Coupled-resonator-induced transparency in two microspheres as the element of angular velocity sensing*

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We proposed a two-coupled microsphere resonator structure as the element of angular velocity sensing under the Sagnac effect. We analyzed the theoretical model of the two coupled microspheres, and derived the coupled-resonator-induced transparency (CRIT) transfer function, the effective phase shift, and the group delay. Experiments were also carried out to demonstrate the CRIT phenomenon in the two-coupled microsphere resonator structure. We calculated that the group index of the two-coupled sphere reaches $n_g = 180.46$, while the input light at a wavelength of 1550 nm.

Keywords: coupled-resonator-induced transparency, spherical resonator, whispering gallery, mode angular velocity

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1. Introduction

Coupled-resonator-induced transparency (CRIT) has been widely studied since it was discovered in coupled whispering-gallery-mode (WGM) resonator structures.[1–3] CRIT renders a coupled-resonator structure transparent over a narrow spectral range because of the classical destructive interference, analogous to electromagnetically-induced-transparency (EIT). CRIT creates extreme dispersion within the transparency window, which leads to a high group index and slow light. Experimental observations of CRIT have been reported in various WGM resonators, such as fiber ring resonators, microspheres, and integrated micro-ring resonators. It has potential applications in various fields, such as slow light, optical buffers, and optical sensors.[4,5]

Coupled resonator optical waveguide (CROW) structures used as angular velocity sensors have attracted much interest in recent years, because their large structural dispersion may enhance the Sagnac phase shift.[6] Zhang *et al.* confirmed that the gyroscope sensitivity has been enhanced in the slow light structure in experiment.[7] Researchers proposed different kinds of integrated CROW structures, the transmission loss of waveguide is much larger than that of the fiber, because of the material and process limitation of the waveguide. In this paper, we proposed a compact CRIT structure composed by two coupled silica microspheres, which can be used in ultra-low transmission loss CRIT gyro devices.

Microspheres are often used as a WGM resonator in experimental devices owing to its ultra-high sensitivity factor (up to 10^{11}) and convenience in preparation.[8] Microspheres are generally fabricated by melting the tip of an optical fiber in a hydrogen–oxygen flame; they possess ultra-high surface smoothness because of the surface tension.[9] The most effective way of coupling to a microsphere is using a tapered fiber, where the coupling efficiency can reach 99%.[10] In our study, we melted a single fiber to obtain silica microspheres as WGM resonators, and placed them on three-dimensional (3D) adjustment stages to control their coupling gap accurately. CRIT phenomena were observed on a test system.

2. Theoretical analysis

We analyzed a model of coupled microspheres, and obtained the theoretical angular velocity sensitivity enhancement factor of the structure. For a passive microsphere resonator, the phase difference between the counter-propagating light is given by the Sagnac effect, that is

$$\frac{\mathrm{d}\phi}{\mathrm{d}\Omega} = \frac{4\omega S}{c^2}, \tag{1}$$

where S is the area of the equatorial plane of the microsphere and Ω is the angular velocity. A large-size microsphere achieves high phase sensitivity, but fabricating a centimeter-diameter microsphere is difficult, as melting the tip of a fiber produces a millimeter-diameter sphere at most. We have proposed a method that increases the phase sensitivity of the Sagnac gyroscope using coupled SiO_2 spheres. According to Ref. [7], when two light waves counter-propagate in a rotating CROW, the Sagnac phase shift $\Delta\Phi$ that they accumulated is given by

$$\Delta\Phi = \frac{n_g}{n_0}\Delta\phi, \tag{2}$$

where $\Delta\Phi$ and $\Delta\phi$ are the Sagnac phase shifts induced by the beams in a CROW and a single resonator, respectively, n_g is

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the CROW group index, and n_0 is the refractive index of the resonators. The CROW group index may be larger than the refractive index of the resonator, so $\Delta\Phi$ may be much larger than $\Delta\phi$. The angular velocity sensitivity hence is enhanced n_g/n_0 times.

For a single microsphere resonator, the theoretical limit $\delta\Omega$ can be expressed as[6]

$$\delta\Omega = \frac{1}{DQ\sqrt{p_{\rm pd}}}\sqrt{\frac{2Bhc^3}{\lambda\eta}}\left(\frac{3600\times 180}{\pi}\right)^{\circ}\frac{1}{h}, \tag{3}$$

where c is the velocity of light in vacuum (c= 3×10^8 m/s), h is Planck's constant (h= 6.62×10^{-34} J/s), $p_{\rm pd}$ is the optical power at the input of the photodetector ($p_{\rm pd}$ = 1 mW), η is photo detector quantum efficiency (η= 0.82), λ is wavelength of the input light (λ= 1550 nm), B is the measurement bandwidth (B =1 Hz), D and Q are diameter and quality factor of the cavity, respectively.

In Ref. [11], the authors analyzed the rotation sensitivity of two coupled resonator structure, the two resonators of which have the same radius. In this bidirectional structure, the light travels in opposite directions in the two resonators, the phase shifts induced by Sagnac effect have opposite signs, the two Sagnac contributions cancel out. However, if the sizes of the two resonators are slightly different, and the same bias and coupling optimization is applied again, the sensitivity increases dramatically. That is because when the resonators are no longer identical, the Sagnac phases in the two resonators no longer have the same amplitude, and they no longer cancel as strongly. In Ref. [12], the authors compared the sensitivity of the CRIT structure with that of a usual passive single resonator. When the parameters of the two are respectively the same, the CRIT structure is more dispersive than the single resonator. The highly dispersive structure is more susceptible to the phase shift induced by the Sagnac effect. And a folded configuration was proposed, in this structure, light travels in the same direction in the two resonators, thus enhancing the sensitivity of rotation.

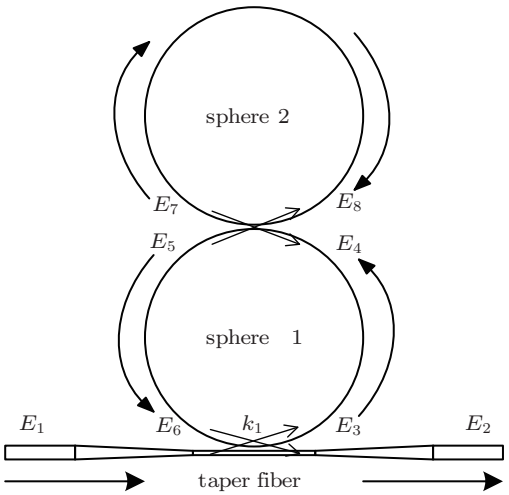


Fig. 1. Two coupled microspheres, coupled to a tapered fiber. The input and output fields are E_1 and E_2; E_3, E_4, E_5, E_6, E_7, and E_8 are optical fields at several points inside the system.

Figure 1 shows a stacked system comprising a microsphere coupled to another microsphere coupled to a tapered fiber. A light wave enters from the left end of the tapered fiber. The transfer function of the coupled microspheres is expressed as

$$T = \frac{E_2}{E_1}. \tag{4}$$

Let k_1 and t_1 be the coefficients for coupling and transmission of the tapered fiber and the first sphere, respectively; similarly, let k_2 and t_2 be the coefficients for coupling and transmission of the first sphere and second sphere, respectively. A coupling matrix formalism introduced by Poon *et al.*[13] is used in the analysis. The relationship among the mode amplitudes can be expressed as

$$\begin{pmatrix} E_3 \\ E_2 \end{pmatrix} = \begin{pmatrix} t_1 & -{\rm i}k_1 \\ -{\rm i}k_1 & t_1 \end{pmatrix}\begin{pmatrix} E_6 \\ E_1 \end{pmatrix}, \tag{5}$$

$$\begin{pmatrix} E_7 \\ E_5 \end{pmatrix} = \begin{pmatrix} t_2 & -{\rm i}k_2 \\ -{\rm i}k_2 & t_2 \end{pmatrix}\begin{pmatrix} E_8 \\ E_4 \end{pmatrix}, \tag{6}$$

$$E_8 = a_2\exp({\rm i}\phi_2)E_7, \tag{7}$$

$$E_6 = a_1\exp\left({\rm i}\frac{1}{2}\phi_1\right)E_5, \tag{8}$$

$$E_4 = a_1\exp\left(i\frac{1}{2}\phi_1\right)E_3, \tag{9}$$

where a_1 and a_2 are the round-trip amplitude attenuation factors depending on the loss coefficient α and the circumferences of the microspheres, L_1 and L_2. By combining Eqs. (3)–(8), the transfer functions are found to be

$$T_2 = \frac{E_5}{E_4} = \frac{t_2 - a_2\exp({\rm i}\phi_2)}{1 - t_2a_2\exp({\rm i}\phi_2)}, \tag{10}$$

$$T_1 = \frac{E_2}{E_1} = \frac{t_1 - a_1T_2\exp({\rm i}\phi_1)}{1 - t_1a_1T_2\exp({\rm i}\phi_1)}. \tag{11}$$

The effective phase shift of the structure is

$$\phi^{\rm eff}(\phi_1,\phi_2) = \arg(T_1). \tag{12}$$

Its group delay is

$$\tau(\omega) = \frac{{\rm d}\phi^{\rm eff}(\phi_1,\phi_2)}{{\rm d}\omega}. \tag{13}$$

The group velocity is defined by

$$V_{\rm g}(\omega) = \frac{L}{\tau(\omega)}, \tag{14}$$

where L is light path length of this structure $L = L_1 + L_2$, L_1 and L_2 are equatorial plane circumferences of sphere 1 and sphere 2, yielding a group index of

$$n_{\rm g}(\omega) = \frac{c}{V_{\rm g}(\omega)}. \tag{15}$$

In Eqs. (10) and (11), $\phi_i = 2\pi n_{\rm eff}L_i/\lambda$ $(i = 1,2)$ is the round-trip phase for each microsphere. The optical pathways

through the first sphere and the second sphere are illustrated in Fig. 1. The $\phi = \phi_1 - \arg(T_2)$ is the phase difference between two optical pathways.

$$\phi = \phi_1 - \left[\arctan\left(\frac{a_2 \sin\phi_2}{t_2 - a_2\cos\phi_2}\right) - \arctan\left(\frac{a_2 t_2 \sin\phi_2}{1 - a_2 t_2 \cos\phi_2}\right)\right]. \quad (16)$$

While ϕ/π is an odd number, these two optical pathways destructively interfere, there is no light localized in the two spheres. Necessary conditions for achieving CRIT in a two-ring system have been discussed by Mario *et al.*[14,15] To achieve CRIT, the following combined conditions must be satisfied: $a_1 < t_1 < t_2 < a_2$ which are also suitable for two coupled microspheres. The simulated curves of transmission, effective phase shift, and group delay are shown in Fig. 2.

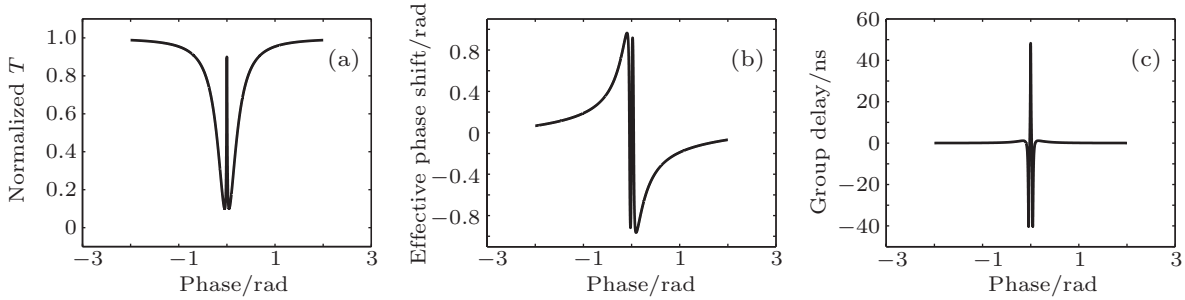


Fig. 2. Results from the simulation model of the two coupled microspheres with $t_1 = 0.9$, $t_2 = 0.999$: $a_1 = 0.88$, $a_2 = 0.999$: (a) transmission, (b) effective phase shift, and (c) group delay.

In the simulation calculations, the impact of the coupler insertion loss was ignored and the resonator attenuation factors $a_1 = 0.88$ and $a_2 = 0.999$ were maintained. Set $t_1 = 0.9$ and let the first microsphere and the tapper in the under-coupled state. Gradually change the coupling factor t_2 between the two microsphere resonators and let the coupling of two microspheres change from the over-coupled state to the under-coupled state. Mathematically calculated resonance spectra under different t_2 are shown in Fig. 3.

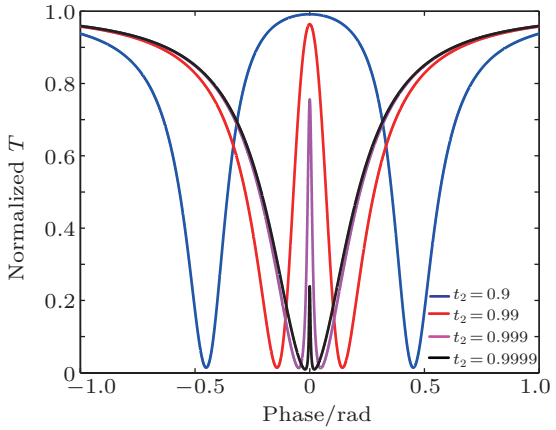


Fig. 3. The transmission change curves with the transmission coefficient of sphere 2 when $t_1 = 0.9$.

As seen in Fig. 3, with the change of t_2, the shape of the resonance peaks does not change and only the resonance peaks shift, while two microspheres are in over-coupled state ($t_2 < a_2$); while when $t_2 > a_2$, the shape of the resonance peaks changes, as the two microspheres are in under-coupled state most of the light propagates in the first microsphere.

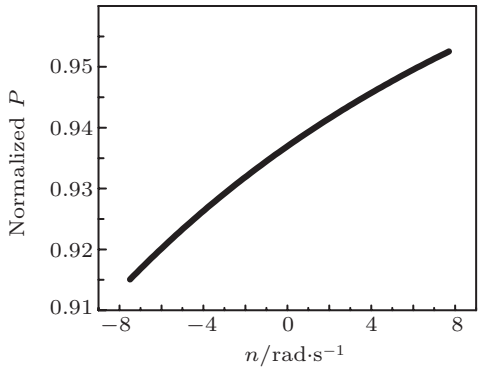


Fig. 4. Rotation-dependent transmitted power P as a function of angular velocity Ω.

When this structure is rotated around an angular velocity Ω, rotating axis perpendicular to the plane of propagation of light in the spherical cavities, the transmitted power of the output is changed with the Sagnac phase shift. The sensitivity reaches the maximum at the CRIT transparent peak. The total complex phase shift experienced by the signal while it travels once around the first microsphere and the second microsphere are given by

$$\phi_1 = \frac{2n\pi R_1 \omega}{c} + \mathrm{i}\pi R_1 \alpha + \frac{2\pi R_1^2 \omega}{c^2}\Omega, \quad (17)$$

$$\phi_2 = \frac{2n\pi R_2 \omega}{c} + \mathrm{i}\pi R_2 \alpha - \frac{2\pi R_2^2 \omega}{c^2}\Omega. \quad (18)$$

Here, the first term is the phase induced by propagation, the second term accounts for the loss, the third term is the Sagnac phase shift induced by rotation. Since the light wave propagates in opposite directions inside the two microspheres, the Sagnac phase shifts in ϕ_1 and ϕ_2 have opposite signs power P

measured by a photodetector is a function of the angular velocity Ω, that is,

$$P_{\text{out}}(\Omega) = |E_2/E_1|^2 = \left| \frac{e^{i(\phi_1+\phi_2)} + \left(1-e^{i\phi_1}\right)\sqrt{1-\kappa} - e^{i\phi_2}(1-\kappa)}{1+\left(e^{i(\phi_1+\phi_2)} - e^{i\phi_2}\right)\sqrt{1-\kappa} - e^{i\phi_1}(1-\kappa)} \right|^2, \tag{19}$$

where κ is the coupling ratio, while the bias and coupling optimization are applied, according to Eq. (19), the relationship between the rotation-dependent transmitted power P and angular velocity Ω is shown in Fig. 4.

3. Experiment and discussion

The experimental setup is schematically shown in Fig. 5. Silica microspheres were fabricated by melting the end of an optical fiber. The tapered fiber was used as a waveguide to excite the WGMs using a tunable laser with the wavelength of 1550 nm and the spectral linewidth of 300 kHz. The function generator was used to linearly tune the laser frequency by setting a triangular voltage signal to the laser. To obtain the CRIT spectrum of the two microspheres, a 3D adjustment stage was used to couple a microsphere to the taper, adjusting the gap between the microsphere and the taper, making them in the critical coupling state. The image of microsphere coupled-resonator-induced transparency structure was captured by a CCD as shown in Fig. 5. The resonance spectrum of the single microsphere was observed on an oscilloscope, and then we adjusted the other microsphere towards the first sphere. In this process, a transparency peak (see Fig. 6) arose at the WGM resonance point in the absorption spectrum of the single microsphere. This resonance peak was split into two modes, the peaks on the right hand side of Fig. 6 are high-order modes of the first microsphere, because the sphere is a 3D structure resonator. We show the emergence of the CRIT spectrum as the second microsphere approaches and couples to the other.

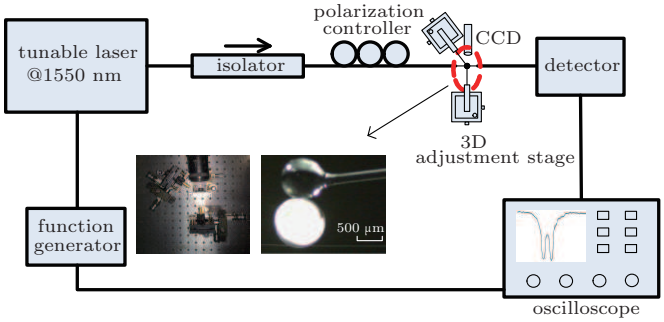


Fig. 5. (color online) Schematic diagram of the two-coupled microsphere CRIT test system.

After the CRIT spectrum obtained, we divided the input light into two equal beams by a 3 dB coupler C1, one beam through the two-coupled microsphere resonator structure, the other through a $LiNbO_3$ phase modulator. Two beams of light through the fiber length are equal to offset the phase shift caused by the fiber length. The phase modulator applied a phase offset $\pi/2$ on the second beam. Two beams through 3 dB coupler C2 interfere to form a fiber MZ interferometer,[16] the expressions of optical fields inside the upper and the lower arms can be written as

$$E_1(\omega) = |T(\omega)| E_{\text{in}} \cos(\phi(\omega) + \phi_{L1}), \tag{20}$$

$$E_2 = \sqrt{1-\alpha} E_{\text{in}} \cos\left(\frac{\pi}{2} + \phi_{L2}\right), \tag{21}$$

where $|T(\omega)|^2$ is the transmission of the two coupled microspheres, can be measured by the system shown in Fig. 5. Here, $\phi(\omega)$ is the effective phase shift of this structure, $\alpha = 0.55$ is the insert loss of the phase modulator. The lengths of fibers in upper and the lower arms of the interferometer are equal $\phi_{L1} = \phi_{L2}$, so we can express the power measured by a photodetector as

$$P_{\text{out}}(\omega) = |E_1(\omega) + E_2|^2 = P_{\text{in}} \left\{ |T(\omega)|^2 + (1-\alpha) + 2|T(\omega)|\sqrt{1-\alpha}\cos\left[\phi(\omega) - \frac{\pi}{2}\right] \right\}. \tag{22}$$

Scan the frequency of laser, and give the effective phase shift of this structure by

$$\phi(\omega) = \frac{\pi}{2} + \arccos\frac{P_{\text{out}}(\omega) - |T(\omega)|^2 P_{\text{in}} - (1-\alpha)P_{\text{in}}}{2|T(\omega)|\sqrt{1-\alpha}}. \tag{23}$$

As shown in Figs. 6(d1) and 6(d4), the transmission spectrum of a single microsphere cavity is an absorption peak, a transparent peak emerges in the absorption peak, due to the destructive interference of light when the other microsphere cavity approaches, which is the CRIT effect. The relative position of the transparent peak and the absorption peak could be tuned by adjusting the relative position of the two microsphere resonators. The effective phase of this CRIT effect could be measured by the experiment system shown in Fig. 6, and the result is shown in Fig. 6(b). Normal dispersion appears at the CRIT transparent peak, the group velocity of light is slowed down, and anomalous dispersion emerges at both sides of the CRIT peak, fast light phenomenon occurs. At CRIT transparent peak, the slow light effect is most evident, the phase change caused by rotation has the greatest impact on the output. From Eq. (12), we obtain the group delay curve of the structure (see Fig. 6), which shows the coupled induced transparency point (wavelength = 1550 nm). The group delay of the structure has a maximum peak at 3.4 ns. The group velocity of light in the structure is calculated to be $V_g = 1.662 \times 10^6$ m/s, the group index of the structure is $n_g = 180.46$ giving a relative index of $n_g/n_0 = 124.46$.

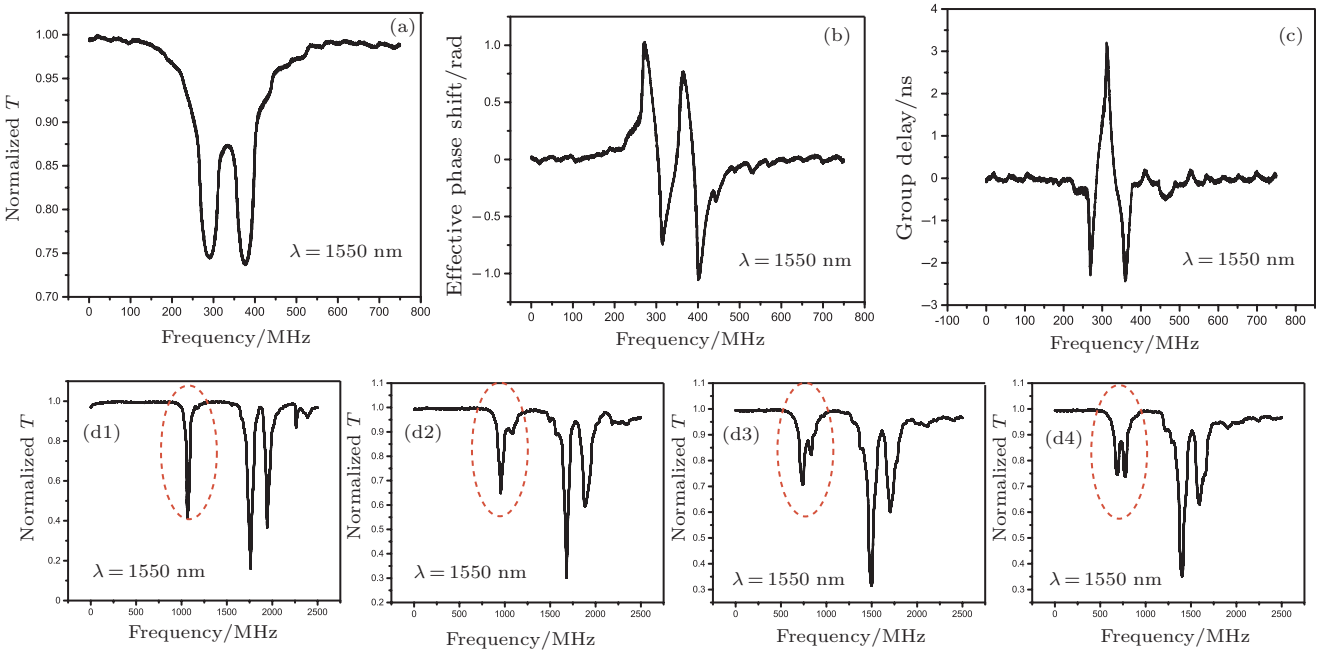


Fig. 6. (color online) Test results of the experiment system shown in Figs. 4(a)–4(c). Transmission, effective phase shift, and group delay with two coupled microspheres, with diameters of two spheres of $d_1 = 1000$ μm and $d_2 = 800$ μm. (d1)–(d4) In drawing near one microsphere to couple to the other, the WGM resonance spectrum is split into two modes.

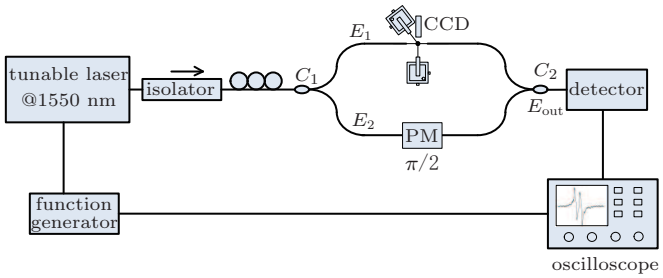


Fig. 7. (color online) Schematic diagram of the two-coupled microsphere phase shift test system.

Compared with that of a single microsphere cavity, the group index of the two-coupled microsphere structure is much larger than the refractive index of the microsphere resonator mainly because of the structure dispersion, the theoretical sensitivity of this structure can reach $8^\circ/\mathrm{h}$.

4. Conclusion

We analyzed a theoretical model of two coupled microspheres, and derived the CRIT transfer function, the effective phase shift, and the group delay. We melted a tip of a single fiber and prepared microsphere resonators, built a precise testing platform, and obtained the CRIT spectrum of the paired microspheres, from which the group velocity and group index could be calculated, giving values $V_\mathrm{g} = 1.662 \times 10^6$ m/s and $n_\mathrm{g} = 180.46$, while $\lambda = 1550$ nm. This low loss and compact CRIT structure can be used in the research of slow light structure gyroscopes.

References

[1] David D S, Hongrok C and Kirk A F 2004 *Phys. Rev. A* **69** 063804
[2] Darmawan S, Tobing L and Zhang D 2011 *Opt. Express* **19** 17813
[3] David D S, Lepeshkin N N and Schweinsberg A 2006 *Opt. Commun.* **264** 163
[4] Totsuka K, Kobayashi N and Tomita M 2007 *Appl. Phys. Lett.* **98** 213904
[5] Zhuang L, Hoekman M and Oldenbeuving R M 2014 *IEEE Photon. Technol. Lett.* **26** 1034
[6] Ciminelli C, Dell'Olio F and Carlo E C 2010 *Adv. Opt. Photonics* 2370
[7] Zhang Y D, Tian H and Zhang X 2010 *Opt. Lett.* **35** 691
[8] Ali A R and Ioppolo T 2014 *Sensors* **14** 7041
[9] Liu J H, Tang J and Shang C L 2015 *Acta Phys. Sin.* **65** 154206 (in Chinese)
[10] Ilchenko V S and Matsko A B 2006 *IEEE J. Sel. Top. Quant.* **12** 15
[11] Terrel M, Digonnet M J F and Fan S 2009 *Laser Photonics Rev* **3** 452
[12] Peng C, Li Z B and Xu A S 2007 *Opt. Express* **15** 3864
[13] Poon J K S, Scheuer S J and Mookherjea G T 2004 *Opt. Express* **12** 90
[14] Landobasa Y M, Desmond C S L and Mee-Koy C 2007 *IEEE Photon. Technol. Lett.* **19** 1688
[15] Landobasa Y M and Mee-Koy C 2008 *Opt. Express* **16** 1796
[16] Dumeige Y, Nguyên T K N and Ghicsa L 2008 *Phys. Rev. A* **78** 13818