SPECIAL TOPIC — Acoustics

An acoustic bending waveguide designed by anisotropic density-near-zero metamaterial*

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Anisotropic metamaterial with only one component of the mass density tensor near zero (ADNZ) is proposed to control the sound wave propagation. We find that such an anisotropic metamaterial can be used to realize perfect bending waveguides. According to a coordinate transformation, the surface waves on the input and output interfaces of the ADNZ metamaterial induces the sound energy flow to be redistributed and match smoothly with the propagating modes inside the metamaterial waveguide. According to the theory of bending waveguide, we realize the "T"-type sound shunting and convergence, as well as acoustic channel selection by embedding small-sized defects. Numerical calculations are performed to confirm the above effects.

Keywords: acoustic bending waveguide, anisotropic metamaterials, mass density tensor near zero

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1. Introduction

The acoustic properties exhibited by metamaterials are remarkable, and various amazing phenomena have been discovered in previous studies on metamaterials,[1–12] such as negative refraction,[1] negative Doppler effect,[2] and acoustic cloaking.[3–7] Metamaterial with parameters near zero are also an important and intriguing material.[13,14] Due to the fact that the equivalent parameters of metamaterial can be changed by simply adjusting the structure parameters, we can obtain not only the negative-index metamaterial with both negative density and bulk modulus,[15–17] but also the zero-index metamaterial (ZIM) with zero density and/or zero reciprocal of bulk modulus.[18,19] Recently, density-near-zero (DNZ) metamaterial and index-near-zero (INZ) metamaterial have aroused growing interest and various applications have been proposed.[20–28] Bongard *et al.* achieved acoustic transmission line metamaterial with zero refractive index by membranes and open channels.[20] Jing *et al.*,[21] Park *et al.*,[22] and Fleury and Alu[23] achieved near-zero-density metamaterial by using membrane structure both in numerical analysis and experimental verification respectively. In addition, Shen *et al.* investigated a type of anisotropic, acoustic complementary metamaterial which consists of unit cells formed by membranes and side branches with open ends.[24] Furthermore, Xu *et al.* designed and fabricated an anisotropic zero-mass acoustic metamaterial lens based on clamped paper membrane units.[25]

Anisotropic DNZ (ADNZ) metamaterial with only one component of the mass density tensor near zero can be used to manipulate the sound transmission in the designed paths. Various prototypes have been demonstrated to manipulate sound transmissions, such as acoustic concentration,[29] and sound transmission through ultranarrow acoustic channels.[30] The bending waveguide effect is one of the especially interesting applications.[31–33] For conventional bending waveguide, reflection and the distortion of the wave front are often complex. Many methods have been proposed to minimize the reflection and distortion of waves.[31]

In this paper, we investigate the properties of the ADNZ metamaterial further. First we find that the ADNZ metamaterial can be introduced into a waveguide system to control sound wave propagation and realize perfect bending waveguides. Then we realize the "T"-type sound shunting and convergence which support the multi-channel communication. Furthermore, acoustic channel selection by embedding small-sized defects is also an interesting finding inspired by the theory of a bending waveguide.

2. Simulation results and discussion

As illustrated in Fig. 1, a two-dimensional (2D) acoustic waveguide structure with hard walls consists of an input rectangular waveguide (region 0), an output rectangular waveguide (region 2), and a waveguide junction (region 1). Region 0 (region 2) is the host medium with an effective density of ρ_0

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and an effective bulk modulus of κ_0. The mass density tensor and bulk modulus of the ADNZ metamaterial in region 1 are (ρ_x, ρ_y) and κ_1 respectively, where ρ_x and ρ_y are the x and y components of the mass density tensor.

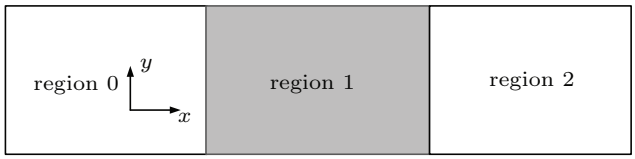


Fig. 1. The schematic graph of a 2D acoustic waveguide structure, whose input and output waveguides (regions 0 and 2) are filled with water with an effective density of $\rho_0 = 1000$ kg/m^3 and an effective bulk modulus of $\kappa_0 = 2.25$ GPa, consists of an ADNZ metamaterial (region 1). The parallel lines in the x direction are hard walls.

For convenience the time variation item $e^{-j\omega t}$ is omitted. The velocity field is given by

$$v_x = -\frac{1}{i\omega\rho_x}\frac{\partial p}{\partial x}$$

and

$$v_y = -\frac{1}{i\omega\rho_y}\frac{\partial p}{\partial y},$$

where p is the pressure field, v_x and v_y are the velocity field components in the x direction and y direction respectively, and ω is the angular frequency. When the condition $\rho_y \approx 0$ is satisfied, $\partial p/\partial y \approx 0$ is required to obtain the finite velocity. This means that the pressure field p in the ADNZ metamaterial is almost uniform in the y direction. Supposing that ρ_x is only dependent on the y coordinate, i.e., $\rho_x(y)$. The impedance of ADNZ metamaterial in the x direction is $\eta_x(y) = \sqrt{\rho_x(y)\kappa_0}$. The forced uniformity of the pressure field in the y direction in the ADNZ metamaterial exerts a strong averaging effect on the inhomogeneous mass density ρ_x. In fact, when $\eta_{x\text{eff}} = \eta_0$, ADNZ metamaterial is impedance matched to the host medium, thereby making the acoustic wave transmit totally. As $\partial p/\partial y \approx 0$, we can obtain $\partial(\rho_x v_x)/\partial y \approx 0$ and the amplitude of the velocity $v_x(y)$ should be proportional to $1/\rho_x(y)$.

In order to verify our theoretical analysis above, the full-wave simulations are carried out by using the finite element method (FEM). Parameters in the ADNZ metamaterial are set as $\rho_x(y) = \rho_0 y/[(y_1+y_2)/2]$, $\rho_y = \rho_0/1000$ and $\kappa_1 = \kappa_0$, where y_1 and y_2 are the upper and lower boundary of the waveguide respectively. Figure 2(a) shows that the pressure field p in the ADNZ metamaterial is almost uniform while $\rho_x(y)$ is a linear function of y. The amplitude of velocity $v_x(y)$ in the ADNZ metamaterial decays almost linearly with y, as shown in Fig. 2(b). Figure 2(c) shows the distribution of the velocity in the y direction.

Now we come to explain the mechanism of the acoustic energy propagating from the input interface to the output interface of the ADNZ metamaterial. The time-averaged acoustic energy flux density, also known as the acoustic intensity in the x direction $I_x = 0.5\text{Re}(v_x p^*)$ should be proportional to $1/\rho_x(y)$ as $v_x(y)$ is proportional to $1/\rho_x(y)$ and the pressure is constant. Therefore, the acoustic energy flow in the x direction is larger than in the lower $\rho_x(y)$ region as shown in Fig. 2(d). One can see that the surface waves on the input and output interfaces of the ADNZ metamaterial are so strong that they can cause the progress of the energy flow redistribution on the interfaces to smoothly connect with the propagating modes inside the ADNZ metamaterial. In order to have a better look at the way the energy flows, figure 2(e) clearly shows the existence of the acoustic intensity in the y direction $I_y = 0.5\,\text{Re}(v_y p^*)$ at the interfaces between the water and the ADNZ slab, and testifies such surface waves. Hence we can utilize the concentration of the acoustic intensity in the small $\rho_x(y)$ region to manipulate the way acoustic energy flows in the whole system.

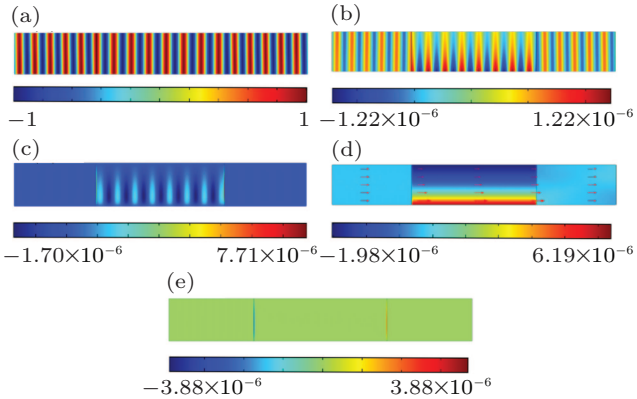


Fig. 2. (color online) Field distributions in the straight channel with an ADNZ metamaterial for (a) pressure field p, (b) velocity field in the x direction, (c) velocity field in the y direction, (d) acoustic intensity in the x direction, and (e) acoustic intensity in the y direction.

With the above results of a straight waveguide of ADNZ metamaterial, it would be interesting to investigate the cases for bending waveguides. As illustrated in Fig. 3(a), a 2D acoustic waveguide structure with hard walls consists of an input rectangular waveguide (region 0), an output rectangular waveguide (region 2), and a semicircular waveguide junction (region 1). Region 0 (region 2) is the host medium with an effective density of ρ_0 and an effective bulk modulus of κ_0. The mass density tensor or bulk modulus of the ADNZ metamaterial in region 1 is (ρ_r, ρ_θ) or κ_0. In order to gain the same phenomenon as that in a rectangular coordinate system, we assume that $g(r) = r$, $g'(r) = 1$, $\rho_y \approx 0$, $R = (y_1+y_2)/2$, $y_1 = 0.5$ m, $y_2 = 1.5$ m, and $\rho_x(y) = (y/R)\rho_{\text{water}}$. According to the coordinate transformation similar to that in Ref. [34], i.e., $x = R\theta$ and $y = g(r)$, we can obtain the following expression: $\rho_r = \rho_y g'(r) r/R$ and $\rho_\theta = \rho_x(y)R/g'(r)r$. Then we can obtain $\rho_r \approx 0$ and $\rho_\theta = \rho_{\text{water}}$. It means that the above straight waveguide is equal to the homogeneous bending waveguide. The impendences of the water and the ADNZ slab are matched in the θ direction, which leads to zero reflection. On the other hand, as $\rho_r \approx 0$, the pressure field is almost uniform in the radial

direction. In the cylindrical coordinate system, we can understand the zero reflection on the interface between the ADNZ metamaterial and water. Then, we make numerical simulations using the FEM. As a consequence, the ADNZ metamaterial has the capability of bending and almost totally transmits waves in a channel shown in Fig. 3(b), however, the wave front is distorted when the ADNZ metamaterial is replaced by water as shown in Fig. 3(c).

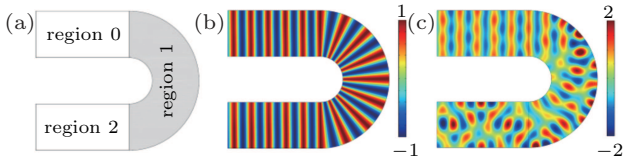


Fig. 3. (color online) (a) Schematic diagram of a 2D acoustic bending waveguide structure. The pressure field distributions with the bending part filled with (b) an ADNZ metamaterial ($\rho_r \approx 0$ and $\rho_\theta = \rho_{\text{water}}$) and (c) water.

Furthermore, it is not limited to the semicircular channel which can make wave propagation perfect, we can make the ADNZ metamaterial into other shapes such as "T" shape as shown in Fig. 4. Ports 1, 2, and 3 are filled with water, and ADNZ metamaterials are placed in the symmetrical corners in Fig. 4(a). When the input source is located in port 1 as shown in Fig. 4(b), it is clear to see that the incident wave successfully separates into two waves, which still keep the shape of the plane wave. More importantly, there is no reflection due to the matched impedances of ADNZ metamaterial and water. Thus the structure should be regarded as a beam splitter. However, if the ADNZ metamaterial is replaced by water, we will find that the reflection and the distortion of wave front are complicated as shown in Fig. 4(c). On the contrary, if the incident waves from ports 2 and 3 are of the same phase, meanwhile the wave numbers are respectively $k_2 = -1$ and $k_3 = 1$, we will find that these two waves merge into one plane wave perfectly as shown in Fig. 4(d). Similarly, if the ADNZ metamaterial is replaced by water, as a common sense, these two waves will conform to the superposition principle and even disappear as shown in Fig. 4(e). With the theory of a bending waveguide, we can realize the "T"-type sound shunting and convergence.

More interestingly, the total reflection can be achieved by embedding ideal soft defects in ADNZ metamaterial, which can be used to realize an acoustic switch as shown in Fig. 5. Figure 4(b) shows that the incident wave will separate into two waves if there are no defects. However, if there is an ideal soft defect in the above ADNZ metamaterial, we will find that the wave below can transmit successfully, and the above wave is blocked on the surface of the defect as shown in Fig. 5(a). Actually, the sound soft boundary condition induces the pressure to be zero on the surface of an ideal soft defect. Due to the uniformity of the pressure field in the ADNZ metamaterial, a small ideal soft defect can cause the pressure field to be zero, and the incident acoustic wave should be totally reflected by the ADNZ metamaterial, no matter how small in size the defect is. Similarly, if the ideal soft defect is placed in the below ADNZ metamaterial, the wave below will be blocked as shown in Fig. 5(b). Obviously, double blocking will take place when two defects are placed in both ADNZ metamaterials. The defect acts as an acoustic switch: if the defect is added into the channel, the channel is in an "off" state; if the defect is taken away from the channel, the channel is in an "on" state.

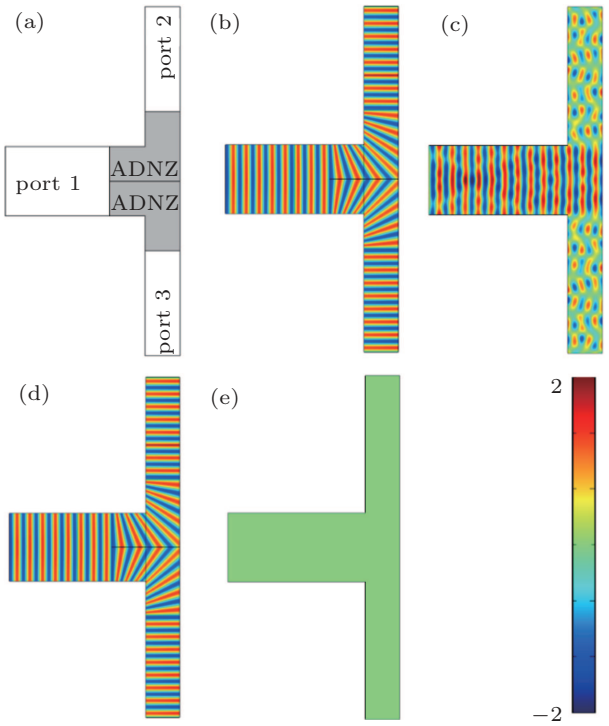


Fig. 4. (color online) Pressure field distribution in a "T"-shape channel filled with ADNZ metamaterial. (a) The schematic diagram of the "T" channel, where ports 1, 2, and 3 are filled with water, symmetrical corners are filled with ADNZ metamaterial, and their boundaries are hard walls. (b) Transmitting from port 1 to ports 2, 3 and (c) transmitting with ADNZ metamaterial replaced by water. (d) Transmitting from ports 2 and 3 to port 1. (e) Transmitting with ADNZ metamaterial replaced by water.

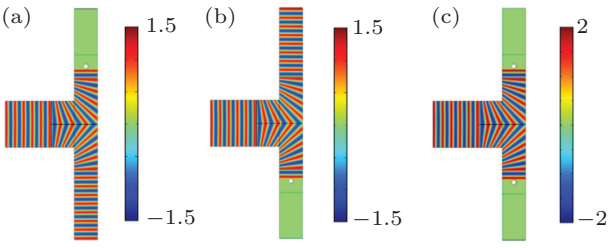


Fig. 5. (color online) Pressure field distributions in a "T"-shape channel filled with ADNZ metamaterial with (a) an ideal soft defect in the above ADNZ metamaterial, (b) an ideal soft defect in the below ADNZ metamaterial, (c) two defects in both ADNZ metamaterials.

3. Conclusions

In this work, perfect bending waveguides with ADNZ metamaterial are realized in theory and in simulation. Theoretical analyses reveal that the key to ensuring the total transmission in bending waveguides is the surface wave, which redistributes the sound energy flow. With the support of the

theory of bending waveguide, "T"-type sound shunting and convergence can support multi-channel communication, and the acoustic channel selection which is affected by the small-sized embedded defects can be used to realize acoustic switch. In practical applications, the bending waveguide achieved by anisotropic near-zero parameters can be possibly implemented by employing membrane-type acoustical metamaterial with properly tuned structural parameters, [24,25] which is the goal of our next work.

References

[1] Zhang S, Yin L and Fang N 2009 *Phys. Rev. Lett.* **102** 194301
[2] Lee S H, Park C M, Seo Y M and Kim C K 2010 *Phys. Rev. B* **81** 241102
[3] Chen H and Chan C T 2007 *Appl. Phys. Lett.* **91** 183518
[4] Cheng Y, Yang F, Xu J Y and Liu X J 2008 *Appl. Phys. Lett.* **92** 151913
[5] Cummer S A, Rahm M and Schurig D 2008 *New J. Phys.* **10** 115025
[6] Farhat M, Guenneau S and Enoch S 2009 *Phys. Rev. Lett.* **103** 024301
[7] Zhang S, Xia C and Fang N 2011 *Phys. Rev. Lett.* **106** 024301
[8] Zhu X F, Liang B, Kan W and Cheng J C 2011 *Phys. Rev. Lett.* **106** 014301
[9] Zhu X F, Ramezani H, Shi C Z, Zhu J and Zhang X 2014 *Phys. Rev. X* **4** 031042
[10] Zhu X F 2013 *Phys. Lett. A* **377** 1784
[11] Dai D D and Zhu X F 2013 *Europhys. Lett.* **102** 14001
[12] Zhao D G, Li Y and Zhu X F 2015 *Sci. Rep.* **5** 9376
[13] Wei Q, Cheng Y and Liu X J 2013 *Appl. Phys. Lett.* **102** 174104
[14] Gu Z M, Liang B, Zou X Y, Yang J, Li Y, Yang J and Cheng J C 2015 *Appl. Phys. Lett.* **107** 213503
[15] Li J and Chan C T 2004 *Phys. Rev. E* **70** 055602
[16] Ding Y, Liu Z, Qiu C and Shi J 2007 *Phys. Rev. Lett.* **99** 093904
[17] Lee S H, Park C M, Seo Y M, Wang Z G and Kim C K 2010 *Phys. Rev. Lett.* **104** 054301
[18] Zhou X and Hu G 2011 *Appl. Phys. Lett.* **98** 263510
[19] Liu F, Huang X and Chan C T 2012 *Appl. Phys. Lett.* **100** 071911
[20] Bongard F, Lissek H and Mosig J R 2010 *Phys. Rev. B* **82** 094306
[21] Jing Y, Xu J and Fang N X 2012 *Phys. Lett. A* **376** 2834
[22] Park J J, Lee K J B, Wright O B, Jung M K and Lee S H 2013 *Phys. Rev. Lett.* **110** 244302
[23] Fleury R and Alu A 2013 *Phys. Rev. Lett.* **111** 055501
[24] Shen C, Xu J, Fang N X and Jing Y 2014 *Phys. Rev. X* **4** 041033
[25] Xu X, Li P, Zhou X and Hu G 2015 *Europhys. Lett.* **109** 28001
[26] Ding E L, Wang Y Y, Liu X Z and Gong X F 2015 *AIP Adv.* **5** 107222
[27] Cheng Q, Cai B G, Jiang W X, Ma H F and Cui T J 2012 *Appl. Phys. Lett.* **101** 141902
[28] Cheng Q, Jiang W X and Cui T J 2012 *Phys. Rev. Lett.* **108** 213903
[29] Liu F, Cai F, Peng S, Hao R, Ke M and Liu Z 2009 *Phys. Rev. E* **80** 026603
[30] Fleury R and Alù A 2013 *Phys. Rev. Lett.* **111** 055501
[31] Wu L Y and Chen L W 2011 *J. Appl. Phys.* **110** 114507
[32] Li X and Liu Z 2005 *Phys. Lett. A* **338** 413
[33] Jiang X, Liang B, Zou X Y, Yin L L and Cheng J C 2014 *Appl. Phys. Lett.* **104** 083510
[34] Liang Z and Li J 2011 *Opt. Express* **19** 16821