Design of a gap tunable flux qubit with FastHenry

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In the preparations of superconducting qubits, circuit design is a vital process because the parameters and layout of the circuit not only determine the way we address the qubits, but also strongly affect the qubit coherence properties. One of the most important circuit parameters, which needs to be carefully designed, is the mutual inductance among different parts of a superconducting circuit. In this paper we demonstrate how to design a gap-tunable flux qubit by layout design and inductance extraction using a fast field solver FastHenry. The energy spectrum of the gap-tunable flux qubit shows that the measured parameters are close to the design values.

Keywords: superconducting circuit, Josephson junction, flux qubit, FastHenry

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1. Introduction

Superconducting quantum circuits based on Josephson junctions[1] are promising candidates for the implementation of quantum computers.[2] This is due to the fact that the complex superconducting quantum circuits can be freely constructed by using well-established micro-fabrication technology in the semiconductor industry. Also the superconducting quantum circuits have advantages, such as being easy to control, scalability, and strong inter-coupling as compared with other quantum systems such as ion traps,[3] nuclear spins,[4,5] electron spins,[6] neutral atoms,[7] and photons.[8] The basic component of a quantum computer is the qubit, which is a two-level system. For a superconducting circuit to function as a qubit, its circuit parameters must be well designed. One parameter which can play an important role in affecting the characteristics of a superconducting qubit is the inductance. In this paper, we demonstrate how to choose proper values for the inductances in designing a gap-tunable flux qubit.[9,10]

A superconducting flux qubit loop is interrupted by three Josephson junctions,[11] the critical current of one of the junctions is α times smaller than those of the other two junctions, which are identical and characterized by the Josephson energy E_J and the charging energy E_c. When the magnetic flux threading the qubit loop Φ_ε is around half flux quantum $\Phi_0/2$, the qubit ground and excited states correspond to clockwise and anticlockwise persistent currents $\pm I_p$ in the qubit loop. When the qubit is biased at the symmetric point or the optimal point $\Phi_\varepsilon = \Phi_0/2$, the persistent current states are degenerate, quantum tunneling lifts this degeneracy and forms an energy gap Δ. The Hamiltonian of a flux qubit is given by:

$$H = \frac{\varepsilon}{2}\sigma_z + \frac{\Delta}{2}\sigma_x. \tag{1}$$

Here,

$$\varepsilon = 2I_p(\Phi_\varepsilon - 1/2\Phi_0),$$

$\Phi_0 = h/2e$ is the flux quantum, while σ_z and σ_x are Pauli matrices. A flux qubit in most cases works at the optimal point for good coherence properties. Therefore, it is desirable to make the energy gap Δ tunable. In order to do this, one needs to replace the smaller junction of the flux qubit by a dc superconducting quantum interference device (SQUID, called an α loop). By applying bias current in an α bias line which is inductively coupled with the α loop, the critical current of the dc superconducting quantum interference device can be modulated, thus the value of α can be changed, making Δ tunable. Since the working point of a flux qubit is determined by the magnetic flux threading the qubit loop Φ_ε, the qubit working point can be changed by applying bias currents in an mw/flux bias line placed in the vicinity of the qubit loop. Microwave for qubit operations is also applied through the mw/flux bias line. The qubit state readout is performed by coupling the qubit to a dc-SQUID to detect the persistent current in the qubit loop. Coupling between two flux qubits or between a qubit and other systems can also be realized through mutual inductance. Thus we can see that for flux qubits the inductance plays an important role. In the following Fig. 1 we show our gap-tunable flux qubit sample with its schematic circuit diagram.

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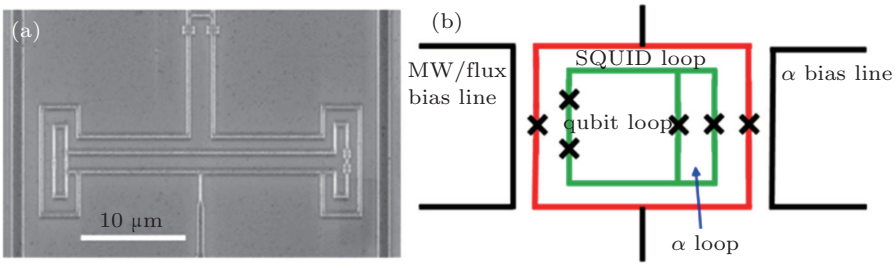


Fig. 1. (color online) SEM image of our gap tunable flux qubit sample (a), and schematic circuit diagram (b), showing the qubit loop (green loop), the SQUID loop (red loop), and the control lines. Crosses represent Josephson junctions.

2. Influence of mutual inductance on qubit coherence

One of the biggest challenges of realizing quantum computers with superconducting qubits is coherence. A qubit in a quantum computer needs to perform a huge number of quantum logic operations to solve a practical problem. Like any other quantum system, a qubit undergoes decoherence due to coupling to noise in its environment. Each noise source with a qubit coupled can be represented by a fluctuation term of some parameter λ in its Hamiltonian $\delta\lambda$ and characterized by its quantum spectral density $s_\lambda(\omega) = 1/(2\pi)\int \mathrm{d}t\langle\delta\lambda(0)\delta\lambda(t)\rangle \mathrm{e}^{-\mathrm{i}\omega t}$.[12]

The effect of decoherence can be separated into two effects, i.e., energy relaxation and dephasing, and characterized by relaxation rate Γ_1 and dephasing rate Γ_ϕ respectively,[13]

$$\Gamma_1 = \frac{1}{2}D^2_{\lambda,\perp}S_\lambda(\omega), \quad (k_\mathrm{B}T \ll \omega), \tag{2}$$

$$\Gamma_\phi = \pi D^2_{\lambda,||}S_\lambda(0). \tag{3}$$

Here $\omega = \sqrt{\varepsilon^2+\Delta^2}$ is the qubit frequency, $D_{\lambda,\perp}$ and $D_{\lambda,||}$ are the transverse and longitudinal term of $\boldsymbol{D}_\lambda = (1/h)\partial\boldsymbol{H}/\partial\lambda$, which is the sensitivity of the qubit Hamiltonian to the noise source, and represented by λ. The most prominent noise sources of a gap-tunable flux qubit are fluctuations of the magnetic flux bias in the qubit loop f_ε and magnetic flux in the α-loop f_α, the sensitivities of the Hamiltonian to these noises are as follows:

$$D_{f_\varepsilon\perp} = -\frac{\Delta}{\hbar\omega}I_\mathrm{p}\Phi_0, \tag{4}$$

$$D_{f_\varepsilon||} = \frac{1}{\hbar}\left|\frac{\varepsilon}{\omega}\frac{\partial\varepsilon}{\partial f_\varepsilon}\right|, \tag{5}$$

$$D_{f_\alpha\perp} = \frac{\Delta}{2\hbar}\left(\frac{\varepsilon}{\omega}\frac{\partial\Delta}{\partial f_\alpha} - \frac{\Delta}{\omega}\frac{\partial\varepsilon}{\partial f_\alpha}\right) \approx \frac{\Delta}{2\hbar}\frac{\varepsilon}{\omega}\frac{\partial\Delta}{\partial f_\alpha}, \tag{6}$$

$$D_{f_\alpha||} = \frac{1}{\hbar}\left|\frac{\Delta}{\omega}\frac{\partial\Delta}{\partial f_\alpha} + \frac{\varepsilon}{\omega}\frac{\partial\varepsilon}{\partial f_\alpha}\right| \approx \frac{1}{\hbar}\frac{\Delta}{\omega}\frac{\partial\Delta}{\partial f_\alpha}. \tag{7}$$

At the optimal point where $(\varepsilon = 0)$, we have $D_{f_\varepsilon||} = D_{f_\alpha\perp} = 0$, $D_{f_\varepsilon\perp}$ and $D_{f_\alpha||}$ are determined only by qubit parameters I_p and Δ, thus in order to minimize decoherence, we can engineer the quantum spectral densities $S_{f_\varepsilon}(\omega)$ and $S_{f_\alpha}(\omega)$. For a superconducting flux qubit, the main noise sources are the readout and control circuits. As we perform qubit state readout by coupling a dc SQUID to the qubit loop, the circuit noise from the readout circuitry is directly coupled to the current through the SQUID bias current I_sqb and the circulating current in the SQUID loop I_cir. Because the SQUID loop is placed symmetrically around the qubit loop, only the fluctuation in I_cir is coupled to the qubit, the coupling of the fluctuation in I_sqb to the qubit loop is zero or negligibly small. The quantum spectral density of the noise from the readout circuit is given by[14]

$$S_{f_\varepsilon,\mathrm{sq}}(\omega) = -\frac{\pi^2}{\hbar\omega}\left(\frac{M_\mathrm{sq}}{\Phi_0}\right)^2 I^2_\mathrm{sqb}\tan^2\left(\frac{\pi\Phi_\mathrm{sq}}{\Phi_0}\right)\mathrm{Re}\{Z_\mathrm{sq}(\omega)\}, \tag{8}$$

where $Z_\mathrm{sq}(\omega)$ is the impedance of the readout circuit, Φ_sq is the magnetic flux threading the SQUID loop, M_sq is the mutual inductance between the SQUID loop and the qubit loop. Current fluctuations in the mw/flux bias line and bias line are also coupled to the qubit, and their quantum spectral density is given by[14]

$$S_{f_\varepsilon,\mathrm{mw}}(\omega) = -\frac{1}{\hbar\omega}\left(\frac{M_\mathrm{mw}}{L_\mathrm{mw}}\right)^2 \mathrm{Re}\{Z_\mathrm{mw}(\omega)\}, \tag{9}$$

$$S_{f_\alpha,\alpha}(\omega) = -\frac{1}{\hbar\omega}\left(\frac{M_\alpha}{L_\alpha}\right)^2 \mathrm{Re}\{Z_\alpha(\omega)\}, \tag{10}$$

where $Z_\mathrm{mw}(\omega)$ and $Z_\alpha(\omega)$ are the impedances of the mw/flux bias line and α bias line respectively, M_mw is the mutual inductance between the mw/flux bias line and the qubit loop, M_α is the mutual inductance between the α bias line and the α-loop. We can see that the quantum spectral densities of all the circuit noise sources are proportional to the squared mutual inductances. Thus in order to minimize the qubit decoherence, mutual inductances between the qubit and the control or readout circuits should be as small as possible.

3. Design of a gap-tunable flux qubit

The readout of a flux qubit state utilizes the factor that the two persistent current states $|\pm I_\mathrm{p}\rangle$ form a basis for the qubit states. Measuring the persistent current in the qubit loop corresponds to a projective measurement of the qubit in the persistent current state basis. Since the SQUID loop is inductively coupled to the qubit loop, the qubit state influences the critical current of the SQUID. Any change of the persistent current is reflected in the SQUID flux bias Φ_sq and modulates its critical

current I_{sq}. We apply pulses to the readout SQUID and detect its switching. The switching is a stochastic process due to quantum tunneling. The pulse amplitude at which the readout SQUID switching takes place has a distribution with typical spread about 1% of I_{sq}, and the center of the distribution is slightly smaller than I_{sq}. In order to obtain sufficient visibility in the readout signal, we need to have about 1% of change in I_{sq} in a change of qubit state from $|-I_p\rangle$ to $|+I_p\rangle$, that is $2M_{sq}I_p \approx 10m\Phi_0$. For a persistent current $I_p = 500$ nA, we have $M_{sq} \approx 20$ pH. While we want the mutual inductance M_{sq} to be as small as possible for minimizing qubit decoherence, the requirement for the readout to be efficient sets a lower limit for the mutual inductance M_{sq}, which as shown above is typically several tens of pico-Henry.

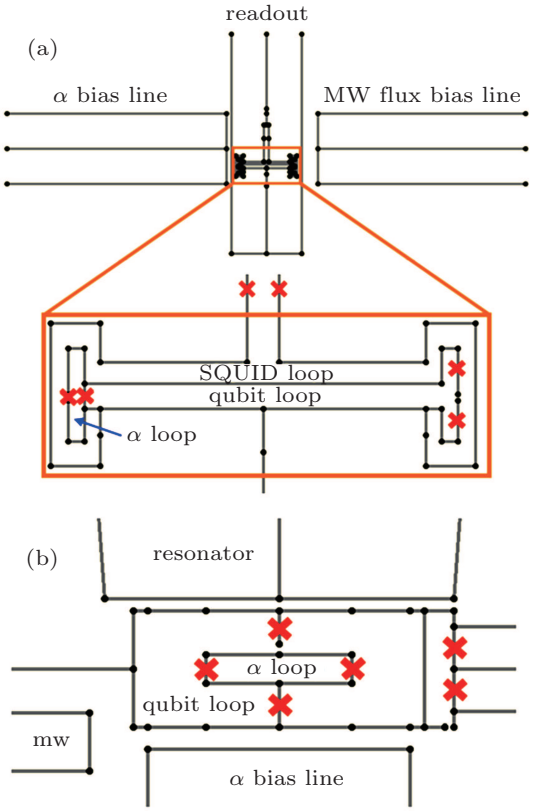


Fig. 2. (color online) Circuit layout for inductance extraction. Red crosses denote Josephson junctions. (a) A gap-tunable flux qubit with readout and control circuits (qubit a) and (b) a flux qubit inductively coupled to a resonator (qubit b).

Just like the mutual inductance between the qubit and the readout SQUID, mutual inductances between the qubit and control lines also have their lower limits. The mw/flux bias line is used for providing fast flux bias to shift the qubit frequency $f_{qb} = \sqrt{\varepsilon^2 + \Delta^2}$ during qubit operations and readout, the maximum amount of this shift is about 30 GHz typically. The maximum current can be applied to the control lines without inducing strong crosstalk or risking the breaking down of superconductivity, and is about 1mA in most samples. Since $\delta f_{qb} \approx 2I_p M_\varepsilon I_\varepsilon$, to induce a qubit frequency change of 30 GHz for a flux qubit with $I_p = 500$ nA the mutual inductance between the mw/flux bias line and the qubit loop M_ε must be at least 0.02 pH. The α bias line is used for tuning the qubit energy gap Δ in a frequency range of several GHz, by numerically calculating the energy gap dependence on the value of α, and we find that this requires a mutual inductance $M_\alpha \approx 0.03$ pH between the α bias line and the α loop.

We develop a dedicated layout computer-aided design (CAD) tool to model the qubit circuit, figure 2(a) shows the modeled circuit layout of a gap-tunable flux qubit (qubit a), figure 2(b) shows the modeled circuit layout of a flux qubit coupled to a resonator (qubit b). Black lines represent metal wires with adjustable width and thickness, black dots denote connections or ports. This CAD tool generates a digital layout file for further numerical inductance extraction.

We use the fast field solver FastHenry[15] to extract the inductances numerically. The FastHenry solver is one of the most popular techniques to extract circuit inductances. It uses the fast multipole method (FMM) to accelerate matrix-vector production. The algorithm used in FastHenry is an acceleration of the mesh formulism approach. The linear system resulting from the mesh formulation is solved by using a generalized minimal residual (GMRES) algorithm with a fast multiple algorithm to efficiently compute iteration. The digital layout file is used as the input of FastHenry to extract the inductances. After the calculation, the result is compared with the desired inductance value and the layout is adjusted accordingly. After a few iterations we obtain a layout with inductances close to the desired values. Table 1 shows the inductances of our sample extracted by FastHenry.

Table 1. Mutual inductances extrated by FastHenry for qubit a and qubit b. Here for qubit b we only extract the mutual inductance between the qubit and the resonator M_{qr}.

	M_{sq}/pH	M_ε/pH	M_α/pH	M_{qr}/pH
Qubit a	26.5	0.023	0.035	–
Qubit b	–	–	–	0.99

4. Experimental results

After the qubit circuit is properly designed the sample is then fabricated and measured. The details about the fabrication and measurement procedures can be found in Refs. [16] and [17]. Figure 3(a) shows the magnetic flux modulation of the readout SQUID critical current I_{sq}. Qubit signals appear as a small shift of the modulation curve $\delta\Phi_{sq} = 15m\Phi_0$. Thus the mutual inductance between the qubit and the readout SQUID is $M_{sq} = \delta\Phi_{sq}/2I_p = 28.4$ pH, very close to the designed value. Figure 3(b) shows the qubit energy spectrum, the qubit flux bias Φ_ε is applied by the mw/flux bias line. By fitting the spectrum line (red dash line) we obtain $I_p = 546$ nA. Figure 3(c) shows the Rabi oscillation result measured at the optimal point, the 2-μs decay time of the oscillation, indicating the good coherent property of our sample.

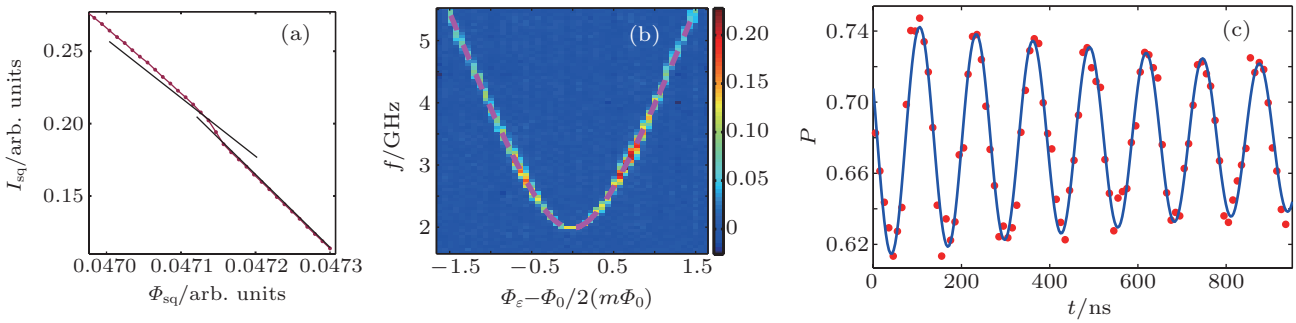


Fig. 3. (color online) Measurement results of sample qubit a. (a) Qubit signal. The qubit signal appears as a small shift of the readout SQUID critical current modulation curve, marked by two black dash lines and two arrows. (b) Energy spectrum by applying qubit flux bias with mw/flux bias line. The red dash line is a fit of the spectrum line. (c) Rabi oscillation of the sample, red dots are data.The decay rate of the oscillation is 1.96 μs, extracted by fitting the data with an exponential decay sinusoidal curve (blue curve on the plot), P is the switching probability of the readout SQUID.

5. Conclusions

In this paper, we demonstrate how to design a gap-tunable flux qubit with proper values of mutual inductances between different circuit parts. It is valuable for ensuring long coherence time. It provides enough control freedom over the qubit state and has sufficient readout efficiency. The method demonstrated here can be generalized to other types of superconducting circuits.

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