Analysis of robustness of urban bus network*

Tao Ren(任涛)†, Yi-Fan Wang(王一帆), Miao-Miao Liu(刘苗苗), and Yan-Jie Xu(徐艳杰)

Software College, Northeastern University, Shenyang 110819, *China*

(Received 18 August 2015; revised manuscript received 25 October 2015; published online 10 January 2016)

In this paper, the invulnerability and cascade failures are discussed for the urban bus network. Firstly, three static models(bus stop network, bus transfer network, and bus line network) are used to analyse the structure and invulnerability of urban bus network in order to understand the features of bus network comprehensively. Secondly, a new way is proposed to study the invulnerability of urban bus network by modelling two layered networks, i.e., the bus stop-line network and the bus line-transfer network and then the interactions between different models are analysed. Finally, by modelling a new layered network which can reflect the dynamic passenger flows, the cascade failures are discussed. Then a new load redistribution method is proposed to study the robustness of dynamic traffic. In this paper, the bus network of Shenyang City which is one of the biggest cities in China, is taken as a simulation example. In addition, some suggestions are given to improve the urban bus network and provide emergency strategies when traffic congestion occurs according to the numerical simulation results.

Keywords: complex network, urban bus network, layered network, invulnerability, cascade failures

PACS: 01.20.+x, 02.60.Nm, 05.10.–a, 05.65.+b **DOI:** 10.1088/1674-1056/25/2/020101

1. Introduction

Owing to the explosion of complex network science,[1,2] a lot of systems, natural and artificial, are considered as complex networks,[3–6] in which the nodes are the elementary components of the system and the edges connect a pair of nodes that mutually exchange information.[1,7–10]

During the last few years, public transportation systems (PTSs) have received increasing attention. Latora and Marchiori introduced the efficiency measure which gives a more general mathematical definition of small worlds in Boston subway.[11] Sen investigated the structural properties of the Indian railway network.[12] Guimera and Amaral proposed a model to analyse the world-wide airport network and found the most connected cities.[13] In addition, some research work concerned about the global cargo ship movements.[14] In the aspect of bus network, Sienkiewicz and Hołyst studied the bus and tramway systems in 22 Polish cities.[15] In addition, Wu *et al.* studied the degree distribution and efficiency of Beijing bus network, which shows the scale-free and small-world properties of urban bus network.[16,17] Zheng *et al.* analysed the topological properties of Beijing bus network in detail.[18] Xu *et al.* reported the statistical properties of three bus networks in three different cities of China.[19] Soh *et al.* studied the structures and properties of Singapore bus networks from weighted complex network.[20] Also there are several other researches about Chinese cities.[21–24] All these studies focused on the complexities hiding in the urban bus network by adopting the complex network theory. However, in these researches the robustness analyses of the urban bus network were not taken into consideration.

Robustness analysis has been considered since the work of Barabasi and Albert,[25] and the results showed that there is a huge difference between the ER network and BA network. Since then, increasing researches focused on the robustness analysis of complex network, which can be divided into static invulnerability analysis and dynamic cascade failures analysis. Callaway *et al.*[26] and Cohen *et al.*[27] focused on the robustness of the network based on percolation theory. Albert *et al.* studied the North American power grid in view of network and analysed its invulnerability.[28] Wang *et al.* studied degree distribution and robustness of cooperative communication network with scale-free model.[29] Not only the static invulnerability research, but also dynamic cascade failures received much attention. Motter and Lai studied the networks where loads can be redistributed among the nodes and the selective attacks can lead to a cascade of overload failures.[30] Kinney *et al.* modelled the cascade failures in the North American power grid.[31] Wang *et al.* took into account two stages of the cascading propagation in the interdependent networks.[32] Xiao *et al.* studied the way to optimize the robustness of complex network against cascade failures.[33] However, for the PTS, especially the urban bus network, most of the researches only focused on the static invulnerability, and their analyses were just for one layer model.[34] In fact, PTS are complex systems with layered structure, and these coexisting topologies interact and depend on each other.[35] On the other hand, PTS have loads

*Project supported by the National Natural Science Foundation of China (Grant Nos. 61473073, 61374178, 61104074, and 61203329), the Fundamental Research Funds for the Central Universities (Grant Nos. N130417006, L1517004), and the Program for Liaoning Excellent Talents in University (Grant No. LJQ2014028).

†Corresponding author. E-mail: chinarentao@163.com

© 2016 Chinese Physical Society and IOP Publishing Ltd

which present the transmitting capabilities on each nodes or links and change dynamically.[35]

So in this paper, we analyse the robustness of urban bus network in both static invulnerability and dynamic cascade failures and choose the Shenyang city (one of the biggest cities in China) as the simulation example throughout the paper. The data of the Shenyang bus network are cited from http://shenyang.8684.cn/ which is a professional urban bus travel web in China. The contributions of this paper are threefold. Firstly, we analyse the urban bus network by modeling three static models (bus stop network, bus transfer network, and bus line network) in order to understand the features of bus network. Secondly, compared with the previous work,[16–24] invulnerability analysis is included. In addition, because different factors (such as bus lines, stops, transfers) will affect each other indirectly in the urban bus network, we propose a new way to study the invulnerability of urban bus network by modeling layered networks, i.e., the bus stop-line network and the bus line-transfer network and then draw a comprehensive conclusion about the invulnerability of urban bus network. Finally, considering the passenger flows in urban bus network, the cascade failures are discussed by modeling a new layered network which can reflect the dynamic passenger flows more objectively than the current results, and then we propose a new load redistribution method to study the robustness of dynamic traffic. Additionally, some suggestions are given for alleviating the congestion. In conclusion, comparing with the previous work about the urban bus network, we not only analyse the structure properties of the urban bus network, but also make a comprehensive robustness analysis in both static and dynamic ways by constructing layered models.

The rest of this paper is organized as follows. In Section 2, the three static models of urban bus network are described and analysed in detail. In Section 3, the invulnerability of urban bus networks is analysed based on both one-layer networks and two-layer networks under two attack strategies. In Section 4, based on the passenger flows, dynamic layered cascade failure model is constructed, and the robustnesses of urban bus network with different capacities are analysed. Finally, some conclusions are drawn from the present studies in Section 5.

2. Network construction and analysis

Urban public transport network is a typical complex network which contains bus stops and bus lines. One bus line contains several bus stops. Also, considering the transfer of the passengers, this paper introduces three types of urban bus network models: bus stop network, bus transfer network, and bus line network. For these three types of models, we only concern the topology of the urban bus network, so all the three types of networks are unweighted and undirected.

2.1. Bus stop network

For bus stop network, the bus stops are chosen as nodes and an edge exists between two nodes if they are consecutive stops on the any bus line.[15] In this way, the node degree k in bus stop network means the number of choices passengers can take to the consecutive stops and the distance d equals the total number of stops passengers need to pass on the shortest path from the start stop to the end stop, but sometimes it ignores the transfer times between two stops. Therefore, the bus stop network can intuitively represent the topology of the urban bus network.

By MATLAB software, we choose the Shenyang bus network as an example, the degree scope of bus stop network is in a range of [2, 13], the average degree of the whole network is 2.7894, and the degrees of most nodes are 2 or 3 since most of stops are in one line and they only connect the previous-stop and the next-stop. A few stops with larger degrees in Shenyang bus stop network are shown in Table 1. From Table 1, we can see that the stop with the largest degree is Shenyang Railway Station which connects 13 other consecutive stops.

Table 1. Top 5 stops with the larger degrees in Shenyang bus stop network.

Bus stops	Shenyang Railway Station	Shengjing Hospital	Huigong Square	Shenyang North Railway Station	Environmental Protection Bureau
Degree	13	12	11	11	11

The distribution of degrees, clustering coefficient and distance of bus stop network are shown in Fig. 1. Figure 1(a) shows that the degree distribution of Shenyang bus stop network obeys power-law distribution $P(k) = 4.469 \cdot k^{(-2.894)}$ and the degrees of most nodes are between 2 to 6. So the bus stop network of this city is a scale-free network.

From Fig. 1(b), the distance d between any nodes ranges from 1 to 78 and the mean path length L is 22.0931. It denotes that reaching one stop from another stop would take an average of 22 stops for passengers in Shenyang bus network. From the studies of other cities,[18,21–24] we find the mean path length L in Shenyang is longer than in Beijing (15), Taiyuan (14), Hangzhou (10), and Shanghai (8). This means that passengers will usually spend more time by bus in Shenyang.

From Fig. 1(c), we find the average clustering coefficient of Shenyang stop network is 0.0692, which is smaller than that of Beijing (0.158). Bigger clustering coefficient means that there are more connections between the nearest neighbor stops, which gives more alternative choices for passengers. So it means that it is easier to be congested and the robustness is worse in Shenyang.[36]

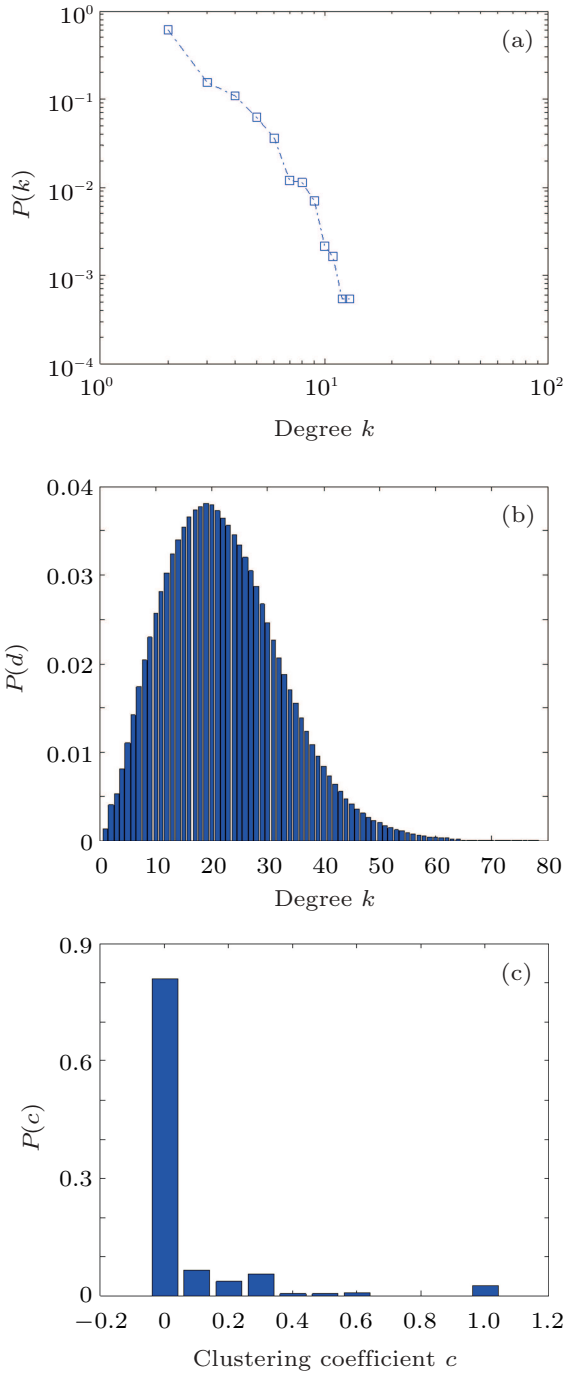


Fig. 1. (color online) Characteristics of Shenyang bus stop network, (a) degree distribution of network, (b) distance distribution of network, (c) clustering coefficient distribution of network.

2.2. Bus transfer network

The least transfer time means more convenient to travel by bus, so the transfer time is also an important factor for bus network. In bus transfer network, bus stops are also defined as nodes, however, an edge between two nodes exits if there is a direct bus line without transfer between two bus stops. In other words, if bus line A consists of nodes a_i, i.e., $A = \{a_1, a_2, ..., a_n\}$ then in the bus transfer network the nearest neighbors of the node a_i are the rest of the nodes in A.[15] Consequently, the node degree k in bus transfer network is the total number of stops which are reachable without transfer from this node. For some bus stops which are contained in only one bus line, $k+1$ is the number of stops in this line. d is the distance between two nodes and $d-1$ equals the number of transfer times from one stop to another stop. Therefore, bus transfer network can represent the actual situation about transfer between bus stops.

The degree of Shenyang bus transfer network is from 2 to 503, the average degree of the whole network is 64.95, and the degrees of the most nodes are in a range from 20 to 61. This means that most of stops in Shenyang can reach other 20 to 61 stops without transfer. A few stops with larger degrees are shown in Table 2. The node with the largest degree is Shenyang North Railway Station, which can reach other 503 stops without transfer. Larger degree means more opportunities for the passengers to reach their destinations without transfer. Meanwhile, we find that the nodes with large degrees in bus transfer network also have large degrees in bus stop network, and these nodes can be denoted as the hub stops which can carry more traffic, so the robustness analysis based on these stops is meaningful.

Table 2. Top 5 stops with the larger degrees in Shenyang bus transfer network.

Bus stops	Shenyang North Railway Station	Wuai Market Station	Shenyang Railway	Xiaojin Bridge Square	King's Temple
Degree	503	460	448	438	413

The distribution of degree, clustering coefficient and distance of bus transfer network are shown in Fig. 2. From Figs. 2(a) and 2(b), the degree distribution of Shenyang bus transfer network is close to the power-law distribution and distance d between nodes is in a range from 1 to 5. Mean path length L in bus transfer network reflects the average transfer time between bus stops. Larger mean path length means more transfer times for passengers by bus. After calculating, we obtain L to be 2.527 in bus transfer network, this means from one stop to another stop passengers will transfer more than one time in average, and it is not convenient for bus travel.

In addition, from Fig. 2(c), we find that the clustering coefficients of most nodes are all 1 because these nodes are in a globally coupled network of one bus line. The average clustering coefficient is 0.741 which is the possibility with which the neighbor stops are in the same bus line. Larger clustering coefficient in bus transfer network means that bus lines link with each other more closely and transfer is more convenient. According to the small mean path length and large clustering coefficient, we can find that Shenyang bus transfer network is a small-world network. In general, the capacity of bus transfer in Shenyang is at a medium level and still needs to be developed.

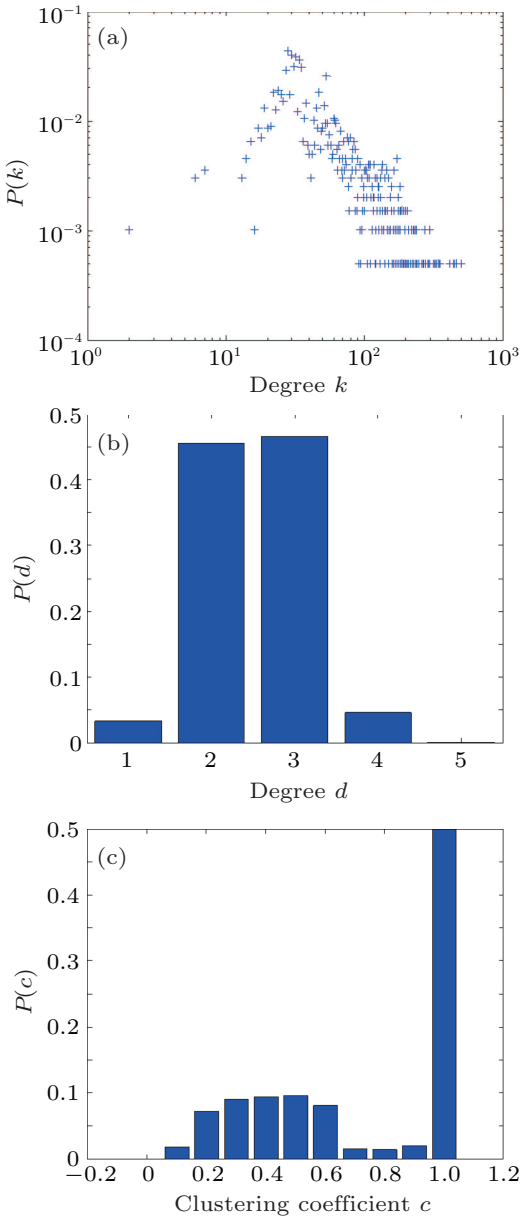


Fig. 2. (color online) Characteristics of Shenyang bus transfer network: (a) degree distribution of network, (b) distance distribution of network, and (c) clustering coefficient distribution of network.

2.3. Bus line network

Usually, passengers may take more than one bus to their destinations, so it is necessary to find the relations between bus lines. For bus line network, we define bus lines as nodes, that is very different from the bus stop network and bus transfer network, and if two different bus lines (nodes) contain one or more the same bus stops, we define that there is an edge between the two lines (nodes), in fact, these bus stops are transfer stops. The node degree k in this topology is the number of the other bus lines to which passengers can transfer. And the distance d can be explained as the number of transfer times through one line to another. Therefore, bus line network can describe the bus transfer between different bus lines.

Also, we take the Shenyang bus network for example, and there are 192 nodes and 5049 edges in bus line network. The degree of this network is in a range of [1, 105], the average degree of the whole network is 52.594, which means that one bus line may intersect with other 52 lines on an average. The top five lines with larger degrees are shown in Table 3, and it is easier for passengers to transfer by these bus lines. The nodes with largest degree are line No. 117 and line No. 152, which shares 105 stops with other lines.

Table 3. Top 5 bus lines with the larger degrees in Shenyang bus line network.

Bus lines	No. 117	No. 152	No. 207	No. 134	No. 328
Degree	105	105	96	92	90

The distribution of degree, clustering coefficient and distance of bus line network are shown in Fig. 3. From the uniform distribution over a wide range shown in Fig. 3(a), we can find that the Shenyang bus line network does not have a scale-free property at all. In a bus line network, smaller mean path

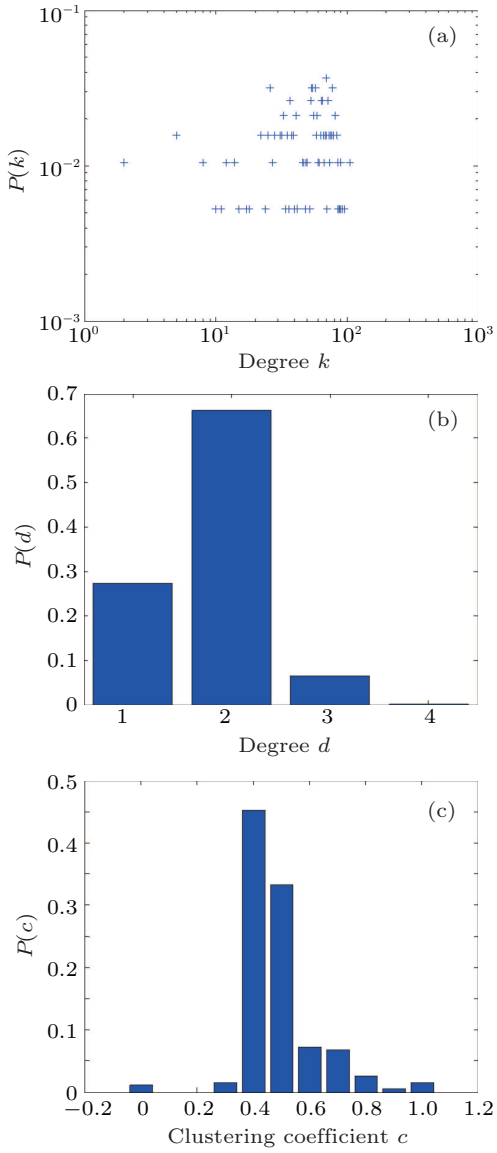


Fig. 3. (color online) Characteristics of Shenyang bus line network: (a) degree distribution of network, (b) distance distribution of network, and (c) clustering coefficient distribution of network.

length means less transfer time between bus lines. As shown in Fig. 3(b), most of distances between two lines are in a range of [1, 3] and we can obtain mean path length L to be 1.791, which means that the passengers who should travel between lines need to transfer two or more times on an average. Compared with the scenarios in Hangzhou (0.89), Taiyuan (0.68), and Xi'an (0.44),[18,21–24] transfer times in Shenyang is large, which means that the passengers usually need more transfers between bus lines in Shenyang. According to the small mean path length and large clustering coefficient, Shenyang bus line network is a small-world network.

Clustering coefficient in bus line network determines the density of the same bus stops shared by the neighbor bus lines. As shown in Fig. 3(c), the average clustering coefficient is 0.537. Compared with the cases in Xi'an (0.71), Shanghai (0.56), and Beijing (0.54),[18,21–24] the same bus stops between bus lines in Shenyang are sparse. Some main routes have much more bus lines while few remote routes have only one line.

3. Invulnerability analysis of urban bus network

In this section, the invulnerabilities of urban bus network are analysed for both static single layer model and layered model based on the real data of Shenyang bus network. We propose a new way to study the robustness of the urban bus network.

3.1. Attack strategies and evaluation indexes

We choose selective attack and random attack for invulnerability analysis. For the selective attack strategy, we remove some nodes with higher degrees according to their degree order from high to low. For the random attack strategy, the nodes are removed randomly. The main evaluation indexes are defined as follows.[25,37,38]

(i) Relative size of the giant component S, which is also called connectivity probability, is defined as

$$S = N'/N, \tag{1}$$

where N' is the number of nodes in the giant component after nodes have been broken down, N is the total number of nodes in original bus network.

(ii) Mean path length L, which is the average distance between any two nodes in the giant component after nodes are broken down, is defined as

$$L = \frac{2}{N'(N'-1)} \sum_{i \neq j} d_{ij}, \tag{2}$$

where d_{ij} is the distance between node i and node j in the network.

(iii) Diameter D, which is the maximum distance between any two nodes in the giant component after nodes have been broken down, defined as

$$D = \max_{i,j}(d_{ij}). \tag{3}$$

(iv) Network performance parameter E, which quantifies the effect on the whole network, is defined as

$$E = \frac{1}{N(N-1)} \sum_{i \neq j} 1/d_{ij}. \tag{4}$$

3.2. Invulnerability of single layer models

3.2.1. Invulnerability of shenyang bus stop network

Based on the bus stop network we built, we take Shenyang bus network for example as shown in Figs. 4(a)–4(d), horizontal axis f represents the proportion of broken stops, and vertical axis represents the relative size of the giant component S, network performance parameter E, diameter D and mean path length L in the giant component respectively. From Fig. 4(a), we can see that S under selective attack strategy decline is faster than under random attack strategy. This means that compared with random attack, selective attack can destroy the bus stop network seriously.

As shown in Fig. 4(b), the performance parameter E in the bus network is 0.0627. The network performance of Shenyang bus stop network is worse than in Seoul (0.81), Tokyo (0.128), Beijing (0.0711), and Nanjing (0.167).[18,21–24] Under selective attack, when 6% stops are broken down in the network, E is 0.0399 (decline to half). It means that the network structure is damaged seriously. When 12% stops are broken down, E tends towards 0, the whole network is destroyed. Under the random attack, it needs to attack 13% of the stops, the performance parameter E can decline half, and when 40% of the stops are destroyed, E is still not 0, the network is still not destroyed totally. This also reflects the scale-free property of Shenyang bus stop network.

As shown in Figs. 4(c) and 4(d), when the increasing number of stops is broken down, diameter D and mean path length L in the giant component increase in the early stage of the selective attack, then they decline rapidly. This is because the bus stop network is divided into many pieces, which causes the decreasing of the size of the giant component S. But by the random attack, D and L fluctuate largely.

3.2.2. Invulnerability of Shenyang bus transfer network

The invulnerability analysis about bus transfer network mainly focuses on the influence of transfer ability under attacks.

From Figs. 5(a) and 5(b), relative size S and network performance E decrease linearly under random attack. What is more, the curves of S under two attack strategies overlapped until 22% of the bus stops are broken down, and then S under the selective attack declines rapidly. Under the selective attack, until 60% of the bus stops are broken down, the whole network is destroyed. All these results mean that compared with the bus stop, transfer between bus stops is harder to be destroyed.

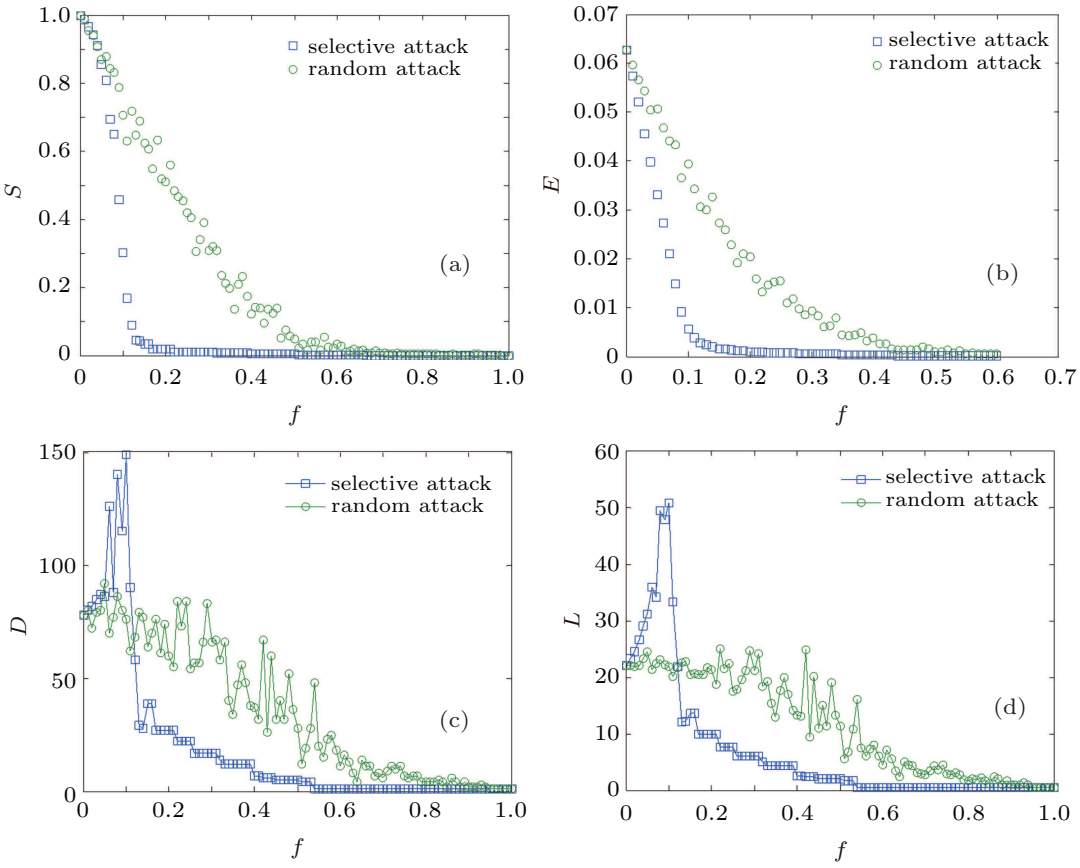


Fig. 4. (color online) Attack results in Shenyang bus stop network: (a) relative size S, (b) network performance parameter E, (c) diameter D, and (d) mean path length L versus proportion of broken stops f.

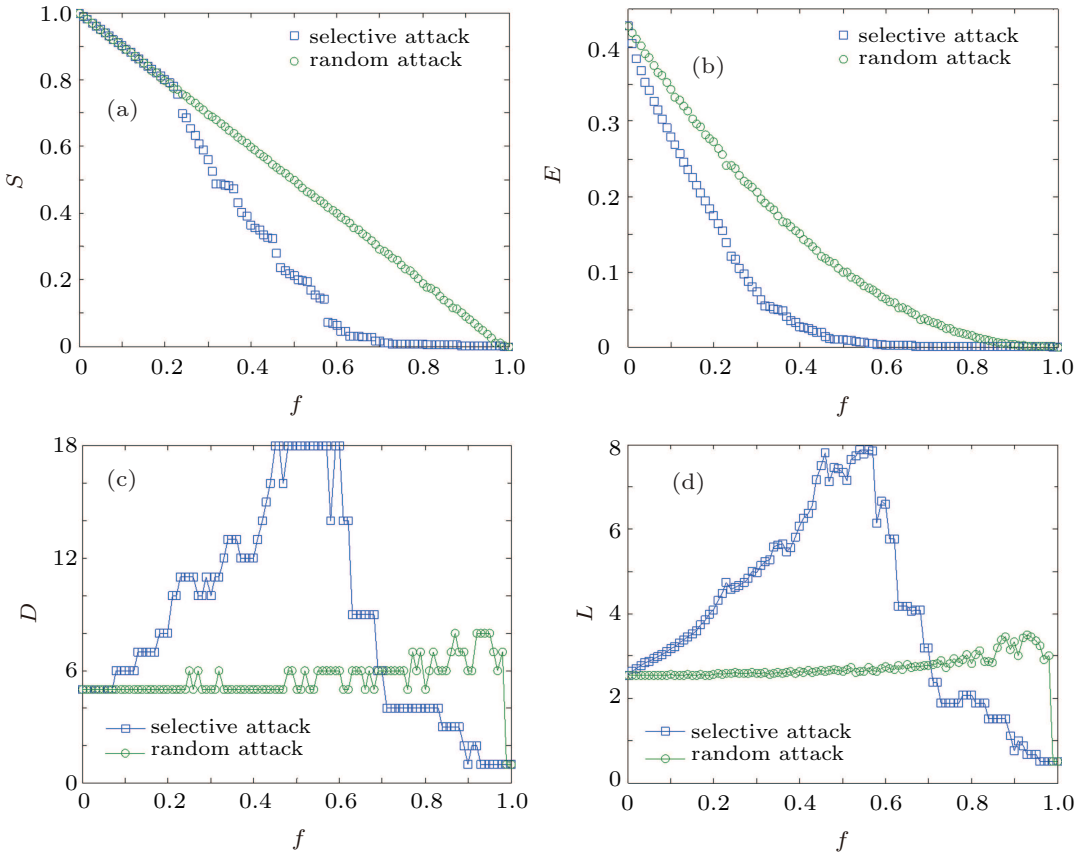


Fig. 5. (color online) Attack results in Shenyang bus transfer network, showing (a) relative size S, (b) network performance parameter E, (c) diameter D, and (d) mean path length L versus proportion of broken stops f.

In addition, as shown in Figs. 5(c) and 5(d), we can see that under both random attack and selective attack, bus transfer network is more robust. Under the random attack, diameter D and mean path length L do not change much until 99% of the stops are broken down. However, under the selective attack, D and L increase until 60% of the stops are broken down, and then decline rapidly, which means that the whole network is destroyed. That is to say, the transfer time between stops does not change too much when some stops do not work anymore, but attacking some hub stops will still experience a huge impact.

3.2.3. Invulnerability of shenyang bus line network

As shown in Figs. 6(a) and 6(b), we can see that relative size S and network performance E decrease linearly under random attack. Curves of S under two attack strategies overlaps until 70% of the bus lines are broken down, and then S declines rapidly under selective attack. For network performance parameter E, two attack strategies receive almost the same results. This means that the Shenyang bus line network does not have the scale-free property at all, and it is almost like a random graph.

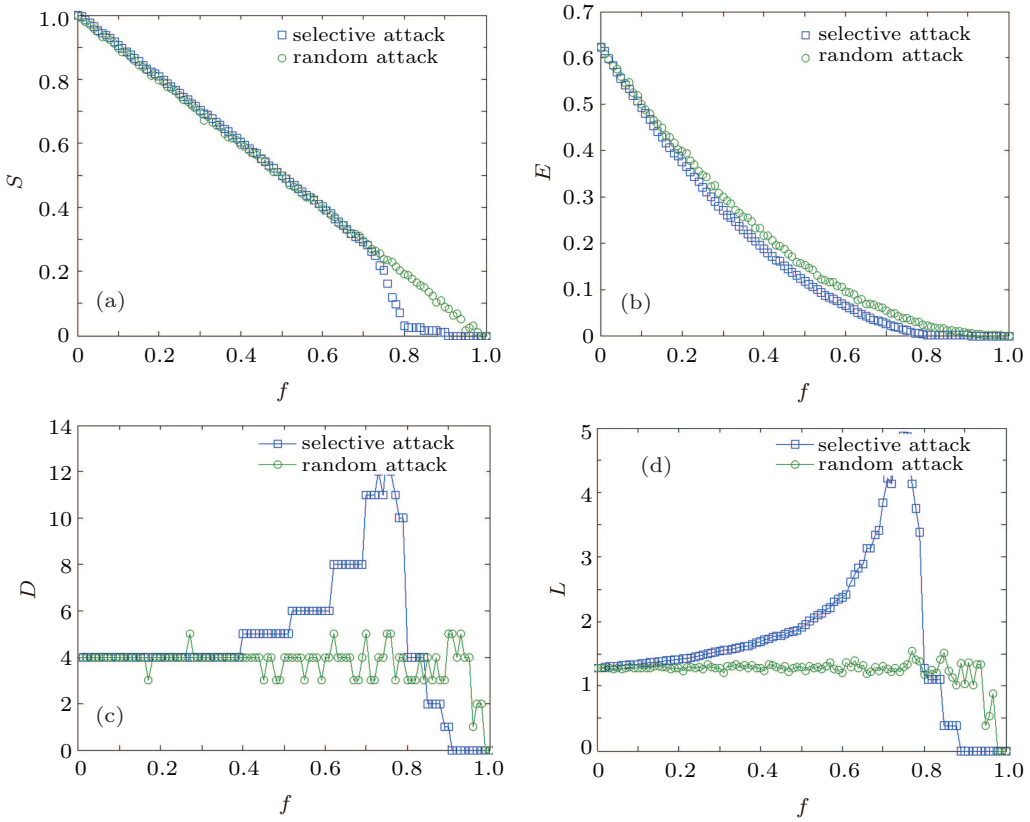


Fig. 6. (color online) Attack results in Shenyang bus line network, showing (a) relative size S, (b) network performance parameter E, (c) diameter D, and (d) mean path length L versus proportion of broken stops f.

As shown in Figs. 6(c) and 6(d), both curves of diameter D and mean path length L under two strategies overlap until about 25% of the bus lines are broken down. And then under the selective attack, D and L increase until 70% of the lines are broken down, and they reach the culminating point and then decline rapidly. Finally, when more than 80% of the lines are broken down under the selective attack, the whole network is destroyed. This means that although bus line network is more robust under the selective attack, the transfer between bus lines still becomes more difficult when too many hub bus lines are broken down. By comparing the above two models, we can see that the bus line network is more robust. In other words, transfer between bus lines is harder to break down.

3.2.4. Invulnerability of layered networks

So far, we have studied the invulnerabilities of three single models respectively. But urban bus network is a complex system which cannot be revealed thoroughly by any network. In fact, the interactive correlation also exists between different models, and the bus stops, bus lines and the bus transfers will have effects on each other indirectly. So, we firstly will consider the interactive correlations between bus stop network and bus line network (named bus stop-line network) then find how the bus stops influence the bus lines; secondly, we will consider the interactive correlations between bus line network and bus transfer network (named bus line-transfer network), then find how the bus lines influence bus transfer. In this paper, both the bus transfer network and bus stop network choose

bus stops as nodes, they cannot influence each other by removing the nodes, so it is unnecessary to consider the "bus stop-transfer network" here.

3.2.4.1. Invulnerability of bus stop-line network

In the bus stop-line model, each bus line in the bus line network depends on several bus stops to transfer to other bus lines. So if these bus stops are broken down, the bus lines may disconnect with other lines in bus line network. In turn, if the bus line is broken down, transfer stops in this line cannot work and then the nodes in bus stop network will be destroyed. By taking Shenyang for example, the invulnerability of this layered model is analyzed.

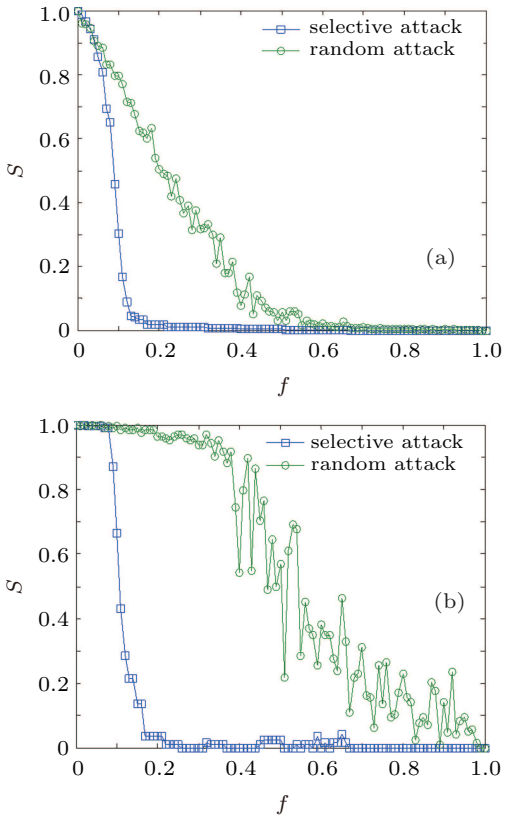


Fig. 7. (color online) Attack results in Shenyang stop-line network: (a) stop network, and (b) line network versus proportion of broken stops f in stop network.

In Fig. 7, horizontal axis f represents the proportion of bus stops attacked in the bus stop network, vertical axes in Figs. 7(a) and 7(b) represent the relative size of giant component S in the bus line network and that in the bus stop network respectively. Compared Fig. 7(a) with Fig. 4(a), the curves of S are very similar under both attack strategies, that is, when bus stops are broken down, bus line network will be affected, and then react the bus stop network. However, the reaction is not obvious. But, as shown in Fig. 7(b), we can see that under the selective attack, S in bus line network changes little until 10% nodes are destroyed, and then it declines faster than under the selective attack in single bus line network (shown in Fig. 6(a)). Under the random attack, S in bus line network declines slower than under the selective attack, but under the random attack, when more than 30% of stops are broken down, S fluctuates largely. So the interaction will make the bus line network less robust.

In addition, compared with the bus line network in the bus stop-line model, the bus stop network shown in Fig. 7(a) is more fragile especially under the selective attack. Therefore, we can know that hub stops can provide most transfers, and once they are broken down, robustness in both bus stop network and line network will decline seriously.

3.2.4.2. Invulnerability of bus line-transfer network

In the bus line-transfer model, each edge in the bus transfer network means that two bus stops are reachable without transfer. So if one node in the bus line network is broken down (one bus line is destroyed), some edges in the bus transfer network will be broken down too, then passengers cannot reach a stop directly due to the failure of this bus line. In turn, if transfer nodes in bus transfer network are broken down, edges which depend on transfer nodes between lines in bus line network will be broken down too. By removing lines in Shenyang bus line network, the invulnerability of this layered model is analysed in Fig. 8.

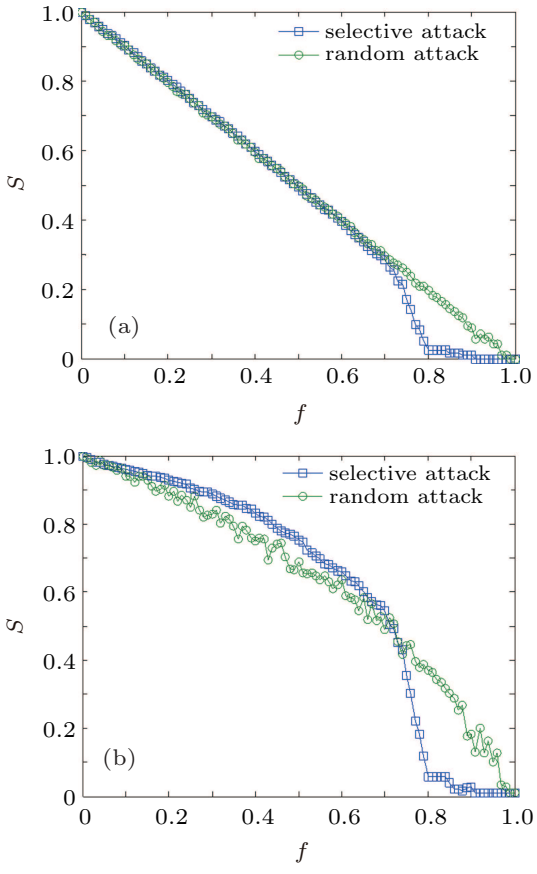


Fig. 8. (color online) Attack results in Shenyang line-transfer network: (a) line network, and (b) transfer network versus proportion of broken stops f in line network.

In Fig. 8, horizontal axis f represents the proportion of broken lines in the bus line network, and vertical axes in Figs. 8(a) and 6(a) represent the relative size of the giant component S in the bus line network and that in the bus transfer network respectively. The comparison of Fig. 8(a) with Fig. 6(a) shows that the curves of S are very similar under both attack strategies. That is to say, when bus lines are broken down, the influence from bus transfer network to bus line network is not obvious under both attack strategies.

But in Fig. 8(b), it is found that under the two attack strategies, collapse of bus transfer network in line-transfer network is less serious than that of the single bus transfer network shown in Fig. 5(a). Another interesting phenomenon is that at the beginning of two attacks in Fig. 8(b), S is higher under the selective attack than under the random attack, when more than 70% of lines are broken, S under the selective attack declines rapidly. This phenomenon denotes that the lines with larger degrees have more transfer chances and most of them are in the urban center. So when one bus line is broken down, several other lines may substitute it for passengers to take without any transfer. But for some suburb area, there may be only one line to take, if this kind of line is destroyed, it may cause more isolated stops which are unconnected in bus transfer network.

3.3. Suggestions for the urban bus network planning

From the above analyses based on both single layer and layered network, we find that for any model, the selective attack always causes more serious consequence than the random attack. Because the selective attack destroys the hub stops or lines, the hub stops or lines can affect the invulnerability of the bus network largely. So the managers should keep the hub bus stops or bus lines work unblocked and effective by reducing some project constructions there.

In addition, the comparison of the bus stop network with bus line network shows that the bus stop network is less robust and will affect the bus lines much more, that is to say, the break-down of bus stops may cause more serious result than that of the bus lines. So managers should pay more attention to the connectivity of bus stops than that of the bus lines

For the bus lines, some suburb area usually have only one bus line which may be isolated easily, therefore, for the bus network planning, bus lines in the suburb area need to be strengthened.

4. Cascade failures analysis of the urban bus network

The static invulnerability analysis only considers the topological structure of the network, however, cascade failures will have more devastating results when the intrinsic dynamics of flows of physical quantities is taken into account.[30,39] And for better representing the characteristics of urban bus network, we construct the urban bus network with layered structure.

4.1. Topology of two-layer network model

Multilayer structure network is well used in many fields. For instance, Each WWW or P2P link is mapped on the underlying IP network,[40] each complete logistic process is determined by the underlying delivery site. So, all the three models above are only a part of large urban bus systems.

In this paper, urban bus network will be viewed as a two-layer network, one layer represents the physical infrastructures while the other layer represents the logical layer, i.e., the passenger flows on the infrastructures. That is, the nodes in the physical layer are the bus stops and the edges are the bus lines connecting neighbor stops. The node in the logical layer is also the stop while the edge is the trip demand which connects the first and the last stop of a particular trip.

Considering the definitions of bus stop network, bus transfer network and bus line network above, the bus transfer network describes a kind of passenger flow while bus stop network represents the information about the bus lines, which is the physical infrastructure of the flow in fact.[41] So the layered bus network is composed of the bus stop network and the bus transfer network, which represents the physical layer $G^{\varphi} = (V^{\varphi}, E^{\varphi})$ and the logical layer $G^{\lambda} = (V^{\lambda}, E^{\lambda})$ respectively.

4.2. Initializing of load and capacity

Because the logical layer represents the trip demand, we consider the logical layer as an undirected graph without the loads nor the capacity. And both layers have the same set of nodes (i.e. bus stops). Edge of logical layer is weighted by assigning it to physical layer, which is denoted by $w(\cdot)$ with $w = 1$. Each logical edge $e^{\lambda} = (u^{\lambda}, v^{\lambda})$ is mapped on the physical graph as a physical path $M(e^{\lambda}) \subset G^{\phi}$ connecting the nodes u^{φ} and v^{φ}, correspondingly the u^{λ} and v^{λ} in the logical layer.[40] This path is based on the least transfer times and the least stops passing from start stop to end stop and if there are several equal choices, we randomly choose one of the paths. The set of paths corresponding to all logical edges is called mapping $M(E^{\lambda})$ from the logical topology to the physical topology. So now the load $L(e^{\varphi})$ of the edges e^{φ} in the physical layer is the sum of weights of all logical edges whose paths traverse this edge, and given as

$$L(e^{\varphi}) = \sum_{e^{\lambda}: e^{\varphi} \in M(e^{\lambda})}^{w(e^{\lambda})} . \tag{5}$$

Load on an edge in physical layer can be seen as the passenger flows traverse over it.[35] Considering economic limitation, capacities of routes are also considered. Following Motter–Lai model,[30] the capacities of an edge C_{ij} in physical layer are defined as

$$C_{ij} = (1+\alpha)L_{ij}, \tag{6}$$

where L_{ij} is the initial load of the edge e_{ij} in the physical layer and $\alpha \geq 0$ is the tolerance parameter which represents the capacity of the edge under economic limitation. When load on an edge exceeds its capacity, that is, $L_{ij} + \Delta L_{ij} > (1+\alpha)L_{ij}$ and ΔL_{ij} represents the additional load, the edge cannot provide the bus transit and will be broken down.

4.3. Cascade failure process and load redistribution

In the physical layer as shown in Fig. 9, if edge e_{ij} is overloaded and cannot provide the bus transit for more passengers, we say it as being broken down. Edges traversing over e_{ij} are all affected in logical layer. So passengers begin to change their paths, affected edge e^{λ} is deleted when there is no path in the physical layer, G^{φ}, otherwise, the logical edge e^{λ} remains in the network, and its mapping is updated by the shortest path in G^{φ}.

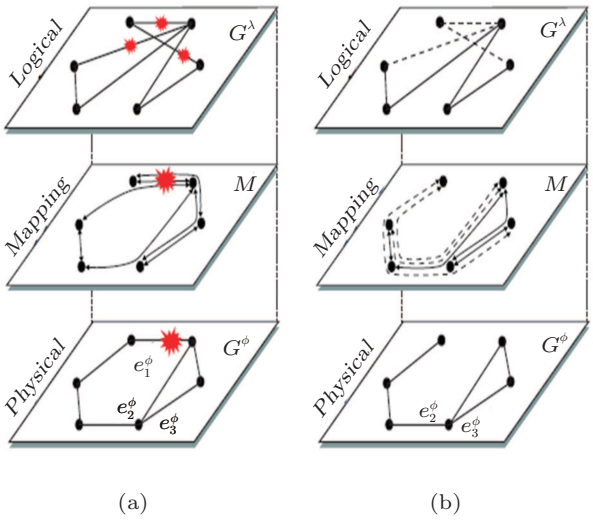


Fig. 9. (color online) Illustrations of the method of redistribution, showing (a) effect propagation in two-layer system when e_{ij} is broken down, and (b) dashed lines are valid in logical layer under rerouting.

Once an edge is broken in the physical layer, loads on this edge should be redistributed and some other edges will receive additional loads. If the total loads on other edges do not exceed their capacities, the cascading failures cannot be triggered. But if loads on the broken edge are high, after redistribution of the loads, some of the other edges are overloaded and then will be broken down. It will result in the further distribution of loads and may trigger cascading failures of the whole network finally. In the urban bus network, higer load means more passenger flows on infrastructures and edges with higher loads are the key routes in physical layer. The key edge is more representative and can influence the cascade failure progress largely.[35,38] So in this paper, we only remove several edges with higher loads in the physical layer.

4.4. Simulation in shenyang bus network

Here, we choose 4 parameters mentioned above, which are the relative size S, diameter D, mean path length L in the giant component, and network performance parameter E. At first, we remove top 10 edges with the highest loads. The results of the attack on the two-layer Shenyang bus network are shown in Fig. 10.

In Fig. 10, horizontal axis represents the tolerance parameter α. Obviously, the higher the value of α for an edge, the larger capacities the edge has and the more difficult the edge to be broken down is, which means that each edge has enough capacity to deal with the additional loads. When α increases from 0 to 1, L monotonically decreases, which means that larger α makes the network more robust.

But considering S, D, and E, the curves are more oscillatory than L. Furthermore, from Fig. 10(a) and 10(b), we can see that the curves are not monotonic obviously. In fact, the curve in Fig. 10(d) is also not monotonic, for example, when $\alpha = 0.8$, E is larger than when $\alpha = 0.9$. So sometimes smaller α may increase robustness of the network. This is because when some edges are broken down in the physical network, the edges in logical layer cannot be mapped on the physical graph, the loads of this path cannot be redistributed to other edges, and for some of other edges in logical layer, they may change their paths so that the loads can be transferred to other uncongested physical edges. We call these broken edges critical edges. With the capacities decreasing, edges are easier to destroy, once the critical edges are broken down, loads of some nodes in network may reduce (We call the phenomenon "load-reducing phenomenon"). That means that in an actual urban bus network, if some of the congested edges between bus stops are closed, which causes passengers unable to make a detour then choose to go out later or change to other lower loads edges, traffic congestion will decrease.

In Fig. 11, relative size S of the giant component in logical layer is the same as the one in physical layer, which means that some stops are not reachable due to the cascade failures. But considering L, D, and E, more load-reducing phenomena emerge. When$\alpha = 0.8$, mean path length L is smaller than with $\alpha = 0.9$, network performance parameter E is larger than with $\alpha = 0.9$, that means more paths are broken down when $\alpha = 0.9$ in logical layer (more broken paths cannot be replaced). When $\alpha = 0.4$, diameter D in giant component is larger than with $\alpha = 0.3$. So this directly confirms our explanation about the load-reducing phenomenon.

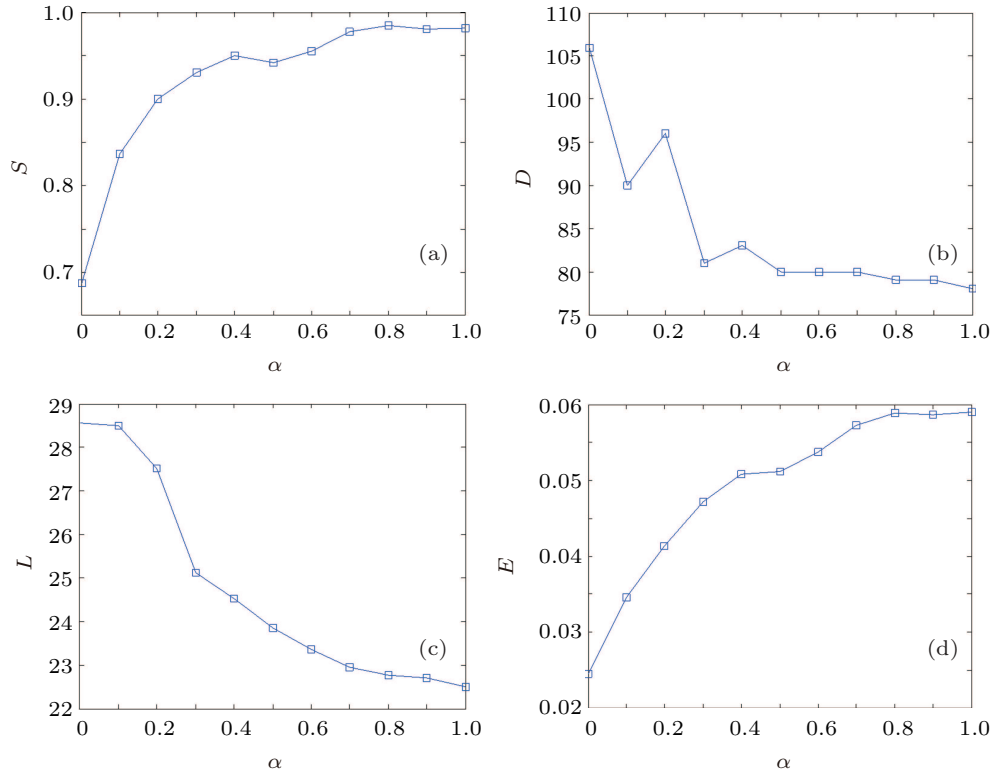


Fig. 10. (color online) Attack results in physical layer (a) relative size S, (b) diameter D, (c) mean path length L, (d) network performance parameter E versus tolerance parameter α.

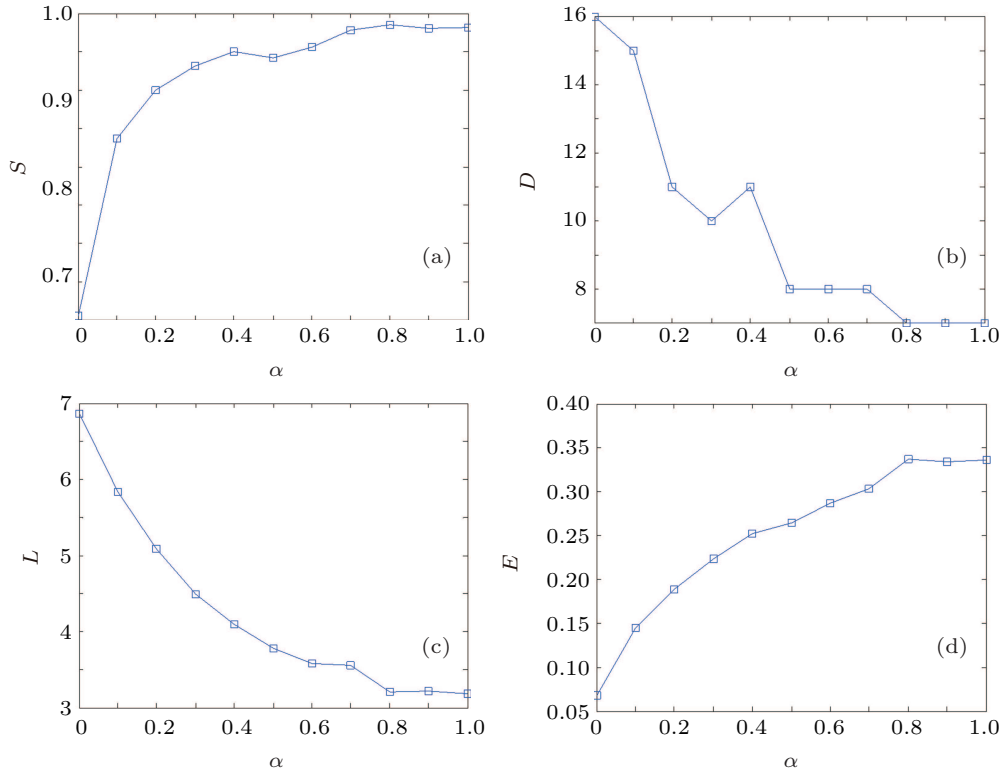


Fig. 11. (color online) Attack results in logical layer, showing (a) relative size S, (b) diameter D, (c) mean path length L, and (d) network performance parameter E versus tolerance parameter α.

4.5. Suggestions for solving the urban bus network congestion

In this subsection, we consider the passenger flows and analyse the dynamic cascade failures in the urban bus network. As a result, "load-reducing phenomenon" is found when some of the critical edges are broken down. According to this conclusion, when the congestion occurs, the managers may close some of the congested bus lines ephemerally between some

bus stops (such as by changing the routes of the bus lines or taking no passengers at these bus stops), so the passengers will choose to go out later or change to other path which will reduce the loads in the congested lines. In fact, it is like the way of "odd-and-even license plate rule" in some big cities of China.

5. Conclusions

In this paper, we construct three static models (bus stop network, bus transfer network, and bus line network) to analyse the structures and the invulnerabilities of urban bus networks under two attack strategies. We find that bus stop network is robust under the random attack, but more fragile under the selective attack. In addition, attack on the bus transfer network shows less influence on transfer time under the random attack compared with on the bus stop network, but has huge impact under the selective attack. On the contrary, difference between two types of attacks in the bus line network is not obvious. Because bus lines, stops and transfers can influence each other. Furthermore, we analyze the invulnerabilities of the stop-line network and line-transfer network, and by using the proposed two layered networks, we study the interactions among three models. Then some suggestions for the urban bus network planning are made.

Finally, the cascades failures of urban bus network are analysed by constructing a two-layer bus network model. We find that properly increasing the capacity of the road will reduce the probability of cascading failure. Additionally, removing some of the critical edges between bus stops may reduce the loads of the bus network. We also give our suggestion for alleviating congestion.

References

[1] Watts D J and Strogatz S H 1998 *Nature* **393** 440
[2] Barabási A L and Albert R 1999 *Science* **286** 509
[3] Wang J L, Liu F A and Zhu Z F 2015 *Acta Phys. Sin.* **64** 045206 (in Chinese)
[4] Barabasi A L, Oltvai Z N 2004 *Nature Reviews Genetics* **5** 101
[5] Montoya J M and Sole R V 2002 *J. Theor. Bio.* **214** 405
[6] Barabási A L, Albert R and Jeong H 2000 *Physica A: Statistical Mechanics and its Applications* **281** 69
[7] Barabási A L, Albert R and Jeong H 1999 *Physica A: Statistical Mechanics and its Applications* **272** 173
[8] Pastor-Satorras R and Vespignani A 2007 *Evolution and structure of the Internet: A statistical physics approach* (Cambridge: Cambridge University Press)
[9] Strogatz S H 2001 *Nature* **410** 268
[10] Zhou T, Bai W J, Wang B H, Liu Z J and Yan G 2005 *Physics* **34** 0 (in Chinese)
[11] Latora V and Marchiori M 2002 *Physica A: Statistical Mechanics and its Applications* **314** 109
[12] Sen P, Dasgupta S, Chatterjee A, Sreeram P A, Mukherjee G and S S 2003 *Phys. Rev. E* **67** 036106
[13] Guimera R and Amaral L A N 2004 *The European Physical Journal B-Condensed Matter and Complex Systems* **38** 381
[14] Kaluza P, Kölzsch A, Gastner M T and Blasius B 2010 *Journal of the Royal Society Interface* **7** 1093
[15] Sienkiewicz J and Hołyst J A 2005 *Phys. Rev. E* **72** 046127
[16] Wu J J, Gao Z Y, Sun H J Huang H J 2004 *Mod. Phys. Lett. B* **18** 1043
[17] Wu J J, Gao Z Yand Sun H J 2006 *Int. J. Mod. Phys. B* **20** 2129
[18] Zheng X, Chen J P, Shao J L and Bie L D 2012 *Acta Phys. Sin* **61** 190510 (in Chinese)
[19] Xu X P, Hu J H, Liu F and Liu L S 2007 *Physica A: Statistical Mechanics and its Applications* **374** 441
[20] Soh H, Lim S, Zhang T Y, Fu X J, Lee G K K, Hung T G G, Di P, Prakasam S, Wong L 2010 *Physica A: Statistical Mechanics and its Applications* **389** 5852
[21] Wang Z and Peng Q Y 2007 *The Traffic and the Computer* **25** 39 (in Chinese)
[22] Li A W 2007 *Journal of North University: Natural Science* **28** 314 (in Chinese)
[23] Zhang C and Zhang N 2006 *Journal of Shanghai University of Science and Technology* **28** 489 (in Chinese)
[24] Wang B, Ke H H and Jiang T F 2011 *Journal of Wuhan University: Engineering Science* **44** 404 (in Chinese)
[25] Albert R, Jeong H and Barabási A L 2000 *Nature* **406** 378
[26] Callaway D S, Newman M E J, Strogatz S H and Watts D J 2000 *Phys. Rev. Lett.* **85** 5468
[27] Cohen R, Erez K, Ben-Avraham D and Havlin S 2000 *Phys. Rev. Lett.* **85** 4626
[28] Albert R, Albert I and Nakarado G L 2004 *Phys. Rev. E* **69** 025103
[29] Wang J R, Wang J P, H Z and Xu H T 2015 *Chin. Phys. B* **24** 060101
[30] Motter A E and Lai Y C 2002 *Phys. Rev. E* **66** 065102
[31] Kinney R, Crucitti P, Albert R and Latora V 2005 *The European Physical Journal B-Condensed Matter and Complex Systems* **46** 101
[32] Wang J W, Jiang C and Qian J F 2013 *Int. J. Mod. Phys. C* **24** 1350076
[33] Xiao Y D, Lao S Y, Hou L L and Bai L 2014 *Chin. Phys. B* **23** 118902
[34] Von Ferber C, Holovatch T and Holovatch Y 2009 *Traffic and Granular Flow'*07 (Berlin: Springer)
[35] Dong Y, Yang X and Chen G 2014 *International Journal of Information Technology and Computer Science (IJITCS)* **6** 30
[36] Derrible S and Kennedy C 2010 *Physica A: Statistical Mechanics and its Applications* **389** 3678
[37] Latora V and Marchiori M 2001 *Phys. Rev. Lett.* **87** 198701
[38] Wang X F, Li X and Chen G R 2005 *Complex network theory and application* (Beijing: Tsinghua University Press) (in Chinese)
[39] Liu W Y and Liu B 2014 *Acta Phys. Sin.* **63** 248901 (in Chinese)
[40] Kurant M and Thiran P 2006 *Phys. Rev. Lett.* **96** 138701
[41] Zou S R, Zhou T, Liu A F, Xu X L and He D R 2010 *Phys. Lett. A* **374** 4406