Evaluation of the frequency instability limited by Dick effect in the microwave $^{199}Hg^{+}$ trapped-ion clock*

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In the microwave $^{199}Hg^{+}$ trapped-ion clock, the frequency instability degradation caused by the Dick effect is unavoidable because of the periodical interrogating field. In this paper, the general expression of the sensitivity function $g(t)$ to the frequency fluctuation of the interrogating field with $N\pi$-pulse (N is odd) is derived. According to the measured phase noise of the 40.5-GHz microwave synthesizer, the Dick-effect limited Allan deviation of our $^{199}Hg^{+}$ trapped-ion clock is worked out. The results indicate that the limited Allan deviations are about $1.75\times10^{-13}/\sqrt{\tau}$ and $3.03\times10^{-13}/\sqrt{\tau}$ respectively in the linear ion trap and in the two-segment extended linear ion trap under our present experimental parameters.

Keywords: Dick effect, $^{199}Hg^{+}$ trapped-ion clock, Allan deviation

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1. Introduction

The first frequency standards based on trapped ions capable of continuously operating were reported separately by the Laboratoire de l'Horloge Atomique (LHA) Laboratory in 1980 and Hewlett–Packard (HP) in 1981.[1] Since 1987, scientists at the Jet Propulsion Laboratory (JPL) have been studying the microwave $^{199}Hg^{+}$ trapped-ion clock. They have developed the microwave $^{199}Hg^{+}$ trapped-ion clock based on the single linear ion trap and two-segment extended linear ion trap successively.[2] JPL's results indicate that the performance of the microwave $^{199}Hg^{+}$ trapped-ion clock is very excellent in microwave frequency standard.[3–7] More noticeably, the demonstration of a mercury ion clock for space at JPL is realized with the Allan deviation about $1.5\times10^{-13}/\sqrt{\tau}$.[6] Our group has been studying the microwave $^{199}Hg^{+}$ trapped-ion clock for many years.[8–10] Recently, the frequency instability of our clock reached about $1.0\times10^{-12}/\sqrt{\tau}$ because of the worse signal-to-noise ratio (SNR) which should be improved in the future. In the microwave $^{199}Hg^{+}$ trapped-ion clock, $^{199}Hg^{+}$ ions are confined in an ion trap, the ^{202}Hg discharged lamp that is practical for continuous operation is used to emit 194.2-nm radiation for state selection, and the helium is used for buffer gas cooling. Since the frequency of the clock transition of $^{199}Hg^{+}$ from $^{2}S_{1/2}$ $(F=0,m_F=0)$ to $^{2}S_{1/2}$ $(F=1,m_F=0)$ is about 40.507 GHz, a frequency synthesizer of 40.507 GHz derived from an ultra-stable 10-MHz oscillator is used to generate the interrogation field. In addition to the microwave interrogation, there are other procedures in the whole locking-loop operation, including ion loading, fluorescence detection and optical pumping. The time for these procedures is the so-called dead time which is a key parameter in the Dick effect.[11]

During the dead time, the local oscillator's frequency is not sensed by the atomic interrogation process, so local oscillator frequency fluctuations during those times are not appropriately corrected, which will cause degradation of frequency instability. This problem was first investigated and analyzed by Dick[11] and Dick *et al.*[12] at the JPL in microwave $^{199}Hg^{+}$ trapped-ion frequency standard, which is now called the Dick effect. In the microwave $^{199}Hg^{+}$ trapped-ion clock, the Dick effect causes the degradation of the frequency instability. Many other frequency standards, such as optical lattice clock,[13] cesium fountain,[14] $^{113}Cd^{+}$ trapped-ion clock,[15] etc., also have this effect. In our group, we are developing a small, compact microwave $^{199}Hg^{+}$ trapped-ion clock, in which the Dick effect is a significant limitation of the frequency instability.

The rest of this paper is structured as follows. In Section 2, the general formulation for the frequency instability degradation induced by the Dick effect in the microwave $^{199}Hg^{+}$ trapped-ion clock is presented, and the general expression of the sensitivity function $g(t)$ to the frequency fluc-

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tuations of the interrogating field with $N\pi$-pulse is derived. In Section 3, the phase noise of the frequency synthesizer for the microwave $^{199}Hg^{+}$ trapped -ion clock is measured, and the experiment parameters of our experiments both in a linear ion trap and in a two-segment extended linear ion trap are described, then the Allan deviations limited by the Dick effect are calculated both in the Ramsey interrogation mode and in the Rabi interrogation mode due to the different types of traps. Brief conclusions are drawn from the present studies in Section 4.

2. Instability degradation induced by Dick effect in the $^{199}Hg^{+}$ trapped-ion clock

In the $^{199}Hg^{+}$ trapped-ion microwave clock, the frequency variation due to the phase fluctuations will cause the transition probability to change during the microwave interrogation time. The sensitivity function $g(t)$ is used to describe the relation between the frequency variation and the variation of transition probability.[11,16]

When the Ramsey interrogation scheme is used, $g(t)$ is given by[11,12,16]

$$g(t)=\begin{cases} a\sin(bt), & 0\le t\le \tau_{\pi/2},\\ a\sin\left(b\tau_{\pi/2}\right), & \tau_{\pi/2}\le t\le T_{\rm s}+\tau_{\pi/2},\\ a\sin\left[b\left(T_{\rm s}+2\tau_{\pi/2}-t\right)\right], & T_{\rm s}+\tau_{\pi/2}\le t\le T_{\rm s}+2\tau_{\pi/2},\\ 0, & T_{\rm s}+2\tau_{\pi/2}\le t\le T_{\rm c}, \end{cases} \tag{1}$$

where $\tau_{\pi/2}$ is the time of the $\pi/2$-pulse , b is the Rabi frequency constant, and $b\tau_{\pi/2}=\pi/2$, $T_{\rm s}$ is the time between the two microwave pulses, $T_{\rm c}$ is the time of one cycle of operation including the time of microwave interrogation and the dead time. $a=\sin(\omega_{\rm m}T_{\rm s})\sin\left(b\tau_{\pi/2}\right)$, with $\omega_{\rm m}$ being the offset frequency from the resonance frequency, usually detuned to the half signal point or the maximum slope signal point in our experiments.

If the π-pulse Rabi interrogation is used, the details of $g(t)$ can be found in Ref. [12]. However, in the microwave $^{199}Hg^{+}$ trapped-ion clock, in order to acquire a narrow linewidth, the 3π-pulse, 5π-pulse or other long time microwave pulse should be used for interrogation. In order to obtain the sensitivity function for this case, we can follow the rotation transformation method in Ref. [11]. If the time for interrogation is τ_N, and

$$\tau_N=N\tau_\pi,\quad N=1,3,5,7,\ldots, \tag{2}$$

where $b\tau_\pi=\pi$. The total rotation angle Ω must be modified into

$$\begin{aligned}\Omega &= \sqrt{b^2+\omega_{\rm m}^2}\cdot\tau_N=N\pi\sqrt{1+\left(\frac{\omega_{\rm m}\tau_N}{N\pi}\right)^2},\\ &N=1,3,5,\ldots,\end{aligned} \tag{3}$$

where

$$\Omega_1(t)=\Omega\left(1-\frac{t}{\tau_N}\right),\quad \Omega_2(t)=\Omega\left(\frac{t}{\tau_N}\right).$$

If $\omega_{\rm m}$ is detuned to the half signal point, that is, $\omega_{\rm m}=0.798685\pi/(N\tau_\pi)$, then the sensitivity function is

$$g(t)=\begin{cases}\sin^2(\theta)\cos(\theta)\left[\sin(\Omega_1(t))\left(1-\cos(\Omega_2(t))\right)+\sin(\Omega_2(t))\left(1-\cos(\Omega_1(t))\right)\right], & 0\le t\le\tau_N,\\ 0, & \tau_N<t\le T_{\rm c},\end{cases} \tag{4}$$

where $\theta=\pi/2+\arctan(\omega_{\rm m}/b)$.

Depending on the harmonic content of sensitivity function $g(t)$, the noise of the oscillator at harmonics frequencies of the operation frequency $1/T_{\rm c}$ will be down-converted to low frequency fluctuations. In a general way, the Allan variance limited by the Dick effect is given by[11,12,17]

$$\sigma_{\rm y}^2(\tau)=\frac{1}{\tau g_0^2}\sum_{m=1}^{\infty}\left(g_{cm}^2+g_{sm}^2\right)S_{\rm y}\left(\frac{M}{T_{\rm c}}\right), \tag{5}$$

where $\sigma_{\rm y}(\tau)$ is the Allan deviation, τ is averaging time, $S_{\rm y}(f)$ is the one-sided power spectral density of the free running interrogation oscillator, and it is very simply related to the phase noise $S_\varphi(f)$ by[18]

$$S_{\rm y}=\frac{f^2}{f_0^2}S_\varphi(f), \tag{6}$$

where f_0 is the carrier frequency, here it is 40.507 GHz. g_0 g_{sm} and g_{cm} are defined as

$$g_0=\frac{1}{T_{\rm c}}\int_0^{T_{\rm c}}g(t)\,{\rm d}t, \tag{7}$$

$$g_{sm}=\frac{1}{T_{\rm c}}\int_0^{T_{\rm c}}g(t)\sin\left(\frac{mt}{T_{\rm c}}\right){\rm d}t, \tag{8}$$

$$g_{cm}=\frac{1}{T_{\rm c}}\int_0^{T_{\rm c}}g(t)\cos\left(\frac{mt}{T_{\rm c}}\right){\rm d}t. \tag{9}$$

3. Phase noise measurement of the frequency synthesizer and calculation of the Dick-effect limited Allan deviations

In the $^{199}Hg^{+}$ trapped-ion clock, the local oscillator is a 10-MHz ultra-stable oven controlled crystal oscillator (OCXO), which is used as a reference to generate the 40.507-GHz clock transition signal. As the time of one cycle operation T_c is several seconds, the power spectral density of frequency fluctuations from $1/T_c$ Hz to several hundred Hz accounts mainly for the Dick effect. However, the phase noise of the synthesizer from $1/T_c$ Hz to several hundred Hz is too small to measure directly by the commercial equipment. There is a method by which two identical microwave synthesizers are mixed as listed in Ref. [15].

In order to reduce the noise, the passive mixer is usually used. However, in the $^{199}Hg^{+}$ trapped-ion clock, the power of the synthesizer is about -13 dBm, which is too small to drive a passive mixer. So in our experiment, the following setup of phase noise measurement is used as shown in the following Fig. 1.

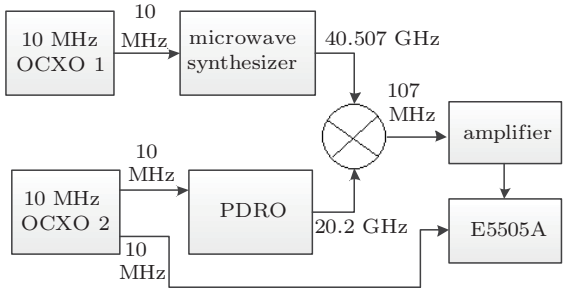


Fig. 1. Schematic diagram of the phase noise measurement setup.

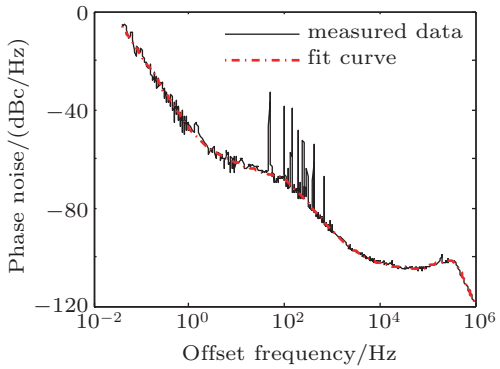


Fig. 2. (color online) Measured phase noise of the 107-MHz amplified output of the sub-harmonic mixer. This phase noise is the upper limitation of noise of the 40.507-GHz output of the microwave synthesizer.

The 40.507-GHz microwave synthesizer is mixed with a 20.2-GHz phase locked dielectric resonator oscillator (PDRO) and the mixer is a sub-harmonic mixer. The power of the PDRO is about 13 dBm. At this power level, the conversion loss of the sub-harmonic mixer is about 13 dB. The output signal of the sub-harmonic mixer is about 107 MHz and it is amplified by an ultra-low noise amplifier, then the phase noise is measured by the E5505A. In the microwave synthesizer, there is a similar PDRO that is mixed with a direct digital synthesizer (DDS) to produce a 40.507-GHz signal, the phase noise of the single PDRO should be smaller than that of the microwave synthesizer, it will still introduce some noises to the 107 MHz. Furthermore, the sub-harmonic mixer and the ultra-low noise amplifier may also add some noises to the 107 MHz, all these additive noises are very small. So the phase noise of the 107 MHz is the upper limitation of noise of the microwave synthesizer. The result is shown in Fig. 2, and it can be expressed as a piecewise function as follows:

$$S_\varphi(f)=\begin{cases}1.04\times10^{-7}+6.32\times10^{-6}f^{-1}-4.50\times10^{-6}f^{-2}\\ \quad+1.68\times10^{-5}f^{-3}-4.40\times10^{-8}f^{-4}, & (0.04\le f\le 100),\\ 3.18\times10^{-11}+1.58\times10^{-7}f^{-1}+8.63\times10^{-4}f^{-2}+0.284f^{-3}-21.75f^{-4}, & (100<f\le 60\times10^{3}),\\ 9.74\times10^{-11}-9.20\times10^{-6}f^{-1}+0.328f^{-2}, & (60\times10^{3}<f\le 350\times10^{3}),\\ 2.37\times10^{-11}-4.31\times10^{-5}f^{-1}+21.16f^{-2}, & (350\times10^{3}<f\le 1.0\times10^{6}).\end{cases}\tag{10}$$

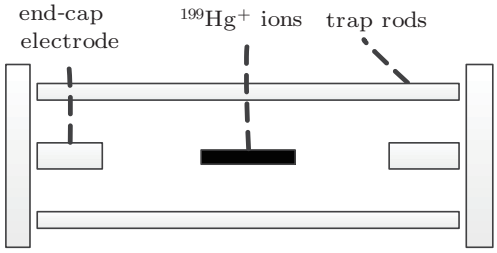


Fig. 3. A simple diagram of the linear ion trap and $^{199}Hg^{+}$ ions in the trap.

In our group, the main experiments are carried out in the linear ion trap (shown in Fig. 3), $^{199}Hg^{+}$ ions are confined in the two transverse directions by the RF pseudopotential alone, and a DC potential is used for confinement in the longitudinal direction.

In this type of trap, the ion-loading, the optical pumping, and the microwave interrogation are in the same region. The Ramsey interrogation scheme is used in our experiment, and the time sequence is shown in Fig. 4.

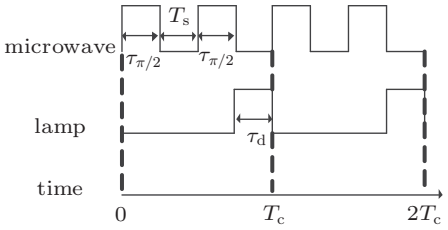


Fig. 4. Time sequence of the Ramsey interrogation in the linear ion trap.

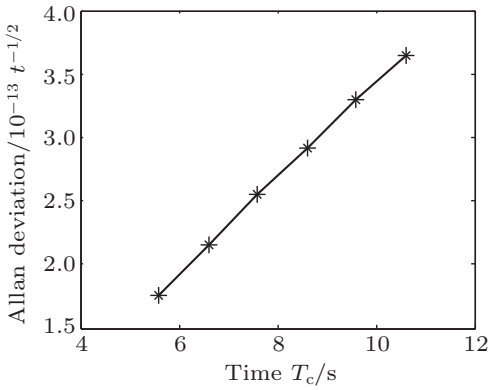


Fig. 5. In the Ramsey interrogation scheme, the calculated Allan deviations limited by the Dick effect versus the cycle time $T_c = 2.6 + \tau_d$.

Two microwave pulses ($\pi/2$-pulse), each with a duration $\tau_{\pi/2}$, interact with the trapped ions, and the two pulses are separated by a time of T_s, then the ^{202}Hg discharged lamp is used for detection and optical pumping, and the time for detection, optical pumping and ion-loading is τ_d which is the dead time. So the total time of one cycle of operation is $T_c = 2 * \tau_{\pi/2} + T_s + \tau_d$. The realistic experimental parameters are $\tau_{\pi/2} = 0.3$ s, $T_s = 2$ s, and $\tau_d = 3$ s then the total time $T_c = 5.6$ s. and the Allan deviation is $1.75 \times 10^{-13}/\sqrt{\tau}$. If the dead time increases, the instability becomes worse, the Allan deviation is shown in Fig. 5 for different cycle times.

In order to reduce the second order Doppler shift, our group is also studying the two-segment extended linear ion trap which is shown in Fig. 6. The distinguishing feature of the two-segment extended linear ion trap is separations of the ion loading and optical pumping region from the microwave resonance region.[2] Ions are moved back and forth between the two regions.

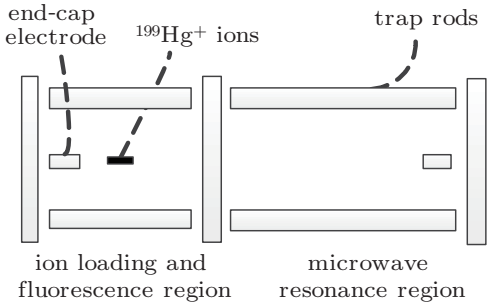


Fig. 6. A simple diagram of the two-segment extended linear ion trap and $^{199}Hg^+$ ions in the trap.

In this trap, the Rabi interrogation scheme is usually used in our experiment, and the time sequence is shown in Fig. 7. A microwave pulse ($N\pi$-pulse, N is odd) for duration of τ_N is used for interrogation in the resonance region, and then the trapped ions are moved to the ion loading and fluorescence region, and the time of this movement is τ_{m1}. After this procedure, the discharged lamp is switched on for detection, ion-loading and optical pumping. The time for detection, ion-loading and optical-pumping is τ_o, then the trapped ions must be moved to the resonance region, which takes a time of τ_{m2}. The dead time is $\tau_d = \tau_o + \tau_{m1} + \tau_{m2}$, and the total time for one cycle of operation is $T_c = \tau_N + \tau_o + \tau_{m1} + \tau_{m2} = \tau_N + \tau_d$.

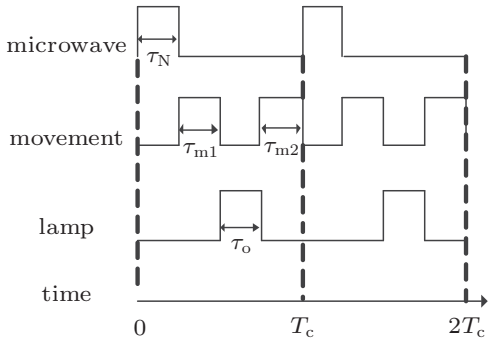


Fig. 7. Time sequence of the Rabi interogation scheme in the two-segment linear ion trap.

In the two-segment extended linear ion trap, the realistic experimental parameters are as follows: $\tau_\pi = 0.6$ s ($N = 1$), $\tau_{m1} = \tau_{m2} = 0.5$ s, and $\tau_o = 3$ s, then $T_c = 4.6$ s and the Allan deviation is $3.03 \times 10^{-13}/\sqrt{\tau}$. If the 3π-pulse is used, then $T_c = 5.8$ s and the Allan deviation is $3.28 \times 10^{-13}/\sqrt{\tau}$. The Allan deviation is shown in Fig. 8 for different dead times in this trap.

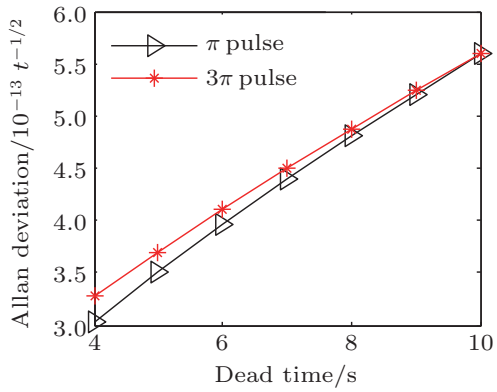


Fig. 8. (color online) In the Rabi interrogation, the calculated Allan deviations limited by the Dick effect versus the dead time τ_d.

When the dead time is the same, the Allan deviation for 3π-pulse interrogation is a little worse than that for π-pulse interrogation, however, the linewidth for 3π-pulse interrogation is one third of that for π-pulse interrogation, which is an advantage in improving the stability.

4. Conclusions

In this work, the sensitivity function for an $N\pi$-pulse (N is odd) is derived, and the Allan deviations limited by the Dick effect are calculated in the microwave $^{199}Hg^+$ trapped-ion clock. In our experiment with the realistic experimental parameters, the Allan deviations are $1.75 \times 10^{-13}/\sqrt{\tau}$ in the linear ion trap and $3.03 \times 10^{-13}/\sqrt{\tau}$ in the two-segment extended linear ion trap, if 3π-pulse microwave interrogation is used in the two-segment extended linear ion trap, the Allan deviation is $3.28 \times 10^{-13}/\sqrt{\tau}$. This performance is adequate

for a compact microwave $^{199}Hg^{+}$ trapped-ion clock. If higher performance is required, there are some methods to improve the performance. One is to build two ion traps for continuous operation to eliminate the dead time,[17] and the other way is to apply a much better LO than the current one, such as a cryogenic sapphire oscillator. In the future, we can try these methods to improve the performance of our microwave $^{199}Hg^{+}$ trapped-ion clock.

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