Lensless ghost imaging through the strongly scattering medium*

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Lensless ghost imaging has attracted much interest in recent years due to its profound physics and potential applications. In this paper we report studies of the robust properties of the lensless ghost imaging system with a pseudo-thermal light source in a strongly scattering medium. The effects of the positions of the strong medium on the ghost imaging are investigated. In the lensless ghost imaging system, a pseudo-thermal light is split into two correlated beams by a beam splitter. One beam goes to a charge-coupled detector camera, labeled as CCD2. The other beam goes to an object and then is collected in another charge-coupled detector camera, labeled as CCD1, which serves as a bucket detector. When the strong medium, a pane of ground glass disk, is placed between the object and CCD1, the bucket detector, the quality of ghost imaging is barely affected and a good image could still be obtained. The quality of the ghost imaging can also be maintained, even when the ground glass is rotating, which is the strongest scattering medium so far. However, when the strongly scattering medium is present in the optical path from the light source to CCD2 or the object, the lensless ghost imaging system hardly retrieves the image of the object. A theoretical analysis in terms of the second-order correlation function is also provided.

Keywords: lensless ghost imaging, second-order correlation, strongly scattering medium, robust property

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1. Introduction

Ghost imaging (GI), also known as correlated imaging, is a nonlocal imaging technology which retrieves an unknown object by measuring the second-order intensity correlation function between the test light field and the reference light field. The test light field goes to the object and then is collected by a bucket detector without spatial resolution, and the reference light field is detected by a spatial resolving detector without interacting with the object. The spatially point-to-point correlation of both entangled light source[1,2] and the thermal light[3–8] have been applied to achieve GI. GI also touches the fundamental principles of physics and aroused heated debates and discussions.[9–12] In addition, many different aspects of the GI have been studied, such as high-order GI,[13–15] compressed sensing GI,[16–20] differential ghost imaging (DGI),[21] normalized ghost imaging (NGI),[22] and so on.[23–30] In recent years, spatial light modulator (SLM) and digital micro-mirror device (DMD) are being developed, and they have also been employed in GI, for instance in the computational ghost imaging (CGI).[31,32] Based on the CGI, three-dimensional image and full-color image of the object have been obtained.[33,34] Furthermore, the influence of atmospheric turbulence, turbid media, or scattering media on GI have also been investigated.[35–42]

The lensless ghost imaging (LGI), where there is no lens in the optical paths, has drawn much attention in recent years.[43–50] In x-ray or γ-ray imaging, LGI possesses many advantages since the corresponding optical components, e.g., lenses, are very scarce, and are vulnerable to radiation damage. The longitudinal coherence length of LGI was studied in Refs. [51]–[55]. Zhang *et al.* studied the LGI of a phase object.[56] LGIs by measuring reflected photons were reported in Refs. [57] and [58]. For practical applications of LGI, the effects of atmospheric turbulence, turbid media, or scattering media in the optical paths must be studied.[59] In 2009, Cheng analytically derived a formula for LGI, taking into account of atmospheric turbulence, and showed that the turbulence can result in the degradation of the imaging quality.[60] In 2011, Shih *et al.* proposed a turbulence-free GI with non-factorizable, point-to-point correlation of thermal light.[61] Later, Bina *et al.* presented a scheme to achieve the backscattering DGI through the turbid media.[62]

It is desirable to study the influences of the strongly scattering medium on LGI. Ground glass is usually used as a scattering medium in optical systems.[63–65] Therefore, we use a pane of ground glass disk, which brings strong scatter and even attenuation, as the scattering medium in this work. We report the first (to the best of our knowledge) experimental investigation of the robustness of LGI in a scattering medium.

The paper is organized as follows. In Section 2, we present the experimental results and theory analysis of LGI

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system. Section 3 encompasses the study of the robust property of LGI when the scattering medium presents in different optical paths. The discussion and summary are given in Section 4.

2. LGI system

The experimental setup is depicted in Fig. 1. A laser beam working at $\lambda = 780$ nm illuminates a rotating ground glass with an angular frequency at 0.1 rotation per minute (RPM) to produce a pseudo-thermal light beam which is spatially incoherent. Then the pseudo-thermal light beam is divided into two beams by a beam splitter (BS). The test beam interacts with an object and is collected by CCD1 which simulates a bucket detector while the reference beam propagates freely to CCD2. d and d_1 are the respective distances from the light source to the object and to CCD2. The distance from the object to the photosensitive array of CCD1 is d_2. In our experiment we choose $d = d_1 = 30$ cm and $d_2 = 2$ cm. The object is a mask of letters THU depicted in Fig. 2(a). To study the robust property of the system, a strongly scattering medium (a pane of 2-mm thick ground glass disk) are placed in three different positions marked A, B, and C which are sketched respectively in Fig. 1. Position A is in the optical path from the object to CCD1, and position B or C is in the optical path from the light source to the object or CCD2. For simplicity, positions A and B are very close to the object and position C is very close to CCD2.

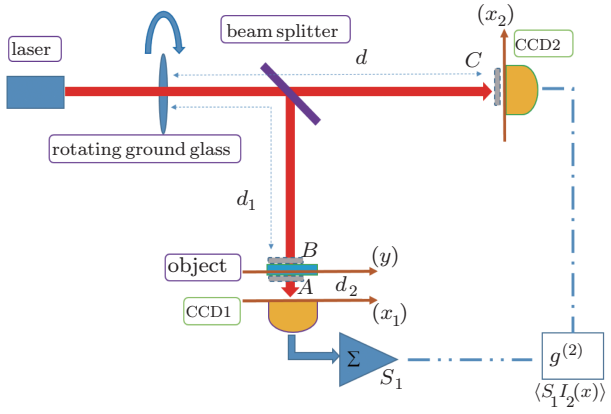


Fig. 1. (color online) Schematic diagram of the lensless ghost imaging system. BS: beam splitter. CCD1, CCD2: charge-coupled device camera (Mintron MTV-1881EX). The ground glass disk is placed in three different positions: (a) behind the object, (b) in front of the object, and (c) in front of CCD2. x_1, x_2 and y are the transverse coordinates on the test plane, the reference plane and the object plane, respectively.

In the classical interpretation of GI,[51] the normalized second-order correlation function between the two spatial points of test and reference beams plays a central role and reads

$$g^{(2)}(x_1,x_2) = \frac{\langle I_1(x_1)I_2(x_2)\rangle}{\langle I_1(x_1)\rangle\langle I_2(x_2)\rangle}, \tag{1}$$

where $I_1(x_1)$ and $I_2(x_2)$ denote the intensities of light fields on the test plane and the reference plane at the transverse coordinates x_1 and x_2, respectively. If the pseudo-thermal light field obeys the Gaussian stochastic process, equation (1) could be rewritten as[46]

$$g^{(2)}(x_1,x_2) = 1 + \frac{|\langle E^*(x_1)E(x_2)\rangle|^2}{\langle I_1(x_1)\rangle\langle I_2(x_2)\rangle}, \tag{2}$$

where $E(x_1)$ and $E(x_2)$ are the light fields on the test detector plane and the reference plane, respectively. By optical coherence theory, the last term of Eq. (2) $|\langle E^*(x_1)E(x_2)\rangle|^2$ takes the form of

$$\left|\langle E^*(x_1)E(x_2)\rangle\right|^2 \propto \left|\int \mathrm{d}u_1\,\mathrm{d}u_2\, G^{(1)}(u_1,u_2) \times h_t^*(x_1,u_1)h_r(x_2,u_2)\right|^2 \tag{3}$$

with $G^{(1)}(u_1,u_2)=I_0\delta(u_1-u_2)$ being the first-order correlation function of the spatially incoherent pseudo-thermal light source, where u_1 and u_2 are the transverse coordinates on light source plane and I_0 is constant. According to Huygens–Fresnel theory and under the paraxial approximation, the point spread function (PSF) of the test beam is

$$h_t(x_1,u_1) = \int \mathrm{d}y \frac{\mathrm{e}^{i2\pi d_1/\lambda}}{\sqrt{\mathrm{i}\lambda d_1}} \exp\left[\frac{\mathrm{i}\pi}{\lambda d_1}(y-u_1)^2\right] t(y) \times \frac{\mathrm{e}^{\mathrm{i}2\pi d_2/\lambda}}{\sqrt{\mathrm{i}\lambda d_2}} \exp\left[\frac{\mathrm{i}\pi}{\lambda d_2}(y-x_1)^2\right], \tag{4}$$

where $t(y)$ is the transmission function of the object. The PSF of the reference beam is

$$h_r(x_2,u_2) = \frac{\mathrm{e}^{\mathrm{i}2\pi d/\lambda}}{\sqrt{\mathrm{i}\lambda d}} \exp\left[\frac{\mathrm{i}\pi}{\lambda d}(x_2-u_2)^2\right]. \tag{5}$$

Combining Eqs. (3)–(5) the second-order correlation function reads[54]

$$|\langle E^*(x_1)E(x_2)\rangle|^2 \propto \left|\int \mathrm{d}y \exp\left[-\frac{\mathrm{i}\pi}{\lambda d_2}(y-x_1)^2 - \frac{\mathrm{i}\pi}{\lambda d}y^2\right] \times t(y)\mathrm{sinc}\left[\frac{\pi D}{\lambda d}(y-x_2)\right]\right|^2, \tag{6}$$

where sinc function is defined as $\mathrm{sinc}(x) = \sin(x)/x$ and the coefficient $\pi D/\lambda d$ determines the spatial resolution of the LGI. Therefore, when the coefficient $\pi D/\lambda d$ is very large, sinc function in Eq. (6) could be approximated as the δ function and the spatial correlation term takes the form of a convolution product

$$|\langle E^*(x_1)E(x_2)\rangle|^2 \propto |t(y)|^2 \otimes \delta(y-x_2), \tag{7}$$

which is the foundation of the LGI. According to Eqs. (6) and (7), the value of d_2 has little influence on LGI as long as it is within the spatial longitudinal coherence length of lensless ghost imaging system with pseudo-thermal light.[52] Therefore, in the optical part from object to CCD1 the LGI is robust against the phase change of the light field.

In this experiment, the traditional ghost imaging algorithm, i.e., normalized iteration algorithm is employed to retrieve the image of the object in the LGI,

$$g^{(2)}(x_2) = \frac{\langle S_1 \cdot I_2(x_2)\rangle}{\langle S_1\rangle \cdot \langle I_2(x_2)\rangle}, \tag{8}$$

where

$$\langle x\rangle = \frac{1}{N}\sum_{i=1}^{N}(x_i)$$

is the average over N-time measured iterations, S_1 is the bucket signal of the test arm and is obtained by summing up intensities of all of the pixels of CCD1.

The classical image of THU is directly captured by CCD1 as depicted in Fig. 2(b), and the image is blurry due to the diffraction of the light field. As showed in Fig. 2(c), the image of THU is retrieved by calculating the second-order correlation function between the test and the reference beam according to Eq. (8), after averaging over $N = 2\times10^4$ iterations. It can be seen from the Figs. 2(b) and 2(c) that the LGI with pseudo-thermal light is robust against the diffraction in the optical path from the object to CCD1 which is consistent with Eqs. (6) and (7).

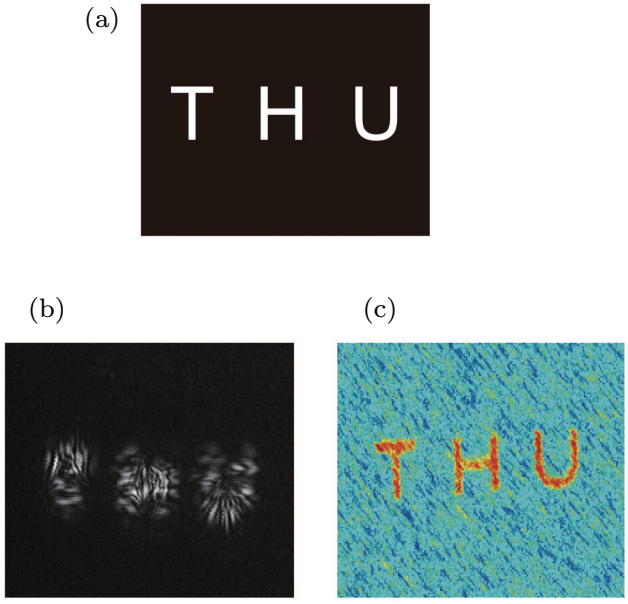


Fig. 2. (color online) (a) The object which is a mask of letters THU. (b) The image captured by CCD1. (c) The ghost image of THU retrieved by calculating the second-order correlation function between CCD1 and CCD2 after averaging over $N = 2\times10^4$ iterations.

3. Robust property of the LGI system in the scattering medium

In the following, we report the investigation of the robust property for the LGI system when the strongly scattering medium (a 2-cm thick pane of ground glass disk) is put in the optical path. Firstly, the strongly scattering medium is placed in position A. Then, CCD1 is totally sheltered by the medium. The picture directly captured by CCD1 is showed in Fig. 3(a), so the classical image of THU is thoroughly destroyed by the strongly scattering medium. However, the ghost image of THU can be clearly retrieved by the LGI system as illustrated in Fig. 3(b). Figures 2(c) and 3(b) make it clear that, in position A, the LGI system with a pseudo-thermal light source is robust against the strongly scattering medium. Moreover, even when the ground glass is rotating with different angular frequencies, 0.01 RPM, 0.1 RPM, and 1 RPM, respectively, the quality of the image for the LGI can also be maintained with total iteration number $N = 2\times10^4$ as presented in Fig. 4.

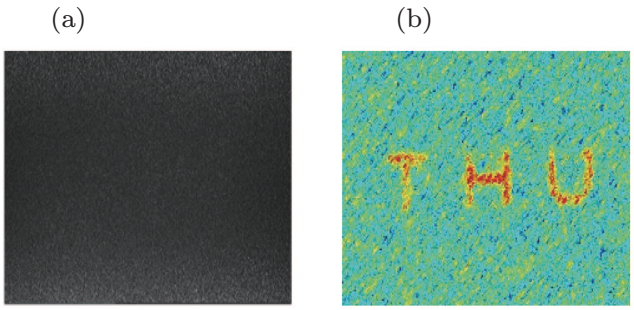


Fig. 3. (color online) The strongly scattering medium present in position A: (a) image captured by CCD1 and (b) the ghost image of the THU with $N = 2\times10^4$ iterations.

The robust property against stationary strongly scattering medium in position A of the LGI system is easy to be understood. According to Eq. (8), when the light field has propagated through the object, only the total intensity of CCD1 camera S_1 is useful. Thus, even if the strongly scattering medium exists in position A and causes the diffuse transmission of the test light field, S_1 only needs to be effectively multiplied by a coefficient factor $\beta < 1$. Then, equation (8) changes to

$$g^{(2)}(x_2) = \frac{\langle \beta S_1 \cdot I_2(x_2)\rangle}{\langle \beta S_1\rangle \cdot \langle I_2(x_2)\rangle}. \tag{9}$$

It is seen that β can be reduced, therefore, the LGI system with pseudo-thermal light is robust against the stationary strongly scattering medium in position A. Although CCD1 cannot collect all of the light scattered by the ground glass disk like the real bucket detector, part of the total light intensity is enough to retrieve the image of the object as shown in Fig. 3(b). If the ground glass rotates with a certain angular frequency, the β will change with time: $\beta(t) = \beta + \delta(t)$, where β is the attenuation term and $\delta(t)$ is the fluctuation due to the rotating of the medium. In Eq. (9), $\beta(t)$ cannot be reduced directly because $\delta(t)$ introducing a new noise to the second-order correlation function. However, as long as the fluctuation of $\beta(t)$ is small and cannot annihilate the fluctuation property of the bucket signal S_1, the LGI system will still work as presented in Fig. 4. In other words, if the roughness of the surface for the strongly scattering medium (ground glass) is dis-

tributed evenly, LGI is also robust against the rotating strongly scattering medium in position A.

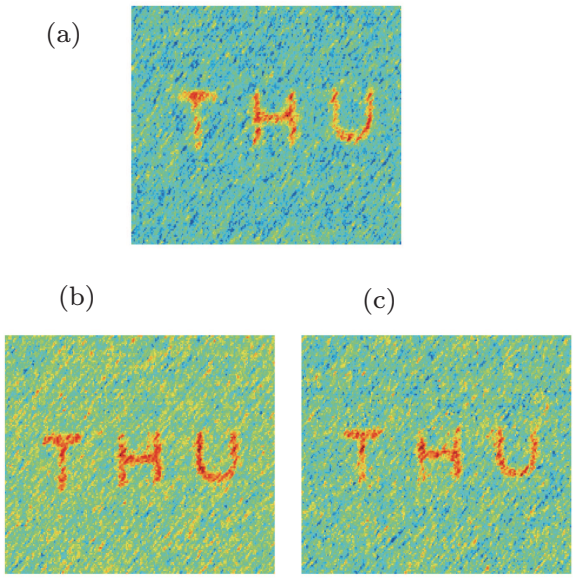


Fig. 4. (color online) The ghost image of THU with the ground glass in position A which rotates in different angular frequencies of (a) 0.01 RPM (b) 0.1 RPM, and (c) 1 RPM respectively.

Furthermore, when the strongly scattering medium is presented in other positions of this part of optical path from the object to CCD1, the only change is the value of β when the ground glass disk is stationary or the mean value of the β in the case that the ground glass disk is rotating. Hence, we can infer that not only position A but also the whole optical path from the object to CCD1 are robust against the strongly scattering medium for the LGI system with pseudo-thermal light.

When the strongly scattering medium is located in position B, no effective image could be retrieved from the LGI system as depicted in Fig. 5(a). The reason is that the PSF $h(x,u)$ from the light source to the object cannot maintain the form of Eq. (4) due to the presence of the scattering medium. Therefore, the second-order correlation function between two points of the test and reference beam will not be a sinc or δ function and the spatially point-to-point correlation is not satisfied. Even through the mean value of S_1 in the case of position B is nearly equal to that in the case of position A, the LGI cannot work anymore. When the ground glass disk is very close to the object, the ghost imaging system will retrieve $t_1(y) = t(y) \times t_{\text{glass}}(y)$, instead of $t(y)$, where $t_{\text{glass}}(y)$ is the transmission function of the ground glass disk, leading to the original $t(y)$ of the object is hard to obtain. Thus, the LGI system is not robust against the strongly scattering medium in position B. When the strongly scattering medium is located in position C, the result is similar to position B as shown in Fig. 5(b). The PSF $h(x,u)$ from the light source to CCD2 will be radically changed by the strongly scattering medium and the spatially point-to-point second-order correlation will also be destroyed, yielding the collapse of the GI. We also could infer that when the strongly scattering medium is present in the other positions of optical paths from the light source to the object or CCD2, LGI will not work.

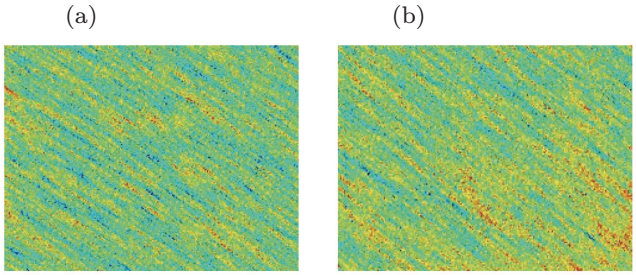


Fig. 5. (color online) The ghost imaging of THU with $N = 2 \times 10^4$ iterations when the ground glass disk (a) in position B and (b) in position C.

4. Summary

In summary, we have studied the robustness of the LGI system with a pseudo-thermal light source using a pane of ground glass as the strongly scattering medium. When such a medium is located at position A, the LGI is robust against both the stationary and rotating strongly scattering medium. In the case that the strongly scattering medium is stationary, the total intensity of the test beam S_1 only needs to be multiplied by an attenuation factor of $\beta < 1$ which could be reduced in the normalized second-order correlation function. When the strongly scattering medium is rotating, the attenuation factor β will vary with time, leading to $\beta(t) = \beta + \delta(t)$. However, as long as the roughness of the surface of the strongly scattering medium is distributed evenly, which means that the fluctuation of $\delta(t)$ is small, the fluctuation of the bucket signal will not be destroyed and the image quality of the LGI system could be maintained. When the strongly scattering medium is present in position B or C, the image of the object cannot be retrieved by the LGI system because the spatially point-to-point correlation function between the test and the reference beams has been destroyed by the strongly scattering medium.

In addition to the three positions reported here, we have also investigated other positions experimentally, finding that the results are similar to these three cases. These show that in the LGI system with a pseudo-thermal light source, the optical path from the object to CCD1 is of good robustness against the strongly scattering medium while the paths from the light source to the object and to CCD2 are not robust against the strongly scattering medium. This work can help us deepen the understanding of the LGI and will be of great benefit to the application of the ghost imaging.

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