Controlled unknown quantum operations on hybrid systems

Yong He(何勇)[1,†] and Ming-Xing Luo(罗明星)[2,3]

[1] *Department of Mathematics and Physics, Chongqing University of Science and Technology, Chongqing* 401331, *China*
[2] *Information Security and National Computing Grid Laboratory, Southwest Jiaotong University, Chengdu* 610031, *China*
[3] *Department of Physics, University of Michigan, Ann Arbor, MI* 48109, *USA*

(Received 13 June 2016; revised manuscript received 24 July 2016; published online 25 October 2016)

Any unknown unitary operations conditioned on a control system can be deterministically performed if ancillary subspaces are available for the target systems [Zhou X Q, *et al.* 2011 *Nat. Commun.* **2** 413]. In this paper, we show that previous optical schemes may be extended to general hybrid systems if unknown operations are provided by optical instruments. Moreover, a probabilistic scheme is proposed when the unknown operation may be performed on the subspaces of ancillary high-dimensional systems. Furthermore, the unknown operations conditioned on the multi-control system may be reduced to the case with a control system using additional linear circuit complexity. The new schemes may be more flexible for different systems or hybrid systems.

Keywords: unknown unitary operations, hybrid systems, multi-control system, circuit complexity

PACS: 03.67.Lx, 03.67.Pp, 03.67.Ac **DOI:** 10.1088/1674-1056/25/12/120304

1. Introduction

Controlled-unitary (CU) gates are elementary operations in quantum information processing based on the quantum circuit model.[1–3] By choosing different Us such as the NOT gate or controlled-NOT gate, various quantum applications may be faithfully completed.[4–11] These circuits have been widely used such as the Kitaev's phase estimation algorithm,[5] Shor's algorithm,[6] and quantum simulation.[7] To realize these schemes, each quantum operation U should be decomposed into a series of time-evolution operators of small physical systems.[3] Unfortunately, the standard decomposition method requires that U should be known,[12,13] which is unsuitable for unknown quantum operations.[5]

Although the traditional quantum circuit model cannot be applied for the controlled unknown gates, fortunately, Zhou *et al.*[14] showed that it may be experimentally realized if ancillary subspaces of the target systems are available. The followed result showed that the high-dimensional encodings are essential for the controlled unknown gates and cannot be replaced.[15] This is ensured by a new quantum no-go theorem, which states that the controlled unknown gates cannot be theoretically performed on the qubit systems.[15] It seems to follow an apparent contradiction between the theory and experiment because the possibility to add control to unknown operations is a common feature of many physical systems.[16] Moreover, they presented the practical setups for adding control to unknown subroutines, which are supplements to previous quantum optical schemes for the black-box control.[17,18]

In this paper, motivated by the experimental schemes,[14–16] we consider general schemes that allow one to add quantum control to unknown gates with different systems. To extend previous photonic schemes,[14,15] we firstly consider a hybrid implementation with an unknown photonic setup. To complete this scheme, the input system should be faithfully fused into a proper photonic system. The unknown photonic gate is implemented on ancillary photonic systems with additional degree of freedoms. The inverse operation of the quantum fusion may be explored to recover the required system. The key of this scheme is to realize an invertible quantum fusion by using proper quantum correlations.[19–21] In this case, our scheme may be used for any input system with unknown photonic setups, if these systems may be faithfully correlated with photons.[21] Moreover, if the unknown operation can only be implemented on the subspace of the high-dimensional system, a probabilistic scheme is proposed to add the control to an unknown unitary gate. Here, high-dimensional systems are used as ancillary systems and can be achieved in other setups.[22–26] Furthermore, the unknown gate conditional on the multi-control system may be reduced to the case of a one control system using additional linear complexity. The present schemes may be flexible for hybrid systems.

The rest of this paper is organized as follows. Deterministic schemes are presented in Section 2 for the case that the unknown operation U is constructed using optical elements. This scheme is useful for different hybrid systems correlated with photonic systems. Probabilistic schemes are proposed in Section 3 when the unknown operation U may be performed on subspaces of ancillary high-dimensional systems. All these schemes are dependent on the control qubit. The multi-control case may be reduced to the single control with additional lin-

*Project supported by the National Natural Science Foundation of China (Grant Nos. 61303039 and 61201253), Chunying Fellowship, and Fundamental Research Funds for the Central Universities, China (Grant No. 2682014CX095).

†Corresponding author. E-mail: heyongmath@163.com

© 2016 Chinese Physical Society and IOP Publishing Ltd

ear resources in Section 4 while Section 5 concludes the paper.

2. Adding control to unknown photonic operations

Previous experimental schemes[14,15] are based on the photonic system with two DoFs. In this section, a general circuit will be constructed for different physical systems. When the unknown operation U is set up with a photonic circuit, the controlled U may be deterministically performed on arbitrary systems that may be perfectly correlated with the photon.

2.1. Unknown qubit operations

The detailed scheme is shown as follows. Firstly, consider an input system of two qubits A and B in the state for simplicity

$$|\phi\rangle_{AB} = \alpha_1|00\rangle + \alpha_2|01\rangle + \alpha_3|10\rangle + \alpha_4|11\rangle, \tag{1}$$

where $\{|0\rangle, |1\rangle\}$ are basis states of one qubit system. This system may be an atomic system, photonic system or other physical system. An ancillary photon is used to realize the controlled-U on the system of A and B as shown in Fig. 1.

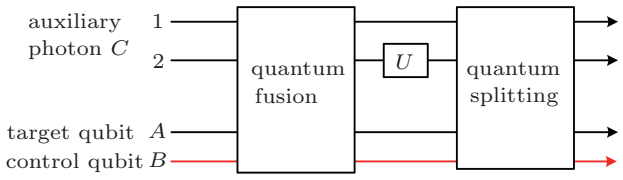


Fig. 1. (color online) A schematic circuit for the controlled unknown qubit operation. An ancillary photon C with two degrees of freedom (DoFs) in the state $|H\rangle_1$ will be used in this scheme. $\{|H\rangle_i, |V\rangle_i\}$ is a polarization basis of the photon C from the spatial mode i, while $|H\rangle$ and $|V\rangle$ denote the horizonal polarization and vertical polarization respectively. U is an unknown qubit operation on the polarization state. Quantum fusion is an invertible transformation or equivalent swapping operations on special systems, i.e., the quantum information of the qubits A and B are swapped to the spatial DoF and polarization DoF of the ancillary photon, respectively. Quantum splitting is the inverse of quantum fusion.

In detail, using an ancillary photon C in the state $|H\rangle_1$ with two spatial modes 1 and 2 in short, the system of A and B may be fused into a single photon in the state

$$|\phi\rangle_C = \alpha_1|H\rangle_1 + \alpha_2|V\rangle_1 + \alpha_3|H\rangle_2 + \alpha_4|V\rangle_2, \tag{2}$$

where $\{|H\rangle_i, |V\rangle_i\}$ is a polarization basis of the photon C from the mode i, and $|H\rangle$ and $|V\rangle$ denote the horizonal polarization and vertical polarization respectively. The output state of the systems A and B is $|00\rangle_{AB}$. Specially, this fusion operation may be realized with two swapping gates on special systems.[19–21] One is to swap the quantum information of qubit A to the spatial state of the photon C while the other is to swap the quantum information of qubit B to the polarization state of photon C.

Now, let photon C from spatial mode 2 pass through unknown optical gate U to get

$$|\phi'\rangle_C = \alpha_1|H\rangle_1 + \alpha_2|V\rangle_1 + \alpha_3(U|H\rangle_2) + \alpha_4(U|V\rangle_2). \tag{3}$$

Finally, the ancillary photon may be split into the input system of systems A and B using the inverses of two swapping gates.[19–21] The final state of systems A and B is

$$|\phi_f\rangle_C = \alpha_1|00\rangle + \alpha_2|01\rangle + \alpha_3|1\rangle(U|0\rangle) + \alpha_4|1\rangle(U|1\rangle). \tag{4}$$

The key of this scheme is the faithful quantum fusion or quantum splitting.[19–21]

2.2. Unknown multi-qubit operations

A similar scheme can be easily followed for a multi-qubit operation U. In detail, suppose that the input systems O (the control qubit system) and $A_1, \ldots, A_n$ are in the state

$$|\Phi\rangle = a_0|0\rangle_O \sum_{j_1,\ldots,j_n=0}^{1} \alpha_{j_1\cdots j_n}|j_1\cdots j_n\rangle_{A_1\cdots A_n} + a_1|1\rangle_O \sum_{i_1,\ldots,i_n=0}^{1} \beta_{i_1\cdots i_n}|i_1\cdots i_n\rangle_{A_1\cdots A_n}. \tag{5}$$

By performing a swapping on each pair of the systems A_j and B_j $(j = 1, \ldots, n)$, the input systems O and $A_1, \ldots, A_n$ will be fused into

$$|\Phi'\rangle = a_0|0\rangle_O \sum_{j_1,\ldots,j_n=0}^{1} \alpha_{j_1\cdots j_n}|j_1\cdots j_n\rangle_{B_1\cdots B_n} + a_1|1\rangle_O \sum_{i_1,\ldots,i_n=0}^{1} \beta_{i_1\cdots i_n}|i_1\cdots i_n\rangle_{B_1\cdots B_n} \tag{6}$$

using ancillary photons $B_1, \ldots, B_n$ in the state

$$\bigotimes_{j=1}^{n} |0\rangle_{B_j}(|0\rangle_{B_j} := |H\rangle_{B_j}, |1\rangle_{B_j} := |V\rangle_{B_j}).$$

The output state of the joint system of $A_1, \ldots, A_n$ is $|0\cdots 0\rangle$.

Now, previous optical implementations[14,15] of the controlled-U may be used for the state $|\Phi'\rangle$ in Eq. (6). In detail, a controlled-swapping gate

$$CS = |1\rangle\langle 1| \otimes (|0\rangle_1\langle 0|_2 + |0\rangle_2\langle 0|_1 + |1\rangle_1\langle 1|_2 + |1\rangle_2\langle 1|_1) + |0\rangle\langle 0| \otimes I_4 \tag{7}$$

is performed on each pair of the systems O and B_j for $j = 1, \ldots, n$, where $\{|0\rangle_1, |1\rangle_1, |0\rangle_2, |1\rangle_2\}$ denotes the basis $\{|H\rangle_1, |V\rangle_1, |H\rangle_2, |V\rangle_2\}$. Then, the unknown gate U is performed on all the photons $A_1, \ldots, A_n$ from their second spatial modes. The followed CS gate is performed on each pair of the systems O and B_j for $j = 1, \ldots, n$. The state $|\Phi'\rangle$ in Eq. (6) is changed to

$$|\Phi''\rangle = a_0|0\rangle_O \sum_{j_1,\ldots,j_n=0}^{1} \alpha_{j_1\cdots j_n}|j_1\cdots j_n\rangle_{B_1\cdots B_n} + a_1|1\rangle_O \sum_{i_1,\ldots,i_n=0}^{1} \beta_{i_1\cdots i_n}U|i_1\cdots i_n\rangle_{B_1\cdots B_n}. \tag{8}$$

Finally, each ancillary photon B_j may be swapped to A_j for $j = 1,\ldots,n$. The final state of O and $A_1,\cdots,A_n$ is

$$|\Phi_f\rangle = a_0|0\rangle_O \sum_{j_1,\ldots,j_n=0}^{1} \alpha_{j_1\cdots j_n}|j_1\cdots j_n\rangle_{A_1\cdots A_n} + a_1|1\rangle_O \sum_{i_1,\ldots,i_n=0}^{1} \beta_{i_1\cdots i_n} U|i_1\cdots i_n\rangle_{A_1\cdots A_n}. \quad (9)$$

In this scheme, the high-dimensional encodings are only available for ancillary photonic systems. Thus an unknown photonic gate U may be controlled for arbitrary physical systems without ancillary subspaces or high-dimensional encoding if these systems can be perfectly correlated with photons. This scheme is different from previous schemes[14–16] and is useful for hybrid systems with the unknown photonic setup.

3. Adding control to unknown operations with ancillary high-dimensional resources

In this section, the ancillary high-dimensional resources (which may not be photons) are used to control unknown operations. This assumption is reasonable in experiments because various physical systems such as photons and atoms have multiple degrees of freedom, which are naturally high-dimensional resources.

3.1. Unknown qubit operations

Consider the input system $|\phi\rangle_{AB}$ defined in Eq. (1). The schematic circuit is shown in Fig. 2 with an ancillary four-dimensional system in the state $|0\rangle_C$. The joint system of A,B, and C is changed to

$$|\Psi_1\rangle_{ABC} = \alpha_1|000\rangle + \alpha_2|011\rangle + \alpha_3|100\rangle + \alpha_4|111\rangle \quad (10)$$

after a CNOT gate being performed on the system of B and C. Then, by using a controlled-S on the system of A and C, $|\Psi_1\rangle_{ABC}$ may be transformed into

$$|\Psi_2\rangle_{ABC} = \alpha_1|002\rangle + \alpha_2|013\rangle + \alpha_3|100\rangle + \alpha_4|111\rangle, \quad (11)$$

where the controlled-S is defined by

$$|0\rangle\langle 0|\otimes(|0\rangle\langle 2| + |2\rangle\langle 0| + |1\rangle\langle 3| + |3\rangle\langle 1|) + |1\rangle\langle 1|\otimes I_4,$$

and $\{|0\rangle,|1\rangle,|2\rangle,|3\rangle\}$ is the basis of the system C. Now, the unknown operation is performed on the subspace [defined by the basis $\{|0\rangle,|1\rangle\}$] of the system C, one can obtain

$$|\Psi_3\rangle_{ABC} = \alpha_1|00\rangle|2\rangle + \alpha_2|01\rangle|3\rangle + \alpha_3|10\rangle(U|0\rangle) + \alpha_4|11\rangle(U|1\rangle). \quad (12)$$

The followed controlled-S is performed on the system of A and C and the swapping gate is performed on the system of B and C. $|\Psi_3\rangle_{ABC}$ can be transformed into

$$|\Psi_4\rangle_{ABC} = \alpha_1|00\rangle|0\rangle + \alpha_2|01\rangle|1\rangle + \alpha_3|1\rangle(U|0\rangle)|0\rangle + \alpha_4|1\rangle(U|1\rangle)|1\rangle. \quad (13)$$

Finally, after an H gate is performed on system C, the measurement is performed on the system C under the computation basis $\{|0\rangle,|1\rangle,|2\rangle,|3\rangle\}$. The collapsed state is

$$|\Psi_f\rangle = \alpha_1|00\rangle + \alpha_2|01\rangle + \alpha_3|1\rangle(U|0\rangle) + \alpha_4|1\rangle(U|1\rangle) \quad (14)$$

for the measurement outcome $|0\rangle$. The success probability is 1/2. The key of this scheme is that the unknown physical apparatus U should be performed on the subspace of the ancillary four-dimensional (4D) system C. This may be reasonable for some cases. For example, assume the unknown physical apparatus U is constructed by optical elements while B is not a photonic system. In this case, an ancillary photon C will be used. The unknown operation U is easily performed on the state restricted in its operated subspace.[13]

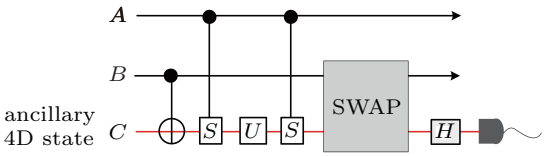


Fig. 2. (color online) A schematic circuit for the controlled unknown qubit operation with an ancillary high-dimensional system. U is an unknown qubit operation. C is an ancillary 4D system. Controlled-S denotes a general controlled swapping gate defined by $|0\rangle\langle 0|\otimes(|0\rangle\langle 2| + |2\rangle\langle 0| + |1\rangle\langle 3| + |3\rangle\langle 1|) + |1\rangle\langle 1|\otimes I_4$, where $\{|0\rangle,|1\rangle,|2\rangle,|3\rangle\}$ is the basis of the system C. SWAP denotes the swapping gate on a two-qubit system. H denotes the Hadamard operation.

3.2. Unknown multi-qubit operations

Consider the input system $|\Phi\rangle_{OA_1\cdots A_n}$ defined in Eq. (5). The schematic circuit is shown in Fig. 3 using ancillary four-dimensional systems in the state $|0\cdots 0\rangle_{B_1\cdots B_n}$. The joint system of $O,A_1,\ldots,A_n$ and $B_1,\ldots,B_n$ will be changed to

$$a_0|0\rangle_O \sum_{j_1,\ldots,j_n=0}^{1} \alpha_{j_1\cdots j_n}|j_1\cdots j_n\rangle_{A_1\cdots A_n}|j_1\cdots j_n\rangle_{B_1\cdots B_n} + a_1|1\rangle_O \sum_{i_1,\ldots,i_n=0}^{1} \beta_{i_1\cdots i_n}|i_1\cdots i_n\rangle_{A_1\cdots A_n}|i_1\cdots i_n\rangle_{B_1\cdots B_n} \quad (15)$$

after a CNOT gate being performed on each pair of A_j and B_j for $j = 1,\ldots,n$. Then, by performing a controlled-S on each pair of O and B_j, the state in Eq. (15) may be transformed into

$$a_0|0\rangle_O \sum_{j_1,\ldots,j_n=0}^{1} \alpha_{j_1\cdots j_n}|j_1\cdots j_n\rangle_{A_1\cdots A_n}|\overline{j_1}\cdots\overline{j_n}\rangle_{B_1\cdots B_n} + a_1|1\rangle_O \sum_{i_1,\ldots,i_n=0}^{1} \beta_{i_1\cdots i_n}|i_1\cdots i_n\rangle_{A_1\cdots A_n}|i_1\cdots i_n\rangle_{B_1\cdots B_n}, \quad (16)$$

where $|\bar{j}_s\rangle = |j_s+2\rangle, s=1,\ldots,n$. Now, the unknown operation is performed on the subspace [defined by the basis $\{|0\rangle,|1\rangle\}$ of B_i] of the joint system of all B_js, it follows that

$$a_0|0\rangle_O \sum_{j_1,\ldots,j_n=0}^{1} \alpha_{j_1\ldots j_n}|j_1\cdots j_n\rangle_{A_1\cdots A_n}|\overline{j_1}\cdots\overline{j_n}\rangle_{B_1\cdots B_n} + a_1|1\rangle_O \sum_{i_1,\ldots,i_n=0}^{1} \beta_{i_1\cdots i_n}|i_1\cdots i_n\rangle_{A_1\cdots A_n}(U|i_1\cdots i_n\rangle_{B_1\cdots B_n}). \tag{17}$$

The followed controlled-S is performed on each system of O and B_j and a swapping gate is performed on each system of A_j and B_j for $j=1,\ldots,n$. The joint system in Eq. (17) is changed to

$$a_0|0\rangle_O \sum_{j_1,\ldots,j_n=0}^{1} \alpha_{j_1\cdots j_n}|j_1\cdots j_n\rangle_{A_1\cdots A_n}|j_1\cdots j_n\rangle_{B_1\cdots B_n} + a_1|1\rangle_O \sum_{i_1,\ldots,i_n=0}^{1} \beta_{i_1\cdots i_n}(U|i_1\cdots i_n\rangle_{A_1\cdots A_n})|i_1\cdots i_n\rangle_{B_1\cdots B_n}. \tag{18}$$

Finally, after an H gate is performed on each system B_j, the measurement is performed on all B_js under the computation. The collapsed state is

$$\begin{aligned}|\Upsilon_f\rangle = {} & a_0|0\rangle_O \sum_{j_1,\ldots,j_n=0}^{1} \alpha_{j_1\cdots j_n}|j_1\cdots j_n\rangle_{A_1\cdots A_n} \\ & + a_1|1\rangle_O \sum_{i_1,\ldots,i_n=0}^{1} \beta_{i_1\cdots i_n}(U|i_1\cdots i_n\rangle_{A_1\cdots A_n})\end{aligned} \tag{19}$$

for all the measurement outcomes $|0\cdots0\rangle_{B_1\cdots B_n}$. The success probability is $1/2^n$. The key of this circuit is that the unknown physical apparatus U should be performed on the subspace of the ancillary system.

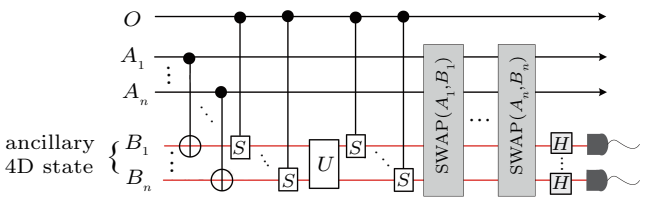


Fig. 3. (color online) A schematic circuit for the controlled multi-qubit operation with ancillary four-dimensional systems. U is an unknown multi-qubit operation which should be performed on the system of $A_1,\ldots,A_n$ conditional on the state of O. $B_1,\ldots,B_n$ are four-dimensional systems. Controlled-S is defined in Fig. 2.

4. Unknown operations conditional on a multi-control system

It has been shown that unknown operations may be controlled in Sections 2 and 3 with a control qubit. In this section, the multi-control system will be considered. In fact, we will show that unknown operations with multi-control may be reduced to the case with a single control using linear quantum resources and circuit complexity.

For the following explanations, define an approximate multi-controlled NOT gate $C^{*k}[\sigma_x]$ as

$$C^{*k}[\sigma_x] = \begin{pmatrix} \sigma_{2,1} & & & \\ & \ddots & & \\ & & \sigma_{2,2^{k-1}-1} & \\ & & & \sigma_x \end{pmatrix}, \tag{20}$$

where $\sigma_{2,j} \in \{I_2, \sigma_z\}$ for $j=1,\ldots,2^{k-1}-1$. This gate is the same as the multi-controlled NOT gate $C^k[\sigma_x]$ on the $k+1$ qubits (or $k+1$-qubit Toffoli gate) except phase differences -1 before some terms. Define R_y as the rotation

$$R_y = \begin{pmatrix} \cos(\pi/8) & -\sin(\pi/8) \\ \sin(\pi/8) & \cos(\pi/8) \end{pmatrix}. \tag{21}$$

Lamma 1 There exists a $C^{*n}[\sigma_x]$ which may be synthesized by the following network shown in Fig. 4.

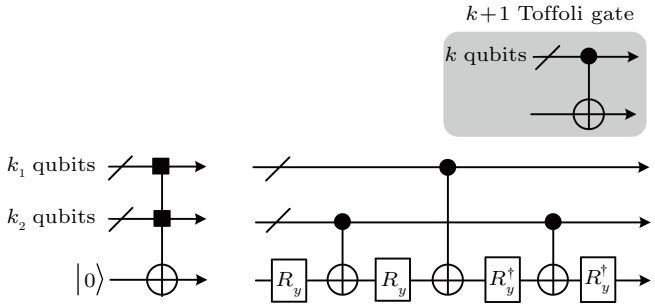


Fig. 4. (color online) A schematic decomposing a multi-qubit Toffoli gate into three multi-qubit Toffoli gates on smaller systems. R_y is a rotation along the y axis defined in Eq. (21).

Proof It is sufficient to prove that the right circuit will result in an $n+1$-qubit approximate multi-control-NOT gate $C^{*n}[\sigma_x]$. It may be completed from four cases.

(i) If some of the first k_1 qubits and some of the second k_2 qubits are in the states $|0\rangle$, the transformation applied to the target qubit is $R_y^\dagger \cdot R_y^\dagger \cdot R_y \cdot R_y = I$ (where $R_y^\dagger = R_y^{-1}$).

(ii) If some of the first k_1 qubits are in the states $|0\rangle$ and all the second k_2 qubits are in the states $|1\rangle$, the transformation applied to the last qubit is $R_y^\dagger \cdot \sigma_x \cdot R_y^\dagger \cdot R_y \cdot \sigma_x \cdot R_y = I$.

(iii) If all the first k_1 qubits are in the states $|1\rangle$ and some of the second k_2 qubits are in the states $|0\rangle$, the transformation applied to the last qubit is $R_y^\dagger \cdot R_y^\dagger \cdot \sigma_x \cdot R_y \cdot R_y = \sigma_z$.

(iv) If all the k_1+k_2 qubits are in the states $|1\rangle$, the transformation applied to the last qubit is $R_y^\dagger \cdot \sigma_x \cdot R_y^\dagger \cdot \sigma_x \cdot R_y \cdot \sigma_x \cdot R_y = \sigma_x$.

Lamma 2 Using an arbitrary gate $C^{*n}[\sigma_x]$, each gate $C^n[U]$ (an unknown gate U conditional on n qubits) may be synthesized by the following network shown in Fig. 5.

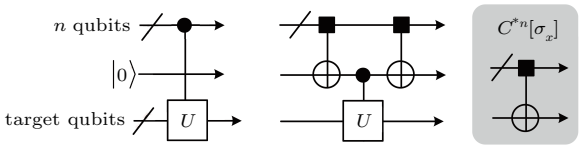


Fig. 5. (color online) A schematic equivalent teleportation circuit of multi-qubit controlled gate with an ancillary qubit in the state $|0\rangle$. Here, $C^n[U]$ is realized with two $C*^n[\sigma_x]$ gates and the gate $C[U]$.

Note that $C^{*n}[\sigma_x]$ is idempotent. The result is easily followed from the ancillary qubit in the state $|0\rangle$.

Theorem 1 Each gate $C^n[U]$ (an unknown gate U conditional on n qubits) may be synthesized with additional linear complexity and one auxiliary qubit in comparison to $C^1[U]$ (an unknown gate U conditional on one qubit).

Proof From Lemmas 1 and 2, $C^n[U]$ may be decomposed into two special gates $C^{*n}[\sigma_x]$ and a controlled-U gate $C^1[U]$. Here, $C^{*n}[\sigma_x]$ may be further realized with four single qubit rotations and two multi-qubit Toffoli gates $C^{k_1}[\sigma_x]$ and $C^{k_2}[\sigma_x]$, where $n/2-1 \leq k_1, k_2 \leq n/2+1$ and $k_1+k_2=n$. Finally, note that each multi-qubit Toffoli gate $C^{k_1}[\sigma_x]$ or $C^{k_2}[\sigma_x]$ may be realized with the linear complexity of CNOT gate and circuit depth using Lemma 7.2 of Ref. [27].

5. Conclusion

In conclusion, we have shown that if the invertible quantum fusion and quantum splitting are applied, previous deterministically optical implementations of controlled-unknown operations may be extended to any physical systems faithfully correlated with photons. This extension requires that the unknown photonic setup is provided. For general cases, a new probabilistic scheme is presented to add a control to unknown operations. To complete the new scheme, the unknown operations should be available for the subspace of an ancillary high-dimensional system. This assumption may be reasonable because various experimental systems have a natural high-dimensional structure from multiple degrees of freedom. Furthermore, with one auxiliary qubit the multiple control of unknown unitary operations can be reduced to the single control with additional linear circuit complexity. This result is similar to the known unitary operation. In comparison with previous photonic implementations, the present schemes may be flexible for different systems or hybrid systems.

References

[1] Deutsch D 1989 *Proc. R. Soc. Lond. A* **425** 73
[2] Deutsch D and Jozsa R 1992 *Proc. R. Soc. London, Ser. A* **439** 553
[3] Nielsen M A and Chuang I L 2000 *Quantum Computation and Quantum Information* (Cambridge: Cambridge University Press)
[4] Knill E, Laffamme R and Milburn G 2001 *Nature* **409** 46
[5] Kitaev A, Shen A and Vyalyi M 2002 *Classical and quantum computation* (New York: American Mathematical Society)
[6] Shor P W 1995 *Phys. Rev. A* **52** 2493
[7] Calderbank A R and Shor P W 1996 *Phys. Rev. A* **54** 1098
[8] Sheng Y B, Zhou L and Cui C 2015 *Chin. Phys. B* **24** 120306
[9] Cao X and Shang Y 2014 *Chin. Phys. Lett.* **31** 110302
[10] Deng F G and Ren B C 2015 *Acta Phys. Sin.* **64** 160303 (in Chinese)
[11] Feynman R P 1982 *Int. J. Theor. Phys.* **21** 467
[12] Barenco A, Bennett C H, Cleve R, DiVincenzo D P, Margolus N, Shor P W, Sleator T, Smolin J A and Weinfurter H 1995 *Phys. Rev. A* **52** 3457
[13] Vartiainen J J, Mottonen M and Salomaa M M 2004 *Phys. Rev. Lett.* **92** 177902
[14] Zhou X Q, Ralph T C, Kalasuwan P, Zhang M, Peruzzo A, Lanyon B P and O'Brien J L 2011 *Nat. Commun.* **2** 413
[15] Araúo M, Feix A, Costa F and Brukner Č 2014 *New J. Phys.* **16** 093026
[16] Friis N, Dunjko V, Dur W and Briegel H J 2014 *Phys. Rev. A* **89** 030303
[17] Chiribella G, D'Ariano G M, Perinotti P and Valiron B 2013 *Phys. Rev. A* **88** 022318
[18] Procopio L M, Moqanaki A, Arau'jo M, Costa F, Calafell I A, Dowd E G, Hamel D R, Rozema L A, Brukner C and Walther P 2015 *Nat. Commun.* **6** 7913
[19] Vitelli C, Spagnolo N, Aparo L, Sciarrino F, Santamato E and Marrucci L 2013 *Nat. Photon.* **7** 521
[20] Passaro E, Vitelli C, Spagnolo N, Sciarrino F, Santamato E and Marrucci L 2013 *Phys. Rev. A* **88** 062321
[21] Luo M X, Ma S Y, Chen X B and Wang X 2015 *Phys. Rev. A* **91** 042326
[22] Briegel H J, Calarco T, Jaksch D, Cirac J I and Zoller P 2000 *J. Mod. Opt.* **47** 415
[23] Duan L M and Kimble H J 2004 *Phys. Rev. Lett.* **92** 127902
[24] Colombe Y, Steinmetz T, Dubois G, Linke F, Hunger D and Reichel J 2007 *Nature* **450** 272
[25] Houck A A, Tureci H E and Koch J 2012 *Nat. Phys.* **8** 292
[26] Pla J J, Tan K Y, Dehollain J P, Lim W H, Morton J J, Jamieson D N, Dzurak A S and Morello A 2012 *Nature* **489** 541
[27] Barenco A, Bennett C H, Cleve R, *et al.* 1995 *Phys. Rev. A* **52** 3457