Synchronization investigation of the network group constituted by the nearest neighbor networks under inner and outer synchronous couplings*

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A new synchronization technique of inner and outer couplings is proposed in this work to investigate the synchronization of network group. Some Haken–Lorenz lasers with chaos behaviors are taken as the nodes to construct a few nearest neighbor complex networks and those sub-networks are also connected to form a network group. The effective node controllers are designed based on Lyapunov function and the complete synchronization among the sub-networks is realized perfectly under inner and outer couplings. The work is of potential applications in the cooperation output of lasers and the communication network.

Keywords: synchronization of network group, inner and outer couplings, Haken–Lorenz laser, nearest neighbor network

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1. Introduction

In recent years, it was found that the complex network is ubiquitous in human society and nature and increasingly plays an important role in many fields, such as communication network, transportation network, and power grid, and so on.[1–6] Therefore, the investigations about the dynamics behaviors of a complex network, especially about the network synchronization (including inner synchronization and outer synchronization), attract more and more comprehensive attention.[7–11] For example, Huys *et al.* reported synchronization research of chaotic networks with multiple delays[12] and Poria *et al.* discussed spatiotemporal synchronization of coupled Ricker maps over a complex network.[13] Correspondingly, the various network synchronization methods have been proposed, for instance, projection synchronization, adaptive synchronization, sliding mode synchronization and backstepping synchronization method, etc.[14,15]

The works about complex network synchronization reported previously focus mainly on the synchronization among all nodes within the network[16–18] (so-called inner synchronization) and on the synchronization between two complex networks (so-called outer synchronization).[19–22] In practice, however, the dynamics characteristics among some sub-networks connected to each other are also needed to analyze in-depth. For example, it is an urgent problem how to obtain the synchronization relationship among local networks or between local network and global network in the all optical communication network. Furthermore, the technique adopted for the synchronization of network group is different from these used in the synchronization within a network or between networks. Hence, the synchronization investigation about the network group which is constituted by a few sub-networks has become a hot topic in many research and application fields.

However, the research about synchronization of lots of complex networks, i.e., the so-called network group, has just started. A few works have been reported, for example, Jalan investigated the impact of the interaction of nodes in a layer of a multiplex network on the dynamical behavior and cluster synchronization of these nodes in the other layers.[23] Singh studied the impact of multiplexing on the global phase synchronizability of different layers in the delayed coupled multiplex networks.[24] Therefore, the research on the dynamics behaviors of the network group, especially on the synchronization principle, is still needed to explore in depth. In our work, a novel technique for realizing the synchronization among multiple networks is proposed and the controller forms of the nodes in every complex network are determined based on Lyapunov stability theorem.[25] In order to demonstrate the effectiveness of our synchronization method, some Haken–Lorenz laser systems with chaos behaviors are selected as the nodes to construct a few nearest neighbor complex networks, at the same time, these sub-networks are also reconnected to form a network group. The complete synchronization among the sub-networks is realized perfectly under inner and outer couplings,

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which also indicates the work is of potential applications in the cooperation output of lasers and the communication network.

2. Synchronization mechanism of network group

In this section, the synchronization mechanism of complex networks within the network group will be investigated. Some nonlinear dynamics systems with chaos behaviors are taken as the notes to construct M sub-networks ($M = 3,4,\ldots$) and to form a network group under inner and outer couplings. Within each sub-network, there are N nodes ($N = 1,2,\ldots$). Hence, the state equations of the i-th node in the 1st sub-network and the q-th sub-network can be respectively described as follows:

$$\begin{aligned}\dot{x}_{i1}(t) &= f_{i1}(x_{i1}(t)) + m_1 \sum_{j=1}^{N} a_{ij1} x_{j1}(t) \\ &\quad + \sum_{p=2}^{M} g_p \sum_{j=1}^{N} b_{ijp} x_{jp}(t), \end{aligned} \tag{1}$$

$$\begin{aligned}\dot{x}_{iq}(t) &= f_{iq}(x_{iq}(t)) + m_q \sum_{j=1}^{N} a_{ijq} x_{jq}(t) \\ &\quad + \sum_{\substack{p=1 \\ p\neq q}}^{M} g_p \sum_{j=1}^{N} b_{ijp} x_{jp}(t), \quad q = 2,3,\ldots,M, \end{aligned} \tag{2}$$

where

$$x_i(t) = \left[x_i^1(t), x_i^2(t), \ldots, x_i^n(t)\right]^{\mathrm{T}} \in R^n$$

and

$$x_{iq}(t) = \left[x_{iq}^1(t), x_{iq}^2(t), \ldots, x_{iq}^n(t)\right]^{\mathrm{T}} \in R^n$$

are the state variables of node equations belonging to the 1st and q-th sub-networks, respectively, and $f(\cdot): R^n \to R^n$ is the nonlinear continuous differential function. N is the node number of each sub-network and M is the network number of the group. The symbols p and q respectively denote different sub-networks in the network group. $A = (a_{ijq}) \in R^{N\times N}$ and $B = (b_{ijq}) \in R^{N\times N}$ are the inner and outer coupling matrices of the sub-networks, and here the matrix elements a_{ijq} and b_{ijq} are defined as follows: If there is a connection between the node i and the node j, $a_{ijq} = 1$ ($i \neq j$); otherwise $a_{ijq} = 0$ ($i \neq j$). Similarly, if there is a coupling between node j in sub-network p and node i in sub-network q, $b_{ijq} = 1$; otherwise $b_{ijq} = 0$. m_q is the inner coupling strength of the sub-network q and g_p is the outer coupling strength between the nodes within different sub-networks q and p.

In this work, the first sub-network is set as target network and the rest of the sub-networks within the network group will track and synchronize with the first one. Under inner and outer couplings, the dynamics equations Eq. (2) of the corresponding nodes at respective controlled sub-networks are rewritten as follows:

$$\begin{aligned}\dot{x}_{iq}(t) &= f_{iq}(x_{iq}(t)) + m_q \sum_{j=1}^{N} a_{ijq} x_{jq}(t) \\ &\quad + \sum_{\substack{p=1 \\ p\neq q}}^{M} g_p \sum_{j=1}^{N} b_{ijp} x_{jp}(t) + u_{iq}(t), \end{aligned} \tag{3}$$

where $u_{iq}(t)$ are the node controllers within the q-th sub-network and the error equations between Eq. (1) and Eq. (3) are defined as

$$e_{iq1}(t) = x_{iq}(t) - x_{i1}(t). \tag{4}$$

Then, the derivatives of errors can be obtained

$$\begin{aligned}\dot{e}_{iq1}(t) &= \dot{x}_{iq}(t) - \dot{x}_{i1}(t) \\ &= \Big(f_{iq}(x_{iq}(t)) + m_q \sum_{j=1}^{N} a_{ijq} x_{jq}(t) \\ &\quad + \sum_{\substack{p=1 \\ p\neq q}}^{M} g_p \sum_{j=1}^{N} b_{ijp} x_{jp}(t) + u_{iq}(t)\Big) - \Big(f_{i1}(x_{i1}(t)) \\ &\quad + m_1 \sum_{j=1}^{N} a_{ij1} x_{j1}(t) + \sum_{p=2}^{M} g_p \sum_{j=1}^{N} b_{ijp} x_{jp}(t)\Big) \\ &= f_{iq}(x_{iq}(t)) - f_{i1}(x_{i1}(t)) + m_q \sum_{j=1}^{N} a_{ijq} x_{jq}(t) \\ &\quad - m_1 \sum_{j=1}^{N} a_{ij1} x_{j1}(t) + g_1 \sum_{j=1}^{N} b_{ij1} x_{j1}(t) \\ &\quad - g_q \sum_{j=1}^{N} b_{ijq} x_{jq}(t) + u_{iq}(t). \end{aligned} \tag{5}$$

For the network group constituted by the complex sub-networks, we select the Lyapunov function as

$$V = \frac{1}{2} \sum_{q=2}^{M} \sum_{i=1}^{N} e_{q1}^{\mathrm{T}}(t)\, e_{q1}(t), \tag{6}$$

and the derivative of Lyapunov function is

$$\dot{V} = \sum_{q=2}^{M} \sum_{i=1}^{N} e_{iq1}^{\mathrm{T}}(t)\, \dot{e}_{iq1}(t). \tag{7}$$

Hence, we can gain:

$$\begin{aligned}\dot{V} &= \sum_{q=2}^{M} \sum_{i=1}^{N} e_{iq1}^{\mathrm{T}}(t) \left(f_{iq}(x_{iq}(t)) - f_{i1}(x_{i1}(t))\right) \\ &\quad + m_q \sum_{j=1}^{N} a_{ijq} x_{jq}(t) - m_1 \sum_{j=1}^{N} a_{ij1} x_{j1}(t) \\ &\quad + g_1 \sum_{j=1}^{N} b_{ij1} x_{j1}(t) - g_q \sum_{j=1}^{N} b_{ijq} x_{jq}(t) + u_{iq}(t)). \end{aligned} \tag{8}$$

Through choosing some appropriate node controllers in the q-th sub-network, the synchronization between the state variables belonging to different sub-networks q and p will be achieved while the conditions $\lim_{x\to+\infty} \left\| x_q(t) - x_p(t) \right\| =$

$0,(i=1,2,\ldots,N)$ are satisfied.[26] Therefore, we design the node controllers as

$$\begin{aligned} u_{iq}(t) &= -f_{iq}(x_{ip}(t))+f_{ip}(x_{iq}(t)) \\ &\quad -m_q\sum_{j=1}^{N}a_{ijq}x_{jp}(t)+m_p\sum_{j=1}^{N}a_{ijp}x_{jq}(t) \\ &\quad -g_p\sum_{j=1}^{N}b_{ijp}x_{jq}(t)+g_q\sum_{j=1}^{N}b_{ijq}x_{jp}(t)-ke_{iqp}(t), \end{aligned} \tag{9}$$

Substituting Eq. (9) into Eq. (8), then the latter can be rewritten as

$$\begin{aligned} \dot{V} &= \sum_{q=2}^{M}\sum_{i=1}^{N}e_{iq1}^{\mathrm{T}}(t)\Big(f_{iq}(x_{iq}(t))-f_{i1}(x_{i1}(t)) \\ &\quad +m_q\sum_{j=1}^{N}a_{ijq}x_{jq}(t)-m_1\sum_{j=1}^{N}a_{ij1}x_{j1}(t) \\ &\quad +g_1\sum_{j=1}^{N}b_{ij1}x_{j1}(t)-g_q\sum_{j=1}^{N}b_{ijq}x_{jq}(t)-f_{iq}(x_{ip}(t)) \\ &\quad +f_{ip}(x_{iq}(t))-m_q\sum_{j=1}^{N}a_{ijq}x_{jp}(t) \\ &\quad +m_p\sum_{j=1}^{N}a_{ijp}x_{jq}(t)-g_p\sum_{j=1}^{N}b_{ijp}x_{jq}(t) \\ &\quad +g_q\sum_{j=1}^{N}b_{ijq}x_{jp}(t)-ke_{iqp}(t)\Big). \end{aligned} \tag{10}$$

Based on Lipschitz condition,[14] that is, the following inequality will be satisfied while there exists a real number $l_i>0$.

$$\left\|f_i(x_{ip}(t))-f_i(x_{iq}(t))\right\|\le l_i\left\|x_{ip}(t)-x_{iq}(t)\right\|, \tag{11}$$

where

$$x_{ip}(t)=\left[x_{ip}^1(t),x_{ip}^2(t),\ldots,x_{ip}^n(t)\right]^{\mathrm{T}}\in R^n$$

and

$$x_{iq}(t)=\left[x_{iq}^1(t),x_{iq}^2(t),\ldots,x_{iq}^n(t)\right]^{\mathrm{T}}\in R^n$$

are time varying vectors. Therefore, equation (10) can be further written as

$$\begin{aligned} \dot{V} &\le \sum_{q=2}^{M}\sum_{i=1}^{N}e_{iq1}^{\mathrm{T}}(t)(L_{q1}e_{iq1}(t)+L_{q2}e_{iq1}(t) \\ &\quad +m_q\sum_{j=1}^{N}a_{ijq}e_{jq1}(t)+m_1\sum_{j=1}^{N}a_{ij1}e_{jq1}(t) \\ &\quad -g_1\sum_{j=1}^{N}b_{ij1}e_{jq1}(t)-g_q\sum_{j=1}^{N}b_{ijq}e_{jq1}(t)-ke_{iq1}(t)) \\ &= \sum_{q=2}^{M}\sum_{i=1}^{N}e_{iq1}^{\mathrm{T}}(t)(L_{q1}+L_{q2}+m_q\sum_{j=1}^{N}a_{ijq}+m_1\sum_{j=1}^{N}a_{ij1} \\ &\quad -g_1\sum_{j=1}^{N}b_{ij1}-g_q\sum_{j=1}^{N}b_{ijq}-k)e_{iq1}(t) \\ &= \sum_{q=2}^{M}\sum_{i=1}^{N}e_{iq1}^{\mathrm{T}}(t)(L_q-k+m_qA_{iq} \\ &\quad +m_1A_{i1}-g_1B_{i1}-g_qB_{iq})e_{iq1}(t), \end{aligned} \tag{12}$$

where the error e_{iq1} can be regarded as the same as that of e_{jq1} because both i and j are the serial numbers of nodes in the network group, $i,\ j=1,2,\ldots,N$. Also, L_{q1}, L_{q2}, and L_q are all positive real numbers and we let $L_q=L_{q1}+L_{q2}$ for convenience. Hence

$$\begin{aligned} \dot{V} &= \sum_{q=2}^{M}\sum_{i=1}^{N}e_{iq1}^{\mathrm{T}}(t)\dot{e}_{iq1}(t) \\ &\le \sum_{q=2}^{M}E_q^{\mathrm{T}}(L_q-k+m_qA_{iq}+m_1A_{i1}-g_1B_{i1}-g_qB_{iq})E_q, \end{aligned} \tag{13}$$

where

$$E_q=(e_{1q}^{\mathrm{T}},\ e_{2q}^{\mathrm{T}},\ \ldots,\ e_{Nq}^{\mathrm{T}})^{\mathrm{T}}\in R^{N\times N}.$$

It is known that the sums of all rows in the configuration matrices are equal to zero, because the matrices $A_{i1}=\sum_{j=1}^{N}a_{ij1}, A_{iq}=\sum_{j=1}^{N}a_{ijq}, B_{i1}=\sum_{j=1}^{N}b_{ij1}$, and $B_{iq}=\sum_{j=1}^{N}b_{ijq}$ need to satisfy the dissipation condition, which leads to $A_{i1}=A_{iq}=B_{i1}=B_{iq}=0$. So the values of parameters m_1, m_q, g_1, and g_q can be adjusted to meet the conditions of the network group synchronization (in our simulations, the following parameters $m_1=0.02$, $m_2=0.05$, $m_3=0.2$, $m_4=0.05$, $g_1=0.05$, $g_2=0.05$, $g_3=0.05$, and $g_4=0.05$ are chosen). In other words, $\dot{V}\le 0$ once k is taken as large enough to satisfy $L_q-k+m_qA_{iq}+m_1A_{i1}-g_1B_{i1}+g_qB_{iq}\le 0$ and according to Lyapunov stability theorem, the error equations between arbitrary two sub-networks will trend quickly to zero, implying that the synchronization of network group constituted by the complex sub-networks can be obtained perfectly. It is worth noting that, in the above discussion, the numbers of sub-networks and nodes within a sub-network can be chosen freely without any limitation, which indicates the synchronization technique of the network group proposed in our work is universal.

It can be found that the theoretic deduction in this section mainly focuses on a directed topology. In fact, this method can be applied in the synchronization of the network group with the bidirectional coupling as well. In this case, the state equations of the i-th nodes in the 1st sub-network and the q-th sub-network can be respectively rewritten as follows:

$$\begin{aligned} \dot{x}_{i1}(t) &= f_{i1}(x_{i1}(t))+m_1\sum_{j=1}^{N}a_{ij1}\left(x_{j1}(t)-x_{i1}(t)\right) \\ &\quad +\sum_{p=2}^{M}g_p\sum_{j=1}^{N}b_{ijp}(x_{jp}(t)-x_{i1}(t)), \\ \dot{x}_{iq}(t) &= f_{iq}(x_{iq}(t))+m_q\sum_{j=1}^{N}a_{ijq}(x_{jq}(t)-x_{iq}(t)) \end{aligned} \tag{14}$$

$$+\sum_{\substack{p=1\\p\neq q}}^{M} g_p \sum_{j=1}^{N} b_{ijp}\left(x_{jp}(t)-x_{iq}(t)\right),$$
$$q=2,3,\ldots,M, \tag{15}$$

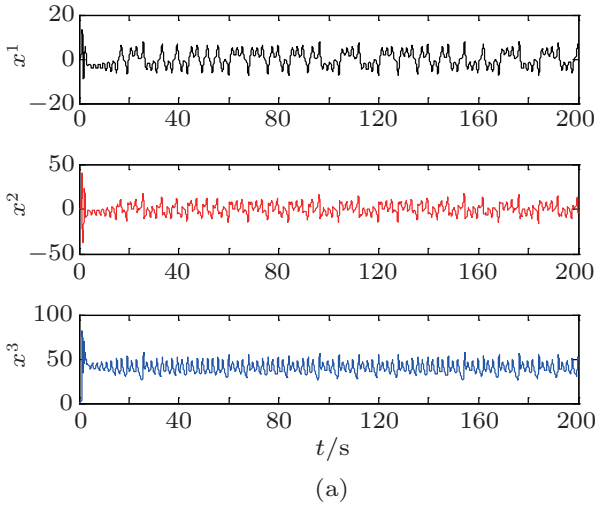
and the synchronization can be achieved by the similar method.

3. Numerical simulation

In our simulations, forty Haken–Lorenz laser chaotic systems with different initial values are taken as the nodes to construct four nearest neighbor networks and there are ten nodes in each sub-network. Haken–Lorenz laser system with chaos behavior can be expressed as follows:[27]

$$\begin{cases} \dot{x}^1 = a\left(x^2 - x^1\right), \\ \dot{x}^2 = rx^1 - x^1x^3 - x^2, \\ \dot{x}^3 = x^1x^2 - bx^3, \end{cases} \tag{16}$$

where x^1, x^2, and x^3 are state variables and parameters a, r, and b are positive real constant. The Haken–Lorenz laser system will be in chaotic state when the parameters are set as $a = 1.4235$, $b = 0.2778$, and $r = 43$. The evolutions of state variables and the phase diagram are shown in Fig. 1 while the initial values of three state variables are chosen in the range [0, 1].



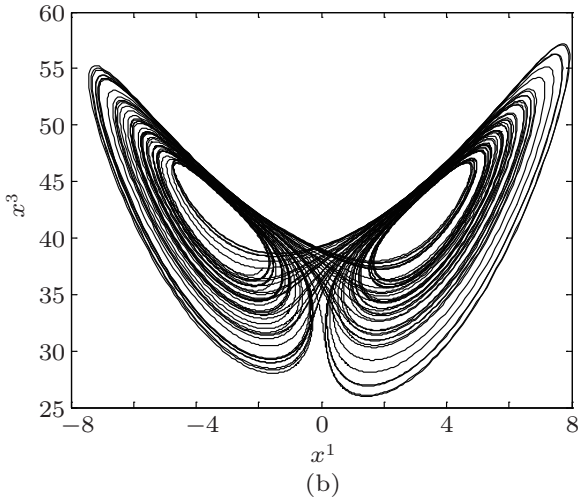


Fig. 1. (color online) Evolutions of three state variables (a) and phase diagram (b) of the chaotic Haken–Lorenz laser.

Figure 2(a) is the topology structure of the nearest network with 10 nodes and figure 1(b) is diagrammatic sketch of the network group constituted by four sub-networks under inner and outer couplings. What needs to be explained is that four sub-networks are the same in our work for convenience, i.e., they are all nearest networks with 10 nodes, meaning the inner coupling coefficient matrices of four sub-networks are also the same and can be expressed as

$$G = \begin{bmatrix} -2 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -2 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -2 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -2 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 \end{bmatrix}. \tag{17}$$

Figure 3 shows all error evolutions of ten nodes between target sub-network 1 and three controlled sub-networks with time, in which the meaning of e^d_{q1i} can be understood as follows: d means the dimension ($d = 1, 2, 3$) of node state variable of the Haken–Lorenz laser; q represents the controlled sub-network (q = 2, 3, 4) and the second subscript 1 describes the target sub-network; i denotes node (i = 1–10) within every sub-network.

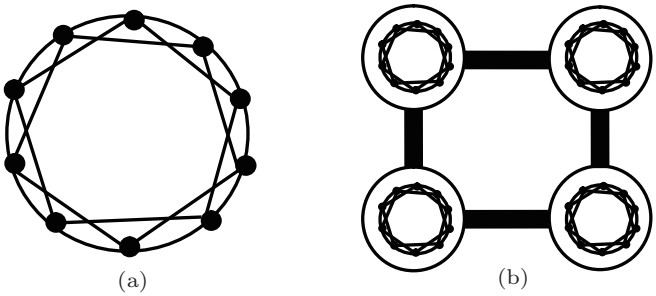


Fig. 2. Topology structure of sub-network (a) and structure sketch of network group (b).

It can be found from Fig. 3 that the error curves will exhibit intense oscillations at the initial stage of evolutions, and tend to zero rapidly and keep zero after a transient oscillation (about 2.7 s) under the effective node controllers, even though the initial values of every Haken–Lorenz lasers at different nodes are diverse, indicating that the synchronization technique proposed in our work is effective.

Hence, it can be seen from Fig. 3, due to the imposed controllers in the controlled network nodes, all errors after transient oscillations all trend to zero quickly, which means that this section of the proposed synchronization scheme is effective.

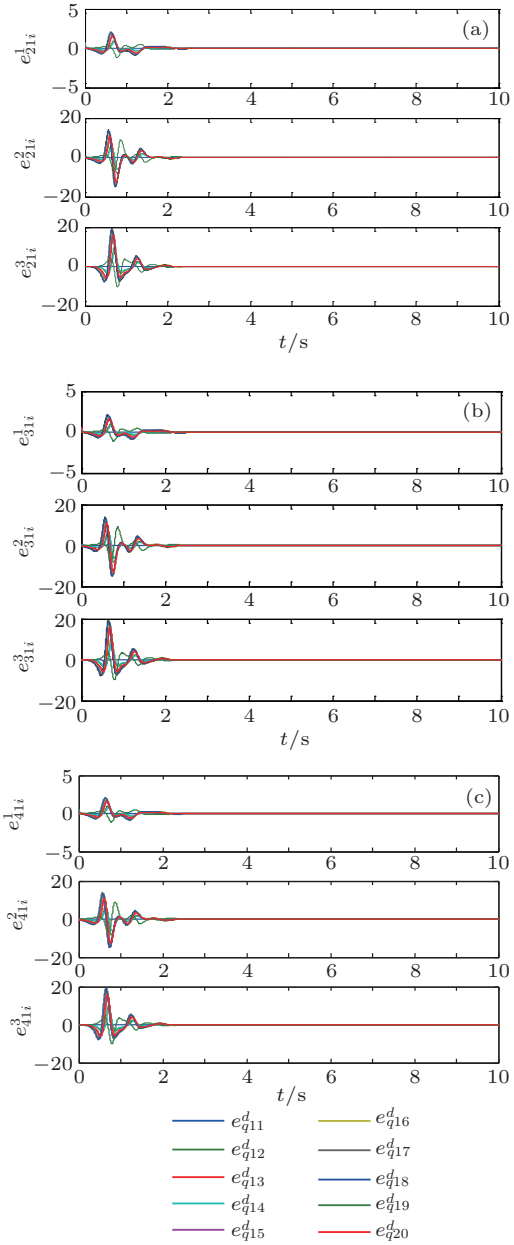


Fig. 3. (color online) Error evolutions of ten nodes with time between (a) network 2 and network 1; (b) network 3 and network 1; (c) network 4 and network 1.

4. Conclusion

In this paper, a new synchronization technique of inner and outer couplings is proposed in order to investigate the synchronization of the network group. The effective node controllers are designed based on Lyapunov function and the complete synchronization among the sub-networks is realized perfectly under inner and outer couplings. For validating the effectiveness of our synchronization method, forty Haken–Lorenz lasers with chaotic behaviors are taken as the nodes to construct four nearest neighbor complex networks and those sub-networks are also connected to form a network group. The simulation results show that all error evolutions between target 1st sub-network and the controlled sub-network tend rapidly to zero after a transient oscillation even though the initial values of all Haken–Lorenz lasers at every nodes are different, which means the synchronization technique proposed in our work is effective. This work is of potential application in the cooperation output of lasers and the communication network.

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