 1/\sqrt{3}$. In Fig. 6, we show the total success probability P_3 altered with β. We let $\gamma = \xi = 1/2$. It shows that P_3 can reach the maximum value 1/8 with $\beta = 1/2$. Meanwhile, the other three coefficients are equal to 1/2.

In our ECPs, the BS plays a key role to realize the concentration. We take the first ECP for example. As shown in Fig. 1, we choose the case that the spatial mode c_2 has no photon. It means that we pick up the even parity states $|\alpha\rangle|\alpha\rangle$ and $|-\alpha\rangle|-\alpha\rangle$, which is similar to the previous ECPs.[34,51] We should point out that a coherent state $|\alpha\rangle$ also has some probability to have no photon. Then, it will induce an error. Fortunately, the error probability is $|\langle 0|\alpha\rangle|^2$. It is about 10^{-5} if $\alpha = 2$, which can be neglected. In our ECPs, the last step is to decrease the amplitude of the coherent state to recover it to the form of the original state. Actually, such an operation is

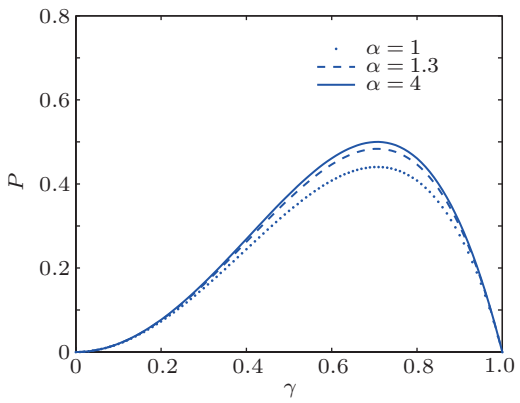


Fig. 4. (color online) The success probability P_1 of obtaining a maximally entangled Bell-type HES. P_1 is altered with the coefficient γ of the initial partially entangled state, where $\gamma \in (0, 1)$. The α is the amplitude of the coherent state.

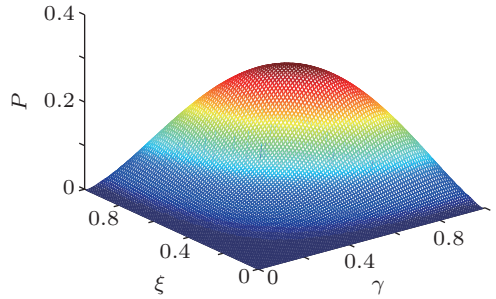


Fig. 5. (color online) The success probability P_2 of obtaining a maximally entangled W-type HES. P_2 is altered with γ and β, which are coefficients of the initial partially entangled state, where $\gamma \in (0,1)$, $\beta \in (0,1)$, and $\gamma^2+\beta^2<1$. The amplitude of the coherent state (α) is set to 2.

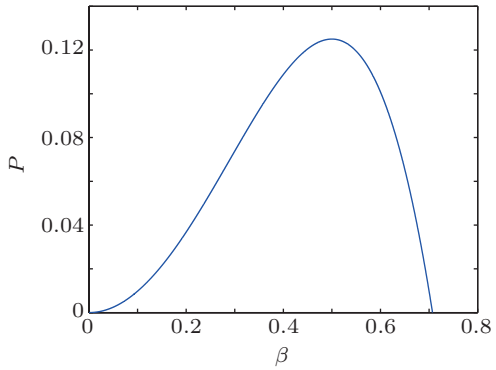


Fig. 6. (color online) The success probability P_3 of obtaining a maximally entangled cluster-type HES. P_3 is altered with β, where we assume that $\xi=1/2$, $\gamma=1/2$, and $\beta \in (0,\sqrt{2}/2)$. The amplitude of the coherent state (α) is set to 2.

not always necessary. In the practical transmission, the coherent state may suffer from the photon loss, which will decrease the amplitude of the coherent state. We should exploit the amplification technology to increase the coherent state. Interestingly, our ECPs can also increase the amplitude of the coherent state. The increased amplitude of the coherent state will become an advantage of these ECPs. In the second and the third ECPs, the whole protocols are both divided into two steps. Actually, in a practical experiment, both steps should be performed simultaneously because of the post-selection principle.

6. Summary

We have proposed three ECPs for three types of HES, respectively. First, we described the ECP for the Bell-type HES with arbitrary coefficients, and the probability of the ECP is $2|N_0\gamma\beta|^2$. Second, we introduced the ECP for the W-type HES. We used two single coherent states to obtain the maximally entangled W-type HES with the probability of $3|N_1N_2\gamma\beta\xi|^2$. Finally, we described the ECP for the cluster-type HES. With the help of a pair of auxiliary particles of ECS and an auxiliary single coherent state, we obtained the maximally entangled cluster-type HES, and this ECP has a success probability $4|N_3N_4\xi\gamma\beta\delta|^2$. These ECPs are all based on the linear optics which is feasible in future experiments. We hope that these ECPs based on HESs are useful in many applications of quantum information.

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