Low band gap frequencies and multiplexing properties in 1D and 2D mass spring structures

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This study reports on the propagation of elastic waves in 1D and 2D mass spring structures. An analytical and computation model is presented for the 1D and 2D mass spring systems with different examples. An enhancement in the band gap values was obtained by modeling the structures to obtain low frequency band gaps at small dimensions. Additionally, the evolution of the band gap as a function of mass value is discussed. Special attention is devoted to the local resonance property in frequency ranges within the gaps in the band structure for the corresponding infinite periodic lattice in the 1D and 2D mass spring system. A linear defect formed of a row of specific masses produces an elastic waveguide that transmits at the narrow pass band frequency. The frequency of the waveguides can be selected by adjusting the mass and stiffness coefficients of the materials constituting the waveguide. Moreover, we pay more attention to analyze the wave multiplexer and DE-multiplexer in the 2D mass spring system. We show that two of these tunable waveguides with alternating materials can be employed to filter and separate specific frequencies from a broad band input signal. The presented simulation data is validated through comparison with the published research, and can be extended in the development of resonators and MEMS verification.

Keywords: mass spring system, multiplexer, DE multiplexer, wave guide, phononic band gap

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1. Introduction

During the past few years, novel materials called phononic crystals (PnCs) have attracted increasing attention due to their implementation as potential materials for waveguides, transducers, filters and all vibration control applications.[1–3] These structures have the ability to control acoustic/elastic waves within some frequency ranges and are called band gaps or stop bands.[4–6] Because of these distinguishing features, PnCs are able to store and control some energies propagating through the structures. Being able to obtain new approaches that manipulate heat energy flow is important for highly efficient technological devices that include thermoelectrics.[7]

Controlling thermal properties at the nanoscale can push us to develop novel energy techniques in thermoelectricity.[8,9] Additionally, PnCs sensors with new trends related to remarkable physical effects were achieved, such as liquid sensors and ionizing particle detectors.[10–13] With the large scalability of mass spring networks in many applications, such as sound transducers, vibration stabilization equipment, resonators and micromachined electrical-mechanical systems (MEMS), interest has risen in establishing a correspondence between continuum structures and discrete lattices.[14–17] Generally, the dispersion relations of a periodic multilayer structure could be equivalent to the discrete lattice of two different atoms connected by weightless elastic springs. The band gap effects of finite periodicity and discrete lattice springs can be represented in mass spring systems. Additionally, finite periodic discrete structures are physically realizable in MEMS.

In the present paper, we report and describe a class of 1D and 2D mass spring networks that incorporate tunable narrow passing bands (NPB) in their band gaps. The idea of acoustic wave multiplexing and DE multiplexing in 2D continuum phononic crystals has been proposed.[18,19] Inspired by the interesting results of multiplexing and DE multiplexing in those 2D composites structures, we try to examine these studies in mass spring structures. The frequency of the passing band is controlled by modifying the types of materials in the waveguides, multiplexer and DE multiplexer inclusions through homogenous mass spring networks with high changes in the location and formation of the transmission gap. Guidance of the waves can be achieved by creating an extended linear row of masses and springs in a perfect mass spring network. Moreover, using materials with low elastic properties (acts as local resonances) in the waveguides and multiplexer, we are able to create several NPBs and transmit along the elastic waveguides with more selected frequencies.[14,20]

Moreover, low frequency band gaps built with locally resonant mechanisms are still a point of discussion for many researchers because of their great importance in the field of engineering vibration control where various conventional structures such as strings, rods, beams, plates, etc., are widely used as waveguides.[21] Actually, focusing on studying PnCs with lower and higher frequency band gaps may lead to applications in mechanical vibrations harness, seismic wave reflection and

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ultrasonics.[22] Hence, we need specific small materials that behave like total reflectors. The pioneering work by Liu *et al.*[22] has led to an additional solution to this problem based on locally resonant mechanisms. They obtained very low frequency band gaps with a smaller lattice and comparable relevant sonic wavelength. Therefore, we will pay more attention to provide low frequency band gaps with built in local resonances through 1D and 2D mass spring structures and we will compare our findings with previous published studies[14,23] for the same systems.

The rest of this paper is organized as follows. The method of calculation is briefly presented in Section 2 for the complex band gap, which is derived to obtain the dispersion curves. Section 3 is devoted to the explanation of the band gap formation mechanism via physical models. Some interesting phenomena in the pass/stop bands are demonstrated and discussed based on variations in mass size, spring stiffness and order. In addition, some approximate formulas for the evaluation of low frequency resonance gaps and NPB behaviors within a resonance gap are derived using an approach different from existing results in mass spring structures and MEMS studies. This is followed by the description of filtering and multiplexing properties of the new NPB waveguides. Finally, some conclusions are given in Section 4.

2. Method of calculation and geometrical models

Here we consider wave propagation through an infinite number of connected 1D and 2D structures. Because the structures are infinite, we can restrict our attention to a unit cell. In the following 1D and 2D unit cells, dispersion relations are obtained by manipulating wave propagation in the infinite periodic lattices. Our goal is to determine the eigenmodes and to look for band gaps in the spectrum. Results are presented in the form of band structures relating the frequency of the propagating waves to the wavenumber or wave vector.[15,23]

2.1. The one-dimensional unit cell

We consider an illustrative example for a 1D unit cell illustrated in Fig. 1. The unit cell has N masses m_j connected by linear elastic springs with stiffness coefficients k_j and anchored to ground with spring k'_j. These unit cells are infinitely repetitive units that are used to build the microstructure (or material).[23]

The small-amplitude displacement of the $(p+j)$th mass is governed by the following equation:[14,24]

$$\ddot{u}_{p+j}m_j = k_j\left(u_{p+j+1}-u_{p+j}\right)-k_{j-1}\left(u_{p+j}-u_{p+j-1}\right) - k'_j u_{p+j}, \tag{1}$$

where p is an arbitrary integer. The traveling wave solution is assumed as

$$u_{p+j}(t) = A_j\,\mathrm{e}^{\mathrm{i}((p+j)\gamma-\omega t)}, \tag{2}$$

where A_j is the wave amplitude, and the wavenumber is $\gamma = |\omega/c| = |k|$. c is the wave velocity, k is the wave vector, and ω is the wave frequency.

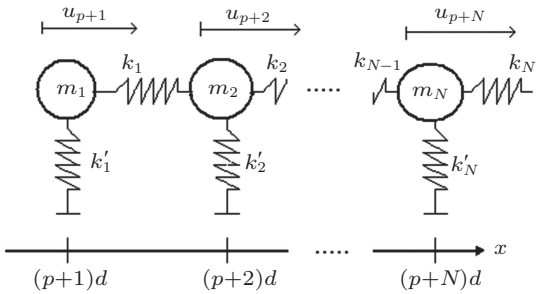


Fig. 1. 1D unit cell with N masses (m_j) and $2\times N$ springs (k_j, k'_j).

Substitute Eq. (2) into Eq. (1) produces a complex eigenvalue problem of the form,

$$\left(\bar{S}(\gamma)-\omega^2\bar{I}\right)\bar{A}=\bar{0}. \tag{3}$$

To construct the band structure of the system, the dispersion relation is obtained from Eq. (3) and plotted for the wavenumber γ versus the eigen frequencies ω Because of the periodicity of the system, it is not necessary to solve Eq. (3) for all values of γ. The dispersion relation is symmetric about the zero wavenumber and restricted in the irreducible Brillouin zone. The wavenumber in such zone $\gamma N = 0,\ldots,\pi$ generates the entire curve for the dispersion relation. The curve would simply be repeated if a larger range of wavenumbers were graphed. At the two points of the zone and π, if the wavenumber is taken to be zero, the frequency is equal to zero. This causes two masses in two neighboring unit cells to have the same amplitude and phase. Moreover, if the wavenumber is taken to be π, then the two masses move at the same amplitude but out of phase.

2.2. The two-dimensional unit cell

Consider a two-dimensional unit cell for an infinite periodic structure as shown in Fig. 2. The unit cell has $N\times N$ masses that are arranged in a square configuration with each mass connected to eight neighboring masses with springs. Mass $m_{j,k}$ is coupled with springs of spring constants $k_{j,k,1}$, $k_{j,k,2}$, $k_{j,k,3}$, and $k_{j,k,4}$ in the 0°, 45°, 90°, and 135° counterclockwise from the positive x-axis. Each mass is also anchored to the mechanical ground with springs $k'_{x,j,k}$ and $k'_{y,j,k}$, and the subscripts x, y denote the spring stiffness in the x and y directions.[14]

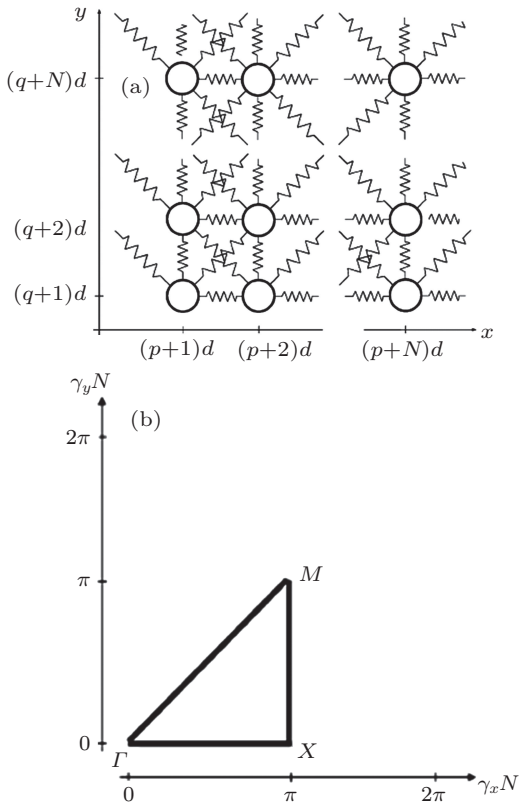


Fig. 2. (a) 2D square lattice with $N \times N$ masses and corresponding springs. The ground springs $k'_{x,j,k}$ and $k'_{y,j,k}$ are not depicted in the figure. (b) Corresponding irreducible Brillouin zone for the structure.[14]

The small-amplitude displacements of the $(p+j)$, $(q+k)$th masses in the x and y directions (uv) are given by the following equations:[23]

$$\begin{aligned}
&m_{j,k}\ddot{u}_{p+j,q+k}\\
&= k_{j,k,1}(u_{p+j+1,q+k} - u_{p+j,q+k})\\
&\quad +0.5k_{j,k,2}(u_{p+j+1,q+k+1} - u_{p+j,q+k}\\
&\quad +v_{p+j+1,q+k+1} - v_{p+j,q+k})\\
&\quad +0.5k_{j,k,4}(u_{p+j-1,q+k+1} - u_{p+j,q+k}\\
&\quad -v_{p+j-1,q+k+1} + v_{p+j,q+k}),\\
&\quad +k_{j-1,k,1}(u_{p+j-1,q+k} - u_{p+j,q+k})\\
&\quad +0.5k_{j-1,k-1,2}(u_{p+j-1,q+k-1} - u_{p+j,q+k}\\
&\quad -v_{p+j-1,q+k-1} + v_{p+j,q+k})\\
&\quad +0.5k_{j+1,k-1,4}(u_{p+j+1,q+k-1} - u_{p+j,q+k}\\
&\quad -v_{p+j+1,q+k-1} + v_{p+j,q+k})\\
&\quad +k'_{x,j,k}(0 - u_{p+j,q+k}),\\
&m_{j,k}\ddot{v}_{p+j,q+k}\\
&= k_{j,k,3}(v_{p+j+1,q+k} - v_{p+j,q+k})\\
&\quad +0.5k_{j,k,2}(v_{p+j+1,q+k+1} - v_{p+j,q+k}\\
&\quad +u_{p+j+1,q+k+1} - u_{p+j,q+k})\\
&\quad +0.5k_{j,k,4}(v_{p+j-1,q+k+1} - v_{p+j,q+k}\\
&\quad -u_{p+j-1,q+k+1} + u_{p+j,q+k})\\
&\quad +k_{j,k-1,3}(v_{p+j,q+k-1} - v_{p+j,q+k})\\
&\quad +0.5k_{j-1,k-1,2}(v_{p+j-1,q+k-1} - v_{p+j,q+k}\\
&\quad -u_{p+j-1,q+k-1} + u_{p+j,q+k})\\
&\quad +0.5k_{j+1,k-1,4}(v_{p+j+1,q+k-1} - v_{p+j,q+k}\\
&\quad -u_{p+j+1,q+k-1} + u_{p+j,q+k})\\
&\quad +k'_{y,j,k}(0 - v_{p+j,q+k}),
\end{aligned} \tag{4}$$

$$\begin{aligned}
&+0.5k_{j-1,k-1,2}(v_{p+j-1,q+k-1} - v_{p+j,q+k}\\
&-u_{p+j-1,q+k-1} + u_{p+j,q+k})\\
&+0.5k_{j+1,k-1,4}(v_{p+j+1,q+k-1} - v_{p+j,q+k}\\
&-u_{p+j+1,q+k-1} + u_{p+j,q+k})\\
&+k'_{y,j,k}(0 - v_{p+j,q+k}),
\end{aligned} \tag{5}$$

where p and q are arbitrary integers.

The traveling wave solution of Eqs. (4) and (5) is assumed in the following forms:

$$u_{p+j,q+k}(t) = A_{j,k}\,\mathrm{e}^{\mathrm{i}((p+j)\gamma_x+(q+k)\gamma_y-\omega t)}, \tag{6}$$

$$v_{p+j,q+k}(t) = B_{j,k}\,\mathrm{e}^{\mathrm{i}((p+j)\gamma_x+(q+k)\gamma_y-\omega t)}. \tag{7}$$

An infinite number of unit cells are considered. Thus, we will consider each unit cell is coupled to the neighboring unit cells by boundary conditions similar to those applied to the 1D structure can be extended easily and found in Ref. [14]. Finally, the corresponding eigenvalue problem is obtained

$$(\boldsymbol{S}(\gamma_x,\gamma_y) - \omega^2\boldsymbol{I})\boldsymbol{A} = \boldsymbol{0}. \tag{8}$$

The solution of this equation gives the relation between the wave frequency ω for known wave vector components γ_x and γ_y. Similar to the 1D structure in Section 2.1, the entire solution and eigen frequencies can be obtained for the wave vector γ_x and γ_y following the triangular path in Fig. 2(b) corresponding to the irreducible Brillouin zone.

3. Results and discussion

First, we briefly discuss the band structure of the 1D and 2D mass spring structures with/without the local resonance property. Second, we introduce our findings concerning the results of the guiding and DE multiplexing properties of waveguides. This provides a suitable basis for the development of waveguides in the subsequent sections.

3.1. 1D structure

As mentioned before, our main goal in this work is to reveal the phenomenon of local resonance inside mass spring structures. Hence, we will exploit new materials in the mass spring structure and compare it with the previous published data where they used aluminum/PMMA as a mass spring network.[23] To calculate the dispersion relation, the 1D model system is built of four masses and springs anchored with four ground springs as depicted in Fig. 3. The infinite periodic structure chosen for verification corresponds to a 0.15 m rod with the center masses and springs representing a material with lower stiffness to mass ratio (Case 1: PMMA /Case 2:

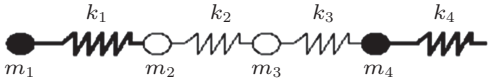


Fig. 3. 1D unit cell with N masses and springs (the ground springs are not depicted in the figure).

rubber) and the two ends of aluminum. The material properties are listed in Table 1.

The masses and springs are chosen as

$$
\begin{aligned}
&m_1 = m_4 = 3.98\ \text{kg}, \quad \text{aluminum},\\
&m_2 = m_3 = 1.69\ \text{kg}, \quad (\text{PMMA}/\text{rubber}),\\
&k_1 = k_4 = 70.9 \times 10^9\ \text{kg/s}^2, \quad \text{aluminum},\\
&k_2 = k_3 = 5.28 \times 10^9\ \text{kg/s}^2, \quad \text{PMMA},\\
&k_2 = k_3 = 1.1840 \times 10^5\ \text{kg/s}^2, \quad \text{rubber}.
\end{aligned}
$$

Table 1. Materials data.

Material	ρ/kg·m^{-3}	Young's modulus $E \times 10^9$/N·m^{-2}
Aluminum	2830	70.9
Rubber	1300	0.0001
PMMA	1200	5.28
Epoxy	1140	4.10
Isoprene	681	0.004

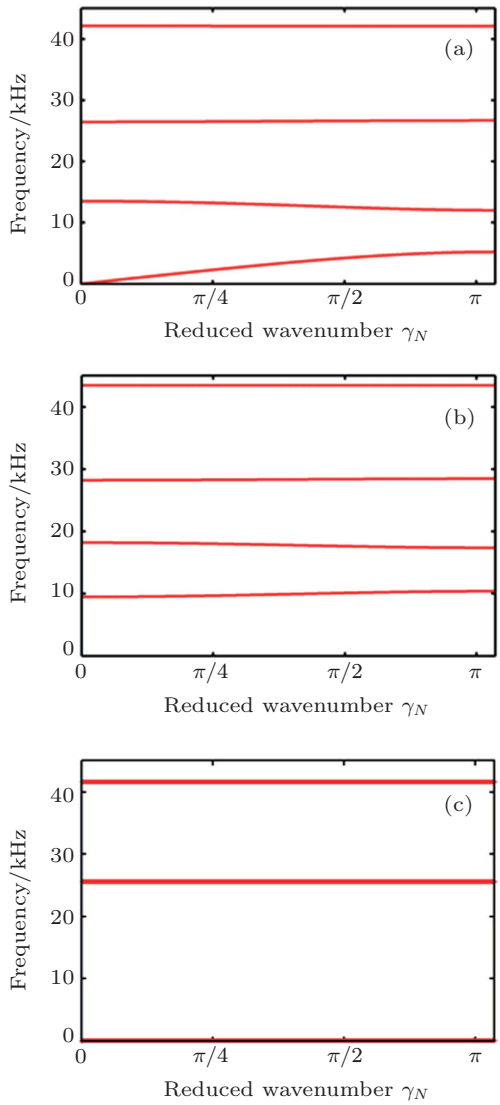


Fig. 4. (color online) Dispersion curves for wave propagation in the 1D mass spring structure depicted in Fig. 3, with unit cell $N = 4$. (a) Case 1, there is a damping force represented by four ground springs; (b) Case 1, without a damping force; (c) Case 2, local resonance curve, the soft material (PMMA) in the center replaced by rubber.

As shown by the Case 1 in Fig. 4(a) the dispersion curve is plotted for the wave propagation in the 1D mass spring structure with the unit cell $N = 4$ masses and springs, and the two ends from aluminum and the soft material from PMMA. There are three large band gaps in the band structure for $\omega \approx 5.2$–12.5, 135–27 and 27–42.3 kHz. In these frequency ranges no waves can propagate through the structure, where the wave number γ is a complex number It can be a positive real or imaginary number $\gamma = \gamma - i\gamma$. Therefore, in the latter case, wave solution in Eq. (2) represents a standing wave with a spatial exponential attenuation in the magnitude and strength proportional to $\gamma \to i\gamma$. In Case 1 of Fig. 4(b), the band gap edges became steeper, shift to higher values and increased in number by one more, due to inserting a damping force in the structure represented by four ground springs from PMMA. The ground springs are not depicted in Fig. 3. In order to compare the band gaps and localization phenomenon in locally resonant structures (Case 2), with those in Case 1 without local resonances, we replaced the PMMA material with a rubber material. Figure 4(c) summarizes the values of these NPB frequencies and low frequency band gaps. As already mentioned before, soft rubber has recently been used[21] as the coating of heavy lead spheres arranged in certain lattice structures in an Epoxy matrix. The low elastic constants of the soft rubber material resulted in a strong resonant band gap band with a frequency lower than the expected one by Bragg's law.[12]

3.2. 2D structure

In this section, we still pose and introduce low frequency band gaps with locally resonant phenomenon in mass spring structures. However, here there is an enhancement in the band gap values and local resonant peaks by applying a 2D structure. The 2D structure here is a 0.02 m×0.02 m unit cell of a soft and flexible epoxy matrix with a heavy and stiff aluminum inclusion. The 2D structure is shown in Fig. 5 with a unit cell of $N \times N = 5 \times 5$ mass spring system with 3×3 center inclusion.

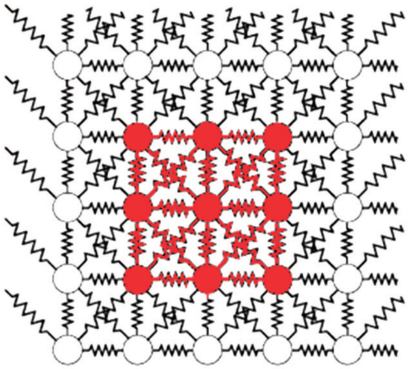


Fig. 5. (color online) 5×5 mass-spring network modeling a 2D unit cell with stiff inclusion 3×3 masses and springs.

The masses and springs are chosen as

$$m_{\text{inclusion}} = 4.53 \times 10^{-2}\ \text{kg}, \quad (\text{Aluminum}),$$
$$m_{\text{matrix}} = 1.82 \times 10^{-2}\ \text{kg}, \quad (\text{Epoxy/Rubber/Isoprene}),$$
$$k_1 = k_3 = 2 \times k_2 = 2 \times k_4 = 70.9 \times 10^9\ \text{kg/s}^2, \quad (\text{Aluminum}),$$
$$k_1 = k_3 = 2 \times k_2 = 2 \times k_4 = 4.10 \times 10^9\ \text{kg/s}^2, \quad (\text{Epoxy}),$$
$$k_1 = k_3 = 2 \times k_2 = 2 \times k_4 = 1.1840 \times 10^5\ \text{kg/s}^2. \quad (\text{Rubber}).$$

As revealed in many studies, one can establish a correspondence between continuum structures and discrete lattices.[15] The dispersion relation of a multilayer periodic structure would be equivalent to a bi-atomic one-dimensional chain of different masses connected by weightless elastic springs. Therefore, we choose springs values k_2 and k_4 to be half the size of the springs k_1 and k_3 to obtain a good qualitative agreement between the 2D mass spring structure and a 2D continuum model with a specific Poisson ratio.

As is well known, band gaps exist only for inhomogeneous structures. Hence, as indicated in Fig. 6(a) by considering the 2D structure in Fig. 5 having the same type of masses, no band gaps appear in the band structure. Next, as shown in Fig. 5, we considered two inhomogeneous structures; in the first one the host matrix has masses connected with springs from epoxy and inclusions from aluminum. In the second case, the host matrix is from rubber and inclusions are from aluminum (ground spring from isoprene in the two cases). Consequently, similar observations are obtained as in the 1D case with more reducing and enhancing in band gap values as shown in Figs. 6(b) and 6(c). By replacing rubber with epoxy, very low band gaps obtained at the range 2138–3366 Hz compared to 46–59 kHz with epoxy inclusions.

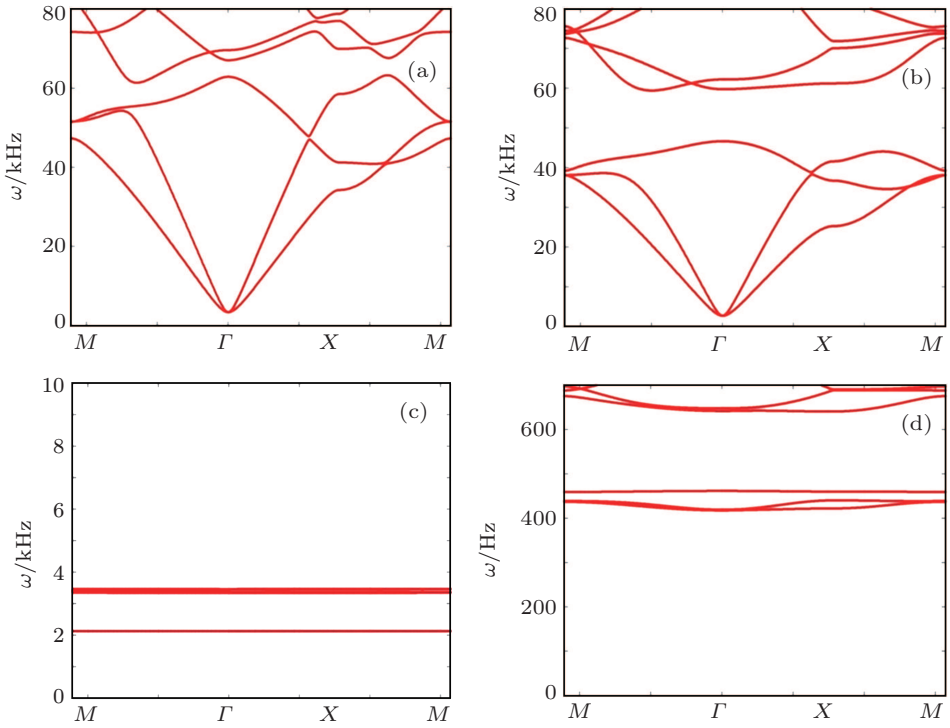


Fig. 6. (color online) Dispersion curves for acoustic wave propagation in the infinite periodic lattice of a 5×5 unit cell of (a) homogenous structure consisting of epoxy with aluminum inclusion springs having the same mass values; (b) epoxy with aluminum inclusion and isoprene as a ground spring; (c) rubber with aluminum inclusion and isoprene as a ground spring; and (d) rubber with aluminum inclusion and rubber as a ground spring.

Moreover, we investigated the modification of the frequency of the narrow band gap at the lower frequencies of the previous cases by changing the ground springs to be rubber as well. We chose rubber springs instead of isoprene because of the large contrast between the elastic parameters (Young modulus and mass density) and those of aluminum. As shown in Fig. 6(d) we have achieved a pronounced band gap frequency at 461–642 Hz which is close to previous studies obtained from the 3D continuum structure based on the local resonance property as well.[22]

Additionally, we have reported in Fig. 7 that the evolution of the band gaps as a function of the mass value. As expected, the width of the band gap increases with the increase in inclusion mass (aluminum) with respect to host matrix (epoxy) until the corresponding mode reaches a significantly wider value as shown in Fig. 7(c). Therefore, we have verified the previous hypothesis in our models and achieved large band gaps by using materials that significantly vary from each other in elastic characteristics

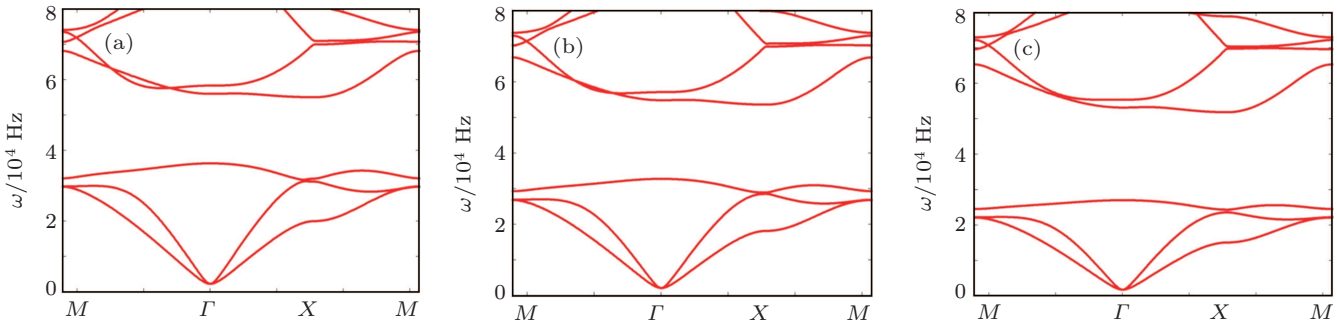


Fig. 7. (color online) Dependence of dispersion curves of acoustic wave propagation in the infinite periodic lattice of a 5×5 unit cell as a function of the mass value (a) mass of inclusion material aluminum $m_{\text{alu}} = 8 \times 10^{-2}$ kg and mass of host matrix epoxy $m_{\text{epoxy}} = 1.82 \times 10^{-2}$ kg; (b) mass of inclusion material aluminum $m_{\text{alu}} = 10 \times 10^{-2}$ kg and mass of host matrix epoxy $m_{\text{epoxy}} = 1.82 \times 10^{-2}$ kg; (c) mass of inclusion material aluminum $m_{\text{alu}} = 15 \times 10^{-2}$ kg and mass of host matrix epoxy $m_{\text{epoxy}} = 1.82 \times 10^{-2}$ kg.

3.3. Multiplexing filtered and waveguide of acoustic waves in 2D structure

In this section, we will discuss the multiplexing, DE multiplexing and waveguide properties of X-shaped waveguides constituting an alternating arrangement of the mass spring systems. Let us consider the geometry sketched in Fig. 8(a) which represents a waveguide constituted of a row of identical masses along the x direction. The homogenous waveguide composed of a mass spring line from a specific material (Case 1: aluminum; Case 2: rubber) with a mass value 4.53×10^{-2} kg is inserted inside a homogenous structure from epoxy with a mass value 1.82×10^{-2} kg.

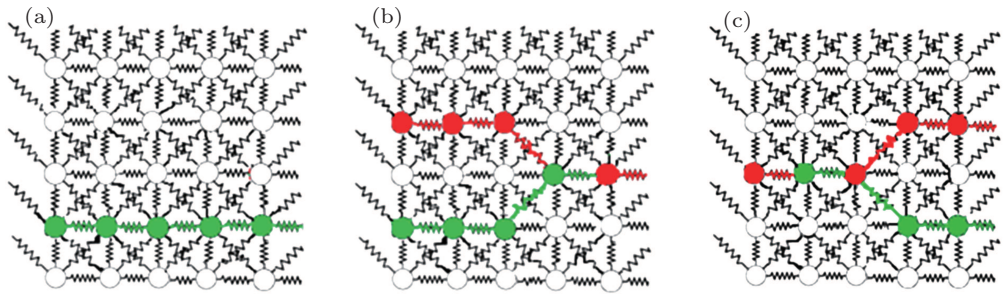


Fig. 8. (color online) (a) Line of a waveguide created inside the 2D mass spring structure; (b) schematic diagram of the X-shaped Multiplexer; (c) schematic diagram of the X-shaped DE Multiplexer. The red part of the Multiplexer and DE Multiplexer represents the mass spring system from rubber with a mass value 8×10^{-2} kg and the green part from aluminum with a mass value 4.53×10^{-2} kg. The host matrix is from epoxy with a mass value 1.82×10^{-2} kg.

As shown in Fig. 9(a), the waveguide will select specific frequencies from a broad band input signal propagating through such a structure. These NPB frequencies correspond to the elastic properties of the material constituting the waveguide. Therefore, a significant difference between the aluminum and rubber waveguides is illustrated in Fig. 9(b). There is an increment in the number of NBPs appear in the dispersion curves in the case of a rubber waveguide. This result reinforced our previous interpretation of materials with low elastic properties in producing large numbers of NPBs in the dispersion curve of mass spring structures. From the above observations, one could design an active device that can transmit acoustic waves selectively at one or two desired frequencies corresponding to the NPBs. Let us consider the geometry sketched in Figs. 9(b) and 9(c) for the X-shaped multiplexer and DE multiplexer, respectively. In order to identify the two NPBs, we have designed those structures from two different waveguides of two mass spring systems. Each branch of the multiplexer and DE multiplexer is designed for the propagation of waves with only specific NPB frequencies corresponding to the elastic properties of each material. The two materials are rubber and aluminum with different mass values embedded in the epoxy host matrix. As shown in Figs. 10(a) and 10(b), the multiplexer and DE multiplexer selected two NPB frequencies from a broad band input signal. The situation is different in each case as indicated in the figure. For instance, the number and position of NPB frequencies transmitted in the multiplexer are different from those propagated in the DE multiplexer. Indeed, in each case, each segment of the waveguides allows the propagation of specific NPBs, while prohibiting propagation of the other NPBs. However, generally the NPBs are observed in the transmission spectrum results from the overlap of several branches in the dispersion curves. Our results here are in a good agreement with those observed for 2D continuum structures in Ref. [19].

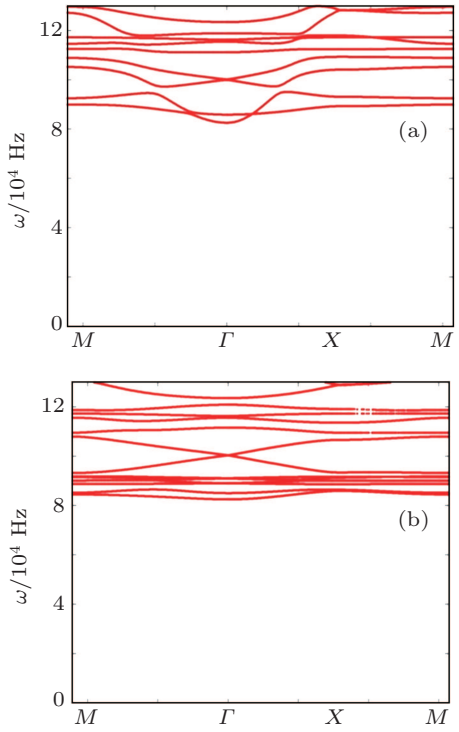


Fig. 9. (color online) Dispersion curves of a line waveguide inserted in the 2D mass spring structure. (a) Case 1, aluminum waveguide; (b) Case 2, rubber waveguide.

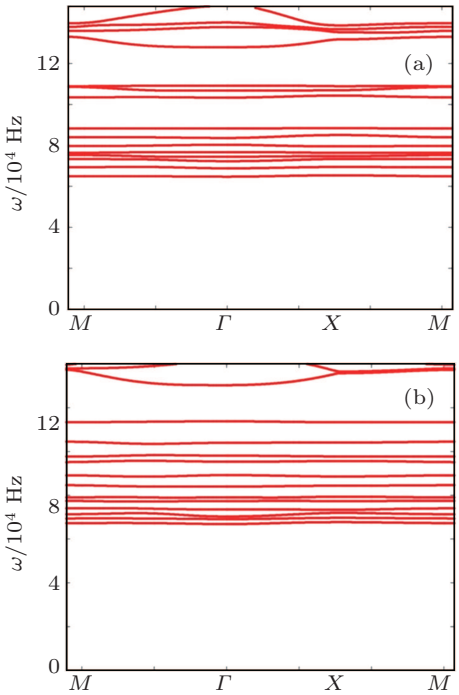


Fig. 10. (color online) Dispersion curves of (a) multiplexer and (b) DE multiplexer in the 2D mass spring structure.

4. Conclusions

In this paper, we have theoretically investigated the propagation of acoustic waves through 1D and 2D mass spring structures. The band gaps can be adjusted by calculating the band gap width as a function of mass value. Additionally, by modeling the structures from host materials with very low elastic properties, we obtained low frequency band gaps at small dimensions compared with the conventional structures. Such systems could be used to transmit acoustic waves at very narrow pass bands inside a broad gap. Moreover, we have studied waveguiding properties of acoustic waves in 2D mass spring systems. This device permits the transmission of NPB of frequencies corresponding to the elastic properties of the material. Additionally, by appropriately designing an active guiding device, we can impose transmission at more than one NPB frequencies. Therefore, we have discussed the model of a multiplexer and DE multiplexer based on the X-shaped waveguide. These systems can be utilized for separating or merging narrow transmission bands with different frequencies.

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