Quantum frequency doubling based on tripartite entanglement with cavities*

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We analyze the entanglement characteristics of three harmonic modes, which are the output fields from three cavities with an input tripartite entangled state at fundamental frequency. The entanglement properties of the input beams can be maintained after their frequencies have been up-converted by the process of second harmonic generation. We have calculated the parametric dependences of the correlation spectrum on the initial squeezing factor, the pump power, the transmission coefficient, and the normalized analysis frequency of cavity. The numerical results provide references to choose proper experimental parameters for designing the experiment. The frequency conversion of the multipartite entangled state can also be applied to a quantum communication network.

Keywords: tripartite entanglement, frequency conversion, quantum communication network

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1. Introduction

Quantum frequency conversion (QFC) has been proposed to connect telecommunication wavelength light (carrier) with atomic transitions (quantum memories)[1,2] and improvement of optical detection,[3] by using the nonlinear effect in nonlinear media $\chi^{(2)}$ and $\chi^{(3)}$. It can be used to change the frequency of a quantum state to a desired one.[4,5] It is proved that the properties of quantum states of light can be maintained during QFC, including non-classical correlation,[6] entanglement,[7] non-classical photon statistics,[8–11] photon coherence,[12,13] and orbital angular momentum.[14] Recently, the successful experiments of spectral compression of single photons,[15] infrared or mid-infrared up-conversion imaging,[16,17] and quantum up-conversion of squeezed states[18,19] has aroused interest in exploring more applications of QFC. A direct quantum interface between photonic qubits of widely different wavelengths was demonstrated in Ref. [7]. In their work, the energy–time entanglement of a photon at 1310 nm with a photon at 1550 nm was transferred to that of another photon at a wavelength of 710 nm via sum-frequency generation (SFG). It is verified that the entanglement is unaffected, though one of the two entangled photons is transferred by an up-conversion process. As 710-nm wavelength is close to alkaline atomic transition and the transmission wavelengths for photons in optical fibers are 1310 nm and 1550 nm,[20,21] the converted entanglement can be used for quantum networks based on atoms and photons.[22]

On the other hand, it is known that when three continuous variable (CV) single-mode squeezed states input on two beam splitters, a tripartite CV entangled state is yielded. In Ref. [23], the squeezing direction of mode 1 is Y, the other modes are squeezed in X as shown in Fig. 1, and the three output modes are all genuinely tripartite entangled states. In this paper, we analyze an intracavity scheme with frequency doubling of the tripartite entangled states. The tripartite entangled fields at 1560 nm are transferred to the three-mode entanglement at 780 nm, by the process of second harmonic generation, which is a special process of up-conversion. As 780 nm is the absorption line of Rb atom, the produced entanglement can be used in an atom system.

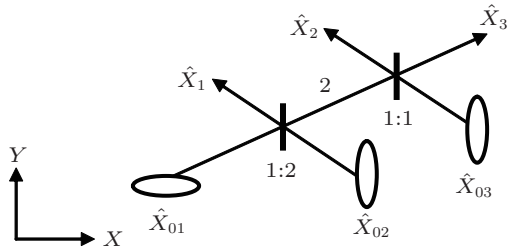


Fig. 1. Scheme for producing the tripartite entangled state of modes 1, 2, and 3 with three single squeezed states (one state is squeezed in Y direction, the other two are squeezed in X direction) and two beam splitters (one is 1:2 BS and the other is 1:1).

2. Theoretical model

The scheme of frequency doubling process is depicted in Fig. 2. The tripartite entangled beams from a three-port device called a tritter[24] (Fig. 1), $\hat{a}_1^{\rm in}(\omega_{\rm l})$, $\hat{a}_2^{\rm in}(\omega_{\rm l})$, and $\hat{a}_3^{\rm in}(\omega_{\rm l})$, are frequency-degenerated and injected into three one-sided cavities. Each cavity consists of a second-order nonlinear crys-

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tal, which provides the interaction of type-I second harmonic generation (SHG). The output fields $\hat{b}_1^{\rm out}(\omega_2), \hat{b}_2^{\rm out}(\omega_2)$, and $\hat{b}_3^{\rm out}(\omega_2)$ are all of frequency $\omega_2 = 2\omega_1$.

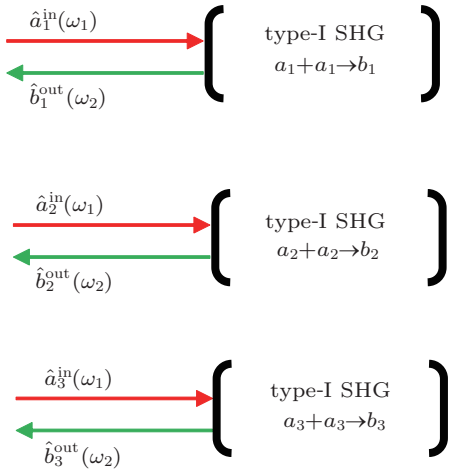


Fig. 2. (color online) Scheme of physical system for frequency doubling using cavity. $\hat{a}_j^{\rm in}(\omega_1)$ are the incoming tripartite entangled fields and $\hat{b}_j^{\rm in}(\omega_2)$ are the three outgoing fields. Type-I SHG takes place in each cavity.

The input–output relations of the amplitude quadrature components $\hat{X}_{(1,2,3)} = \hat{a}_{(1,2,3)} + \hat{a}^{\dagger}_{(1,2,3)}$ and phase quadrature components $\hat{Y}_{(1,2,3)} = [\hat{a}_{(1,2,3)} - \hat{a}^{\dagger}_{(1,2,3)}]/{\rm i}$ on the field modes from the tritter are given in Ref. [23]:

$$\begin{aligned}
\hat{X}_1 &= \sqrt{\frac{1}{3}}\,{\rm e}^{r_1}\hat{X}_{01} + \sqrt{\frac{2}{3}}\,{\rm e}^{-r_2}\hat{X}_{02},\\
\hat{Y}_1 &= \sqrt{\frac{1}{3}}\,{\rm e}^{-r_1}\hat{Y}_{01} + \sqrt{\frac{2}{3}}\,{\rm e}^{r_2}\hat{Y}_{02},\\
\hat{X}_2 &= \sqrt{\frac{1}{3}}\,{\rm e}^{r_1}\hat{X}_{01} - \sqrt{\frac{1}{6}}\,{\rm e}^{-r_2}\hat{X}_{02} + \sqrt{\frac{1}{2}}\,{\rm e}^{-r_2}\hat{X}_{03},\\
\hat{Y}_2 &= \sqrt{\frac{1}{3}}\,{\rm e}^{-r_1}\hat{Y}_{01} - \sqrt{\frac{1}{6}}\,{\rm e}^{r_2}\hat{Y}_{02} + \sqrt{\frac{1}{2}}\,{\rm e}^{r_2}\hat{Y}_{03},\\
\hat{X}_3 &= \sqrt{\frac{1}{3}}\,{\rm e}^{r_1}\hat{X}_{01} - \sqrt{\frac{1}{6}}\,{\rm e}^{-r_2}\hat{X}_{02} - \sqrt{\frac{1}{2}}\,{\rm e}^{-r_2}\hat{X}_{03},\\
\hat{Y}_3 &= \sqrt{\frac{1}{3}}\,{\rm e}^{-r_1}\hat{Y}_{01} - \sqrt{\frac{1}{6}}\,{\rm e}^{r_2}\hat{Y}_{02} - \sqrt{\frac{1}{2}}\,{\rm e}^{r_2}\hat{Y}_{03},
\end{aligned} \tag{1}$$

where r_1 and $r_2 = r_3$ are the squeezing factors of the three input lights respectively. The variances of the relative amplitude quadratures and total phase quadratures of three modes are calculated as

$$\begin{aligned}
&\left\langle \delta^2(\hat{X}_1 - \hat{X}_2)\right\rangle = \left\langle \delta^2(\hat{X}_1 - \hat{X}_3)\right\rangle = \left\langle \delta^2(\hat{X}_2 - \hat{X}_3)\right\rangle = 2\,{\rm e}^{-2r_2},\\
&\left\langle \delta^2(\hat{Y}_1 + \hat{Y}_2 + \hat{Y}_3)\right\rangle = 3\,{\rm e}^{-2r_1}.
\end{aligned} \tag{2}$$

It is found that it is easy for the variances to be below the shot noise limit (SNL) and the perfect entanglement can be obtained with infinite squeezing ($r_1 = r_2 \rightarrow \infty$) in single-mode squeezed states.

We will discuss the process of SHG and analyze the preserved tripartite entanglement among the output fields $\hat{b}_1^{\rm out}(\omega_2), \hat{b}_2^{\rm out}(\omega_2), \hat{b}_3^{\rm out}(\omega_2)$ in the next section.

3. Frequency doubling process

Under the assumption of perfect phase matching and zero detuning, the Hamiltonian describing the interaction in the cavity can be written as

$$\hat{H}_{\rm int} = \frac{{\rm i}\hbar\kappa}{2}(\hat{a}_j^{\dagger 2}\hat{b}_j - \hat{b}_j^{\dagger}\hat{a}_j^2), \tag{3}$$

where $\hat{a}_j^{\dagger}$ ($j = 1, 2, 3$) is the creation operator of the cavity mode with fundamental frequency and $\hat{b}_j$ is the annihilation operator of the harmonic mode; κ represents the effective nonlinear parameter and is proportional to the nonlinear susceptibility $\chi^{(2)}$.

Assuming that nonlinear coefficient κ and loss coefficient γ are the same for all cavities, the quantum Langevin equations of motion for the cavity modes can be expressed as

$$\begin{aligned}
\tau\frac{{\rm d}\hat{a}_1}{{\rm d}t} &= -\gamma_1\hat{a}_1 + \kappa\hat{a}_1^{\dagger}\hat{b}_1 + \sqrt{2\gamma_1}\hat{a}_1^{\rm in},\\
\tau\frac{{\rm d}\hat{b}_1}{{\rm d}t} &= -\gamma_2\hat{b}_1 - \frac{\kappa}{2}\hat{a}_1^2 + \sqrt{2\gamma_2}\hat{b}_1^{\rm in},\\
\tau\frac{{\rm d}\hat{a}_2}{{\rm d}t} &= -\gamma_1\hat{a}_2 + \kappa\hat{a}_2^{\dagger}\hat{b}_2 + \sqrt{2\gamma_1}\hat{a}_2^{\rm in},\\
\tau\frac{{\rm d}\hat{b}_2}{{\rm d}t} &= -\gamma_2\hat{b}_2 - \frac{\kappa}{2}\hat{a}_2^2 + \sqrt{2\gamma_2}\hat{b}_2^{\rm in},\\
\tau\frac{{\rm d}\hat{a}_3}{{\rm d}t} &= -\gamma_1\hat{a}_3 + \kappa\hat{a}_3^{\dagger}\hat{b}_3 + \sqrt{2\gamma_1}\hat{a}_3^{\rm in},\\
\tau\frac{{\rm d}\hat{b}_3}{{\rm d}t} &= -\gamma_2\hat{b}_3 - \frac{\kappa}{2}\hat{a}_3^2 + \sqrt{2\gamma_2}\hat{b}_3^{\rm in},
\end{aligned} \tag{4}$$

where γ_1 and γ_2 represent the total loss parameters for the fundamental mode and the harmonic mode respectively and are related to the amplitude reflection coefficients r_j and amplitude transmission coefficients t_j by $r_j = 1 - \gamma_j$, $t_j = \sqrt{2\gamma_j}$; τ denotes the cavity round trip time and is assumed to be the same for all fields; $\hat{a}_j^{\rm in}$ and $\hat{b}_j^{\rm in}$ ($j = 1, 2$) denote the operators of the input light fields.

In the symmetric case, we set the steady-state amplitudes of the fundamental modes $\alpha_1 = \alpha_2 = \alpha_3 = \alpha$ and $\sqrt{2\gamma_1}\alpha_1^{\rm in} = \sqrt{2\gamma_1}\alpha_2^{\rm in} = \sqrt{2\gamma_1}\alpha_3^{\rm in} = \varepsilon$. Following the semiclassical method, the threshold to the beginning of the self-pulsing is given by $\varepsilon_{\rm th}$ and the steady-state solutions below the threshold are found to be

$$\begin{aligned}
\varepsilon_{\rm th} &= (2\gamma_1 + \gamma_2)\frac{\sqrt{2\gamma_2(\gamma_1 + \gamma_2)}}{\kappa},\\
\alpha &= -\frac{2\gamma_1\gamma_2}{D} + \frac{D}{3\kappa^2}, \quad \beta = -\frac{\kappa}{2\gamma_2}\alpha^2,
\end{aligned} \tag{5}$$

where

$$D = \left(27\gamma_2\varepsilon\kappa^4 + 3\sqrt{3}\sqrt{8\gamma_1^3\gamma_2^3\kappa^6 + 27\gamma_2^2\varepsilon^2\kappa^8}\right)^{1/3}$$

and $\beta = \beta_1 = \beta_2 = \beta_3$ denote the steady-state amplitudes of the harmonic modes $\hat{b}_1$, $\hat{b}_2$, and $\hat{b}_3$.

The linearized equations of motion for quadrature-phase amplitude may be written in a matrix form by setting the following relations $\hat{a}_j = \alpha_j + \delta\hat{a}_j$, $\hat{b}_j = \beta_j + \delta\hat{b}_j$, and $\hat{X} = \delta\hat{a} + \delta\hat{a}^\dagger$, $\hat{Y} = (\delta\hat{a} - \delta\hat{a}^\dagger)/\mathrm{i}$,

$$\begin{aligned}\tau\frac{\mathrm{d}\delta\tilde{\hat{x}}}{\mathrm{d}t} &= M_x\delta\tilde{\hat{x}} + N\delta\tilde{\hat{x}}^{\mathrm{in}},\\ \tau\frac{\mathrm{d}\delta\tilde{\hat{y}}}{\mathrm{d}t} &= M_y\delta\tilde{\hat{y}} + N\delta\tilde{\hat{y}}^{\mathrm{in}},\end{aligned} \tag{6}$$

with

$$\begin{aligned}\delta\tilde{\hat{x}} &= [\delta\hat{X}_{a1}, \delta\hat{X}_{b1}, \delta\hat{X}_{a2}, \delta\hat{X}_{b2}, \delta\hat{X}_{a3}, \delta\hat{X}_{b3}]^{\mathrm{T}},\\ \delta\tilde{\hat{x}}^{\mathrm{in}} &= [\delta\hat{X}^{\mathrm{in}}_{a1}, \delta\hat{X}^{\mathrm{in}}_{b1}, \delta\hat{X}^{\mathrm{in}}_{a2}, \delta\hat{X}^{\mathrm{in}}_{b2}, \delta\hat{X}^{\mathrm{in}}_{a3}, \delta\hat{X}^{\mathrm{in}}_{b3}]^{\mathrm{T}},\\ \delta\tilde{\hat{y}} &= [\delta\hat{Y}_{a1}, \delta\hat{Y}_{b1}, \delta\hat{Y}_{a2}, \delta\hat{Y}_{b2}, \delta\hat{Y}_{a3}, \delta\hat{Y}_{b3}]^{\mathrm{T}},\\ \delta\tilde{\hat{y}}^{\mathrm{in}} &= [\delta\hat{Y}^{\mathrm{in}}_{a1}, \delta\hat{Y}^{\mathrm{in}}_{b1}, \delta\hat{Y}^{\mathrm{in}}_{a2}, \delta\hat{Y}^{\mathrm{in}}_{b2}, \delta\hat{Y}^{\mathrm{in}}_{a3}, \delta\hat{Y}^{\mathrm{in}}_{b3}]^{\mathrm{T}}\end{aligned}$$

and

$$M_x = \begin{pmatrix} -\gamma_1+\kappa\beta_1 & \kappa\alpha_1^* & 0 & 0 & 0 & 0\\ -\kappa\alpha_1 & -\gamma_2 & 0 & 0 & 0 & 0\\ 0 & 0 & -\gamma_1+\kappa\beta_2 & \kappa\alpha_2^* & 0 & 0\\ 0 & 0 & -\kappa\alpha_2 & -\gamma_2 & 0 & 0\\ y0 & 0 & 0 & 0 & -\gamma_1+\kappa\beta_3 & \kappa\alpha_3^*\\ 0 & 0 & 0 & 0 & -\kappa\alpha_3 & -\gamma_2 \end{pmatrix},$$

$$M_y = \begin{pmatrix} -\gamma_1-\kappa\beta_1 & \kappa\alpha_1^* & 0 & 0 & 0 & 0\\ y-\kappa\alpha_1 & -\gamma_2 & 0 & 0 & 0 & 0\\ y0 & 0 & -\gamma_1-\kappa\beta_2 & \kappa\alpha_2^* & 0 & 0\\ 0 & 0 & -\kappa\alpha_2 & -\gamma_2 & 0 & 0\\ 0 & 0 & 0 & 0 & -\gamma_1-\kappa\beta_3 & \kappa\alpha_3^*\\ 0 & 0 & 0 & 0 & -\kappa\alpha_3 & -\gamma_2 \end{pmatrix},$$

$$N = \begin{pmatrix} \sqrt{2\gamma_1} & 0 & 0 & 0 & 0 & 0\\ 0 & \sqrt{2\gamma_2} & 0 & 0 & 0 & 0\\ 0 & 0 & \sqrt{2\gamma_1} & 0 & 0 & 0\\ 0 & 0 & 0 & \sqrt{2\gamma_2} & 0 & 0\\ 0 & 0 & 0 & 0 & \sqrt{2\gamma_1} & 0\\ 0 & 0 & 0 & 0 & 0 & \sqrt{2\gamma_2} \end{pmatrix}.$$

According to the boundary condition of the coupling mirrors $\delta\tilde{\hat{x}}^{\mathrm{out}} = N\delta\tilde{\hat{x}} - \delta\tilde{\hat{x}}^{\mathrm{in}}$, $\delta\tilde{\hat{y}}^{\mathrm{out}} = N\delta\tilde{\hat{y}} - \delta\tilde{\hat{y}}^{\mathrm{in}}$, we can obtain the fluctuation of output fields at the analysis frequency ω[25]

$$\delta\tilde{\hat{x}}^{\mathrm{out}}(\omega) = [N(\mathrm{i}\omega\tau E - M_x)^{-1}N - E]\delta\tilde{\hat{x}}^{\mathrm{in}}(\omega), \tag{7}$$

$$\delta\tilde{\hat{y}}^{\mathrm{out}}(\omega) = [N(\mathrm{i}\omega\tau E - M_y)^{-1}N - E]\delta\tilde{\hat{y}}^{\mathrm{in}}(\omega), \tag{8}$$

where E is the 6×6 identity matrix.

4. Entanglement characteristics

According to Eq. (1), the input fluctuation of $\hat{a}_1$, $\hat{a}_2$, and $\hat{a}_3$ can be expressed as:

$$\begin{pmatrix} \delta\hat{X}^{\mathrm{in}}_{a1}\\ \delta\hat{X}^{\mathrm{in}}_{a2}\\ \delta\hat{X}^{\mathrm{in}}_{a3} \end{pmatrix} = \begin{pmatrix} \sqrt{\frac{1}{3}}\mathrm{e}^{r_1} & \sqrt{\frac{2}{3}}\mathrm{e}^{-r_2} & 0\\ \sqrt{\frac{1}{3}}\mathrm{e}^{r_1} & -\sqrt{\frac{1}{6}}\mathrm{e}^{-r_2} & \sqrt{\frac{1}{2}}\mathrm{e}^{-r_2}\\ \sqrt{\frac{1}{3}}\mathrm{e}^{r_1} & -\sqrt{\frac{1}{6}}\mathrm{e}^{-r_2} & -\sqrt{\frac{1}{2}}\mathrm{e}^{-r_2} \end{pmatrix} \begin{pmatrix} \delta\hat{X}_{01}\\ \delta\hat{X}_{02}\\ \delta\hat{X}_{03} \end{pmatrix},$$

$$\begin{pmatrix} \delta\hat{Y}^{\mathrm{in}}_{a1}\\ \delta\hat{Y}^{\mathrm{in}}_{a2}\\ \delta\hat{Y}^{\mathrm{in}}_{a3} \end{pmatrix} = \begin{pmatrix} \sqrt{\frac{1}{3}}\mathrm{e}^{-r_1} & \sqrt{\frac{2}{3}}\mathrm{e}^{r_2} & 0\\ \sqrt{\frac{1}{3}}\mathrm{e}^{-r_1} & -\sqrt{\frac{1}{6}}\mathrm{e}^{r_2} & \sqrt{\frac{1}{2}}\mathrm{e}^{r_2}\\ \sqrt{\frac{1}{3}}\mathrm{e}^{-r_1} & -\sqrt{\frac{1}{6}}\mathrm{e}^{r_2} & -\sqrt{\frac{1}{2}}\mathrm{e}^{r_2} \end{pmatrix} \begin{pmatrix} \delta\hat{Y}_{01}\\ \delta\hat{Y}_{02}\\ \delta\hat{Y}_{03} \end{pmatrix}. \tag{9}$$

By substituting Eq. (9) into Eqs. (7) and (8), we can obtain the correlation spectra to verify the entanglement among $\hat{b}_1^{\mathrm{out}}$, $\hat{b}_2^{\mathrm{out}}$, and $\hat{b}_3^{\mathrm{out}}$. Based on the sufficient inseparability criteria for CV tripartite entanglement proposed by van Loock and Furusawa,[26] the following three inequalities should be satisfied at the same time:

$$\begin{aligned} S_1 &= \delta^2(\hat{X}^{\mathrm{out}}_{b1} - \hat{X}^{\mathrm{out}}_{b2}) + \delta^2(\hat{Y}^{\mathrm{out}}_{b1} + \hat{Y}^{\mathrm{out}}_{b2} + g_3\hat{Y}^{\mathrm{out}}_{b3}) < 4,\\ S_2 &= \delta^2(\hat{X}^{\mathrm{out}}_{b2} - \hat{X}^{\mathrm{out}}_{b3}) + \delta^2(g_1\hat{Y}^{\mathrm{out}}_{b1} + \hat{Y}^{\mathrm{out}}_{b2} + \hat{Y}^{\mathrm{out}}_{b3}) < 4,\\ S_3 &= \delta^2(\hat{X}^{\mathrm{out}}_{b1} - \hat{X}^{\mathrm{out}}_{b3}) + \delta^2(\hat{Y}^{\mathrm{out}}_{b1} + g_2\hat{Y}^{\mathrm{out}}_{b2} + \hat{Y}^{\mathrm{out}}_{b3}) < 4, \end{aligned} \tag{10}$$

where g_1, g_2, and g_3 are scaling factors.

Because the variances of the relative amplitude quadratures and total phase quadratures of the input tripartite entanglement are identical and we have assumed the parameters of the three cavities to be the same, the correlation spectra $S_1 = S_2 = S_3 = S$. The dependences of S on the normalized analysis frequency $\Omega = \omega\tau/\gamma_1$ under different squeezed factor are shown in Fig. 3. Obviously, the maximum correlation (S is minimum) is obtained at low frequency. For the low frequency, if the initial squeezing factor $r_2 = r_1 = r$ becomes larger, the preserved correlation among b_1, b_2, and b_3 is better. But for the high frequency, the smaller the squeezing factor, the better the preserved correlation is, and we have a wider frequency range to measure the entanglement corresponding to smaller r. Figure 4 shows the correlation fluctuation spectra S versus

pump parameter $\sigma = \varepsilon/\varepsilon_{\rm th}$ for different values of squeezing factor r. The tripartite entanglement is retained at low pump power. It is obvious that the better correlation is obtained with larger r. But for a given r, we have an optimal pump parameter (σ around 0.12) where S reaches a minimum. In addition, the optimal cavity parameters should be chosen to meet the smallest S for preserving the quantum correlation as shown in Fig. 5. If the amplitude transmission coefficient of harmonic mode is twice that of fundamental mode, the maximum entanglement is maintained.

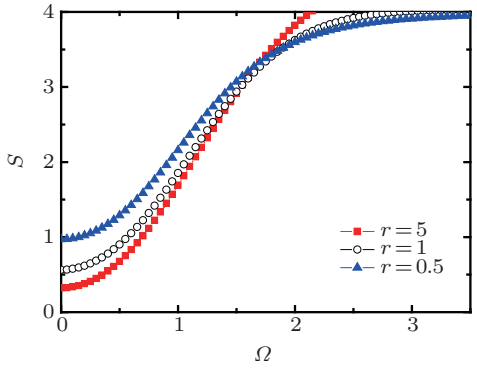


Fig. 3. (color online) Correlation fluctuation spectra S versus the normalized analysis frequency Ω, pump parameter $\sigma = 0.1$, transmission of the output coupler mirror $\gamma_2/\gamma_1 = 2$. The squares, circles, and triangles are for squeezing factor $r = 5$, 1, 0.5 respectively.

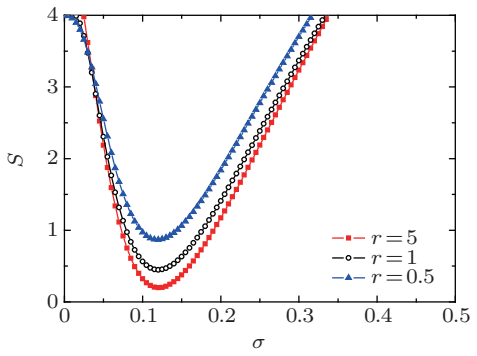


Fig. 4. (color online) Correlation fluctuation spectra S at $\Omega = 0$ versus pump parameter σ for different values of squeezing factor r, the transmission of the output coupler mirror $\gamma_2/\gamma_1 = 2$. The squares, circles, and triangles are for squeezing factor $r = 5$, 1, 0.5 respectively.

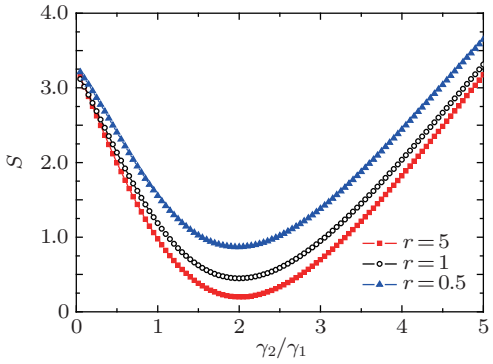


Fig. 5. (color online) Correlation fluctuation spectra S at $\Omega = 0$ versus transmission of the output coupler mirror γ_2/γ_1 for different values of squeezing factor ($r = 5$, 1, 0.5), the pump parameter $\sigma = 0.12$. The squares, circles, and triangles are for squeezing factor $r = 5$, 1, 0.5 respectively.

5. Conclusions

We propose a scheme to preserve the tripartite entanglement based on the process of SHG in three cavities, and it can be realized when the cavity operates far below the threshold. The dependences of the correlation spectrum on the parameters of cavity, the analysis frequency, and the initial squeezing are discussed. The entangled state conversion between different frequencies can be used to connect the atom with light in a quantum communication network, and the experimental scheme can be realized with the matured entanglement technology.

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