Amplifying and freezing of quantum coherence using weak measurement and quantum measurement reversal*

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We analyze universal conditions where the l_1 norm and relative entropy of coherence are amplified and frozen under identical bit-flip channels; that is, using pre-measurements (quantum weak measurements or quantum measurement reversals) on the systems before undergoing local bit-flip channels. With the option of quantum weak measurements or quantum measurement reversals, the measurement strength and the success probability are all determined by the initial state of the quantum system.

Keywords: frozen coherence, weak measurement, quantum measurement reversal

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1. Introduction

Quantum coherence is a fundamental feature of quantum mechanics, describing the quantum interference phenomena of a quantum system. It has been widely used as a resource and root concept, such as quantum information processing,[1] quantum metrology,[2–5] entanglement creation,[6,7] thermodynamics,[8–12] and quantum biology.[13–16]

Only very recently have there been achievements towards developing a rigorous theory of coherence as a physical resource,[17–19] and necessary constraints have been put forward to assess the valid quantifiers of coherence.[17] A number of coherence measures based on various physical contexts have been put forward, such as the l_1 norm of coherence and the relative entropy of coherence,[17] skew information,[20,21] and coherence measures based on entanglement.[22]

Quantum coherence is a useful physical resource but the coherence of a quantum state is often destroyed by noise. A challenge in exploiting the resource is to protect coherence from the decoherence caused by noise. Studies on this topic have been started in Ref. [23], where the authors found that the coherence measurements based on distances are frozen for some initial states of a quantum system undergoing local identical bit-flip channels. This finding illustrates that there exist such quantum states of which some coherence measures remain constant in certain noisy channels. However, the conditions of frozen coherence are not same for single qubit and multi-qubit and nontrivial universal freezing of the relative entropy of coherence is impossible for the dynamics of a single qubit under paradigmatic incoherent maps. Then, we ask whether there is a universal method to amplify and freeze coherence for single-qubit and multi-qubit, and for the l_1 norm and the relative entropy of coherence measure.

Intuitively, both coherence and entanglement capture the quantumness of a physical system, and it is well known that entanglement stems from the superposition principle, which is also the essence of coherence. In practice, decoherence can severely undermine the quantum features in quantum information processing, and quantum entanglement is fragile to environmental noises. An idea is proposed to protect quantum states and quantum entanglements from decoherence using a weak measurement and measurement reversal.[24,25] Recently, reference [26] proved that using a weak measurement and measurement reversal can suppressing the amplitude damping decoherence.

Therefore, we can optimize quantum states and freeze coherence from decoherence using a weak measurement and measurement reversal.

The rest of this paper is organized as follows. In Section 2, we briefly discuss the measures of quantum coherence used in this study, describe the methodology for investigating their dynamics in the presence of local noise, introduced the weak measurement and quantum measurement reversal. In Section 3, we conclude the necessary and sufficient conditions for amplifying and freezing the quantum coherence of quantum systems undergoing local identical bit-flip channels using a weak measurement and measurement reversal. Section 4 compares the different methods of using measurement to protect the quantum coherence and entanglement when a quantum system undergoes incoherent channels. Section 5 contains the concluding remarks.

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2. Incoherent operations and coherence measures

We first need to review some notions, such as incoherent states, incoherent operations, and coherence measures. For an N-qubit system associated with a Hilbert space C^{2^N}, we fix the computational basis $\{|0\rangle,|1\rangle\}^{\otimes N}$ as the reference basis, and we define incoherent states as those whose density matrix δ is diagonal in such a basis,

$$\delta=\sum_{i_1,\ldots,i_N}^{1} d_{i_1,\ldots,i_N}|i_1,\ldots,i_N\rangle\langle i_1,\ldots,i_N|. \tag{1}$$

The Markovian dynamics of an open quantum system are described by a completely positive trace-preserving (CPTP) map Λ, i.e., a quantum channel, whose action on the state ρ of the system can be characterized by a set of Kraus operators $\{K_j\}$ such that $\Lambda(\rho)=\sum_j K_j\rho K_j^+$, where $\sum_j K_j^+K_j=I$. Incoherent quantum channels (ICPTP maps) constitute a subset of quantum channels that satisfy the additional constraint $K_j\tau K_j^+\subset\tau$ for all j, where τ is the set of incoherent states.[17] This implies that ICPTP maps transform incoherent states into incoherent states, and no creation of coherence could be witnessed, even if an observer had access to individual outcomes.

We will consider paradigmatic instances of incoherent channels that embody typical noise sources in quantum information processing,[1,17] whose action on a single qubit is described as follows, in terms of a parameter $q\in[0,1]$, that encodes the strength of the noise and depends explicitly on time t. The bit-flip, bit + phase-flip, and phase-flip channels are represented in Kraus form by

$$K_0=\sqrt{1-\frac{q}{2}}I,\quad K_1^i=\sqrt{\frac{q}{2}}\sigma^i, \tag{2}$$

where σ^i, $i=1,2,3$, are the Pauli spin matrices. Here, K_1^i for $i=1,2,3$ correspond to the BF, BPF, and PF channels, respectively. The action of N independent and identical local noisy channels on each qubit of an N-qubit system maps the system state ρ into the evolved state

$$\Lambda_q^{\otimes N}(\rho)=\sum_{j_1,\ldots,j_N}(K_{j_1}\otimes\cdots\otimes K_{j_N})\rho(K_{j_1}^+\otimes\cdots\otimes K_{j_N}^+). \tag{3}$$

We now recall known measures of coherence.[17] The l_1 norm quantifies coherence in an intuitive way via the off-diagonal elements of a density matrix ρ in the reference basis $C_{l_1}(\rho)=\sum_{i\neq j}|\rho_{ij}|$. The relative entropy of coherence $C_{\rm r}$ is given by $C_{\rm r}(\rho)=S(\rho_{\rm d})-S(\rho)$ for any state ρ, where $\rho_{\rm d}$ is the matrix containing only the leading diagonal elements of ρ in the reference basis and $S(\rho)=-{\rm Tr}(\rho\log\rho)$ is the von Neumann entropy.

Theoretically, a weak measurement on a qubit can be done by monitoring its environment via an ideal detector. Whenever there is an excitation in the environment, the result is discarded. In the case of no excitation, the qubit state is just renormalized and allowed to evolve. The null-result weak measurement on a qubit maps as follows:

$$|0_j\rangle|\bar{0}\rangle\rightarrow|0_j\rangle|\bar{0}\rangle, \tag{4a}$$
$$|1_j\rangle|\bar{0}\rangle\rightarrow\sqrt{1-p}|1_j\rangle|\bar{0}\rangle. \tag{4b}$$

The corresponding non-unitary quantum operation can be written as

$$M_{\rm wk}=\begin{pmatrix}1&0\\0&\sqrt{1-p}\end{pmatrix}, \tag{5}$$

where p $(0\leq p\leq 1)$ is the weak measurement strength. Because the weight of the component $|1_j\rangle$ is reduced and the computational basis state $|0_j\rangle$ does not couple to the environment, the qubit after the weak measurement is less vulnerable to decoherence. In contrast, the quantum measurement reversal on a qubit maps as

$$|1_j\rangle|\bar{0}\rangle\rightarrow|1_j\rangle|\bar{0}\rangle, \tag{6a}$$
$$|0_j\rangle|\bar{0}\rangle\rightarrow\sqrt{1-p_{\rm r}}|0_j\rangle|\bar{0}\rangle, \tag{6b}$$

and the reversing measurement is also a non-unitary operation that can be written as

$$M_{\rm rev}=\begin{pmatrix}\sqrt{1-p_{\rm r}}&0\\0&1\end{pmatrix}, \tag{7}$$

where $p_{\rm r}$ $(0\leq p_{\rm r}\leq 1)$ is the quantum measurement strength.

3. Amplifying and freezing of quantum coherence

We now analyze conditions such that the l_1 norm and the relative entropy of coherence are all invariant during the evolution of a quantum system under any of the noisy channels Λ_q described above.

Bromley *et al.* proved that only the bit and bit + phase-flip channels allow for nonzero frozen coherence (in the computational basis), while all the other considered incoherent channels leave coherence invariant only trivially when the initial state is already incoherent.[23] We focus our attention on the bit-flip channel.

3.1. One qubit

For a single-qubit state in general as $\rho=(I+\sum_j n_j\sigma_j)/2$, in terms of its Bloch vector $\boldsymbol{n}=\{n_1,n_2,n_3\}$, the bit-flip channel Λ_q maps an initial Bloch vector $\boldsymbol{n}(0)$ to an evolved one $\boldsymbol{n}(q)=\{n_1(0),(1-q)n_2(0),(1-q)n_3(0)\}$. The necessary and sufficient freezing conditions for C_{l_1} under a single-qubit bit-flip channel amount to $n_2(0)=0$ in the initial state; however, the freezing condition is not suitable for the relative entropy of coherence measure. When $n_1(0)=n_2(0)=0$ (the initial state is incoherent) or $n_2(0)=n_3(0)=0$ (the initial state is invariant under the bit-flip channel), both coherence measures are

frozen under the bit-flip channel but there is trivial freezing of the relative entropy of coherence under the bit-flip channel.[23]

We consider a single-qubit pure state

$$|\psi\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle, \tag{8}$$

with $\theta \in [0,\pi]$. The state corresponds to the general state $\{n_1 = \sin\theta\cos\theta, n_2 = 0, n_3 = \cos^2\theta - \sin^2\theta\}$. If $\theta = \{0,\pi/2,\pi\}$, then $n_1(0) = n_2(0) = 0$, the initial state is incoherent. If $\theta = \{\pi/4, 3\pi/4\}$, the state becomes a maximally coherent state, and $n_2(0) = n_3(0) = 0$, then the initial state is invariant under the bit-flip channel.

We now analyze the universal freezing of coherence for specific initial states (excluding the $\theta = \{0,\pi/4,\pi/2,3\pi/4,\pi\}$ in the above state, only $n_2(0) = 0$) under bit-flip channels using a prior weak measurement or prior measurement reversal.

On the one hand, we perform prior weak measurements on a single-qubit state $|\psi\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle$, and the initial state of the qubit becomes

$$|\psi\rangle_{\rm wk} = \frac{1}{\sqrt{\cos^2\theta + (1-p)\sin^2\theta}} \times (\cos\theta|0\rangle + \sqrt{1-p}\sin\theta|1\rangle). \tag{9}$$

The success probability is

$$P_{\rm wk} = \cos^2\theta + (1-p)\sin^2\theta. \tag{10}$$

We can see that if $\theta \in (\pi/4,\pi/2)$ or $\theta \in (\pi/2,3\pi/4)$ (because the weak measurement strength is $0 \le p \le 1$), when $p = 1 - \cot^2\theta$, the state becomes

$$|\psi\rangle_{\rm wk} = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle). \tag{11}$$

This coincides with the maximally coherent state, which is $n_2(0) = n_3(0) = 0$. Then, we let the qubit enter the bit-flip channel. The state coherence is frozen for both coherence measurements under a single-qubit bit-flip channel, and the success probability corresponds to

$$P_{\rm wk} = 2\cos^2\theta. \tag{12}$$

On the other hand, we perform a prior measurement reversal operator on a single-qubit state $|\psi\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle$, the state of the qubit becomes

$$|\psi\rangle_{\rm rev} = \frac{1}{\sqrt{(1-p_{\rm r})\cos^2\theta + \sin^2\theta}} \times (\sqrt{1-p_{\rm r}}\cos\theta|0\rangle + \sin\theta|1\rangle). \tag{13}$$

The success probability is

$$P_{\rm rev} = (1-p_{\rm r})\cos^2\theta + \sin^2\theta. \tag{14}$$

We can see that if $\theta \in (0,\pi/4)$ or $\theta \in (3\pi/4,\pi)$ (because the quantum measurement strength is $0 \le p_{\rm r} \le 1$), when $p_{\rm r} = 1 - \tan^2\theta$, then the state becomes

$$|\psi\rangle_{\rm rev} = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle). \tag{15}$$

This also coincides with the maximally coherent state, which is $n_2(0) = n_3(0) = 0$. Then, we let the qubit enter the bit-flip channel. The state coherence is frozen for both coherence measures under a single-qubit bit-flip channel, and the success probability corresponds to

$$P_{\rm rev} = 2\sin^2\theta. \tag{16}$$

We now consider a single-qubit mixed state $\rho = (I + \sum_j n_j\sigma_j)/2$, and assume its Bloch vector is $\boldsymbol{n} = \{n_1, n_2 = 0, n_3\}$, with the condition $n_1^2 + n_3^2 < 1$. We performed prior weak measurements on single-qubit mixed state ρ, and the initial state of the qubit becomes

$$\rho_{\rm wk} = \frac{1}{P}\begin{pmatrix} 1+n_3 & \sqrt{1-p}n_1 \\ \sqrt{1-p}n_1 & (1-p)(1-n_3) \end{pmatrix}, \tag{17}$$

where $P = 1 + n_3 + (1-p)(1-n_3)$. The success probability is

$$P_{\rm wk} = \frac{1}{2}[1 + n_3 + (1-p)(1-n_3)]. \tag{18}$$

We can see that if $-1 \le n_3 \le 0$ (because the weak measurement strength is $0 \le p \le 1$), when $p = 1 - (1+n_3)/(1-n_3)$, the state becomes

$$\rho_{\rm wk} = \frac{1}{2}\begin{pmatrix} 1 & \sqrt{\dfrac{n_1^2}{1-n_3^2}} \\ \sqrt{\dfrac{n_1^2}{1-n_3^2}} & 1 \end{pmatrix}, \tag{19}$$

for $n_2(0) = n_3(0) = 0$. Then, we let the qubit enter the bit-flip channel, and the state coherence is frozen for both coherence measurements under a single-qubit bit-flip channel. The success probability corresponds to

$$P_{\rm wk} = 1 + n_3. \tag{20}$$

On the other hand, we perform prior measurement reversal operator on one qubit of state ρ and the state of the qubit becomes

$$\rho_{\rm rev} = \frac{1}{P}\begin{pmatrix} (1-p_{\rm r})(1+n_3) & \sqrt{1-p_{\rm r}}n_1 \\ \sqrt{1-p_{\rm r}}n_1 & 1-n_3 \end{pmatrix}, \tag{21}$$

where $P = 1 - n_3 + (1-p_{\rm r})(1+n_3)$, the success probability is

$$P_{\rm rev} = \frac{1}{2}[1 - n_3 + (1-p_{\rm r})(1+n_3)]. \tag{22}$$

We can see that if $0 \leq n_3 \leq 1$ (because the quantum measurement strength is $0 \leq p_r \leq 1$), when $p_r = 1-(1-n_3)/(1+n_3)$, the state becomes

$$\rho_{wk} = \frac{1}{2}\begin{pmatrix} 1 & \sqrt{\frac{n_1^2}{1-n_3^2}} \\ \sqrt{\frac{n_1^2}{1-n_3^2}} & 1 \end{pmatrix}, \tag{23}$$

for $n_2(0) = n_3(0) = 0$. Then, we let the qubit enter the bit-flip channel, the state coherence is frozen for both coherence measurements under a single-qubit bit-flip channel, and the success probability corresponds to

$$P_{rev} = 1 - n_3. \tag{24}$$

For the single-qubit mixed state ρ, $n_1^2 + n_3^2 < 1$, we conclude $\sqrt{n_1^2/(1-n_3^2)} > n_1$. Thus, using prior weak measurements or prior measurement reversal can lead to amplified and frozen coherence of both coherence measurements under a single-qubit bit-flip channel.

In conclusion, when using prior weak measurements or prior measurement reversal, nontrivial universal freezing of coherence is possible for the dynamics of a single qubit under paradigmatic incoherent maps. Furthermore, this method can amplify the coherence of the state.

3.2. Two qubits

We will show that the l_1 norm and relative entropy of coherence manifest freezing forever in the case of two qubits undergoing local identical bit-flip channels starting from conditions of using prior weak measurements or prior measurement reversals.

For two-qubit states with maximally mixed marginals (M_{32} states), also known as Bell-diagonal states,[27] which are identified by a triple $\boldsymbol{c} = \{c_1, c_2, c_3\}$ in their Bloch representation $\rho = (I^A \otimes I^B + \sum_{j=1}^{3} c_j \sigma_j^A \otimes \sigma_j^B)/4$. Local bit-flip channels on each qubit map initial M_{32} states with $\boldsymbol{c}(0) = \{c_1(0), c_2(0), c_3(0)\}$ to M_{32} states with $\boldsymbol{c}(q) = \{c_1(0), (1-q)^2 c_2(0), (1-q)^2 c_3(0)\}$. The subset of M_{32} states supporting frozen coherence for both coherence measures is given by the initial condition,[23]

$$c_2(0) = -c_1(0)c_3(0). \tag{25}$$

We consider the two-qubit Bell-like state

$$|\varphi\rangle = \cos\theta|00\rangle + \sin\theta|11\rangle, \tag{26}$$

with $\theta \in [0, \pi]$. The local bit-flip channels on each qubit map the initial state $|\varphi\rangle$ to the state

$$\begin{aligned}\rho(q) =\ & a(q)|00\rangle\langle 00| + b(q)|11\rangle\langle 11| \\ & + c(q)(|01\rangle\langle 01| + |10\rangle\langle 10|) \\ & + d(q)(|00\rangle\langle 11| + |11\rangle\langle 00|) \\ & + e(q)(|01\rangle\langle 10| + |10\rangle\langle 01|),\end{aligned} \tag{27}$$

where

$$\begin{aligned} a(q) &= \left(1-\frac{q}{2}\right)^2 \cos^2\theta + \left(\frac{q}{2}\right)^2 \sin^2\theta, \\ b(q) &= \left(\frac{q}{2}\right)^2 \cos^2\theta + \left(1-\frac{q}{2}\right)^2 \sin^2\theta, \\ c(q) &= \frac{q}{2}\left(1-\frac{q}{2}\right), \\ d(q) &= \left[\left(1-\frac{q}{2}\right)^2 + \left(\frac{q}{2}\right)^2\right] \sin\theta\cos\theta, \\ e(q) &= q\left(1-\frac{q}{2}\right)\sin\theta\cos\theta. \end{aligned}$$

If $\theta = \{0, \pi/2, \pi\}$, then the initial state is the incoherent state, the evolved state is also an incoherent state. If $\theta = \{\pi/4, 3\pi/4\}$, then the initial state is the Bell state $c_2(0) = -c_1(0)c_3(0) = -1$. The state coherence is frozen for both coherence measurements under bit-flip channels, if θ is other values that manifest frozen coherence as measured by C_{l_1} under local bit-flip channels; however, the same does not hold for C_r.[23]

We apply prior weak measurements or prior measurement reversals in specific initial states $|\varphi\rangle$ (excluding $\theta = \{0, \pi/4, \pi/2, 3\pi/4, \pi\}$) to realize the freezing of the coherence of both coherence measurements under bit-flip channels. We perform prior weak measurements on one qubit of state $|\varphi\rangle = \cos\theta|00\rangle + \sin\theta|11\rangle$, and the initial two-qubit state becomes

$$\begin{aligned}|\varphi\rangle_{wk} =\ & \frac{1}{\sqrt{\cos^2\theta + (1-p)\sin^2\theta}} \\ & \times (\cos\theta|00\rangle + \sqrt{1-p}\sin\theta|11\rangle),\end{aligned} \tag{28}$$

and the success probability is

$$P_{wk} = \cos^2\theta + (1-p)\sin^2\theta. \tag{29}$$

We can see that if $\theta \in (\pi/4, \pi/2)$ or $\theta \in (\pi/2, 3\pi/4)$, when $p = 1 - \cot^2\theta$, then the state becomes

$$|\psi\rangle_{wk} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle), \tag{30}$$

which coincides with the Bell state $c_2(0) = -c_1(0)c_3(0) = -1$. Then, we let the two-qubit enter the bit-flip channels. The state coherence is frozen for both coherence measurements under bit-flip channels, and the success probability corresponds to

$$P_{wk} = 2\cos^2\theta. \tag{31}$$

Similar to single-qubit case, if $\theta \in (0, \pi/4)$ or $\theta \in (3\pi/4, \pi)$, we perform the prior measurement reversal operator on one qubit of state $|\varphi\rangle = \cos\theta|00\rangle + \sin\theta|11\rangle$. When $p = 1 -$

$\tan^2\theta$, the state becomes $|\psi\rangle_{\rm rev} = (|00\rangle + |11\rangle)/\sqrt{2}$. This coincides with the Bell state $c_2(0) = -c_1(0)c_3(0) = -1$. Then, we let the two-qubit enter the bit-flip channels. The state coherence is frozen for both coherence measurements under bit-flip channels, and the success probability corresponds to $P_{\rm rev} = 2\sin^2\theta$.

As can be seen from above, using prior weak measurements or prior measurement reversals, the two-qubit state can be amplified and frozen into the coherence of both coherence measurements under local identical bit-flip channels.

3.3. *N* qubits

We will show that both measurements of quantum coherence manifest freezing forever in the case of N qubits undergoing local identical bit-flip channels on the conditions of using prior weak measurements or prior measurement reversals.

For N-qubit states

$$|\phi\rangle = \cos\theta|0^{\otimes N}\rangle + \sin\theta|1^{\otimes N}\rangle, \tag{32}$$

with $\theta \in [0,\pi]$ and $N \geq 3$, if $\theta = \{0,\pi/2,\pi\}$, the initial states are incoherent states. When the system is evolving according to identical and independent local bit-flip channels acting on each qubit, the state is also an incoherent state. If $\theta = \{\pi/4, 3\pi/4\}$, then the initial states are just the N-qubit GHZ states. Yu *et al.* proved that all coherence measures manifest freezing forever for the N-qubit systems initially in the N-qubit GHZ states undergoing local bit-flip channels.[28]

For $\theta \in (\pi/4,\pi/2)$ or $\theta \in (\pi/2, 3\pi/4)$, we perform prior weak measurements on one qubit of state $|\phi\rangle = \cos\theta|0^{\otimes N}\rangle + \sin\theta|1^{\otimes N}\rangle$, and the initial N-qubit state becomes

$$\begin{aligned}|\varphi\rangle_{\rm wk} = & \frac{1}{\sqrt{\cos^2\theta + (1-p)\sin^2\theta}} \\ & \times (\cos\theta|0^{\otimes N}\rangle + \sqrt{1-p}\sin\theta|1^{\otimes N}\rangle).\end{aligned} \tag{33}$$

The success probability is

$$P_{\rm wk} = \cos^2\theta + (1-p)\sin^2\theta. \tag{34}$$

We can see that when $p = 1 - \cot^2\theta$, the state becomes

$$|\psi\rangle_{\rm wk} = \frac{1}{\sqrt{2}}(|0^{\otimes N}\rangle + |1^{\otimes N}\rangle). \tag{35}$$

This coincides with N-qubit GHZ states, and the success probability corresponds to

$$P_{\rm wk} = 2\cos^2\theta. \tag{36}$$

Similarly to single qubit case, for $\theta \in (0,\pi/4)$ or $\theta \in (3\pi/4,\pi)$, we perform a prior measurement reversal on one qubit of state $|\phi\rangle = \cos\theta|0^{\otimes N}\rangle + \sin\theta|1^{\otimes N}\rangle$. When $p = 1 - \tan^2\theta$, the state becomes $|\psi\rangle_{\rm rev} = (|0^{\otimes N}\rangle + |1^{\otimes N}\rangle)/\sqrt{2}$. This also coincides with N-qubit GHZ states and the success probability corresponds to $P_{\rm rev} = 2\sin^2\theta$.

Similar to one qubit and two qubits, using prior weak measurements or prior measurement reversals, the l_1 norm and relative entropy of coherence can be amplified and frozen forever in the case of N qubits undergoing local identical bit-flip channels.

4. Coherence versus quantum entanglement

Quantum coherence and quantum entanglement are all useful physical resources; furthermore, they are often destroyed by noise. The amplifying and freezing conditions established here for coherence have been identified. Now, we need to determine if these conditions can promote and protect entanglement. Focusing on the two-qubit case for clarity, we note the state $|\varphi\rangle = \cos\theta|00\rangle + \sin\theta|11\rangle$, using prior weak measurements on the one qubit of state $|\varphi\rangle$, we let the two-qubit enter the bit-flip channels. If $\theta \in (0,\pi/4)$ or $\theta \in (3\pi/4,\pi)$, when $p = 1 - \tan^2\theta$, the state evolves to

$$\begin{aligned}\rho(q) = & a(q)(|00\rangle\langle 00| + |11\rangle\langle 11| + |00\rangle\langle 11| + |11\rangle\langle 00|) \\ & + b(q)(|01\rangle\langle 01| + |10\rangle\langle 10| + |01\rangle\langle 10| \\ & + |10\rangle\langle 01|),\end{aligned} \tag{37}$$

where

$$\begin{aligned}a(q) &= \frac{1}{2}\left[\left(1-\frac{q}{2}\right)^2 + \left(\frac{q}{2}\right)^2\right], \\ b(q) &= \frac{q}{2}\left(1-\frac{q}{2}\right).\end{aligned}$$

The coherence of the initial state has been proven to be amplified and frozen under these conditions. Using Wootters concurrence[29] to quantify the entanglement, one can exactly derive the form of concurrence of qubits' entanglement from Eq. (37),

$$E[\rho(q)] = (1-q)^2. \tag{38}$$

That is to say, using prior weak measurements cannot protect entanglement under bit-flip channels on two-qubit. Similar conclusions can be drawn for using prior quantum measurement reversal. It has been shown that the weak measurement and quantum measurement reversal can effectively suppress the entanglement decay of qubits in Refs. [25], [30], and [31]. For the case of two qubits, the weak measurement and quantum measurement reversal can remarkably increase the entanglement, and even avoid the sudden death of the entanglement,[25,32,33] where their method is to apply the weak measurement as a pre-measurement on each qubit before it undergoes decoherence. When the qubits interact with incoherent channels, the proper post-measurements are performed on both qubits.

It is thus evident that coherence is easier to control and protect than entanglement when the qubits undergo identical bit-flip channels. Therefore, quantum coherence may be a more useful physical resource than entanglement for performing communication and computational tasks.

5. Conclusion

We have determined conditions that any coherence measurements of quantum coherence are dynamically amplified and frozen. This occurs for an arbitrary number of qubits, initialized in a particular class of states using the pre-measurements (weak measurements or quantum measurement reversals) on one qubit of the quantum states before undergoing local independent and identical nondissipative flip channels. We find that the option of quantum weak measurements or quantum measurement reversals is determined by the initial state and the measurement strength is also determined by the initial state of the quantum system. We have shown that general agreements on amplifying and freezing conditions are both for the l_1 norm and the relative entropy of coherence. These universal freezing conditions ensure a durable physical exploitation of coherence, regardless of how it is quantified. It will be interesting to explore the practical realizations of such dynamical conditions. Compared with increasing entanglement and trapping conditions by using the weak measurement and quantum measurement reversal, we can conclude that quantum coherence may be a more useful physical resource than entanglement.

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